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Stabilization of Nonlinear System with Input Delay and Biased Sinusoidal Disturbance

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Abstract: We present a new stabilization approach for nonlinear plants with a long input delay. The stabilization problem is complicated by the influence of an uncertain biased sinusoidal disturbance. The problem of control design is solved with using measurable state variables. The adaptive scheme allows to identify the frequency and other parameters of the external disturbance that are used in a rejection loop. The main result provides delay compensation for a class of nonlinear systems and asymptotic stability for the closed-loop system..

Keywords: asymptotic stability, time-delay system, disturbance rejection, nonlinear systems.

1. INTRODUCTION

The paper is devoted to the development of control methods in terms of delay and disturbances. Over the past 60 - 70 years a variety of approaches to the control of systems with time delay (see, e.g., Bobtsov [2008], Krstic and Smysklyaev [2008], Krstic [2009], Niculescu and Annaswamy [2004], Parsheva and Tsykunov [2001], Pyrkin et al. [2010 a], Pyrkin [2010], Pyrkin et al. [2011], Pyrkin and Bobtsov [2011], Smith [1959]) were given by researchers from around the world. A large number of works devoted to the analysis of closed systems using Lyapunov-Krasovskii functional, which for systems with delayed state is analogous to the classical Lyapunov functions (see, for example Bobtsov [2008], Pyrkin et al. [2011], Pyrkin and Bobtsov [2011]). More difficult case, in the authors' opinion, is the synthesis of controllers for the systems with delay in the control. For such systems often the so-called Smith predictor (Smith [1959]) and its expansion, proposed, in particular, in (Arstein, [1982], Krstic and Smysklyaev [2008], Krstic [2009], Kwon and Pearson [1980], Manitus and Olbrot [1979]) are used. In (Krstic [2009]) the exponential stability of the closed-loop system with the predictor was proved using Lyapunov functions, which is useful for the stabilization of systems with input delay. Obviously, the disadvantage of this approach is that it requires accurate estimates of all parameters of the system and is not applicable for nonlinear systems. However, this approach has been extended for asymptotically stable linear parametrically uncertain systems. For example, the method (Parsheva and Tsykunov [2001]) allows us to solve the problem of tracking the reference signal for a certain class of parametrically undefined objects.

We can also highlight works (Krstic and Smysklyaev [2008], Pyrkin et al. [2010 a]), in which the approach (Krstic [2009]) was extended to the stabilization of linear time-invariant system in conditions of uncertain sinusoidal disturbance.

In this paper we consider the problem of stabilization of a nonlinear system

$$\dot{x}(t) = Ax(t) + B(u(t-D) + \delta(t)) + \Psi(y(t)), \quad (1)$$

where $\delta(t)$ is an unmeasured biased sinusoidal disturbance with unknown parameters.

In serial of works the frequency estimation approaches were investigated (Bobtsov and Pyrkin [2012], Bobtsov et al. [2012], Pyrkin et al. [2010 a,b], Pyrkin [2010], Pyrkin and Bobtsov [2011]). In (Pyrkin et al. [2010 a,b], Pyrkin [2010]) the exponential convergence of frequency estimator has been proved. In (Bobtsov and Pyrkin [2010]) the frequency estimation scheme was extended to a multisinusoidal case and in (Bobtsov et al. [2011 b]) for the multisinusoidal signals with nonzero offset. In (Bobtsov et al. [2012], Pyrkin and Bobtsov [2011]) the estimation result was investigated for the biased sinusoidal disturbance corrupted by irregular noise. Moreover, the frequency estimation scheme proposed in (Pyrkin and Bobtsov [2011]) is the simplest one in comparison with known analogues

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(Hsu et al. [1999], Marino et al. [2007, 2008], Marino and Tomei [2011, 2013], Xia [2002]).

The control algorithm proposed in this paper will develop the results (Pyrkin et al. [2010 a,b]) for the case of nonlinear plant. Also we can note that this paper develops a fairly extensive self-direction associated with the compensation of disturbances having a sinusoidal nature.

2. STATEMENT OF THE PROBLEM

In this paper we will consider the plant (1) in the following form

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) + \psi_1(y(t-\tau_1)) + a_1 y(t),\\ &\dots\\ \dot{x}_n(t) &= u(t-D) + \psi_n(y(t-\tau_n)) + a_n y(t) + \delta(t),\\ y(t) &= x_1(t),\end{aligned} \tag{2}$$

where $x(t) = col\{x_1, \ x_2, ..., x_n\}$ is the known vector of state variables, $u(t)$ is the scalar input with initial condition $u(t-D) = 0$ for $t < D$, $y(t)$ is measurable scalar output, $D \geq 0$ is known constant delay, a_i are known parameters, $\psi_i(y(t-\tau_i))$ and τ_i are corresponding, known nonlinear functions and positive numbers, $\delta(t) = \sigma_0 + \sigma\sin(\omega t + \vartheta)$ is an unknown disturbance.

