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Nonlinear Control and Observation of a Boost Converter Associated with a Fuel-Cell Source in Presence of Model Uncertainty

A.Tahri*, H. El Fadil, F. Giri***, F.Z. Chaoui***

**ENSET, Mohamed V University -Souissi, Rabat, Morocco*

***EISEI Team LGS Lab. ENSA, Ibn Tofail University, Kénitra, 14000, Morocco*

****GREYC Lab, UMR CNRS, University of Caen Basse-Normandie, 14032 Caen, France.*

Abstract: This study deals with the problem of controlling DC-to-DC switched power converter of boost type associated with a fuel cell generator. The system is modelled using the PWM technique. The aim is to tightly regulate the output voltage of the converter to a desired reference. The voltage-current characteristic is subject to parameter uncertainty. The system load is in turn time-varying. A nonlinear adaptive controller is designed using the Lyapunov approach, based on the nonlinear model of the system. The controller is formally shown to meet its control objectives.
Keywords: Fuel cell, boost converter, adaptive control, nonlinear observer, Lyapunov approach

1. INTRODUCTION

A combination of high prices of fossil fuels and the increased awareness of their negative environmental impact has influenced the development of new cleaner energy sources. Among various viable technologies, fuel cells have emerged as one of the most promising sources for both portable and stationary applications.

Fuel cell stacks produce DC voltage with a variation in output voltage with load conditions. Hence, to increase the utilization efficiency a power conditioner consisting of DC-DC converters is required for load interface. In this paper, the design and analysis for the control of a Boost converter (BC) is presented in detail.

Design of controllers for boost power converter used in FC generation system presents interesting challenges. There is an increasing need for a good controller design to perform tight regulation under load variation due to the most of power electronic application operated with parameter variation, non linearity, load disturbance, etc.

Much effort has been spent to define small-signal linear approximation of power converter and fuel cell characteristic so that classical control theory could be applied to the design (Chen et al., 2006, Phatiphat and Davat, 2010, Thounthong et al., 2008, Rajashekara, 2005).

The point is that, both the boost converter and the fuel cell exhibit highly nonlinear behaviour making linear controllers only effective within around specific operation points.

In (Spinetti et al., 2009) a Lyapunov based control is used to design a controller that regulates the output voltage to v_e involving a nominal load value R_0. However the voltage regulation performances fail when the boost converter feeds an unknown load R. In (El Fadil 2007) a backstepping design approach is used in orde,to estimate the load resistance, and to regulate the output voltage.) A Lyapunov based control principle in a hybrid energy storage system associating a FC and a supercapacitor is dealt with in (El Fadil *et al.* 2012. In (El Fadil *et al* 2011) adaptive output feedback control of interleaved parallel boost converters is used based on Lyapunov stability tools.

All the above studies consider that FC voltage is known or measured. In (Sira-Ramirez et al 1995) and in (Sira-Ramirez et al 1996) the adaptive backstepping is used to update all the parameters of the boost converter (BC), among of this, the unknown constant voltage source.

In this paper, the problem of controlling fuel cell boost converter (FCBC) system is dealt with based on a more accurate model that really accounts for the system nonlinearities and varying FC voltage. Doing so, the model turns out to be well representative of both the boost converter large-signal dynamic behaviour and the fuel-cell nonlinear characteristics.

The uncertainty and the control are “matched” if they appear in the same equation (Kristic et al 1995), in this case one can apply the Lyapunov approach to adaptive nonlinear control. Consequently, according to the system plant (1), a Lyapunov based control is used to achieve two objectives: (i) asymptotic

stability of the closed loop system and (ii) tight output DC link voltage regulation, in spite of bus impedance changes and fuel cell characteristics uncertainties. Accordingly, the controller involves online estimation of the FC characteristic. It is formally shown, using theoretical analysis and simulations, that the developed adaptive controller actually meets its control objectives.
The paper is organized as follows. Section 2 is devoted to presentation and modelling the boost converter while section 3 is devoted to determine the FC voltage estimator and current controller. The error convergence analysis will be shown in section 4. Section 5 is devoted to the simulation results. A conclusion and list of references end the paper

2. PRESENTATION AND MODELLING OF FUEL CELL BOOST CONVERTER SYSTEM

The boost converter is a circuit that is constituted of power electronics components connected as shown in figure 1. The circuit operating mode is the so-called Pulse Width Modulation (PWM).

