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Robust Control of Aircraft Lateral Movement[1]

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Abstract: The paper deals with robust output feedback continuous control design for time-continuous linear plants under parametric uncertainties and external bounded disturbance. A parallel reference model (auxiliary loop) to the plant is used for obtaining the uncertainties acting on the plant. The proposed algorithm tracks the output of the plant to the reference output with the required accuracy. We apply the algorithm to the control of lateral movement of an aircraft under parametric and external disturbances. We also compare the proposed algorithm with the H^∞ control and speed-gradient algorithm. The simulation results illustrate the efficiency and robustness of the suggested control system.

1. INTRODUCTION

Modern highly maneuverable aircrafts, such as fighters, operate over a wide range of flight conditions, which vary with altitude, Mach number, angle of attack, and engine thrust. The mechanical characteristics of an airframe, such as the centre of gravity, change as well. The aircraft autopilot has to be able to produce a response that is accurate and fast despite sever variations in speed and altitude of the airframe or, in other words, in the face of large parametric uncertainty (Belkharraz and Sobel, 2007; Gurfil, 2001; Singh, Steinberg, and Page, 2003; Tsourdos and White, 2001) and external disturbances (Bukov, 2006). The adaptive and robust methods have to meet the conflicting requirements on the tuning rate and performance quality under conditions of lack of aircraft state measurements (Ben Yamin, Yaesh, and Shaked, 2007; Schumacher and Kumar, 2000; Singh et al., 2003).

The auxiliary loop method was introduced in Tsykunov (2007) for control of continuous-time plant under parametric uncertainties and external disturbance. Later related structures were used for control of nonlinear plants with structure uncertainty (Furtat and Tsykunov, (2008)), robust suboptimal control (Furtat, (2009)), control of dynamical networks (Furtat (2011), Furtat, Fradkov, and Tsykunov (2011), Furtat, Fradkov, and Tsykunov (2013), Furtat, (2014)). Usage of auxiliary loop algorithm based flight control is motivated by its simplicity and its properties of good compensation of perturbations. The idea of this method consists of implementation of an auxiliary loop with desired parameters parallel to a plant. The difference between the output of plant and the output of auxiliary loop gives a function which depends on parametric and external disturbances. This function is then used as input the the control law that guaranties required accuracy of the control system.

In this study, the auxiliary loop algorithm is applied for robust control of lateral movement of an aircraft in landing mode. For illustration of the effectiveness of this algorithm we compare the simulation results with H^∞ control and speed-gradient algorithm.

The paper is organized as follows. The problem statement is presented in Section 2. Some essentials of the auxiliary loop method are in Section 3. Section 4 describes the given model of the aircraft. In Section 5 the control of the model of the aircraft without disturbances is considered. Section 6 is devoted to the application of the auxiliary loop method for robust control of flexible aircraft. In Section 7 the comparison of the simulation results for auxiliary loop algorithm, H^∞ control, and speed-gradient algorithm are presented. Concluding remarks are given in Section 8. Appendix A gives the proof of the auxiliary loop algorithm.

2. PROBLEM STATEMENT

Consider the plant model

$$\begin{aligned} &Q(p)y(t) = kR(p)u(t) + f(t), \\ &p^{i-1}y(0) = y_{0i}, \quad i = 1, \dots, n, \end{aligned} \tag{1}$$

where $y(t) \in R$ is an output, $u(t) \in R$ is an input, $f(t) \in R$ is a uncontrollable bounded disturbance, $Q(p)$, $R(p)$ are linear differential operators with unknown coefficients, $\deg Q(p) = n$, $\deg R(p) = m$, $\gamma = n - m \geq 1$, γ is a relative degree, $k > 0$, y_{0i} are unknown initial conditions.

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We study (1) under the following assumptions.

Assumptions.