The objective is to find the control $u(t)$ that achieves regulation of the output

$$\lim_{t\to\infty} y(t) = 0 \tag{3}$$

under the following assumptions:

Assumption 1. Parameters $\tau_i \geq D$ for all $i = \overline{1,n}$.

Assumption 2. Parameters σ_0, σ, ω, ϑ of the disturbance $\delta(t)$ are unknown.

3. MAIN RESULT

Let us assume $\Psi(y) = 0$, $\delta(t) = 0$ and $D = 0$. Then we have the trivial control:

$$u(t) = Kx(t), \tag{4}$$

For the case $D > 0$ rewrite the control (4) in the following form:

$$u(t) = Kx(t+D), \tag{5}$$

Obviously, the control law of the form (5) is not realizable, since the vector $x(t+D)$ is not available for direct measurement. However, following (Krstic [2009]), the vector $x(t+D)$ can be calculated as follows:

$$\begin{aligned}x(t+D) &= e^{A(t+D)}x(0) + \int_0^{t+D} e^{A(t+D-\tau)}Bu(\tau-D)d\tau\\ &= e^{AD}e^{At}x(0) + e^{AD}\int_0^t e^{A(t-\tau)}Bu(\tau-D)d\tau\\ &+ \int_t^{t+D} e^{A(t+D-\tau)}Bu(\tau-D)d\tau\\ &= e^{AD}x(t) + \int_{t-D}^t e^{A(t-\tau)}Bu(\tau)d\tau.\end{aligned}$$

Then the control algorithm, which provides stabilization of systems with delay is

$$u(t) = Ke^{AD}x(t) + K\int_{t-D}^t e^{A(t-\tau)}Bu(\tau)d\tau. \tag{6}$$

However, in terms of the problem the object is non-linear control and is affected by disturbance

$$\delta(t) = \sigma_0 + \sigma\sin(\omega t + \vartheta).$$

Feasible solution of the problem can be found in a few steps

Step 1. Find the estimation of disturbance $\delta(t) = \sigma_0 + \sigma\sin(\omega t + \vartheta) = \sigma_0 + \sigma_1\sin\omega t + \sigma_2\cos\omega t$. Construct the following observer

$$\dot{\hat{x}}_n(t) = u(t-D) + \psi_n(y) + a_n y(t) + k_n\tilde{x}_n(t), \tag{7}$$

where $\hat{x}_n(t)$ is an estimation of $x_n(t)$, $\tilde{x}_n(t) = x_n(t) - \hat{x}_n(t)$ and parameter $k_n > 0$, $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$.

Consider the following error

$$\tilde{x}_n(t) = x_n(t) - \hat{x}_n(t). \tag{8}$$

After differentiating the equation (8) by force (2) and (7) we obtain

$$\dot{\tilde{x}}_n(t) = -k_n\tilde{x}_n(t) + \delta(t).$$

Since we are dealing with a differential equation of the first order, the signal $\tilde{x}_n(t) = x_n(t) - \hat{x}_n(t)$ is a sinusoidal function of the same frequency ω as the disturbance $\delta(t) = \sigma_0 + \sigma\sin(\omega t + \vartheta)$ and can be represented in the form

$$\tilde{x}_n(t) = \mu_0 + \mu\sin(\omega t + \bar{\vartheta}) + \varepsilon_1(t) = \frac{1}{p + k_n}\delta(t),$$

where $p = d/dt$ and $\varepsilon_1(t)$ is an exponentially decaying function.

Following the idea presented in (Bobtsov and Pyrkin [2012], Bobtsov et al. [2012], Pyrkin et al. [2010 a,b], Pyrkin [2010]) we use the signal $\tilde{y}_1$ to estimate the frequency of the disturbance. We start by introducing the linear second-order filter

$$\xi(s) = \frac{\lambda_0^2}{\gamma(s)}\tilde{x}_n(s), \tag{9}$$

where $\lambda_0 > 0$, $\gamma(s) = s^2 + \gamma_1 s + \gamma_0$ is a Hurwitz polynomial with two different eigenvalues λ_1 and λ_2. Let $\gamma_0 = \lambda_0^2$ and $\lambda = \min_{i=1,2}\{|\mathrm{Re}\,\lambda_i|\}$.

