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A Bernoulli Model of Selective Assembly Systems

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Abstract: In many assembly systems, components may exhibit different quality behaviors. By selecting the mating part whose characteristics match with those of the main part, high precision assemblies can be achieved. Such a process is referred to as *selective assembly*. Most of the studies on selective assembly only consider the case where machines are reliable and the buffer capacity is infinite. However, unreliable machines and finite buffers are commonly observed in many assembly systems. This paper studies a two-component assembly system with unreliable Bernoulli machines and finite buffers. Analytical methods based on a two-level decomposition procedure are developed to analyze the system performance. The convergence of the procedure is justified analytically. Numerical study shows that such a method provides a high accuracy in performance evaluation.

Keywords: Selective assembly, Bernoulli reliability model, production rate, match degree.

1. INTRODUCTION

Assembly systems are designed for many manufacturing processes where two or more components are assembled to produce a finished product. Traditional assembly systems merely fetch parts in a default or random order assuming all parts are identical. However, the characteristics of subcomponents may not necessarily remain the same from part to part, though they are all regarded as conforming parts. In order to compensate such inevitable difference and maximize the use value of all parts, *selective assembly* is employed to select the mating part according to the characteristic of the other part (main part) to be assembled. Thus, high-precision assemblies could be obtained from relatively low-precision components.

The idea of selective assembly is first presented by Mansoor (1961) to define the natural process tolerance and establish the relationship to the tolerance specification. Since then, mounting research has be devoted to analyzing the partitioning or binning strategies to reduce mismatches mostly in dimensional issues, such as ball bearing manufacturing, scroll compressor shell manufacturing, sleeve-and-shaft assembly, etc. (see, for instance, representative papers by Fang and Zhang (1996); Coullard *et al.* (1998); Mease *et al.* (2004); Matsuura and Shinozaki (2007); Kannan *et al.* (2008); Tan and Wu (2012)). However, most research typically either neglects the issue of machine reliability and storage capacity or only focuses on the batch operation environment where the production rate requirement is not a concern. Nevertheless, selective assembly could also find its applicability in automatic transfer lines with limited buffer capacities. To tackle the problem in finite buffer cases, several simulation studies have been carried out by Thesen and Jantayavichit (1999); Akansel *et al.* (2011).

The merit of selective assembly lies in utilizing low valued parts to produce relatively high valued finished products. It is also suitable for quality improvement. For example, in battery assembly manufacturing for electric vehicles (Ju *et al.* (2013)), due to the structure of battery packs, multiple battery cells need to be welded within a tight product envelope. Cells will firstly be sorted and stacked into sections (or modules), and then connected by welds or mechanical joints. In order to create durable and conductive bond between workpieces, battery cells need to be perfectly aligned thus imposing a strict requirement for cell dimensions within a single section (or module). As cells can be categorized into groups according to their dimensions, selective assembly can help to select parts so that cells from the same group can be assembled.

Nevertheless, the selection is not limitless, especially in production environment where buffers are finite or even relatively small. With small inventory carried, the mating part is not guaranteed to be observed and putting off the assembly process until a desirable mating part comes in place will dramatically impede the throughput of assembly systems and may even result in a deadlock. The unreliable machines will make the issue worse. In such systems, mismatches are inherently inevitable, and the resulting products may exhibit lower quality grade but still pass inspections. Therefore, a heuristic policy for such a selective assembly system could be selecting the

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closest matching of mating part as possible and keeping the assembly process running smoothly. However, questions arise naturally, for instance, what is the ratio of matching products in such systems and what is the potential improvement using such selection policy. The answers to these questions still remain unclear. Therefore, to study the selective assembly systems with unreliable machines and finite buffers is of critical importance, yet the existing literature remains surprisingly silent about this problem. No analytical model on selective assembly in production systems with unreliable machines and finite buffers has been found in the current literature. The goal of this study is intended to contribute to this end.

Although analytical methods on evaluating assembly systems with unreliable machines and finite buffers have been investigated extensively in existing literature (see monographs by Gershwin (1994); Li and Meerkov (2009) and representative papers by Mascolo *et al.* (1991); Gershwin (1991); Kuo *et al.* (1996); Chiang *et al.* (2000); Ching *et al.* (2008)). However, the issue of selective assembly is not addressed. Therefore, in this paper, we study a two-component selective assembly system with unreliable machines and finite buffers, and propose a novel method to evaluate/predict efficiency and efficacy measures for selective assembly systems analytically.

