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Collective Target Tracking Mean Field Control for Markovian Jump-Driven Models of Electric Water Heating Loads

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Abstract: Load control has traditionally been viewed as a useful tool for peak load reduction in power systems. With the increasing renewable energy penetration in the grid, load control is also considered as a tool to exploit the storage in dispersed devices naturally present in power systems such as electric water heaters to mitigate generation variability. Tapping into the storage dispersed across the power system is challenging because of the large number of devices that need to be coordinated to produce desirable system level behavior. In this paper a mean field game theoretic based control architecture is proposed as a load control mechanism to limit the required flows of information, and produce local constraints conscious decentralized individual controls which aggregate to a desired mean behavior. A Markovian jump-driven model of individual electric water heating loads is employed where the mean field effect is mediated through the quadratic cost function parameters under the form of an integral error. The corresponding system of mean field Nash equilibrium inducing equations is developed and numerical simulation results are presented.

1. INTRODUCTION

In this paper the potential of energy storage in dispersed devices naturally present in electric water heaters is employed as a tool to mitigate renewable generation variability and reduce peak load. The envisioned control architecture is hybrid: (i) centralized in terms of target trajectory generation for homogeneous groups of energy storage capable electric devices, so as to preserve overall optimality characteristics, (ii) decentralized at the implementation level so as to locally enforce safety and comfort constraints, as well as to minimize communication requirements. More specifically, we mention the following implementation principles, and argue that a class of decentralized control schemes based on a so-called mean field game (MFG) setup (see Lasry and Lions (2006), Huang et al. (2003, 2006, 2007)) can actually meet all the requirements.

(1) Each controller has to be situated locally.

A completely centralized control architecture micromanaging every individual device to be controlled requires a significant bandwidth as well as a very large computational power. Moreover, in the event of a loss of communication, users might face difficulties in the sense of comfort and safety. When the controller is situated locally, these worries are void since the controller can locally enforce comfort and safety constraints. Even if the communication with the central authority is lost, it is able to maintain the safety and comfort requirements of the user.

(2) Data exchanges should be kept to a minimum both with the central authority and among users.

(3) User disturbance should be kept at minimum.

Since the centralized authority is solely interested in the aggregate consumption, individual trajectories do not need to necessarily follow the targets set by the authority. On the contrary, in fact in the case of a population of controlled electric space heaters, or air conditioners, or electric water heaters for example, it is desirable to shape the mean temperature of the population with least disturbance; ideally without customers even noticing the effects of the control actions on their comfort level. Also, it is important to maintain some measure of fairness among the users when it comes to sharing the control effort. We shall show that the recent developments in Kizilkale and Malhamé (2013) allow the flexibility to address these issues.

In Kizilkale and Malhamé (2013) we introduced *Collective Target Tracking Mean Field Control*, where the presence of large numbers of space heating electric devices is employed to develop a decentralized mean field control based approach to the problem of these devices following a desired mean trajectory. The proposed solution deviates from the classical formulation which would have each element track the desired mean temperature thus introducing unnecessary control actions. The solution made possible by mean field theory enforces collective mean temperature tracking while leaving individual devices freer to remain, if possible within their comfort zone. In this context, the mean field effect is mediated by quadratic cost function parameters under the form of an integral error, as compared

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to currently prevailing mean field control formulations (e.g.Huang et al. (2007)) where the mean field effect is concentrated on the tracking signal.

The model in Kizilkale and Malhamé (2013) applies to space heaters where temperature evolves according to linear dynamics subject to random processes of heat addition and heat losses stemming from human activity within the dwelling. The random heat processes are modeled as Brownian motion. This diffusion model is not sufficient to characterize the dynamics of a water heater where the system is mostly isolated, and dynamical parameter changes take place intermittently with respect to the extraction of water from the water heater tank. An alternative model that captures the essence of water heater dynamics is a Markovian jump process where dynamical parameters randomly switch between states (Malhamé (1990)). In this paper, we extend the model introduced in Kizilkale and Malhamé (2013) to the system of populations where the dynamical parameters are time varying and characterized by a sequence of independent Markov chains with identical generators.

Statistical mechanics inspired models of large aggregates of energy storage associated loads, particularly space heating and cooling loads, to be controlled within peak shaving and valley filling load management programs by direct control were presented in Malhamé and Chong (1985), whereas the modeling methodology was applied to electric water heaters in Laurent and Malhamé (1994).