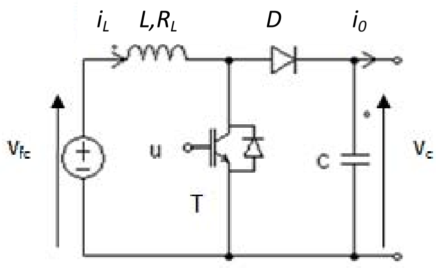

Fig.1: Fuel cell boost converter (FCBC)

A typical fuel-cell polarization curve is shown in Fig.2. This shows that the FC voltage v_{fc} is a decreasing nonlinear function of load current density i_L. For future analytical treatment, the polarization curve is given the following polynomial approximation (El Fadil et al., 2011):

$$v_{fc} = \sum_{k=0}^{n} b_k i_L^k \tag{1}$$

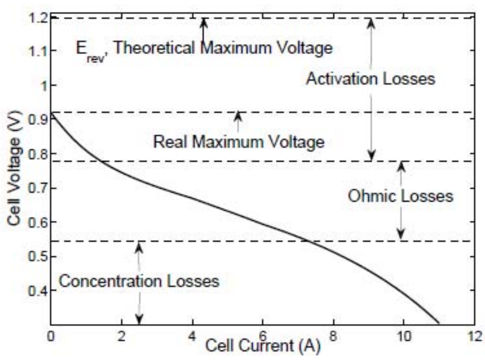

Fig.2: Fuel cell polarization zones

It is shown in many places that the averaged model of the power converter is the following (El Fadil et al., 2011, El Fadil et al., 2013)

$$\dot{x}_1 = \frac{-R_L}{L}x_1 - \frac{1-\mu}{L}x_2 + \frac{1}{L}\sum_{k=0}^{n} b_k x_1^k \tag{2a}$$

$$\dot{x}_2 = \frac{1-\mu}{C}x_1 - \frac{i_0}{C} \tag{2b}$$

Where x_1 and x_2 denote the average input current i_L and the average output capacitor voltage v_c respectively. The control input for the above model is the function μ, called duty ratio function.

3. CONTROLLER DESIGN

The V-I fuel cell characteristic is not unique as it depends on temperature, pressure and flow of hydrogen etc. In Fig3, two polarization curves are plotted in the case of two different pressure levels.

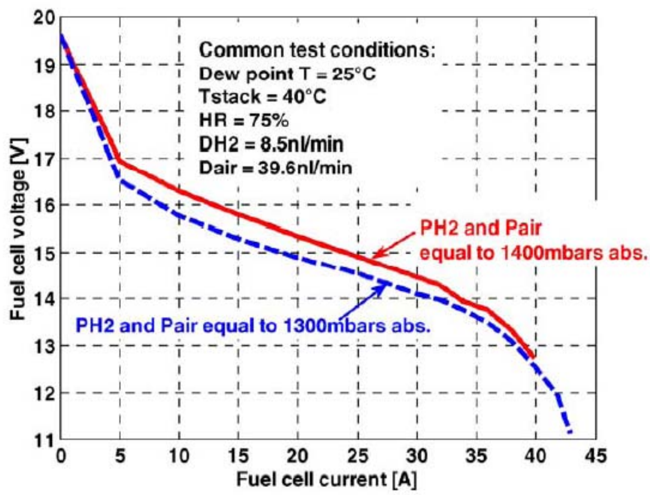

Fig.3: Polarisation curves recorded for two hydrogen and air pressures.