Lemma 1. For the filter (9) and the input signal $\tilde{x}_n(t)$ the relation

$$\dddot{\xi}(t) = \theta\dot{\xi}(t) + \varepsilon(t) \tag{10}$$

holds, where functions $\dot{\xi}(t)$ and $\dddot{\xi}(t)$ are derivatives of the output variable of the linear filter (9)

$$\dot{\xi}(s) = \frac{\lambda_0^2 s}{\gamma(s)}\tilde{x}_n(s), \qquad \dddot{\xi}(s) = \frac{\lambda_0^2 s^3}{\gamma(s)}\tilde{x}_n(s), \tag{11}$$

$\theta = -\omega^2$, function $|\varepsilon(t)| \leq \rho_0 e^{-\lambda t}$ and its derivatives are bounded by an exponentially decaying function.

Proof 1. It is well known (Pyrkin et al. [2010 a,b], Pyrkin [2010], Pyrkin and Bobtsov [2011]) that signal $\tilde{x}_n$ is the solution of the system

$$\dddot{\tilde{x}}_n(t) = -\omega^2\dot{\tilde{x}}_n(t) + \dot{\varepsilon}_1(t) + \dddot{\varepsilon}_1(t). \tag{12}$$

Taking the Laplace transformation in (12) we obtain

$$s^3\tilde{x}_n(s) = \theta s\tilde{x}_n(s) + d(s), \tag{13}$$

where $d(s)$ denotes initial conditions and terms caused by $\varepsilon_1(t)$. Multiplying (13) on $\frac{\lambda_0^2}{\gamma(s)}$ with respect to (9) yields

$$s^3\xi(s) = \theta s\xi(s) + \frac{\lambda_0^2}{\gamma(s)}d(s). \tag{14}$$

After inverse Laplace transformation in (14) we have necessary equation (10)

$$\dddot{\xi}(t) = \theta\dot{\xi}(t) + \varepsilon(t), \tag{15}$$

where $\varepsilon(t) = \mathcal{L}^{-1}\{\frac{\lambda_0^2 d(s)}{\gamma(s)}\}$. By force the polynomial $\gamma(s)$ the function $\varepsilon(t)$ can be represented as a sum of decaying exponents. Therefore, derivatives of these functions are also exponentially decaying.

The adaptive scheme for frequency estimation is presented in the following theorem (Pyrkin and Bobtsov [2011]).

Theorem 2. The update law

$$\hat{\omega} = \sqrt{\left|\hat{\theta}\right|}, \tag{16}$$

$$\hat{\theta} = \chi + k\dot{\xi}\ddot{\xi}, \tag{17}$$

$$\dot{\chi} = -k\dot{\xi}^2\hat{\theta} - k\ddot{\xi}^2, \tag{18}$$

where $k > 0$, guarantees that the estimation error $\tilde{\omega} = \omega - \hat{\omega}$ exponentially converges to zero:

$$|\tilde{\omega}(t)| \le \rho_1 e^{-\beta_1 t}, \quad \rho_1, \beta_1 > 0, \quad \forall t \ge 0. \tag{19}$$

Proof 2. Using Lemma 1, we compute the derivative of the estimation error $\tilde{\theta} = \theta - \hat{\theta}$:

$$\begin{aligned}
\dot{\tilde{\theta}}(t) &= \dot{\theta} - \dot{\hat{\theta}}(t) \\
&= -\dot{\chi}(t) - k\ddot{\xi}^2(t) - k\dot{\xi}(t)\dddot{\xi}(t) \\
&= k\dot{\xi}^2(t)\hat{\theta}(t) + k\ddot{\xi}^2(t) - k\ddot{\xi}^2(t) - k\dot{\xi}(t)\dddot{\xi}(t) \\
&= k\dot{\xi}^2(t)\hat{\theta}(t) - k\dot{\xi}^2(t)\theta - k\dot{\xi}(t)\varepsilon(t) \\
&= -k\dot{\xi}^2(t)\tilde{\theta}(t) - k\dot{\xi}(t)\varepsilon(t).
\end{aligned} \tag{20}$$