1. Unknown coefficients of the operators $Q(p)$, $R(p)$ and unknown coefficient k belong to a known bounded set Ξ.

2. Only signals $y(t)$ and $u(t)$ are available for measurement.

3. The plant (1) is minimum phase.

Consider the reference (nominal) model

$$Q_N(p)y_N(t) = R_N(p)u_0(t), \tag{2}$$

where $y_N(t) \in R$ is an output, $u_0(t) \in R$ is a command bounded signal, $Q_N(p)$ and $R_N(p)$ are linear differential operators with known coefficients, $R_N(\lambda)$ is Hurwitz polynomial, λ is a complex variable, deg $Q_N(p) = n$, deg $R_N(p) = m$.

The goal is to design a control law which provides the following condition

$$|y(t) - y_N(t)| < \delta \text{ for } t > T, \tag{3}$$

where $\delta > 0$ is a small enough scalar.

3. ROBUST ALGORITHM

Let us represent the system in terms of error with the nominal model. Represent the operators $R(p)$ and $Q(p)$ in the form

$$R(p) = R_N(p) + \Delta R(p),\ Q(p) = Q_N(p) + \Delta Q(p). \tag{4}$$

Here $\Delta R(\lambda)$ and $\Delta Q(\lambda)$ are polynomials with parametric uncertainties of (1), deg $\Delta Q_i(p) < n$, deg $\Delta R_i(p) < m$. Taking into account (1) and (4) write the tracking error $e(t) = y(t) - y_N(t)$ in the form

$$Q_N(p)e(t) = kR_N(p)u(t) + \sigma(t), \tag{5}$$

where

$$\sigma(t) = \tilde{\sigma}(t) - Q_N(p)y_N(t),$$
$$\tilde{\sigma}(t) = k\Delta Ru(t) - \Delta Q(p)y(t) + f(t).$$

The function $\sigma(t)$ depends on parametric uncertainty and external disturbances on (1). If $\tilde{\sigma}(t) = 0$ in (5), then we obtain a nominal plant (2). However, $\tilde{\sigma}(t) \neq 0$ in (1) from Problem statement. Therefore, we adopt the method from Tsykunov (2007) for compensation of $\sigma(t)$. Introduce the auxiliary loop

$$Q_0(p)e_a(t) = \beta R_N(p)u(t), \tag{6}$$

where $e_a(t)$ is an output of the auxiliary loop, $\beta > 0$ is a designed parameter, $Q_0(\lambda)$ is a desired polynomial for the closed-loop system. The auxiliary loop is a parallel model which characterizes the desired behaviour of the transients in the closed-loop system. Therefore, taking into account (5) and (6), form the error function $\zeta(t) = e(t) - e_a(t)$ as

$$Q_0(p)\zeta(t) = R_N(p)\varphi(t), \tag{7}$$

where

$$\varphi(t) = (k - \beta)u(t) + \frac{Q_0(p) - Q_N(p)}{R_N(p)}e(t) + \frac{1}{R_N(p)}\sigma(t)$$

is a new function which depends on a parametric uncertainty and disturbances on (1).

If derivatives of the signal $\zeta(t)$ are accessible for measurement, then the ideal control law $u(t)$ is defined by

$$u(t) = -\beta^{-1}R_N^{-1}(p)Q_0(p)\zeta(t). \tag{8}$$

Substituting (8) in (5) we obtain the equation of a closed-loop system

$$Q_0(p)e(t) = 0. \tag{9}$$

However, according to assumption 2, the derivatives of the signal $\zeta(t)$ are not available for measurement. Therefore, we rewrite the ideal control law (8) as

$$u(t) = -\beta^{-1}\left(\overline{Q}_0(p)\bar{\zeta}(t) + R_N^{-1}(p)\tilde{Q}_0(p)\zeta(t)\right), \tag{10}$$

where $\bar{\zeta}(t)$ is an estimate of the function $\zeta(t)$, $\deg \overline{Q}_0(p) = \gamma$, $\deg \tilde{Q}_0(p) < m$.

Substituting (10) in (5) and taking into account (8) rewrite the equation of a closed-loop system of the form

$$Q_0(p)e(t) = R_N(p)\overline{\Delta}(t) \tag{11}$$

where $\overline{\Delta}(t) = \bar{\zeta}(t) - \zeta(t)$.

