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Resonant FCS Predictive Control of Power Converter in Stationary Reference Frame ⋆

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Abstract: This paper proposes resonant finite control set (FCS)-predictive control of a two level power converter. In the stationary reference frame, firstly the traditional FCS-predictive control system is shown to be closed-loop feedback proportional control system. In the presence of constraints, the objective function is a weighted error function between the desired optimal and candidate voltage control signals. The weighting coefficient in the objective function is the ratio between the sampling interval and the inductance. Secondly, it is proposed in this paper to design a discrete-time resonant controller for eliminating the sinusoidal error using a cascaded control system structure. The proposed method is simple in design and implementation with an analytical solution for the resonant controller gain. Experimental results are used to demonstrate the efficacy of the proposed approach and much improved closed-loop control performance.

Keywords: power converter, resonant control, predictive control

1. INTRODUCTION

The predictive control of power converter has been studied by many researchers in last two decades. Kawabata et al. (1990) and Kukrer (1996) present dead beat control with PWM modulation for a three phase PWM inverter. Finite control set predictive control is first introduced by Rodriguez et al. (2004) in 2004. Direct predictive control with explicit solution by Linder and Kennel (2005). To address issue of the steady-state error, Aguilera et al. (2013) has proposed a cost function with integral term to improve steady-state performance.

This paper is organized as follows. In Section 2, the original finite control set is discussed. In Section 3, a new FCS with resonant controller is presented to address the sinusoidal error. Experimental results are used in Section 4 to illustrate the performance of proposed resonant FCS predictive control.

2. THE ORIGINAL FCS-PREDICTIVE CONTROL OF CURRENT FOR POWER CONVERTER

The original FCS-predictive control of current in the $\alpha-\beta$ reference frame is designed identically to the controller in the $d-q$ reference frame. This means that the sum of squares errors between the current reference signals (i_α^*, i_β^*) and the one-step ahead prediction of the current signals (i_α, i_β) is minimized at the sampling time t_i to obtain the optimal voltage control signals (v_α, v_β). Receding horizon control principle is applied leading to feedback control.

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2.1 Predictive Control using One-step-ahead Prediction

The mathematical model of a power converter in the stationary frame is described by

$$\frac{di_\alpha(t)}{dt} = -\frac{R_s}{L_s}i_\alpha(t) - \frac{1}{L_s}v_\alpha(t) + \frac{1}{L_s}e_\alpha(t) \tag{1}$$

$$\frac{di_\beta(t)}{dt} = -\frac{R_s}{L_s}i_\beta(t) - \frac{1}{L_s}v_\beta(t) + \frac{1}{L_s}e_\beta(t) \tag{2}$$

$$\frac{dv_{dc}(t)}{dt} = \frac{3}{4C}(S_\alpha(t)i_\alpha(t) + S_\beta(t)i_\beta(t)) - i_L \tag{3}$$

where e_α and e_β are the stationary frame grid voltages, and $i_\alpha(t)$ and $i_\beta(t)$ are the stationary frame grid currents. v_{dc} is the DC bus voltage.

The manipulated variables for the current control are the voltages v_α, v_β in the α - β reference frame, and they are related to S_α ,S_β and $v_{dc}(t)$ via the following relationships:

$$v_\alpha = \frac{S_\alpha(t)v_{dc}(t)}{2} \tag{4}$$

$$v_\beta = \frac{S_\beta(t)v_{dc}(t)}{2} \tag{5}$$

In the $\alpha - \beta$ reference frame, there are seven pairs of candidate voltage values v_α and v_β, which are dependent on the DC bus voltage $v_{dc}(t)$. Their exact values are characterized by the values listed below:

$$\begin{bmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & -1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \frac{2}{3}v_{dc}(t) \tag{6}$$