Consider the Lyapunov function

$$V(t) = \frac{1}{2}f(t)\tilde{\theta}^2(t), \tag{21}$$

where $f(t)$ is a bounded positive function that will be chosen later. We get

$$\begin{aligned}
\dot{V} &= \frac{1}{2}\dot{f}\tilde{\theta}^2 + f\tilde{\theta}\dot{\tilde{\theta}} \\
&= \frac{1}{2}\dot{f}\tilde{\theta}^2 - kf\dot{\xi}^2\tilde{\theta}^2 - kf\dot{\xi}\tilde{\theta}\varepsilon \\
&\le \frac{1}{2}\dot{f}\tilde{\theta}^2 - \frac{k}{2}f\dot{\xi}^2\tilde{\theta}^2 + \frac{k}{2}f\varepsilon^2.
\end{aligned} \tag{22}$$

From (15) we have

$$\dot{\xi}(t) = \mu_1\sin(\omega t + \phi_1) + \varepsilon_1(t), \tag{23}$$

where μ_1 is an amplitude, ϕ_1 is a phase shift, $\varepsilon_1(t)$ is an exponentially decaying function as well as its derivatives. From (23) we obtain

$$\begin{aligned}
\dot{\xi}^2(t) &= (\mu_1\sin(\omega t + \phi_1) + \varepsilon_1(t))^2 \\
&= \frac{1}{2}\mu_1^2 - \frac{1}{2}\mu_1^2\cos(2\omega t + 2\phi_1) \\
&\quad + 2\mu_1\sin(\omega t + \phi_1)\varepsilon_1(t) + \varepsilon_1^2(t).
\end{aligned} \tag{24}$$

Substituting (24) into (22) we get

$$\begin{aligned}
\dot{V} \le{}& \frac{1}{2}\dot{f}\tilde{\theta}^2 - \frac{1}{2}k\mu_1^2\frac{1}{2}f\tilde{\theta}^2 \\
&- \frac{1}{2}kf\tilde{\theta}^2\left(-\frac{1}{2}\mu_1^2\cos(2\omega t + 2\phi_1)\right. \\
&\left.+2\mu_1\sin(\omega t + \phi_1)\varepsilon_1 + \varepsilon_1^2\right) + \frac{1}{2}kf\varepsilon^2.
\end{aligned} \tag{25}$$

Let $f(t) = e^{g(t)}$, where

$$\begin{aligned}
g(t) = {}& k\int_0^t \left(2\mu_1\sin(\omega\tau + \phi_1)\varepsilon_1(\tau) + \varepsilon_1^2(\tau)\right)d\tau \\
&- \frac{1}{4}k\mu_1^2(\sin(2\omega t + 2\phi_1) - \sin(2\phi_1)).
\end{aligned} \tag{26}$$

Since $g(t)$ depends on bounded harmonic and exponential functions, it is bounded, $|g(t)| \le C_1$, where C_1 is a positive constant. Therefore, $e^{-C_1} \le f(t) \le e^{C_1}$ and the Lyapunov function V is well defined.

Using (26), from (25) after a number of simple transformation we obtain

$$\begin{aligned}
\dot{V}(t) &\le -\frac{1}{2}k\mu_1^2\frac{1}{2}f(t)\tilde{\theta}^2(t) + \frac{1}{2}kf(t)\varepsilon^2(t) \\
&\le -C_2V(t) + \rho_2 e^{-\beta_2 t},
\end{aligned} \tag{27}$$

where $C_2 = \frac{1}{2}k\mu_1^2$, $\rho_2 = \frac{1}{2}ke^{C_1}\rho_0^2$, and $\beta_2 = 2\lambda$ are positive constants. Using comparison principle (Khalil [2002]), it is easy to show that

$$V(t) \le \rho_3 e^{-\beta_3 t}, \tag{28}$$

where $\rho_3 > 0$ and $\beta_3 = \min\{C_2, \beta_2\}$. From (21) and (28) we get

$$\left|\tilde{\theta}(t)\right| \le \sqrt{\frac{2\rho_3}{e^{-C_1}}}e^{-\frac{1}{2}\beta_3 t}. \tag{29}$$

Using (16), it is straightforward to show that

$$|\tilde{\omega}(t)| \le \sqrt{\left|\tilde{\theta}(t)\right|} \le \rho_1 e^{-\beta_1 t}, \tag{30}$$

where $\rho_1 = \sqrt[4]{2\rho_3 e^{C_1}}$ and $\beta_1 = \beta_3/4$.