For implementation of (10) consider the following observer (Atassi and Khalil (1999))

$$\dot{\xi}(t) = G_0\xi(t) + D_0\left(\bar{\zeta}(t) - \zeta(t)\right),\quad \bar{\zeta}(t) = L\xi(t), \tag{12}$$

where $\xi(t) \in R^{\gamma}$, $G_0 = \begin{bmatrix} 0 & I_{\gamma-1} \\ 0 & 0 \end{bmatrix}$, $I_{\gamma-1}$ is an identity matrix of order $\gamma - 1$, $D_0 = -\left[d_1\mu^{-1}, d_2\mu^{-2}, \ldots, d_\gamma\mu^{-\gamma}\right]^{\mathrm{T}}$, the coefficients $d_1, d_2, \ldots, d_\gamma$ are chosen such than the matrix $G = G_0 - DL$ is Hurwitz, $D = \left[d_1, d_2, \ldots, d_\gamma\right]^{\mathrm{T}}$, $\mu > 0$.

Consider the vector $\bar{\eta}(t) = \Gamma^{-1}\left(\xi(t) - \theta(t)\right)$, where $\Gamma = diag\left\{\mu^{\gamma-1}, \mu^{\gamma-2}, \ldots, \mu, 1\right\}$, $\theta(t) = \left[\zeta(t), \dot{\zeta}(t), \ldots, \zeta^{(\gamma)}(t)\right]^{\mathrm{T}}$. From (12), take the derivative of $\bar{\eta}(t)$

$$\dot{\bar{\eta}}(t) = \mu^{-1}G\bar{\eta}(t) + \tilde{b}\,\zeta^{(\gamma+1)}(t),\ \overline{\Delta}(t) = \mu^{\gamma-1}L\bar{\eta}(t),$$

where $\widetilde{b} = [0, \ldots, 0, 1]^{\mathrm{T}}$. Rewrite last equations in the form

$$\dot{\eta}(t) = \mu^{-1} G\eta(t) + b\dot{\zeta}(t), \; \overline{\Delta}(t) = \mu^{\gamma-1} L\eta(t). \tag{13}$$

Here $\eta_i(t) = \overline{\eta}_i(t) - \mu^{1+i-\gamma}\zeta^{(i-1)}(t)$, $i = \overline{2, \gamma}$, $\eta_1(t) = \overline{\eta}_1(t)$, $b = \left[\mu^{2-\gamma}, 0, \ldots, 0\right]^{\mathrm{T}}$. The two last equations are equivalent because $\eta_1(t) = \overline{\eta}_1(t)$ and $\left(p^{\gamma} + d_1\mu^{-1}p^{\gamma-1} + \ldots + d_{\gamma}\,\mu^{-\gamma}\right)\overline{\eta}_1(t) = p^{\gamma}\zeta(t)$.

Taking into account (13) rewrite equation (11) as

$$\begin{aligned} \dot{\varepsilon}(t) &= A_0\varepsilon(t) + \mu^{\gamma-1}\overline{b}g\Delta(t), \\ e(t) &= L_1\varepsilon(t), \quad \varepsilon(0) = 0. \end{aligned} \tag{14}$$

where A_0, $\overline{b}$ and L_1 are obtained from transformation of the transfer function $R_N(\lambda) / Q_0(\lambda)$ to system (14), $\Delta(t) = \left[\eta_1(t), \dot{\eta}_1(t), \ldots, \eta_1^{(\gamma)}(t)\right]^{\mathrm{T}}$, g^{T} is a vector consisting of coefficients of the operator $\overline{Q}_0(p)$.

Theorem (Tsykunov (2007)). Let Assumptions 1-3 hold. Then there exist a scalar $\mu_0 > 0$ such that for $\mu \le \mu_0$ the control systems (6), (10), (12) provide the goal (3).