The remainder of the article is organized as follows: Section 2 formulates the problem and introduces the assumptions. In Section 3, based on the proposed two levels of decomposition, analytical procedures are formulated and numerical experiments are conducted to justify their accuracy. Finally, the conclusions are formulated in Section 4. Due to space limitation, all the proofs are omitted and can be found in Ju and Li (2013).

2. PROBLEM FORMULATION

In this paper, a two-component selective assembly system is considered (shown in Figure 1). Here the circles represent machines and the rectangles are buffers. The following assumptions address the characteristics of machines, buffers and product quality behaviors.

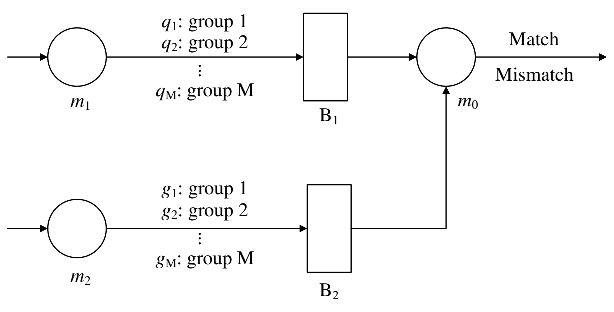

Fig. 1. Selective assembly system

1) The assembly system consists of a main line (machine m_1 and buffer B_1), a mating line (machine m_2 and buffer B_2), and an assembly machine m_0.
2) Machines m_1 and m_2 produce subcomponents categorized into groups 1 to M. The probability of producing a group i main part and mating part are independent and characterized by q_i and g_i respectively, $i = 1, 2, \ldots, M$. Therefore,

$$\sum_{i=1}^{M} q_i = \sum_{i=1}^{M} g_i = 1.$$

3) Selection will only be conducted within the mating line. Any part in the mating line could be selected corresponding to the main part. When two subcomponents with the same group number are assembled, the finished product is regarded as a matching product (assembled product with matching components). Otherwise, it is identified as a mismatch.
4) All machines have an identical cycle time T, thus the time axis is slotted with slot duration T.
5) Machine m_i, $i = 0, 1, 2$, is characterized by probability p_i to be up and $1 - p_i$ to be down in each cycle.
6) Buffer B_i, $i = 1, 2$, is limited by a finite capacity $0 < N_i < \infty$.
7) Machine m_1 or m_2 is blocked if its downstream buffer B_1 or B_2 is full, respectively, and the assembly machine m_0 fails to take a part at the beginning of the current time slot. Machine m_0 itself will never be blocked.
8) Machine m_0 is starved if either buffer is empty at the beginning of the current time slot. Machines m_1 and m_2 are assumed to be never starved.
9) No scrap is produced during the whole process.

Remark 1. Assumptions 5) formulates the Bernoulli reliability model of the machines, which has been thoroughly studied in Li and Meerkov (2009). Although it is relatively simple comparing to other reliability models, it does represent many natures of production systems. Such models are typically suitable for assembly type systems where the machine is comparable to the cycle time. The Bernoulli models have been successfully applied in many manufacturing system studies (see case studies in Li and Meerkov (2009)). In this paper, we focus on selective assembly system with Bernoulli machines. In future work, exponential and other reliability models will be investigated.

In practice, such a selective assembly line is typically automatic and could collect the group tag for all parts stored in buffers. Taking advantage of such information, an effective selection policy is proposed below which is intuitive to practitioners and easy to be implemented.

a) Select the mating part with the same group number as the current main part to be assembled if available.
b) If there is no matching part, the mating part to be selected is prioritized in the order of $1 > 2 > \ldots > M$.

Remark 2. Following the above policy, even though no matching part exists in the buffer, the assembly machine will still take mating part in a specific order. Therefore, no production lost will take place due to the unavailability of matching parts.