Now we construct the estimation of the disturbance. Consider the equation

$$\begin{aligned}
\dot{\tilde{x}}_n(t) &= -k_n\tilde{x}_n(t) + \sigma_1\sin\omega t + \sigma_2\cos\omega t \\
&= -k_n\tilde{x}_n(t) + \bar{\sigma}^T\upsilon.
\end{aligned} \tag{31}$$

where $\bar{\sigma} = \begin{bmatrix}\sigma_0\\ \sigma_1\\ \sigma_2\end{bmatrix}$ and $\upsilon = \begin{bmatrix}\upsilon_0\\ \upsilon_1\\ \upsilon_2\end{bmatrix} = \begin{bmatrix}1\\ \sin\omega t\\ \cos\omega t\end{bmatrix}$.

For regressor $\upsilon = \begin{bmatrix}1\\ \sin\omega t\\ \cos\omega t\end{bmatrix}$ substitute $\hat{\omega}(t) = \sqrt{\left|\hat{\theta}(t)\right|}$ and prove

$$\lim_{t\to\infty}(\upsilon - \hat{\upsilon}) = 0,$$

where $\hat{\upsilon} = \begin{bmatrix}1\\ \hat{\upsilon}_1\\ \hat{\upsilon}_2\end{bmatrix} = \begin{bmatrix}1\\ \sin\hat{\omega}t\\ \cos\hat{\omega}t\end{bmatrix}$.

Consider separate term

$$\hat{\upsilon}_1 = \sin\hat{\omega}t = \sin(\omega t - \tilde{\omega}t) = \sin\omega t\cos\tilde{\omega}t - \cos\omega t\sin\tilde{\omega}t.$$

By force

$$\begin{aligned}
\tilde{\omega}t &= \omega t - \hat{\omega}t \\
&= \omega t - \sqrt{\left|\hat{\theta}t^2\right|} \\
&= \omega t - \sqrt{\left|\theta t^2 - \tilde{\theta}t^2\right|} \\
&= \omega t - \sqrt{|\theta t^2 - \beta t^2 e^{-\alpha t}|}
\end{aligned}$$

it is easy to see

$$\lim_{t\to\infty} \tilde{\omega}t = \omega t - \sqrt{|\theta t^2|} = 0$$

and

$$\lim_{t\to\infty} \sin\tilde{\omega}t = 0, \qquad \lim_{t\to\infty} \cos\tilde{\omega}t = 1.$$

So

$$\lim_{t\to\infty} (\upsilon_1(t) - \hat{\upsilon}_1(t)) = 0.$$

Similarly, we can show

$$\lim_{t\to\infty} (\upsilon_2(t) - \hat{\upsilon}_2(t)) = 0.$$

For identification vector $\bar{\sigma}$ write an ideal algorithm

$$\dot{\hat{\bar{\sigma}}} = -\gamma_\sigma \upsilon\upsilon^T \hat{\bar{\sigma}} + \gamma_\sigma \upsilon\upsilon^T \bar{\sigma}, \tag{32}$$

where $\gamma_\sigma > 0$ is any positive constant.

It easy to show that algorithm (32) by force PE condition of vector $\upsilon(t)$, provides

$$\lim_{t\to\infty} \left(\bar{\sigma} - \hat{\bar{\sigma}}(t)\right) = 0.$$

However the vector $\bar{\sigma}$ contains unknown components, and therefore, the algorithm (32) is unrealizable. Using equation (31), for (32) we obtain

$$\dot{\hat{\bar{\sigma}}} = -\gamma_\sigma \upsilon\upsilon^T \hat{\bar{\sigma}} + \gamma_\sigma \upsilon(\dot{\tilde{x}}_n + k_n\tilde{x}_n).$$

Consider the new variable $\chi = \hat{\bar{\sigma}} - \gamma_\sigma \upsilon\tilde{x}_n$. Then

$$\begin{aligned}
\dot{\chi} &= \dot{\hat{\bar{\sigma}}} - \gamma_\sigma(\dot{\upsilon}\tilde{x}_n + \upsilon\dot{\tilde{x}}_n) \\
&= -\gamma_\sigma \upsilon\upsilon^T \hat{\bar{\sigma}} + \gamma_\sigma \upsilon(\dot{\tilde{x}}_n + k_n\tilde{x}_n) - \gamma_\sigma(\dot{\upsilon}\tilde{x}_n + \upsilon\dot{\tilde{x}}_n) \\
&= -\gamma_\sigma \upsilon\upsilon^T \hat{\bar{\sigma}} + \gamma_\sigma k_n\tilde{x}_n - \gamma_\sigma\dot{\upsilon}\tilde{x}_n.
\end{aligned}$$