Remark. It is seen from the theorem's proof in Appendix that the value δ in (3) can be overbounded as follows

$$\delta \le \sqrt{\lambda_{\min}^{-1}(P)\left(e^{-\chi t}V(0) + \left(1 - e^{-\chi t}\right)\chi^{-1}\mu_0\overline{\varphi}\right)}, \tag{15}$$

where $V(t) = \varepsilon^{\mathrm{T}}(t)P\varepsilon(t) + \eta^{\mathrm{T}}(t)H\eta(t)$, $P = P^{\mathrm{T}} > 0$, $H = H^{\mathrm{T}} > 0$ are solutions of equations $A_0^{\mathrm{T}}P + PA_0 = -Q_1$, $G^{\mathrm{T}}H + HG = -Q_2$, $Q_1 = Q_1^{\mathrm{T}} > 0$, $Q_2 = Q_2^{\mathrm{T}} > 0$, $\chi = \min\left\{\dfrac{\lambda_{\min}(Q_3)}{\lambda_{\max}(P)}, \dfrac{\lambda_{\min}(Q_4)}{\lambda_{\max}(H)}\right\}$,
$Q_3 = Q_1 - 2\mu_0^{\gamma-1}P\overline{b}g\left(P\overline{b}g\right)^{\mathrm{T}}$, $Q_4 = Q_2 - \mu_0^2 Hbb^{\mathrm{T}}H$, $\overline{\varphi} = 2\left(k_1^2 + \mu_0^{\gamma-2}k_2^2\right)$, $k_1 = \sup_t\left|\dot{\zeta}(t)\right|$, $k_2 = \sup_t\left|\Delta(t)\right|$, $\mu \le \mu_0$.

It follows from (15) that the value δ explicitly depend on β and μ. Moreover, the value δ in (3) can be reduced by decreasing of the values β and μ.

Results are illustrated in the following on the aircraft example.

4. MODEL OF THE LATERAL MOTION OF AN AIRCRAFT IN LANDING MODE

Consider the linearized model of the lateral motion of an aircraft in landing mode (Letov (1969), Bukov (2006))

$$\begin{aligned} \dot{x}(t) &= \left(A_N + B_N c^{\mathrm{T}}\right)x(t) + \vartheta B_N u(t) + kB_N f(t), \\ y(t) &= Lx(t), \quad x(0) = x_0, \end{aligned} \tag{16}$$

where $x(t) = \left[\Delta z(t), \Delta\psi(t), \Delta\gamma(t), \Delta\omega_x(t)\right]^{\mathrm{T}}$ and $u(t)$ are state vector and control of the linearized model of the aircraft without sliding, $f(t)$ is an unaccounted disturbance, $\Delta z(t)$ is a value of the lateral deviation of the aircraft of a mass center of a longitudinal axis of a landing strip, $\Delta\psi(t)$ is an angle between a longitudinal axis of the landing strip and the horizontal projection of a aircraft velocity vector, $\Delta\gamma(t)$ is a change on the roll angle of the aircraft, $\Delta\omega_x(t)$ is a change in angular velocity of the aircraft relative to the longitudinal axis, $u(t)$ is the aileron deviation from balancing position. The matrix A_N and the vector B_N in (16) are equal to

$$A_N = \begin{bmatrix} 0 & 85 & 0 & 0 \\ 0 & 0 & 0.12 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}, \quad B_N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3.4 \end{bmatrix}. \tag{17}$$

The matrices (17) are obtained in Letov (1969) for an aircraft speed equal to 85 m/s, $c = 0$, $\vartheta = 1$, and $k = 0$. According to Bukov (2006), parametric uncertainties are presented by

$$\begin{aligned} &c = [0 \;\; 0 \;\; 0 \;\; \rho], \; -20/17 \le \rho \le 20/17, \\ &-0.5 \le \vartheta \le 0, \quad -1 \le k \le 1, \quad |f(t)| \le 1. \end{aligned} \tag{18}$$

According to (Letov (1969), Bukov (2006)), it is enough to use the aileron deviation from balancing position for control of the lateral motion of an aircraft in landing mode.