Using dispersed storage for accommodating renewable intermittency is a growing area of research. Dispersed energy storage for frequency regulation in the presence of wind energy is investigated in Callaway (2009). This work uses the aggregate load modeling framework in Malhamé and Chong (1985) and extends it for improved transient analysis. Dynamic pricing for controlling the load of aggregates of large commercial buildings is analyzed in Mathieu et al. (2010), and domestic heating systems are employed as heat buffers in Tahersima et al. (2011). A decentralized charging control strategy for large populations of plug-in electric vehicles (PEVs) using the mean field methodology is presented in Ma et al. (2010).

In Section 2 the Markovian jump linear quadratic Gaussian multi-agent mean state tracking problem is reviewed. Then in Section 3 we present the linearly controlled water heater dynamics together with a cost function that has been formulated to penalize the deviation of the population mean temperature from the desired mean temperature. A classical linear quadratic solution is presented, followed by the collective target tracking Markovian jump mean field solution. L^2 stability of the individual systems and equilibrium properties of the population are given. Lastly, in Section 4, we provide associated simulation results together with comparisons to a simplistic target tracking control formulation.

2. BACKGROUND ON MARKOVIAN JUMP LINEAR QUADRATIC GAUSSIAN MEAN FIELD CONTROL

In order to meet the three requirements above, the principles of the mean field control methodology (Huang et al. (2007)) are employed. This framework is based on a decentralized scheme whereby each agent calculates its individual best response to the anticipated action profile of the population. The anticipation of the population response curve is possible because the numbers are sufficiently large that the law of large numbers applies. Under technical constraints together with the assumption of each agent's individual rationality, the approach looks for a Nash Equilibrium as the number of users goes to infinity when agents implement their stabilizing best response actions. The anticipation of the action profile is carried out offline and locally with statistical information obtained based on a parsimonious population measurement scheme and available at the start of the control horizon. The control scheme is fully decentralized; i.e., *no communication is required amongst individual controllers throughout the horizon.*

A Review of the Markovian Jump LQG Multi-Agent Heterogeneous Population Mean State Tracking Problem (Wang and Zhang (2012))

A large population of N stochastic dynamic agents is considered where agents are stochastically independent, but which shall be cost coupled and such that the individual dynamics are defined by

$$dx_t^i = \left(A^{\theta_t^i} x_t^i + B^{\theta_t^i} u_t^i + c^{\theta_t^i}\right) dt + D^{\theta_t^i} dw_t^i, \quad t \geq 0, \quad (1)$$

$1 \leq i \leq N$, where for agent $\mathcal{A}_i$, $x^i \in \mathbb{R}^n$ is the state, $u^i \in \mathbb{R}^m$ is the control input; $w^i \in \mathbb{R}^r$ is a standard Wiener process on a sufficiently large underlying probability space $(\Omega, \mathcal{F}, P)$ such that w^i is progressively measurable with respect to $\mathcal{F}^{w^i,\theta^i} := \{\mathcal{F}_t^{w^i} \times F_t^{\theta^i}; t \geq 0\}$. $\{\theta_t^i, 1 \leq i \leq N\}$ is a sequence of independent continuous time Markov chains taking values in $\Theta = \{1, 2, ..., p\}$ with the identical infinitesimal generator $\Lambda = \{\lambda_{ij}, i, j, = 1, ..., p\}$ progressively measurable with respect to $\mathcal{F}^{w^i,\theta^i}$. We denote the population average state by $x^{(N)} = (1/N) \sum_{i=1}^N x^i$.

The cost function for agent $\mathcal{A}_i, 1 \leq i \leq N$, is given by

$$J_i^N(u^i, u^{-i}) = \mathbb{E} \int_0^\infty e^{-\delta t} \Big[\|x_t^i - m_t^N\|_Q^2 + \|u_t^i\|_R^2\Big] dt, \quad (2)$$

where $Q \geq 0$ and $R > 0$. The cost-coupling is assumed to be in the form of an averaging function $m_t^N := m(x_t^{(N)} + \eta)$, $\eta \in \mathbb{R}^n$. The term u^i is the control input of the agent $\mathcal{A}_i$ and u^{-i} denotes the control inputs of the complementary set of agents $\mathcal{A}_{-i} = \{\mathcal{A}_j, j \neq i, 1 \leq j \leq N\}$.