The variations of the candidate voltage values are caused by the changes of DC bus voltage v_{dc}. It is seen from this model that in the $\alpha - \beta$ reference frame, there is no interaction between the currents i_α and i_β, which will effectively reduce the current controller in this reference frame to two single-input and single- output controllers. To calculate the control variables, at the sampling time t_i, the objective function is chosen as sum of the squared errors between the desired and predicted signals:

$$
\begin{aligned}
J &= (i_\alpha^*(t_i) - i_\alpha(t_i+\Delta t))^2 + (i_\beta^*(t_i) - i_\beta(t_i+\Delta t))^2 \\
&= \left(\begin{bmatrix} i_\alpha^*(t_i) \\ i_\beta^*(t_i) \end{bmatrix} - \begin{bmatrix} i_\alpha(t_i+\Delta t) \\ i_\beta(t_i+\Delta t) \end{bmatrix} \right)^T \\
&\times \left(\begin{bmatrix} i_\alpha^*(t_i) \\ i_\beta^*(t_i) \end{bmatrix} - \begin{bmatrix} i_\alpha(t_i+\Delta t) \\ i_\beta(t_i+\Delta t) \end{bmatrix} \right)
\end{aligned} \tag{7}
$$

where $i_\alpha(t_i+\Delta t)$ and $i_\beta(t_i+\Delta t)$ are one-step-ahead predictions of $i_\alpha(t_i)$ and $i_\beta(t_i)$, respectively. The one-step-ahead predictions of the $i_\alpha(t_i+\Delta t)$ and $i_\beta(t_i+\Delta t)$ are expressed in matrix and vector forms:

$$
\begin{aligned}
\begin{bmatrix} i_\alpha(t_i+\Delta t) \\ i_\beta(t_i+\Delta t) \end{bmatrix} &= (I + \Delta t A_m) \begin{bmatrix} i_\alpha(t_i) \\ i_\beta(t_i) \end{bmatrix} \\
&+ \Delta t B_m \begin{bmatrix} v_\alpha(t_i) \\ v_\beta(t_i) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} e_\alpha \Delta t \\ \frac{1}{L_s} e_\beta \Delta t \end{bmatrix}
\end{aligned} \tag{8}
$$

where I is the identity matrix with dimension 2×2 and the system matrices A_m and B_m are defined as

$$
A_m = \begin{bmatrix} -\frac{R_s}{L_s} & 0 \\ 0 & -\frac{R_s}{L_s} \end{bmatrix}; B_m = \begin{bmatrix} -\frac{1}{L_s} & 0 \\ 0 & -\frac{1}{L_s} \end{bmatrix}
$$

By substituting the one-step-ahead prediction given by (8) into the objective function J (7), we obtain,

$$
J = [\, f_\alpha(t_i) \;\; f_\beta(t_i) \,] \begin{bmatrix} f_\alpha(t_i) \\ f_\beta(t_i) \end{bmatrix} \tag{9}
$$

$$
\begin{aligned}
&- 2 [\, v_\alpha(t_i) \;\; v_\beta(t_i) \,] \Delta t B_m^T \begin{bmatrix} f_\alpha(t_i) \\ f_\alpha(t_i) \end{bmatrix} \\
&+ [\, v_\alpha(t_i) \;\; v_\beta(t_i) \,] \Delta t^2 B_m^T B_m \begin{bmatrix} v_\alpha(t_i) \\ v_\beta(t_i) \end{bmatrix}
\end{aligned} \tag{10}
$$

where the functions $f_\alpha(t_i)$ and $f_\beta(t_i)$ are defined as

$$
\begin{aligned}
\begin{bmatrix} f_\alpha(t_i) \\ f_\beta(t_i) \end{bmatrix} &= \begin{bmatrix} i_\alpha^*(t_i) \\ i_\beta^*(t_i) \end{bmatrix} - (I + \Delta t A_m) \begin{bmatrix} i_\alpha(t_i) \\ i_\beta(t_i) \end{bmatrix} \\
&- \begin{bmatrix} \frac{1}{L_s} e_\alpha \Delta t \\ \frac{1}{L_s} e_\beta \Delta t \end{bmatrix}
\end{aligned} \tag{11}
$$

We obtain the optimal control signals $v_\alpha(t_i)$ and $v_\beta(t_i)$ that minimize the objective function J:

$$
\begin{aligned}
\begin{bmatrix} v_\alpha(t_i)^{opt} \\ v_\beta(t_i)^{opt} \end{bmatrix} &= (\Delta t^2 B_m^T B_m)^{-1} \Delta t B_m^T \begin{bmatrix} f_\alpha(t_i) \\ f_\beta(t_i) \end{bmatrix} \\
&= -\frac{1}{\Delta t} \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} \begin{bmatrix} f_\alpha(t_i) \\ f_\beta(t_i) \end{bmatrix}
\end{aligned} \tag{12}
$$