In this study, the coefficients b_k $(k = 0,..n)$ in (1) are let to be (what they are in practice i.e.) uncertain parameters. Also, the equivalent serial resistance, i.e. the parameter R_L in (2a), is let to be unknown. To cope with such model uncertainty the controller will be given a learning capacity. Specifically, the controller includes an on-line estimator of the unknown parameter vector:

$$\theta = \frac{1}{L}\left[b_0, b_1 - R_L, b_2 \cdots, b_n\right]^T \tag{3}$$

Accordingly, the model (2a-b) rewrites as follows:

$$\dot{x}_1 = -\frac{1-\mu}{L}x_2 + \phi^T\theta \tag{4a}$$

$$\dot{x}_2 = \frac{1-\mu}{C}x_1 - \frac{i_0}{C} \tag{4b}$$

with

$$\phi = \left[1, x_1, x_1^2, \cdots, x_1^n\right]^T \tag{4c}$$

The unknown parameter vector θ should be estimated based on equation (4a). One usual way to reach this is to introduce a state observer of the system consisting of (4a) and the additional equation $\dot{\theta} = 0$ with the current x_1 being the measurable output. The first equation of the observer is just a copy of (4a) augmented by a correction term in the estimation error, $\tilde{x}_1 = x_1 - \hat{x}_1$ .Specifically, one has

$$\dot{\hat{x}}_1 = -\frac{1-\mu}{L}x_2 + \phi^T\hat{\theta} + k\ \tilde{x}_1 \tag{5}$$

where $\hat{\theta}$ denotes the online estimate of the uncertain parameter vector θ. Such online estimation will be performed with an adaptive law yet to be determined, $k > 0$ is a design parameter. Comparing (5) and (4a), it follows that the current estimation error undergoes the following differential equation

$$\dot{\tilde{x}}_1 = \phi^T\tilde{\theta} - k\ \tilde{x}_1 \tag{6}$$

with $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter estimation error.
The control objective one seeks is that the load voltage v_c must be tightly regulated to a given reference value v_d. It is pointed out in many places (Sira-Ramirez, 1987, Sira-Ramirez, 2012), (El Fadil and Giri, 2007) that the zero-dynamics of the system (2a-b), taking the voltage x_2 as output, is unstable i.e. the system seen from the output voltage is non minimum phase. The usual practice is to reformulate the control objective in terms of the current x_1. Specifically, the latter must track a reference x_{1ref} where this is selected so that, if $x_1 = x_{1ref}$ then $x_2 = v_d$ where v_d is the desired output voltage chosen by the user. A relation between the current and the voltage references is readily obtained by writing the total power balance in the BC. Doing so, one gets using Fig. 1:

$$0 = -\frac{i_0}{x_{1ref}}v_d + L\sum_{k=0}^{n}\hat{\theta}_k x_{1ref}^k \tag{7}$$

Since, the parameters $\theta_k\ (k = 1\ldots n)$ are unknown; they have been replaced by their estimates in (7).
The next step, in the controller design is to determine a control that meets the control objective i.e. driving the following current tracking error to the origin:

$$e = x_1 - x_{1ref} \tag{8a}$$

It is readily seen from (4a) that e undergoes the following differential equation:

$$\dot{e} = -\frac{1-\mu}{L}x_2 + \phi^T(\hat{\theta} + \tilde{\theta}) - \dot{x}_{1ref} \tag{8b}$$

Let us consider the following Lyapunov function:

$$V_1 = \frac{\tilde{x}_1^{\ 2}}{2} + \frac{e^{\ 2}}{2} + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} \tag{9}$$

Clearly, V_1 is a positive definite function of the errors $(\tilde{x}_1, e_1, \tilde{\theta})$. Its derivative along the error-trajectory with (6) and (8b):

$$\dot{V}_1 = -k\ \tilde{x}_1^{\ 2} + e\ \psi + \tilde{\theta}^T(\phi\tilde{x}_1 + e\phi + \frac{\dot{\tilde{\theta}}}{\gamma}) \tag{10a}$$

with

$$\psi = -(1-\mu)\frac{x_2}{L} + \phi^T\hat{\theta} - \dot{x}_{1ref} \tag{10b}$$

Equation (10a) suggests the following control law and parameter adaptive law:

$$\mu = 1 - \frac{L}{x_2}(\phi^T\hat{\theta} - \dot{x}_{1ref} + c_1 e\ ) \tag{11}$$

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} = -\gamma\ \phi(\tilde{x}_1 + e) \tag{12}$$

where $c_1 > 0$ and $\gamma > 0$ are design parameters. Using (11) and (8b), one gets the following equation that describes the trajectory of the tracking error e_1:

$$\dot{e} = -c_1 e \tag{13}$$

Equations (6), (12) and (13) describe together the closed-loop system in terms of the error vector $(\tilde{x}_1, e\ , \tilde{\theta})$. Clearly, $(\tilde{x}_1, e\ , \tilde{\theta}) = (0,0,0)$ is an equilibrium of the system. The adaptive controller thus established is constituted of the control law (11) and the parameter adaptive law (5) and (12). The performances of this controller are analyzed in the next section.