Each agent $\mathcal{A}_i, 1 \leq i \leq N$, obtains the positive definite solution to the coupled algebraic Riccati equations

$$\Pi^j \left(A^j - \frac{\delta}{2} I\right) + \left(A^j - \frac{\delta}{2} I\right)^\top \Pi^j$$
$$- \Pi^j B^j R^{-1} {B^j}^\top \Pi^j + \sum_{k=1}^p \lambda_{jk} \Pi^j + Q, \quad 1 \leq j \leq p. \quad (3)$$

For a given posited *mass tracking signal* $x^* \in \mathbf{C}_b[0, \infty)$ the mass offset function s^i is generated by the coupled set of differential equations

$$-\frac{ds_t^j}{dt} = (A^j - \delta I - B^j R^{-1} {B^j}^\top \Pi_t^j)^\top s_t^j - Q x_t^* + \Pi^j c^j + \sum_{k=1}^{p} \lambda_{jk} s_t^k, \quad 1 \le j \le p, \quad (4)$$

for $t \in [0, \infty)$. Then, the optimal tracking control law is given by

$$u_t^\circ = -\sum_{j=1}^{p} \mathbb{I}_{[\theta_t = j]} R^{-1} {B^j}^\top (\Pi^j x_t + s_t^j), \quad t \ge 0. \quad (5)$$

Note that x^* is assumed to be fixed although unknown. For that x^* to be sustainable, it must be collectively replicated by the agents implementing their best responses to that signal. Thus, system (4), (5) must be complemented by a fixed point requirement leading to the mean field equation system in Definition 1 below.

Definition 1. Markovian Jump (MJ) Mean Field (MF) Equation System on $t \in [0, \infty)$:

$$\begin{aligned}
-\frac{ds_t^j}{dt} &= (A^j - \delta I - B^j R^{-1} {B^j}^\top \Pi^j)^\top s_t^j - Q x_t^* + \Pi^j c^j \\
&\quad + \sum_{k=1}^{p} \lambda_{jk} s_t^k, \quad 1 \le j \le p, \\
\frac{d\bar{x}_t^j}{dt} &= (A^j - B^j R^{-1} {B^j}^\top \Pi^j) \bar{x}_t^j + \sum_{k=1}^{p} \lambda_{kj} \bar{x}_t^j + \zeta^j c^j \\
&\quad - \zeta^j B^j R^{-1} {B^j}^\top s_t^j, \quad 1 \le j \le p, \\
\bar{x}_t &= \sum_{j=1}^{p} \bar{x}_t^j, \\
x_t^* &= m(\bar{x}_t + \eta), \quad t \in [0, \infty),
\end{aligned} \quad (6)$$

where

- $\bar{x}_t^j := \mathbb{E}(\bar{x}_t \mathbb{I}_{[\theta_t = j]})$, and,
- $\zeta = [\zeta_1, ..., \zeta_p]$ is the steady state distribution of the Markov chain.

Global Observation Control set $\mathcal{U}_g^N$: The set of control inputs $\mathcal{U}_g^N$ consists of all feedback controls adapted to $\mathcal{F}_t^N$ where $\mathcal{F}_t^N$ is the σ-field generated by the set $\{x_\tau^j, \theta_\tau^j; 0 \le \tau \le t, 1 \le j \le N\}$.

Theorem 2. MJ MF Stochastic Control Theorem (Wang and Zhang (2012))

Under technical assumptions (see Wang and Zhang (2012)), the MJ MF Stochastic Control Law (5) generates a set of controls $\mathcal{U}_{MF}^N \triangleq \{(u^i)^0; 1 \le i \le N\}$, $1 \le N < \infty$, with

$$u_t^0 = -\sum_{j=1}^{p} \mathbb{I}_{(\theta_t = j)} R^{-1} {B^j}^\top (\Pi^j x_t + s_t^j), \quad t \ge 0, \quad 1 \le j \le p, \quad (7)$$

such that

(i) the MF equations (6) have a unique solution;
(ii) all agent system trajectories x^i, $1 \le i \le N$, are L^2 stable;
(iii) $\{\mathcal{U}_{MF}^N; 1 \le N < \infty\}$ yields an ϵ-Nash equilibrium for all $\epsilon > 0$; i.e., for all $\epsilon > 0$, there exists $N(\epsilon)$ such that for all $N \ge N(\epsilon)$

$$J_i^N\left((u^i)^0, (u^{-i})^0\right) - \epsilon \le \inf_{u^i \in \mathcal{U}_g^N} J_i^N\left(u^i, (u^{-i})^0\right) \le J_i^N\left((u^i)^0, (u^{-i})^0\right).$$