Thus, we implemented the algorithm identifying the disturbance. For identification of vector $\bar{\sigma}$ we use the following algorithm

$$\hat{\delta}(t) = \hat{\bar{\sigma}}^T \hat{\upsilon}, \tag{33}$$

$$\hat{\bar{\sigma}} = \hat{\chi} + \gamma_\sigma \hat{\upsilon}\tilde{x}_n, \tag{34}$$

$$\dot{\hat{\chi}} = -\gamma_\sigma \hat{\upsilon}\hat{\upsilon}^T \hat{\bar{\sigma}} + \gamma_\sigma k_n \hat{\upsilon}\tilde{x}_n - \gamma_\sigma \dot{\hat{\upsilon}}\tilde{x}_n, \tag{35}$$

where the function $\hat{\upsilon}(t)$ can be calculated by formula

$$\hat{\upsilon}(t) = \begin{bmatrix} 0 \\ \hat{\upsilon}_1(t) \\ \hat{\upsilon}_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{\omega}\cos\hat{\omega}t \\ -\hat{\omega}\sin\hat{\omega}t \end{bmatrix}.$$

Since we provide $\lim_{t\to\infty} (\omega - \hat{\omega}(t)) = 0$, it is straightforward to show

$$\lim_{t\to\infty} \left(\dot{\upsilon}(t) - \dot{\hat{\upsilon}}(t)\right) = 0.$$

So we have the estimation $\hat{\delta}(t)$ of disturbance $\delta(t)$ such that

$$\lim_{t\to\infty} \tilde{\delta}(t) = 0,$$

where $\tilde{\delta}(t) = \delta(t) - \hat{\delta}(t)$.

Step 2. On the next step we need to design predictor-based observer for the disturbance $\hat{\delta}(t+D)$ that is necessary in the compensation task. Consider predictor-based observer in the following form:

$$\begin{aligned}
\hat{\delta}(t+D) &= \hat{\sigma}_0 + \hat{\sigma}_1\sin(\hat{\omega}t + \hat{\omega}D) + \hat{\sigma}_2\cos(\hat{\omega}t + \hat{\omega}D) \\
&= \hat{\sigma}_0 + \hat{\sigma}_1\sin(\hat{\omega}t)\cos(\hat{\omega}D) + \hat{\sigma}_1\cos(\hat{\omega}t)\sin(\hat{\omega}D) \\
&\quad + \hat{\sigma}_2\cos(\hat{\omega}t)\cos(\hat{\omega}D) - \hat{\sigma}_2\sin(\hat{\omega}t)\sin(\hat{\omega}D) \\
&= \hat{\sigma}_0 + \hat{\sigma}_3\sin(\hat{\omega}t) + \hat{\sigma}_4\cos(\hat{\omega}t),
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
\hat{\sigma}_3 &= \hat{\sigma}_1\cos(\hat{\omega}D) - \hat{\sigma}_2\sin(\hat{\omega}D), \\
\hat{\sigma}_4 &= \sigma_1\sin(\hat{\omega}D) + \hat{\sigma}_2\cos(\hat{\omega}D).
\end{aligned} \tag{37}$$

From condition $\lim_{t\to\infty} (\omega - \hat{\omega}(t)) = 0$ we obtain:

$$\lim_{t\to\infty} \left(\delta(t+D) - \hat{\delta}(t+D)\right) = 0.$$

Consider control in the following form

$$u(t) = u_0(t) - \hat{\delta}(t+D), \tag{38}$$

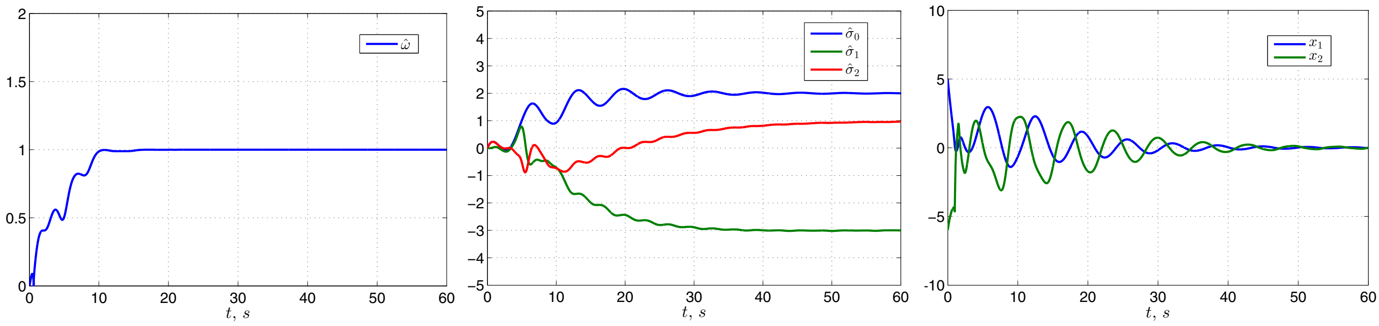
where u_0 is a stabilizing term.