In such a selective assembly system, the key performance measurement is the **match degree (MD)** between the main parts and the mating parts, and it is defined as:

$$MD = \frac{PR_{match}}{PR}, \tag{1}$$

where PR represents the overall production rate, while PR_{match} characterizes the production rate of finished products with matching components.

Therefore, the problem to be addressed in this paper is formulated as follows: *Given the assembly system defined by assumptions 1)-9), develop a method to evaluate/predict MD as a function of system parameters and investigate its potential improvement compared with random assembly.*

Given Equation (1), the problem of evaluating MD could be divided into two sub-problems where PR and PR_{match} are investigated individually.

3. PERFORMANCE ANALYSIS

In this section, we focus the case where each subcomponent is categorized into two groups.

First consider the assembly system without selection, i.e., random selection. In this case, subcomponents reside in the buffer according to a random order and the assembly machine will take the part from the beginning of the queue to produce a finished product. So the assembly process will not change the sequence of subcomponents. The main part and the mating part to be assembled, in consequence, carry group number i independently with probability q_i and g_i, respectively, $i = 1, 2$. Therefore, the match degree MD for such a random assembly system could be expressed as follows.

Lemma 1. In an assembly system defined by assumptions 1)-9) with parts being categorized into two groups, if selection is carried out randomly, the estimated match degree, $\widehat{MD}^{rand}$, could be expressed as

$$\widehat{MD}^{rand} = q_1 g_1 + q_2 g_2.$$

When the selection procedure is involved, the problem becomes more complicated since selections could change the sequence of subcomponents which leads to a lack of information about the part to be assembled. Keeping track of the buffer occupancy in a close form requires tremendous efforts, let alone the case where the group information needs to be recorded as well. Therefore, rather than explicating the states of buffers and machine status, we introduce a new approach to estimate the marginal probability of each group of parts in the buffer based on two levels of decomposition. The decomposition procedures are described next.

3.1 Decomposition Level 1

Considering the selective assembly system described in Section 2, the assembly machine will continue fetching parts as long as the upstream buffer is not empty, even though no matching mating part is available in the buffer. In other words, the unavailability of matching mating parts will not impede the overall production rate. Therefore, PR of the selective assembly system is identical to an assembly system without selection and categorical subcomponents.

To analyze such a system, Kuo *et al.* (1996) introduce an approximation method by decomposing the assembly system into two overlapped serial lines (shown in Figure 2), where machines m_0^1 and m_0^2 represent the assembly machine when it is not starved by buffers B_1 and B_2 respectively. Based on such a decomposition method, a

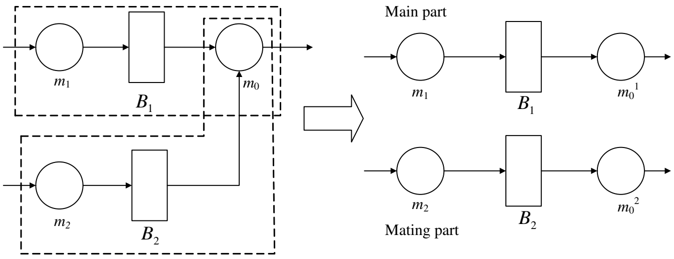

Fig. 2. Decomposition level 1

recursive procedure is formulated and its convergence is guaranteed. The estimates of probabilities that buffers B_1 and B_2 are empty can be obtained, denoted as $\widehat{x}_1$ and $\widehat{x}_2$, respectively. Then the estimate of production rate $\widehat{PR}$ could be formulated in the following manner.

Theorem 1. In an assembly system defined by assumptions 1)-9) with random selection, the production rate can be estimated as

$$\widehat{PR} = p_0(1 - \widehat{x}_1)(1 - \widehat{x}_2). \tag{2}$$

When the estimated production rate $\widehat{PR}$ is obtained, the rest of the problem is to find the production rate of matching products PR_{match}. Therefore, we introduce the second level of decomposition to estimate PR_{match}.