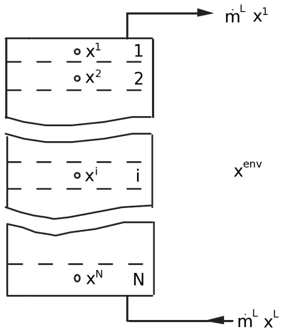

Fig. 1. Stratification in a water heater

Table 1. Parameters for Water Heater Dynamics

x^i	temperature of the ith segment
u^i	control action at the ith segment
$\dot{m}^L$	fluid mass flow rate to the load
$\dot{Q}^i$	rate of energy input by the heating element
x^{env}	temperature of the environment
x^L	temperature of the inlet fluid
M^i	mass of the fluid in the ith segment
A^i	surface area of the ith segment
C^{pf}	specific heat of the fluid
U	loss coefficient betwen the tank and its environment

In essence Theorem 2 states that the Markovian Jump MF equation system produces a set of decentralized control policies for each agent, which collectively become arbitrarily close in performance to a Nash equilibrium in the space of feedback strategies, provided the number of agents increases sufficiently.

3. ELECTRIC WATER HEATER MODELS

The dynamics of the temperature of a water heater tank subject to thermal stratification can be modeled by assuming that the tank consists of n fully mixed equal volume segments as shown in Figure 1. For the thermal dynamics we adopt the linear model given in Klein (1976) which is below with the nomenclature specified in Table 1.

$$\begin{aligned}
M^i C^{pf} \frac{dx_t^i}{dt} &= U A^i (x^{env} - x_t^i) + \dot{m}_t^L C^{pf} (x_t^{i+1} - x_t^i) \\
&\quad + \dot{Q}^i u_t^i, \quad t \ge 0, \quad i \ne n, \\
M^i C^{pf} \frac{dx_t^i}{dt} &= U A^i (x^{env} - x_t^i) + \dot{m}_t^L C^{pf} (x_t^L - x_t^i) \\
&\quad + \dot{Q}^i u_t^i, \quad t \ge 0, \quad i = n.
\end{aligned} \quad (8)$$

Note that in (8) $\dot{m}^L$ represents the amount of water being pulled from the tank per time unit. We will treat water demand as a stochastic process denoted by θ_t, $t \ge 0$, which is a continuous time Markov chain taking values in $\Theta = \{1, 2, ..., p\}$, where the states represent different types of water demand such as dishwashing, hand-washing, shower etc. Note that, in general, electric heating can only be contributed in the top and bottom segments.

The dynamics of a water tank therefore can be written in linear form as:

$$\frac{dx_t}{dt} = A^{\theta_t} x_t + Bu_t + c^{\theta_t}, \quad t \geq 0.$$

3.1 Classical MJ Linear Quadratic (MJLQ) Tracking

Dynamics for a population of N water heaters is given as

$$\frac{dx_t^i}{dt} = A^{\theta_t^i} x_t^i + Bu_t^i + c^{\theta_t^i}, \quad t \geq 0, \quad 1 \leq i \leq N. \tag{9}$$

Following the results of a global optimization analysis, it is assumed that the central authority has determined that it is best that the mean temperature of a given population of water heater tracks some fixed target temperature signal y, thus creating for example a temporary power relief on the system. In the MJ Linear Quadratic (MJLQ) tracking formulation each agent's cost function is defined as

$$J_i^N(u^i, u^{-i}) = \mathbb{E} \int_0^\infty e^{-\delta t} \left[(Hx_t^i - y)^2 q + \|u_t^i\|_R^2 \right] dt, \tag{10}$$

where $H = [1/n, ..., 1/n]$.

The problem with this approach is that each agent minimizes its own cost function and tracks the same signal. Even though the central authority is only interested in aggregate behaviour and in particular the *mean* temperature, this control approach causes *all agents* to track the target signal, which is undesirable.