On the next step we need to design u_0. Differentiating variable $y(t) = x_1(t)$ we obtain

$$\begin{aligned}
\dot{y}(t) &= \dot{\varsigma}_1(t) \\
&= x_2(t) + a_1 y(t) + \psi_1(y(t-\tau_1)) \\
&= \varsigma_2(t), \\
\ddot{y}(t) &= \dot{\varsigma}_2(t) \\
&= x_3(t) + a_1\dot{y}(t) + \frac{\partial\psi_1(y(t-\tau_1))}{\partial y(t-\tau_1)}\dot{y}(t-\tau_1) \\
&\quad + a_2 y(t) + \psi_2(y(t-\tau_2)) \\
&= \varsigma_3(t), \\
&\;\;\ldots \\
y^{(n)}(t) &= \dot{\varsigma}_n(t) \\
&= u_0(t-D) + \frac{\partial^{n-1}\psi_1(y(t-\tau_1))}{\partial y(t-\tau_1)^{n-1}}\dot{y}^{n-1}(t-\tau_1) \\
&\quad + \ldots + \frac{\partial\psi_1(y(t-\tau_1))}{\partial y(t-\tau_1)}y^{(n-1)}(t-\tau_1) \\
&\quad + \ldots + \psi_n(y(t-\tau_n)) \\
&\quad + a_1 y^{(n-1)}(t) + \ldots + a_n y(t) + \tilde{\delta}(t) \\
&= u_0(t-D) + \frac{\partial^{n-1}\psi_1(y(t-\tau_1))}{\partial y(t-\tau_1)^{n-1}}\varsigma_2^{n-1}(t-\tau_1) \\
&\quad + \ldots + \frac{\partial\psi_1(y(t-\tau_1))}{\partial y(t-\tau_1)}\varsigma_n(t-\tau_1) \\
&\quad + \ldots + \psi_n(y(t-\tau_n)) \\
&\quad + a_1\varsigma_{n-1}(t) + \ldots + a_n y(t) + \tilde{\delta}(t).
\end{aligned} \tag{39}$$

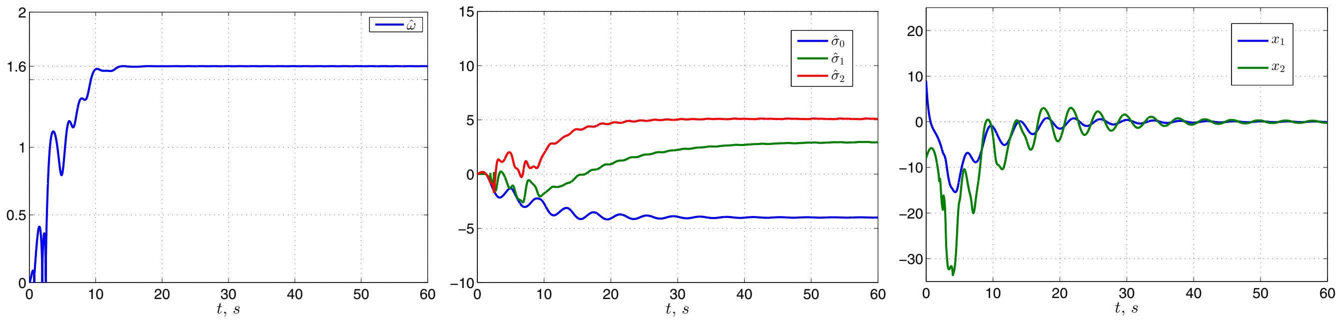
Because variables of the vector $x(t) = col\{x_1,\ x_2, \ldots, x_n\}$ are known then all variables $\varsigma_1(t),\ \varsigma_2(t), \ldots,\ \varsigma_n(t)$ of the model (39) also are known. In this case we obtain the control in the following form

$$\begin{aligned}
u_0(t) &= u_1(t) \\
&\quad - \Bigg(\frac{\partial^{n-1}\psi_1(y(t+D-\tau_1))}{\partial y(t+D-\tau_1)^{n-1}}\varsigma_2^{n-1}(t+D-\tau_1) \\
&\quad + \ldots + \frac{\partial\psi_1(y(t+D-\tau_1))}{\partial y(t+D-\tau_1)}\varsigma_n(t+D-\tau_1) \\
&\quad + \ldots + \psi_n(y(t-\tau_n)).
\end{aligned} \tag{40}$$