3.2 Decomposition Level 2

Since the selection will only be carried out in the mating line buffer, the group number for the main part could be still estimated by probability q_i, $i = 1, 2$. As for the mating line, virtual machine m_0^2 has been introduced in Subsection 3.1, we further decompose it into two serial lines. Specifically, machine m_2 splits into two virtual machines $m_2^{t_1}$ and $m_2^{t_2}$, producing group 1 and group 2 mating parts respectively (shown in Figure 3). Therefore, their parameters $p_2^{t_1}$ and $p_2^{t_2}$ could be expressed as,

$$p_2^{t_1} = p_2 g_1,$$
$$p_2^{t_2} = p_2 g_2.$$

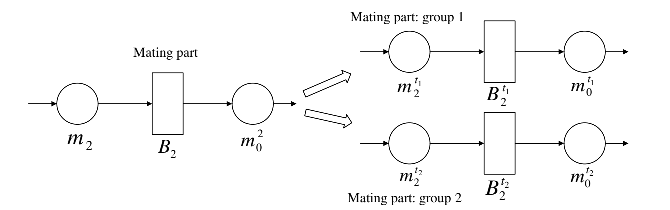

Fig. 3. Decomposition level 2

Moreover, the virtual machine m_0^2 is divided into posterity virtual machines $m_0^{t_1}$ and $m_0^{t_2}$, which fetch groups 1 and 2 mating parts respectively. Machine $m_0^{t_1}$ can "work" on three conditions: 1) machine m_0 is up, 2) buffer B_1 is nonempty and 3) the current main part is in group 1 or it is a group 2 main part but there is no matching mating part available in buffer B_2 (so that a group 1 part has to be selected). In other words, its parameter $p_0^{t_1}$ could be described as:

$$p_0^{t_1} = p_0(1-\widehat{x}_1)(q_1 + q_2 Pr\{\text{no group 2 part in } B_2\}),$$

where $\widehat{x}_1$ comes from the estimate introduced in Kuo *et al.* (1996). Similarly, machine $m_0^{t_2}$ could be characterize as:

$$p_0^{t_2} = p_0(1-\widehat{x}_1)(q_2 + q_1 Pr\{\text{no group 1 part in } B_2\}).$$

As for the buffer, B_2 is decomposed into virtual buffers $B_2^{t_1}$ and $B_2^{t_2}$ containing groups 1 and 2 parts separately. Their capacities, nevertheless, are highly dependent on the interactions between the two decomposed lines. Specifically, given there are i group 2 parts in buffer B_2 during a cycle, the capacity of buffer $B_2^{t_1}$ should be $N_2 - i$ accordingly. Such a buffer capacity could be regarded as a conditional capacity. In this case, we could estimate the conditional probability of the buffer occupancy of group 1 parts in a close formula as follows:

$$\begin{aligned}
&Pr\{\text{no group 1 part in } B_2 | i \text{ group 2 parts in } B_2\}\\
&\quad = Q(p_2^{t_1}, p_0^{t_1}, N_2), \quad i = 1,\dots,N_2, \qquad (3)\\
&Pr\{j \text{ group 1 parts in } B_2 | i \text{ group 2 parts in } B_2\}\\
&\quad = \frac{Q(p_2^{t_1}, p_0^{t_1}, N_2 - j)}{1 - p_0^{t_1}}[\alpha(p_2^{t_1}, p_0^{t_1})]^j,\\
&\quad i = 1,\dots,N_2, j = 1,\dots,N_2 - i,
\end{aligned}$$

where $Q(\cdot)$ and $\alpha(\cdot)$ are defined as (see Li and Meerkov (2009))

$$Q(p_1,p_2,N) = \begin{cases} \dfrac{(1-p_1)(1-\alpha(p_1,p_2))}{1-\frac{p_1}{p_2}\alpha^N(p_1,p_2)}, & \text{if } p_1 \neq p_2,\\ \dfrac{1-p}{N+1-p}, & \text{if } p_1 = p_2 = p, \end{cases} \qquad (4)$$

$$\alpha(p_1,p_2) = \frac{p_1(1-p_2)}{p_2(1-p_1)}. \qquad (5)$$

If the marginal probabilities of group 2 parts are known, combining them with the conditional probabilities of buffer occupancy, the marginal probabilities of group 1 parts could be obtained by the law of total probability, i.e.,

$$\begin{aligned}
&Pr\{j \text{ group 1 parts in } B_2\}\\
&= \sum_{i=0}^{N_2-j} Pr\{i \text{ group 2 parts in } B_2\}\\
&\quad \cdot Pr\{j \text{ group 1 parts in } B_2 | i \text{ group 2 parts in } B_2\}.
\end{aligned}$$

Likewise, the same method could be applied in turn to estimate the marginal probabilities of group 2 parts. Using such a method, the buffer capacities are probabilistic, rather than deterministic, and can be investigated by conditioning.