3.2 Collective Target Tracking Mean Field Model

We employ the dynamics for the heaters given in (9). We also assume that the initial mean temperature of the population of the heaters is above the control center dictated target y for that mean (power relief required). The infinite horizon cost function for agent $\mathcal{A}_i, 1 \leq i \leq N$, is defined instead as follows:

$$J_i^N(u^i, u^{-i}) = \mathbb{E} \int_0^\infty e^{-\delta t} \Big[(Hx_t^i - z)^2 q_t^z + (Hx_t^i - Hx_0^i)^2 q^{x_0} + \|u_t^i\|_R^2 \Big] dt, \tag{11}$$

where $H = [1/n, ..., 1/n]$, and where z, is a *direction* assigned to each agent in the population and the mean of the layers of each agent's deviation from this direction is penalized by the deviation penalty coefficient q_t^z, $t \in [0, \infty)$, which captures the mean field information and calculated as the following *integrated error signal*:

$$q_t^z = \left| \int_0^t (Hx_t^{(N)} - y) dt \right|.$$

The justification for the above cost function is that for a system where $z < y < x_0^{(N)}$, by pointing individual agents towards what is considered as the minimum comfort temperature z, it dictates a global decrease in their individual temperatures. This pressure for decrease persists as long as the differential between the mean temperature and the mean target y is high. The role of the integral controller is to mechanically compute the *right* level of penalty coefficient q which, in the steady-state, should maintain the mean population temperature at y. When this happens, due to the second term on the cost function which penalizes each individual's temperature from its initial temperature, individual agents reach themselves their own steady state (in general different from y and closer to their comfort zone than classical MJLQ tracking would dictate). In order to derive the limiting infinite population MF equation system, and analogously to the more classical MJ MF LQ case in Section 2, we start this time assuming a given (although unknown) cost penalty trajectory $q^z \in \mathbf{C}_b[0, \infty)$ and a constant q^{x_0}. Given q_t^z, $t \geq 0$, individual agents $\mathcal{A}_i$, $1 \leq i \leq N$, solve a classical target tracking MJLQ problem with time varying cost coefficient with Riccati gain Π_t^i, $t \geq 0$, evolving as follows:

$$-\frac{d\Pi_t^j}{dt} = \Pi_t^j \left(A^j - \frac{\delta}{2} I \right) + \left(A^j - \frac{\delta}{2} I \right)^\top \Pi_t^j - \Pi_t^j BR^{-1} B^\top \Pi_t^j + \sum_{k=1}^p \lambda_{jk} \Pi_t^k + (q_t^z + q^{x_0}) H^\top H,$$

$1 \leq j \leq p$ for $t \in [0, \infty)$. Moreover, for a given direction z, the Markov state dependent offset function s_t^i is generated by the differential equation

$$-\frac{ds_t^j}{dt} = (A^j - \delta I - BR^{-1} B^\top \Pi_t^j)^\top s_t^j - q_t^z H^\top z - q^{x_0} H^\top x_0^i + \Pi_t^j c^j + \sum_{k=1}^p \lambda_{jk} s_t^k, \quad 1 \leq j \leq p.$$

Then, the optimal tracking control law is given by

$$u_t^\circ = -\sum_{j=1}^p \mathbb{I}_{[\theta_t = j]} R^{-1} B^\top (\Pi_t^j x_t + s_t^j), \quad t \geq 0. \tag{12}$$

The calculation of the unknown q_t^z, $t \geq 0$, is obtained by requiring that $q_t^z, t \geq 0$, must be such that the individual agents carrying their corresponding optimal responses must collectively replicate q_t^z, $t \geq 0$, itself. This fixed point requirement leads to the last equation in Definition 3 below for the Collective Target Tracking MJ MF Equation System.