Note that in the $\alpha - \beta$ reference frame, the system matrix A_m is diagonal and there is no interaction between the i_α and i_β currents. The calculations of $v_\alpha(t_i)^{opt}$ and $v_\beta(t_i)^{opt}$ signals are scalar operations. More specifically, since the matrix $I + A_m \Delta t$ has the form:

$$
I + A_m \Delta t = \begin{bmatrix} 1 - \frac{R_s \Delta t}{L_s} & 0 \\ 0 & 1 - \frac{R_s \Delta t}{L_s} \end{bmatrix}
$$

from (12), we have

$$
v_\alpha(t_i)^{opt} = -(\frac{L_s}{\Delta t} i_\alpha(t_i)^* - \frac{L_s}{\Delta t}(1 - \frac{R_s}{L_s}\Delta t) i_\alpha(t_i) - \frac{1}{\Delta t} e_\alpha) \tag{13}
$$

$$
v_\beta(t_i)^{opt} = -(\frac{L_s}{\Delta t} i_\beta(t_i)^* - \frac{L_s}{\Delta t}(1 - \frac{R_s}{L_s}\Delta t) i_\beta(t_i) - \frac{1}{\Delta t} e_\beta) \tag{14}
$$

where $i_\alpha(t_i)^*$ and $i_\beta(t_i)^*$ are current reference signals in the $\alpha - \beta$ reference frame, and $i_\alpha(t_i)$ and $i_\beta(t_i)$ are the measured current of the power converter. It is clearly seen that predictive controller uses a proportional feedback control with a feedforward compensation. Furthermore, the feedback control gain is dependent on the sampling interval of the current control system with the value

$$
k_{fcs}^\alpha = k_{fcs}^\beta = -\frac{L_s}{\Delta t}(1 - \frac{R_s}{L_s}\Delta t) \tag{15}
$$

In order to ensure a negative feedback in the current control, the quantity $1 - \frac{R_s}{L_s}\Delta t > 0$, that is $\frac{R_s}{L_s}\Delta t < 1$. Here the feedback controllers have a negative gain because of the negative diagonal elements in B_m matrix.

2.2 FCS Current Control in $\alpha - \beta$ Reference Frame

It is easy to show that the objective function J (10) can also be expressed in terms of the optimal voltage signals in the $\alpha - \beta$ reference frame as

$$
\begin{aligned}
J &= \left(\begin{bmatrix} v_\alpha(t_i) \\ v_\beta(t_i) \end{bmatrix} - \begin{bmatrix} v_\alpha(t_i)^{opt} \\ v_\beta(t_i)^{opt} \end{bmatrix} \right)^T (\Delta t^2 B_m^T B_m) \\
&\times \left(\begin{bmatrix} v_\alpha(t_i) \\ v_\beta(t_i) \end{bmatrix} - \begin{bmatrix} v_\alpha(t_i)^{opt} \\ v_\beta(t_i)^{opt} \end{bmatrix} \right) \\
&= \frac{\Delta t^2}{L_s^2}(v_\alpha(t_i) - v_\alpha(t_i)^{opt})^2 + \frac{\Delta t^2}{L_s^2}(v_\beta(t_i) - v_\beta(t_i)^{opt})^2
\end{aligned} \tag{16}
$$

With both objective function and the optimal control signals defined, the next step in the FCS predictive control is to find the control signal $v_\alpha(t_i)$ and $v_\beta(t_i)$ that will minimize the objective function subject to the limited number of choices of voltage variables as mentioned in (6).

In FCS current control proposed in the $\alpha - \beta$ reference frame, the seven pairs of v_α and v_β values from (6) are used to evaluate the objective function (16). The pair of v_α and v_β that has yielded a minimum of the objective function J will be chosen as the FCS current control signals in the $\alpha - \beta$ reference frame.

It is worthwhile to emphasis that because the reference signals in the $\alpha - \beta$ frame are sinusoidal signals, the error signals $i_\alpha(t)^* - i_\alpha(t)$ and $i_\beta(t)^* - i_\beta(t)$ will not converge to zero as $t \to \infty$. However, as sampling interval Δt reduces, the $|i_\alpha(t)^* - i_\alpha(t)|$ and $|i_\beta(t)^* - i_\beta(t)|$ will reduce as the feedback controller gain $\frac{L_s}{\Delta t}(1 - \frac{R_s}{L_s}\Delta t)$ increases.