3.3 Recursive Procedure for Decomposition Level 2

Given the decomposition method described above, the problem arises that information for marginal probabilities is not available. Therefore, a recursive procedure is presented by assuming the initial value of parameters and updating them by iterations. Finally the procedure converges. Such an idea is illustrated as follows:

Step 1 Set initial values for the estimates of marginal probabilities for each group of parts in B_2.

Step 2 Update the parameters of decomposed assembly machines $m_0^{t_1}$ and $m_0^{t_2}$ using marginal probabilities for each group of parts in B_2.

Step 3 Given i group 1 (or group 2) parts in B_2, calculate the conditional probability that there is no group 2 (respectively, group 1) part in B_2.

Step 4 Based on Step 3, calculate the conditional probability of buffer occupancy for each group of parts.

Step 5 Combining Steps 3 and 4 and using the marginal probabilities of group 1 (respectively, group 2) parts, calculate the marginal probabilities of group 2 (respectively, group 1) parts.

Step 6 If the updated estimates of marginal probabilities are close enough to those in the previous iteration, stop. Otherwise, return to Step 2.

Let $P_i^{t_r}$ denote the probability of i parts from group r in buffer B_2, $i = 0,1,\dots,N_2$, $r \in \{1,2\}$, and $P_{j,t_s=i}^{t_r}$ be the probability of j group r parts in buffer B_2 given i group s parts occupied, $i = 0,1,\dots,N_2$, $j = 0,1,\dots,N_2-i$, $r \in \{1,2\}$, $s \in \{2,1\}$. Then

Procedure 1.

$$\begin{aligned}
p_0^{t_1}(n+1) &= p_0(1-\widehat{x}_1)(q_1 + q_2 P_0^{t_2}(n)), && (6)\\
P_{0,t_2=i}^{t_1}(n+1) &= Q(p_2^{t_1}, p_0^{t_1}(n+1), N_2 - i), && (7)\\
&\qquad i = 0,1,\dots,N_2,\\
P_{j,t_2=i}^{t_1}(n+1) &= \frac{P_{0,t_2=i}^{t_1}(n+1)}{1-p_0^{t_1}(n+1)}[\alpha(p_2^{t_1}, p_0^{t_1}(n+1))]^j, && (8)\\
&\qquad i = 0,1,\dots,N_2-1, \quad j = 1,\dots,N_2-i,\\
P_j^{t_1}(n+1) &= \sum_{i=0}^{N_2-j} P_i^{t_2}(n)\cdot P_{j,t_2=i}^{t_1}(n+1), && (9)\\
&\qquad j = 0,1,\dots,N_2,\\
p_0^{t_2}(n+1) &= p_0(1-\widehat{x}_1)(q_2 + q_1 P_0^{t_1}(n+1)), && (10)\\
P_{0,t_1=i}^{t_2}(n+1) &= Q(p_2^{t_2}, p_0^{t_2}(n+1), N_2 - i), && (11)\\
&\qquad i = 0,1,\dots,N_2,\\
P_{j,t_1=i}^{t_2}(n+1) &= \frac{P_{0,t_1=i}^{t_2}(n+1)}{1-p_0^{t_2}(n+1)}[\alpha(p_2^{t_2}, p_0^{t_2}(n+1))]^j, && (12)\\
&\qquad i = 0,1,\dots,N_2-1, \quad j = 1,\dots,N_2-i,\\
P_j^{t_2}(n+1) &= \sum_{i=0}^{N_2-j} P_i^{t_1}(n+1)\cdot P_{j,t_1=i}^{t_2}(n+1), && (13)\\
&\qquad j = 0,1,\dots,N_2,\\
&\qquad n = 0,1,2,\dots,
\end{aligned}$$

where $Q(\cdot)$ and $\alpha(\cdot)$ are defined in Equations (4) and (5), respectively, with initial conditions

$$\begin{aligned}
P_0^{t_2}(0) &= 1,\\
P_i^{t_2}(0) &= 0, \quad i = 1,\dots,N_2.
\end{aligned}$$

Based on extensive numerical studies, we observe that Procedure 1 always leads to convergent results. Thus, we formulate this as a numerical fact.