Definition 3. Collective Target Tracking MJ MF Equation System on $t \in [0, \infty)$:

$$\begin{aligned}
-\frac{d\Pi_t^j}{dt} &= \Pi_t^j \left(A^j - \frac{\delta}{2} I \right) + \left(A^j - \frac{\delta}{2} I \right)^\top \Pi_t^j \\
&\quad - \Pi_t^j BR^{-1} B^\top \Pi_t^j + \sum_{k=1}^p \lambda_{jk} \Pi_t^k + (q_t^z + q^{x_0}) H^\top H, \\
-\frac{ds_t^j}{dt} &= (A^j - \delta I - BR^{-1} B^\top \Pi_t^j)^\top s_t^j - q_t^z H^\top z \\
&\quad - q^{x_0} H^\top \bar{x}_0 + \Pi_t^j c^j + \sum_{k=1}^p \lambda_{jk} s_t^k, \\
\frac{d\bar{x}_t^j}{dt} &= (A^j - BR^{-1} B^\top \Pi_t^j) \bar{x}_t^j + \sum_{k=1}^p \lambda_{kj} \bar{x}_t^j + \zeta^j c^j \\
&\quad - \zeta^j BR^{-1} B^\top s_t^j, \\
\bar{x}_t &= \sum_{k=1}^p \bar{x}_t^j, \\
q_t^z &= \left| \int_0^t (H\bar{x}_t - y) dt \right|.
\end{aligned} \tag{13}$$

One recalls

- $\bar{x}_t^j = \mathbb{E}(\bar{x}_t I_{[\theta_t = j]})$,
- $\Lambda = \{\lambda_{ij}, i, j = 1, ..., p\}$ is the infinitesimal generator of the Markov chain, and,
- $\zeta = [\zeta_1, ..., \zeta_p]$ is the steady state distribution of the Markov chain.

Note that the MF Equations for this model is significantly different from (6). Indeed system (6) is amenable to analysis within a linear systems framework while system (13) is fundamentally nonlinear (because of the form of q_t^z, $t \geq 0$) and special arguments have to be developed for analysis of existence and uniqueness of solutions.

3.3 ϵ-Nash Theorem

Here we present the main theorem of the paper. It provides an MF stochastic control law that achieves a Nash equilibrium at the population limit when applied by all agents in the system. Moreover, an ϵ-Nash equilibrium property holds for any finite population.

Theorem 4. Collective Target Tracking MJ MF Stochastic Control Theorem

Under technical conditions (Kizilkale and Malhamé (2014)), the Collective Target Tracking MJ MF Stochastic Control Law (12) generates a set of controls $\mathcal{U}_{col}^N \triangleq \{(u^i)^0; 1 \leq i \leq N\}$, $1 \leq N < \infty$, with

$$u_t^\circ = -\sum_{j=1}^{p} I_{[\theta_t = j]} R^{-1} B^\top (\Pi_t^j x_t + s_t^j), \quad t \geq 0,$$

such that

(i) all agent system trajectories x^i, $1 \leq i \leq N$, are L^2 stable;
(ii) $\{\mathcal{U}_{col}^N; 1 \leq N < \infty\}$ yields an ϵ-Nash equilibrium for all $\epsilon > 0$.

4. SIMULATIONS

Here in the first simulation we simulate a population of 200 identical water heaters with 2 stratification layers where the mean initial temperature in the population is 57°C. The central authority sets the target temperature to 55°C over a 5 hours horizon, and provides the target temperature trajectory to each controller while local controllers solve an MJLQ tracking problem as provided in Section 3.1.

For the simulations we use the dynamics given in (8), where $\dot{m}_t^L := K\theta_t$, $t \geq 0$, and $\theta_t \in \{0, 1\}$, $t \geq 0$, is a two-state continuous time Markov chain with infinitesimal generator

$$L = \begin{bmatrix} -0.5 & 0.5 \\ 7 & -7 \end{bmatrix}.$$

Note that these values correspond to an average of 288 liters of water consumption in 24 hours. The selected parameter values are provided in Table 2. We employ the cost function provided in (11) where $q^{x_0} = 10000$ and $R = 0.05 \times I_{2\times2}$.

In this experiment the central authority sets the target temperature to 55°C, and all agents are assigned to track 50°C by applying Collective Target Tracking MJ MF control. The trajectories for only 10 agents are presented and the calculated mean field signal is shown in Figure 2. It can be seen that while the mean temperature still settles at 55°C, the population is disturbed much less than the MJ LQ tracking implementation.