4. CONTROLLER ANALYSIS

Definition 1. The vector function ϕ is said to be persistently exciting if, there exist positive real constants (T, ε) such that, one has for all t:

$$\int_t^{t+T}\phi(\tau)\phi^T(\tau)d\tau > \varepsilon I_n \tag{14}$$

Where I_n denotes the identity matrix of dimension n □

Theorem 1. Consider the FC-BC system, represented by the model (2a-b), in closed-loop with the adaptive output-feedback controller described by equations (11), (5) (12). Then, one has the following properties:

i) The equilibrium $(\tilde{x}_1, e, \tilde{\theta}) = (0,0,0)$ is stable.

ii) The errors $\tilde{x}_1$ and e are globally asymptotically vanishing.

iii) If ϕ is persistently exciting then, the parameter estimation error $\tilde{\theta}$ is globally asymptotically vanishing □

Proof. Part 1. It follows, substituting (11)-(12) in (10a):

$$\dot{V}_1 = -k\,\tilde{x}_1^{\,2} - c_1 e^{\,2} \tag{15}$$

This implies using the equilibrium point theorem that $(\tilde{x}_1, e_1, \tilde{\theta}) = (0,0,0)$ is stable in the Lyapunov sense (Khalil H 1996).

Proof of Part 2. For convenience, the closed-loop system equations (6), (11) and (12) are rewritten:

$$\dot{e} = -c_1 e \tag{16}$$

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} = -\gamma\,\phi(\tilde{x}_1 + e) \tag{17}$$

$$\dot{\tilde{x}}_1 = \phi^T \tilde{\theta} - k\,\tilde{x}_1 \tag{18}$$

Applying the Lasalle's invariance principle, it follows from (15) that, whatever the initial position $(\tilde{x}_1(0), e_1(0), \tilde{\theta}(0))$, the state vector $(\tilde{x}_1, e_1, \tilde{\theta})$ converges to M, the largest invariance set of the system (16-18), that is contained in the following set:

$$Z = \left\{ \left[\tilde{x}_1, e, \tilde{\theta}\right]^T \in R^{n+2} : \dot{V}_1 = 0 \right\}$$

It readily follows from (15) and (17) that:

$$Z = \left\{ [0,0,\alpha]^T : \alpha \in R^n \right\} \tag{19}$$

Part 2 is an immediate consequence of *(19)*.

Proof of Part 3. Now, let $[0,0,\alpha]^T \in M$ for some $\alpha \in R^n$. The aim is to show that $\alpha = 0$ i.e. M is reduced to the null vector. To this end, let $[\tilde{x}_1(0), e(0), \tilde{\theta}(0)] = [0,0,\alpha]$. As M is an invariance set, it follows that $[\tilde{x}_1(t), e(t), \tilde{\theta}(t)] \in M$, for all $t \geq 0$. As $M \subset Z$, one gets from (19) that:

$$\tilde{x}_1(t) = 0 \text{ and } e(t) = 0, \text{ for all } t \geq 0 \tag{20}$$

Then, it follows from (17) that, $\dot{\tilde{\theta}} = 0$ and so,

$$\tilde{\theta}(t) = \tilde{\theta}(0) = \alpha, \text{ for all } t \geq 0 \tag{21}$$

Also, using (20), one gets from (18) that:

$$\phi^T \tilde{\theta} = \phi^T \alpha = 0, \text{ for all } t \geq 0$$

Multiplying the above equation by ϕ and integrating over $[t\ t+T]$ yields

$$\left(\int_t^{t+T} \phi(\tau)\phi^T(\tau) \right) \alpha = 0$$

which together with (14) implies that $\alpha = 0$. We have proved that M is actually reduced to the null vector which establishes Part 3, using the invariance principle ■

5. SIMULATION RESULTS

In practice the system studied operate between to power limits P_{min} and P_{max}. That is, one can approximate the FC curves to straight lines. Then we get $\theta = [\theta_0, \theta_1]^T$ and $\phi = [1, x_1]^T$.