Numerical Fact 1. Recursive Procedure 1 is convergent, i.e., the following limits exist:

$$\widehat{P}_i^{t_1} = \lim_{n\to\infty} P_i^{t_1}(n), \quad i = 0, 1, \ldots, N_2, \tag{14}$$

$$\widehat{P}_i^{t_2} = \lim_{n\to\infty} P_i^{t_2}(n), \quad i = 0, 1, \ldots, N_2. \tag{15}$$

3.4 Performance Evaluation

The selective assembly system with 2 groups of subcomponents can produce a matching finished product under the following conditions: (i) The assembly machine m_0 is functioning; (ii) Buffer B_1 is not empty; (iii) Buffer B_2 contains the matching mating part corresponding to the current main part to be assembled. Using the marginal probabilities of each group of parts in the mating line estimated from recursive procedure 1, the approximation of production rate for matching products, $\widehat{PR}_{match}^{sele}$, can be obtained.

Theorem 2. In an assembly system with two groups of subcomponents defined by assumptions 1)-9) and selection policy a), b),

$$\widehat{PR}_{match}^{sele} = p_0(1-\widehat{x}_1)[q_1(1-\widehat{P}_0^{t_1}) + q_2(1-\widehat{P}_0^{t_2})], \tag{16}$$

where calculation of $\widehat{x}_1$ is provided in Kuo *et al.* (1996), and $\widehat{P}_0^{t_i}$, $i = 1, 2$, are obtained from Procedure 1.

According to Equations (1), (2) and (16), the following corollary can be obtained.

Corollary 1. In an assembly system with two groups of subcomponents defined by assumptions 1)-9) and selection policy a), b),

$$\widehat{MD}^{sele} = \frac{q_1(1-\widehat{P}_0^{t_1}) + q_2(1-\widehat{P}_0^{t_2})}{1-\widehat{x}_2}. \tag{17}$$

To study the estimation accuracy, we implement both the analytical and simulation approaches in MATLAB. More than 100 experiments are conducted with system parameters generated uniformly from the following sets.

$$\begin{aligned} &p_i \in [0.5, 0.99], \quad i = 1, 2, \\ &p_0 \in [0.8, 0.99], \\ &q_1 \in [0.5, 0.8], \\ &g_1 \in [0.5, 0.8], \\ &N_i \in [3, 13], \quad i = 1, 2. \end{aligned}$$

Then q_2 and g_2 could be obtained as $1 - q_1$ and $1 - g_1$ respectively. Each simulation experiment has a warmup period of 10,000 time units and the following 50,000 time units are used for result collection. Moreover, 20 replications are carried out for each scenario. The resulting 95% confidence interval is typically around ± 0.008. Based on the experiments, we study the relative error of PR_{match} and MD. In addition, work-in-process (WIP) is another critical performance measurement which reflects the average number of parts contained in the buffer in the steady state. Using the marginal probabilities obtained from Equations (14) and (15), the estimate of WIP for each group of parts is formulated as follows.

$$\widehat{WIP}_{t_j}^{sele} = \sum_{i=0}^{N_2} i\widehat{P}_i^{t_j}, \quad j = 1, 2.$$

The relative errors for all the performance measurements in the ith experiment are defined as follows.

$$\begin{aligned} \delta_{PR_{match}}(i) &= \frac{\widehat{PR}_{match}^{sele}(i) - PR_{match,sim}^{sele}(i)}{PR_{match,sim}^{sele}(i)} \times 100\%, \\ \delta_{MD}(i) &= \frac{\widehat{MD}^{sele}(i) - MD_{sim}^{sele}(i)}{MD_{sim}^{sele}(i)} \times 100\%, \\ \delta_{WIP_{t_1}}^{sele}(i) &= \frac{\widehat{WIP}_{t_1}^{sele}(i) - WIP_{t_1,sim}^{sele}(i)}{N_2(i)} \times 100\%, \\ \delta_{WIP_{t_2}}^{sele}(i) &= \frac{\widehat{WIP}_{t_2}^{sele}(i) - WIP_{t_2,sim}^{sele}(i)}{N_2(i)} \times 100\%, \\ &i = 1, \ldots, 100, \end{aligned}$$