Table 2. Parameter Values for Water Heater Dynamics

K	$0.05\ l/sec$
$\dot{Q}^i$	$4500\ J/s$
x^{env}	25°C
x^L	15°C
M^i	$139\ l$
A^i	$1.715\ m^3$
C^{pf}	$4190\ J/(kg°C)$
U	$1.05\ J/(sm^2K)$

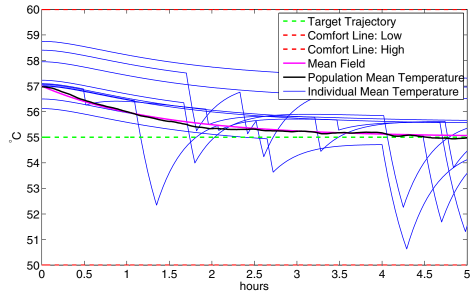

Fig. 2. Agents Applying Collective Target Tracking MJ MF Control: All Agents Following the Low Comfort Level Signal

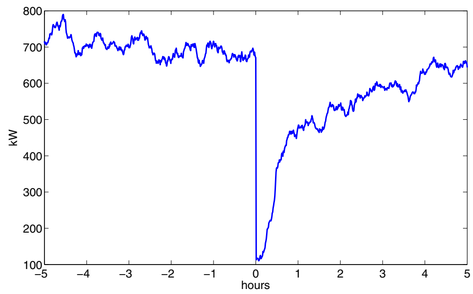

Fig. 3. Aggregate Power Consumption

The aggregate power consumption plot is provided in Figure 3 for a system of 1000 water heaters. Not only the algorithm provides immense relief at the early stages of the horizon, but it also provides a smooth transition to the steady state power consumption profile without a delayed payback peak typical of direct control schemes of thermostats.

In Figure 4 for the same simulation we plot the iterations of a Collective Target Tracking MJ MF successive substitutions algorithm to identify the fixed point mean temperature trajectory until convergence occurs.

For the next experiment, we study a population of initial mean temperature at 57°C. We separate the population in two groups where the first group consists of the agents above 57°C initial temperature and the second group consists of the ones below 57°C. Both groups are assigned to track 50°C. In order to achieve a level of fairness, the first group is assigned a smaller control penalty coefficient R. Collective Target Tracking MJ MF control is applied

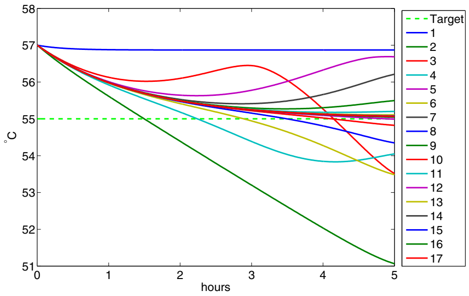

Fig. 4. Collective Target Tracking MJ MF Iterations

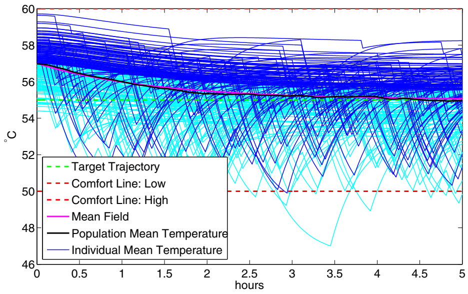

Fig. 5. Agents Applying Collective Target Tracking MJ MF Control: Different r for subpopulations

to these groups, and the simulation result is provided in Figure 5. It can be seen that the MF based integral error control scheme leads the mean temperature of the whole water heaters population to 55°C while soliciting the agents with higher initial temperatures more intensely than those with colder initial temperatures. This illustrates how one could shape the collective tracking response for greater fairness.

5. CONCLUDING REMARKS

In this paper we extended the *collective target tracking MF control* methodology to the cases where the dynamics are modeled through Markovian jump processes. This formulation allows the characterization of the dynamics of a water heater where the water consumption event is modeled as a Markovian jump process. The presence of large numbers of electric devices associated with energy storage is employed to develop a decentralized mean field control based approach to the problem of these devices following a desired mean trajectory. Given the statistics of the hot water consumption events and provided with the mean temperature target trajectory as well as the initial mean temperature in the controlled group, the devices generate their own control locally, and thus enforce their safety and comfort constraints locally as well.

In future work we shall concentrate on the analysis of existence and uniqueness conditions for solutions of the fixed point MJ MFG system in Definition 3. Also, we shall consider the impact of temperature comfort and safety constraints on the aggregate control performance. Finally, the case of periodic target trajectories will also be considered.

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