The whole simulated controller is summarized in Table 1.

Table1: Proposed adaptive nonlinear regulator

$$\dot{\hat{x}}_1 = -\frac{1-\mu}{L} x_2 + \hat{\theta}_1 x_1 + \hat{\theta}_0 + k\,\tilde{x}_1 \tag{22a}$$

$$\dot{\hat{\theta}}_0 = \gamma_1(\tilde{x}_1 + e) \tag{22b}$$

$$\dot{\hat{\theta}}_1 = \gamma_2 x_1(\tilde{x}_1 + e) \tag{22c}$$

$$\tilde{x}_1 = x_1 - \hat{x}_1 \tag{22d}$$

$$e = x_1 - \hat{i}_d \tag{22e}$$

$$\hat{i}_d = \frac{-\hat{\theta}_0 - \sqrt{\hat{\theta}_0^{\,2} + 4\hat{\theta}_1 \frac{i_0}{L} v_d}}{2\hat{\theta}_1} \tag{22f}$$

$$\mu = 1 - \frac{L}{x_2}(\hat{\theta}_1 x_1 + \hat{\theta}_0 - \dot{\hat{i}}_d + c_1 e) \tag{22g}$$

where $c_1 > 0$, $k > 0$ and $\gamma_{1,2} > 0$ are design parameters.

The simulated system is constituted of a FC source of 50KW, a boost converter that guaranties 4A and 20V of maximum ripples in fuel cell current and output voltage, respectively. The aim is to tightly regulate the output voltage of the converter to 750V.

The boost converter associated with the fuel cell (FC-BC) and the proposed adaptive controller are simulated using Matlab-Simulink software. The controlled system parameters and design parameters are summarized in Table 2.
Figure 4 illustrates the controller performances in presence of step changes in the load current. One can clearly shows that the output voltage v_c is regulated to its reference v_d=750V.

Figure 6 illustrates the controller behavior in presence of changes in fuel and air pressures of the FC. These changes induce the variation in the V-I characteristic of the FC from curve 1 to curve 2 (see Fig. 5). Figure 6 shows that, despite these uncertainties, the output voltage vc is regulated to its desired value vd=750V.

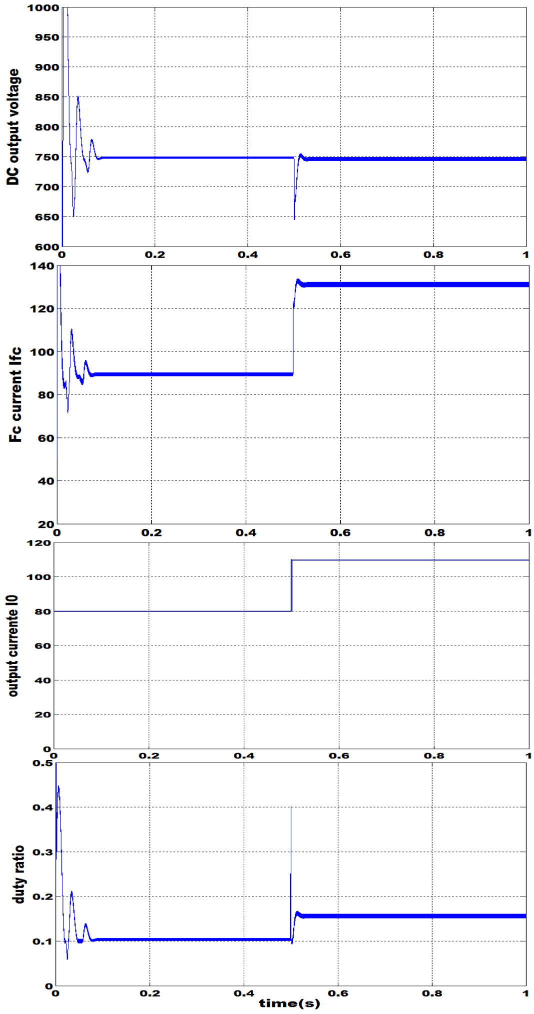

Fig. 4: Controller behavior in presence of output voltage reference v_d=750V and load current changes