5. NOMINAL MODEL OF THE AIRCRAFT

Here we explain the choice of the parameters of the nominal plant (2). According to Bukov (2006), consider the following performance index

$$J = \int_0^{\infty}\left(Qy^2(t) + ru_0^2(t)\right)dt, \tag{19}$$

where

$$Q = \begin{bmatrix} 6.25 \cdot 10^{-6} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 31 \end{bmatrix} \text{ and } r = 93. \tag{20}$$

For minimization of the quadratic coast (19) the optimal control $u_0(t)$ to be used in (2) is (Athans and Falb (1966))

$$u_0(t) = -K_0 x_N(t),$$

where $K_0 = r^{-1}B_N^{\mathrm{T}}H$ and the matrix $H = H^{\mathrm{T}} > 0$ is a solution of the Riccati equation $A_N^{\mathrm{T}}H + HA_N - r^{-1}HB_N B_N^{\mathrm{T}}H = -Q$. With respect to (17) and (20), we calculate the matrix K_0 and get

$$K_0 = [0.0003 \quad 0.3737 \quad 0.3218 \quad 0.3438]. \tag{21}$$

6. AUXILIARY LOOP ALGORITHM

Let $\varepsilon_a^{\mathrm{T}}(0) = [0\,0\,0\,0]$, $\beta = 0.4$. According to (17) and (21), we take the matrix A_0 as

$$A_0 = A_N - B_N K_0 = \\ = \begin{bmatrix} 0 & 85 & 0 & 0 \\ 0 & 0 & 0.12 & 0 \\ 0 & 0 & 0 & 1 \\ -0.0009 & -1.2707 & -1.0941 & -3.1688 \end{bmatrix}.$$

Rewrite auxiliary loop (5) as follows

$$\left(p^4 + 3.1688p^3 + 1.0941p^2 + 0.1525p + 0.009\right)e_a(t) \\ = 0.4 \cdot 34.68u(t).$$

Let $\overline{D} = [20\,150\,500\,625]^{\mathrm{T}}$ and $\mu = 0.01$. Then the observer (12) is defined by

$$\dot{\xi}_1(t) = -\xi_2(t) - 2\cdot 10^3\left(\xi_1(t) - \zeta(t)\right),$$
$$\dot{\xi}_2(t) = -\xi_3(t) - 1.5\cdot 10^6\left(\xi_1(t) - \zeta(t)\right),$$
$$\dot{\xi}_3(t) = -\xi_4(t) - 5\cdot 10^8\left(\xi_1(t) - \zeta(t)\right),$$
$$\dot{\xi}_4(t) = -6.25\cdot 10^{10}\left(\xi_1(t) - \zeta(t)\right),$$
$$\xi(0) = 0.$$

Finally, the control law (10) is given by

$$u(t) = -2.5\cdot 34.68\left(\dot{\xi}_4(t) + 3.1688\xi_4(t) + 1.0941\xi_3(t) \\ + 0.1525\xi_2(t) + 0.0009\xi_1(t)\right).$$

7. SEMULATION RESULTS

We assume that initial conditions $\Delta z_N(0) = \Delta z(0)$ and $\Delta\psi(0) = \Delta\gamma(0) = \Delta\omega_x(0) = 0$ are set before landing mode is started. Consider three uncertainty cases (Bukov (2006)).

Case 1. Let $\rho = -20/17$, $\vartheta = 0$ and $f(t) = 0.05 + 0.01\sin 0.1t$ in (18).

Case 2. Let $\rho = 20/17$, $\vartheta = 0$, $k = 1$ and $f(t) = 0.05 + 0.05\sin 0.2t$ in (18).

Case 3. Let $\rho = 20/17$, $\vartheta = -0.5$ (it means that the effectiveness of the ailerons is half of its nominal value), $k = 1$ and $f(t) = 0.01\sin 0.05t$ in (18).

For comparison we also consider the synthesis of control systems using H^∞ control and speed-gradient algorithm.

The H^∞ control is obtained by using Matlab procedures for the transfer function

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{34.68}{p^4 + 2p^3} \begin{bmatrix} \frac{1}{3.4} & 1 \end{bmatrix}.$$

It allows minimization of the error in equation (5).