where $PR_{match}^{sele}(i)$, $MD^{sele}(i)$, $WIP_{t_1}^{sele}(i)$, $WIP_{t_2}^{sele}(i)$ are calculated from the model in ith experiment, and $PR_{match,sim}^{sele}(i)$, $MD_{sim}^{sele}(i)$, $WIP_{t_1,sim}^{sele}(i)$, $WIP_{t_2,sim}^{sele}(i)$ are obtained from the ith simulation experiment.

By observing the experiment results, $\delta_{PR_{match}}$ is typically below 5%, with rare cases up to 13%. For δ_{MD}, it is observed that the error dynamics are approximately aligned with $\delta_{PR_{match}}$. This is because δ_{PR} is relatively small comparing to $\delta_{PR_{match}}$. Thus δ_{MD} is dominated by $\delta_{PR_{match}}$ according to Equation (1).

As for the WIP estimation, the minor difference between simulation and estimation are typically observed in small WIP cases. The error tends to increase in scenarios where WIP is relatively large. Nevertheless, the difference is still mostly less than 1 part, which is acceptable in practical applications.

To integrate all the experiment results, the average absolute relative error for the above performance measures are listed in Table 1, which ensures that such a method could deliver acceptable accuracy for performance evaluation.

Table 1. Average relative error of estimates

$\bar{\delta}_{PR_{match}}$	$\bar{\delta}_{MD}$	$\bar{\delta}_{WIP_{t_1}}$	$\bar{\delta}_{WIP_{t_2}}$
2.37%	2.36%	6.35%	5.34%

3.5 Structural Property

Corollary 2. In an assembly system with two groups of subcomponents defined by assumptions 1)-9) and selection policy a), b),

$$\widehat{MD}^{sele} \geq \widehat{MD}^{rand}.$$

Corollary 2 illustrates the fact that selective assembly is guaranteed to improve MD of the system compared with random assembly. It is shown that the improvement by employing selective assembly varies from 3% up to more than 70% depending on the parameters setting of the system. Generally speaking, selection within the assembly system could increase MD dramatically.

3.6 Extension to General Case

In systems with multiple groups, a similar decomposition method as the two-group case can be applied.

When $M = 3$, the problem scales up since three groups of parts interact with each other in a single buffer. Estimating their individual marginal probability of buffer occupancy involves two conditioning evidences, which increase the complexity of analysis. To make the estimation tractable, a similar decomposition method based on two-group case is introduced. For instance, to estimate the marginal probability of group 1 parts, a combinatorial group 2&3 is introduced, which can be used to condition the group 1 probability in the buffer. In such a manner, the mating line is decomposed into two sub-lines for group 1 and group 2&3 parts. Then, the marginal probability of buffer occupancy for group 1 parts can be estimated by applying the procedure of the two-group case. Similarly, group 1&3 and group 1&2 are introduced to evaluate the marginal probability for groups 2 and 3, respectively. Finally, a recursive procedure for these three steps of the second level decomposition is formulated, and its convergence is proved and estimation accuracy has been justified.

When $M > 3$, similar to the three-group case, the mating line could be decomposed into two sub-lines by extracting one group and combining the rest into a single group, e.g., group i and group $1\&2\&\ldots\&(i-1)\&(i+1)\&\ldots\&M$, $i = 1, 2, \ldots, M$. This type of decomposition needs to be implemented for each group of parts in order to estimate the marginal probability density explicitly. Therefore, a total of M steps of decomposition are needed.

The detailed derivations and results can be found in Ju and Li (2013).

4. CONCLUSIONS

In this paper, a selective assembly system with unreliable machines and finite buffers is studied. Bernoulli machine reliability model is assumed. Analytical methods, based on a two-level decomposition approach, are introduced to evaluate the performance (production rate and match degree). The convergence of the iteration procedures has been justified analytically. It is shown that such a method results in high accuracy in performance approximation. Such a method provides a quantitative tool to study selective assembly in production environment.

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