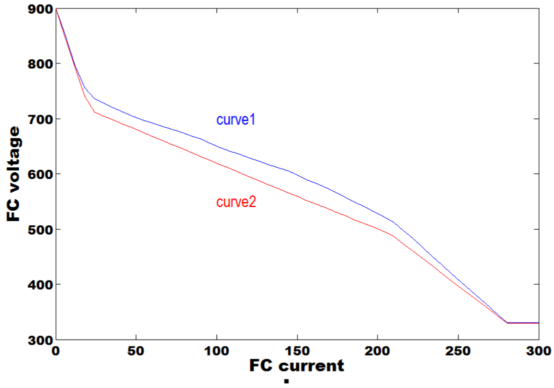

Fig. 5: simulated FC curves for two pressures.

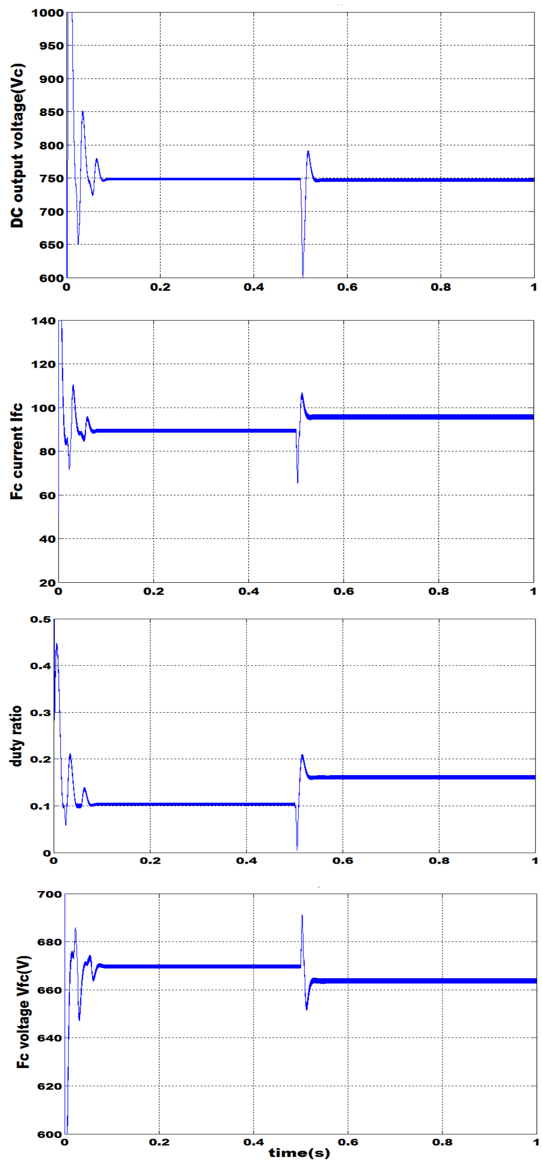

Fig. 6: Controller behaviour in presence of output voltage reference v_d=750V and FC pressure changes.

Table 2: FC-IBC characteristics

Parameter	Symbol	Value
Design parameters	c_1	300
	γ_1	0.01
	γ_2	10^5
	k	10^4
Inductance	L	$4.7mH$
Output Capacitor	C	$300\,\mu F$
Switching frequency	f_s	$10kHz$

6. CONCLUSION

Theoretical analysis and simulations studies prove that the closed loop realize the main aim which is to tightly regulate the FC-BC output voltage to the desired reference, in spite of the load current variation and the operating fuel and air pressures of the FC shift. The controller design is based on the average PWM model of the BC. The adaptive Lyapunov based approach allows a sensorless learning capability about the FC voltage necessary to calculate the indirect control law. The FC voltage-current function assumed to be unknown and depends of the operating conditions.

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