According to Fradkov and Andrievsky (2011), the speed-gradient algorithm is presented by the following equations

$$e_c(t) = \frac{(0.1p+1)^2}{(p+1)^3}u(t),$$

$$\sigma_t = [1 \quad 3 \quad 3 \quad 1]^{\mathrm{T}}\left(e(t) + e_c(t)\right),$$

$$\dot{k}(t) = -10^3\sigma_t\left(e(t) + e_c(t)\right),$$

$$u(t) = -k^{\mathrm{T}}(t)\left(e(t) + e_c(t)\right) - 0.1\operatorname{sgn}\sigma_t.$$

Let $\Delta z(0) = 400$ m. In Fig. 1-6 the transients are presented for the tracking errors $e(t)$ and control signals $u(t)$ which are obtained by auxiliary loop algorithm, H^∞ control, and speed-gradient algorithm for each of three cases. In Fig.1-6 black curve, blue curve, and red curve correspond to the cases 1, 2, and 3 respectively. In Fig. 2, 4, 6 green line corresponds to the optimal control $u_0(t)$ for the nominal plant.

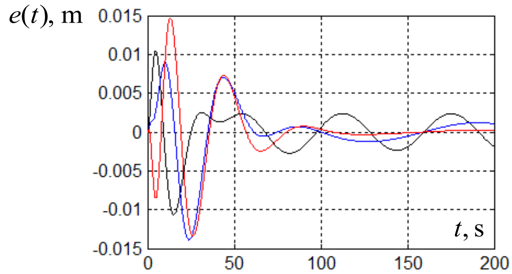

Fig. 1. Transients of $e(t)$ with using auxiliary loop method

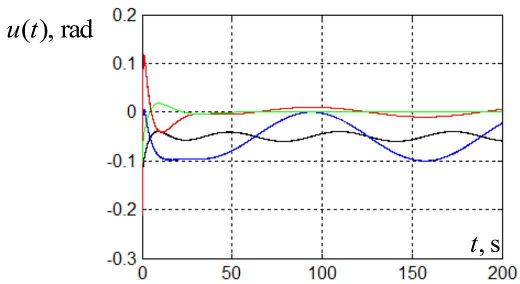

Fig. 2. Transients of $u(t)$ with using auxiliary loop method

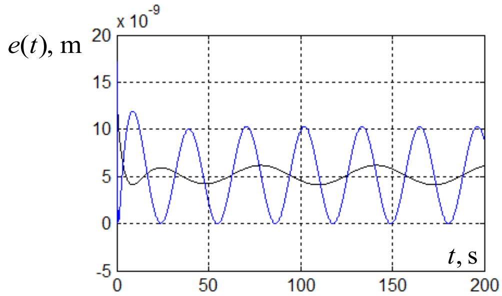

Fig.3. Transients of $e(t)$ with using H^∞ control (red line is not in Figure because the closed-loop system is unstable for case 3)

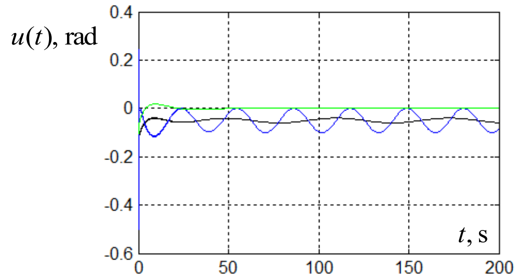

Fig. 4. Transients of $u(t)$ with using H^{∞} control (red line is not in Figure because the closed-loop system is unstable for case 3)