2.3 Generating Current Reference Signals in $\alpha - \beta$ Frame

The reference signals to the FCS current control in the $\alpha - \beta$ reference signals are sinusoidal signals in which their frequency is determined by the frequency of the grid ω_g. In the applications, the desired operational performance of a power converter in a closed-loop current control is specified via the desired values of i_d and i_q currents. For instance, the desired value for the i_d current is related to real power in demand and i_q is chosen to be 0 for unity power factor. The reference signals to the i_d and i_q currents are transparent to the applications and easy to choose. For these reasons, the reference signals to i_α and i_β currents are calculated using the inverse Park Transform:

$$\begin{bmatrix} i_\alpha(t)^* \\ i_\beta(t)^* \end{bmatrix} = \begin{bmatrix} \cos\omega_g t & -\sin\omega_g t \\ \sin\omega_g t & \cos\omega_g t \end{bmatrix} \begin{bmatrix} i_d(t)^* \\ i_q(t)^* \end{bmatrix} \quad (17)$$

In the majority of applications where $i_q(t)^* = 0$, simply, $i_\alpha(t)^* = \cos\omega_g t i_d(t)^*$, and $i_\beta(t)^* = \sin\omega_g t i_d(t)^*$.

Figure 1 shows the configuration of the feedback control system to generate optimal control signals $v_\alpha(t)^{opt}$ and $v_\beta(t)^{opt}$.

3. RESONANT FCS CURRENT CONTROL

To reduce the steady-state errors in the i_d and i_q currents, the feedback errors $i_\alpha(t)^* - i_\alpha(t)$ and $i_\beta(t)^* - i_\beta(t)$ in the current control system need to be reduced.

3.1 Control System Configuration

It is known from internal model control principle that in order for the feedback control system to track a periodic signal, the signal generator needs to be embedded in the controller. For the case of the current control in the $\alpha-\beta$ reference frame, because the reference current signals are sinusoidal signals, the generator of a sinusoidal signal should be embedded in the feedback control system so that the output current signals $i_\alpha(t)$ and $i_\beta(t)$ would track their reference signals without steady-state errors. In short, the controller should have a polynomial factor $1-2\cos(\omega_d)z^{-1}+z^{-2}$, which is $(1-e^{j\omega_d}z^{-1})(1-e^{-j\omega_d}z^{-1})$, contained in its denominator where ω_d is the discrete frequency of the sinusoidal reference signal. In other words, there is a pair of complex poles contained in the controller, where the locations of the poles are at $e^{\pm j\omega_d}$ on the complex plane.

The controller that has the capability to track a sinusoidal reference signal or to reject a sinusoidal disturbance signal is called resonant controller. The resonant FCS current controller is proposed to have the feedback structure as illustrated in Figure 2. In the proposed control system structure, the feedback controllers k^α_{fcs} and k^β_{fcs} derived from the one-step-ahead prediction and optimization shown in Section 2 are used in the inner-loops for fast dynamic response, while two resonant controllers are used in the outer-loops to provide further compensations for the tracking errors between the reference and feedback current signals in the $\alpha-\beta$ reference frame.

The frequency ω_d is the discrete frequency, having the unit of radian. Assuming that a sinusoidal signal has a period of T, with a sampling interval Δt, the number of samples within this period T is $N_T = \frac{T}{\Delta t}$. The discrete frequency ω_d is calculated as

$$\omega_d = \frac{2\pi}{N_T} = \frac{2\pi\Delta t}{T}$$

Suppose that the frequency of the grid is 50 Hz and the sampling interval is $\Delta t = 80 \times 10^{-6}$ second. Then the frequency parameter ω_d is then

$$\omega_d = \frac{2\pi \times \Delta t}{0.02} = 0.0251$$

The frequency parameter ω_d is time invariant in the design because the grid frequency is generally assumed unchanged. In reality, it may change with respect to time. However, when the sampling interval Δt is small, this variation has a small effect on the locations of the complex poles in the controller. Let us say that the grid frequency varies from 50 to 50.5 Hz. When $\Delta t = 80 \times 10^{-6}$ second, the corresponding ω_d to 50 Hz is approximately 0.0251 rad and 50.5 Hz is 0.0254 rad. The controller poles for the former case are approximately $0.9997 \pm j0.0251$ and the latter case approximately 0.9997 ± 0.0254. Therefore, there is no need to track the change of the grid frequency and incorporate it in the resonant controller.