(a) Frequency estimate $\hat{\omega}(t)$ (b) Disturbance parameters estimates $\hat{\bar{\sigma}}_1, \hat{\bar{\sigma}}_2$ (c) State variables $x_1(t), x_2(t)$

Fig. 1. Transients for the closed-loop system with $D = 0.5$, $\psi_1(y) = \sin(y(t-0.8))$, $\psi_2(y) = y^2(t-1)\sin^2(y(t-1))$ and the disturbance $\delta(t) = 2 - 3\sin(t) + \cos(t)$



(a) Frequency estimate $\hat{\omega}(t)$ (b) Disturbance parameters estimates $\hat{\bar{\sigma}}_1, \hat{\bar{\sigma}}_2$ (c) State variables $x_1(t), x_2(t)$

Fig. 2. Transients for the closed-loop system with $D = 1.5$, $\psi_1(y) = \sin(y(t-1.9))$, $\psi_2(y) = (y(t-2.5)-2)^2(1+\sin(y(t-2.5)))$ and the disturbance $\delta(t) = -4 + 3\sin(1.6t) + 5\cos(1.6t)$

Then, substituting (40) into equation (39) we have the linear time-invariant system

$$\begin{aligned}\dot{\varsigma}_1(t) &= \varsigma_2(t),\\ \dot{\varsigma}_2(t) &= \varsigma_3(t),\\ &\dots\\ \dot{\varsigma}_n(t) &= a_1\varsigma_{n-1}(t) + \dots + a_n\varsigma_1(t) + \tilde{\delta}(t)\\ &\quad + u_1(t-D).\end{aligned} \tag{41}$$

Now rewrite (41) in the form

$$\begin{aligned}\dot{\varsigma}(t) &= G\varsigma(t) + qu_1(t-D) + \tilde{\delta}(t),\\ y(t) &= h^T\varsigma(t),\end{aligned}$$

where $\varsigma(t) = \begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\\ \vdots\\ \varsigma_n(t)\end{bmatrix}$, $G = \begin{bmatrix}0 & 1 & \dots & 0\\ 0 & 0 & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ a_n & a_{n-1} & \dots & a_1\end{bmatrix}$, $q = \begin{bmatrix}0\\ 0\\ \vdots\\ 1\end{bmatrix}$

and $h^T = \begin{bmatrix}1\\ 0\\ \vdots\\ 0\end{bmatrix}$.

Write the control law $u_1(t)$ using the equation (6)

$$u_1(t) = Le^{GD}\varsigma(t) + L\int_{t-D}^{t} e^{G(t-\tau)}qu_1(\tau)d\tau, \tag{42}$$

where vector L is such that the matrix $F = G + qL$ is Hurwitz.

4. NUMERICAL EXAMPLE

In Figs. 1 and 2 we present simulation results for two second-order systems like (2) with different parameters and initial conditions.

In Fig. 1 transients are shown for the closed-loop system with parameters $a_1 = 0.1$, $a_2 = 0.1$, nonlinearities

$$\psi_1(y) = \sin(y(t-0.8)),$$

$$\psi_2(y) = y^2(t-1)\,\sin^2(y(t-1)),$$

the input delay $D = 0.5$, and the disturbance

$$\delta(t) = 2 - 3\sin(t) + \cos(t).$$

In Fig. 2 transients for the closed-loop system with parameters $a_1 = -2$, $a_2 = 0.5$, nonlinear functions

$$\psi_1(y) = \sin(y(t-1.9)),$$

$$\psi_2(y) = (y(t-2.5)-2)^2\,(1+\sin(y(t-2.5))),$$

the input delay $D = 1.5$, and the disturbance

$$\delta(t) = -4 + 3\sin(1.6t) + 5\cos(1.6t)$$

are also illustrate the efficiency of the proposed control algorithm.

For both cases we choose matrix

$$L = [-5 \quad -5]$$

in the control loop (42). One can check that such gain L provide the stability of closed-loop state matrix

$$F = G + qL.$$

5. CONCLUSION

This paper presents the new control approach for the nonlinear systems with delay and unknown disturbance. The synthesized control law consists of the loop of adaptive estimation of frequency, the loop of feedforward compensation and stabilization algorithm object. The exponential convergence of the estimation error to zero, frequency perturbations and asymptotic stability of the zero equilibrium position of a closed system are shown.

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