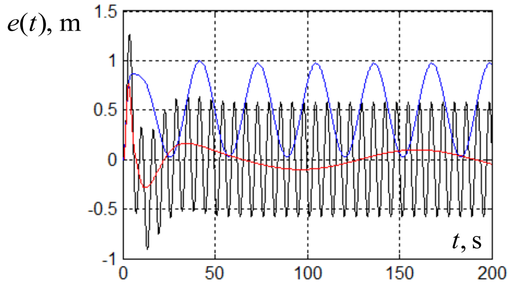

Fig. 5. Transients of $e(t)$ with using the speed-gradient method

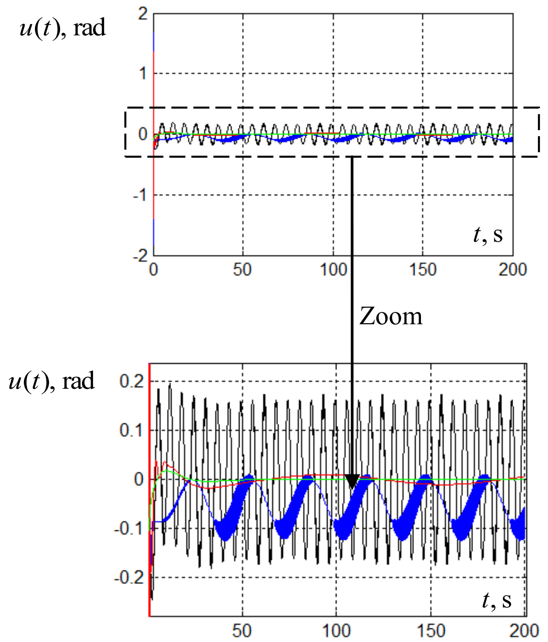

Fig. 6. Transients of $u(t)$ with using the speed-gradient method

The analysis of simulation results shows that the smallest amplitude of the tracking error is obtained by H^{∞} control. However, the closed-loop system is unstable for case 3; it follows that a control system is sensitive to parametric uncertainty in effectiveness of ailerons. The speed-gradient algorithm is stable for all parametric uncertainty but is highly sensitive to external disturbances. The auxiliary loop algorithm takes the middle place between H^{∞} control and speed-gradient algorithm. It allows to compensate parametric and external disturbances with moderate amplitude of the tracking error $e(t)$ and moderate deviation of the control signal. It follows from simulations that decrease of μ and increase of β the value δ in (3) can be reduced.

8. CONCLUSIONS

The robust method for disturbances compensation is applied for robust flight control system design. It is shown that an auxiliary loop allows to extract plant uncertainties and process these simolfuneously with perturbations. An example is considered illustrating a typical design procedure for control of the lateral motion of an aircraft in landing mode. Simulation results demonstrate the efficiency and robustness of the suggested control method.

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APPENDIX A.

Proof of Theorem. Rewrite equations (13) and (14) in the form

$$\begin{aligned} \dot{\varepsilon}(t) &= A_0\varepsilon(t) + \mu_2^{\gamma-1}\bar{b}g\Delta(t), \\ \mu_1\dot{\eta}(t) &= G\eta(t) + \mu_2 b\dot{\zeta}(t), \end{aligned} \tag{22}$$

where $\mu_1 = \mu_2 = \mu$. Consider following Lemma (Furtat, Fradkov, and Tsykunov (2013)).

Lemma (Furtat, Fradkov, and Tsykunov (2013)). Let the system be described by the following differential equation

$$\dot{x} = f(x, \mu_1, \mu_2, t), \tag{23}$$

where $x \in R^{s_1}$, $\mu = \mathrm{col}(\mu_1, \mu_2) \in R^{s_2}$, $f(x, \mu_1, \mu_2, t)$ is Lipchitz continuous function in x. Let (23) have a bounded closed set of attraction $\Omega = \{x \mid P(x) \le C\}$ for $\mu_2 = 0$, where $P(x)$ is piecewise-smooth, positive definite function in R^{s_1}. In addition let there exist some scalars $C_1 > 0$ and $\bar{\mu}_1 > 0$ such that the following condition holds

$$\sup_{|\mu_1|\le\bar{\mu}_1}\left[\left\langle \left[\frac{\partial P(x)}{\partial x}\right]^{\mathrm{T}}, f(x,\mu_1,0,t)\right\rangle \middle| \quad P(x) = C\right] \le -C_1 .$$

Then there exists $\mu_0 > 0$ such that the system (23) has the same set of attraction Ω for $\mu_2 \le \mu_0$.

According to Lemma, consider system (22) when $\mu_2 = 0$. Since the matrixes A_0 and G are Hurwitz solutions of system (22) are asymptotically stable. According to Lemma, the signals $\eta(t)$, $\varepsilon(t)$, $\Delta(t)$, $\dot{\zeta}(t)$ are bounded. Therefore, all signals in the closed-loop system are bounded.

However, from asymptotic stability of (22) when $\mu_2 = 0$ it does not follow asymptotic stability of (22) for $\mu_2 > 0$. Let $\mu_1 = \mu_2 = \mu_0$ in (22). Choose Lyapunov function $V(t)$ as in Remark. Take the derivative of $V(t)$ along the trajectories (22), we get

$$\begin{aligned} \dot{V}(t) &= -\varepsilon^{\mathrm{T}}(t)Q_1\varepsilon(t) + 2\mu_0^{\gamma-1}\varepsilon^{\mathrm{T}}(t)P\bar{b}g\Delta(t) \\ &- \mu_0^{-1}\eta^{\mathrm{T}}(t)Q_2\eta(t) + 2\mu_0\eta^{\mathrm{T}}(t)Hb\dot{\zeta}(t). \end{aligned} \tag{24}$$

Consider the following estimates

$$\begin{aligned} &2\mu_0^{\gamma-1}\varepsilon^{\mathrm{T}}(t)P\bar{b}g\Delta(t) \\ &\le 2\mu_0^{\gamma-1}\varepsilon^{\mathrm{T}}(t)P\bar{b}g\left(P\bar{b}g\right)^{\mathrm{T}}\varepsilon(t) + 2\mu_0^{\gamma-1}|\Delta(t)|^2 \\ &\le 2\mu_0^{\gamma-1}\varepsilon^{\mathrm{T}}(t)P\bar{b}g\left(P\bar{b}g\right)^{\mathrm{T}}\varepsilon(t) + 2\mu_0^{\gamma-1}k_2^2; \end{aligned}$$

$$\begin{aligned} 2\mu_0\eta^{\mathrm{T}}(t)Hb\dot{\zeta}(t) &\le 2\mu_0\eta^{\mathrm{T}}(t)Hbb^T H\eta(t) + 2\mu_0\left|\dot{\zeta}(t)\right|^2 \le \\ &\le 2\mu_0\eta^{\mathrm{T}}(t)Hbb^T H\eta(t) + 2\mu_0 k_1^2. \end{aligned}$$

Taking into account the estimates, (24) rewrites as

$$\dot{V}(t) \le -\varepsilon^{\mathrm{T}}(t)Q_3\varepsilon^{\mathrm{T}}(t) - \eta^{\mathrm{T}}(t)Q_4\eta(t) + \bar{\varphi}, \tag{25}$$

Obviously, there exist $\mu_0 > 0$ such that $Q_3 > 0$ and $Q_4 > 0$.

Rewrite (25) if the form

$$\dot{V}(t) \le -\chi V(t) + \mu_0\bar{\varphi} .$$

Solving this inequality with respect to $V(t)$, we get

$$V(t) \le e^{-\chi t}V(0) + \chi^{-1}\left(1 - e^{-\chi t}\right)\mu_0\bar{\varphi} ,$$

and $\lim_{t\to\infty} V(t) \le \chi^{-1}\bar{\varphi}$. Taking into account the structure of $V(t)$, we get

$$|e(t)| \le |\varepsilon(t)| \le \sqrt{\lambda_{\min}^{-1}(P)\left(e^{-\chi t}V(0) + \left(1 - e^{-\chi t}\right)\chi^{-1}\mu_0\bar{\varphi}\right)}. \tag{24}$$

From (24) the inequality (15) is obtained. Obviously, the right hand side of (15) depends on the value of μ_0. Therefore, the error $e(t)$ can be reduced by decreasing of the value of μ_0. The theorem is proved.