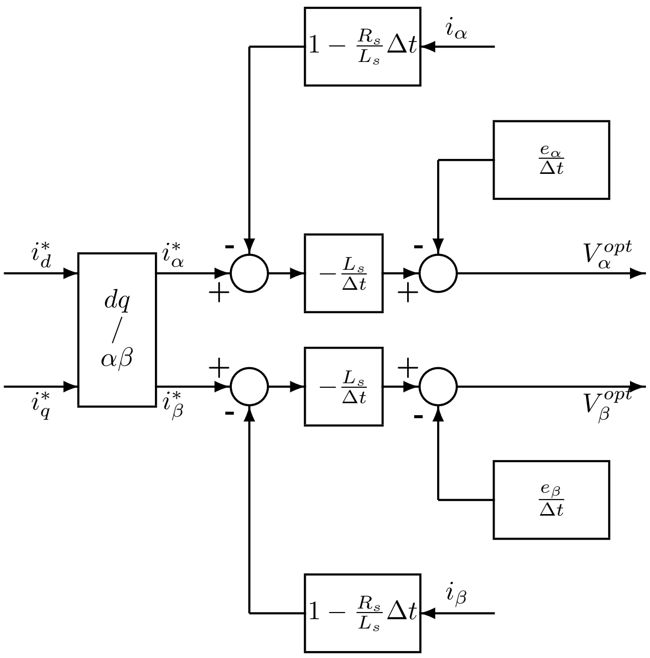


Fig. 1. Finite Control Set Current Control in $\alpha-\beta$ frame

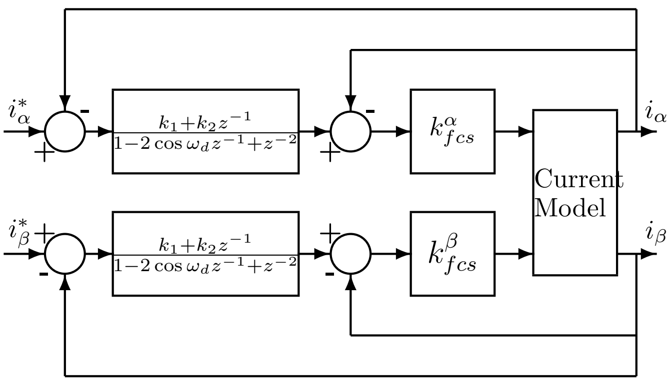


Fig. 2. Finite Control Set (FCS) resonant current control in $\alpha-\beta$ frame

3.2 Outer-loop Controller Design

It is easy to show that the one-step-ahead predictive controller for the inner-loop system result in a closed-loop system with a transfer function z^{-1} (Wang and Gan (2014)).

The same procedure is applied here to obtain the same result in the $\alpha - \beta$ reference frame.

With the inner-loop system modeled as one sample of delay z^{-1}, the task of designing the resonant controller in the outer-loop becomes straightforward. Figure 3 illustrates the outer-loop system for controlling current i_α with the resonant controller and the inner-loop approximated by the transfer function z^{-1}.

It is clearly seen that the closed-loop system from the reference signal i_α^* to i_α is described by the z-transfer function

$$T(z) = \frac{k_1 z^{-1} + k_2 z^{-2}}{1 - 2cos\omega_d z^{-1} + z^{-2} + k_1 z^{-1} + k_2 z^{-2}} \quad (18)$$

This is a second order discrete-time system with two closed-loop poles. Thus, the two coefficients from the resonant controller, k_1 and k_2, can be uniquely determined by using the technique of pole-assignment controller design.

For simplicity, assuming that the desired closed-loops are identical, denoted as $0 \leq \lambda < 1$, the desired closed-loop polynomial for the discrete system is given by

$$(1 - \lambda z^{-1})^2 = 1 - 2\lambda z^{-1} + \lambda^2 z^{-2}$$

By comparing the desired closed-loop polynomial with the actual closed-loop polynomial given by the denominator of (18), we obtain the following equalities:

$$k_1 - 2\cos\omega_d = -2\lambda$$
$$k_2 + 1 = \lambda^2$$

These equalities lead to the solutions for the gains of the resonant controller where

$$k_1 = 2\cos\omega_d - 2\lambda \quad (19)$$
$$k_2 = \lambda^2 - 1 \quad (20)$$

The performance tuning parameter for the resonant FCS controller is the location of the pair of desired discrete closed-loop poles $0 \leq \lambda < 1$. This parameter is selected in the design to reflect the closed-loop bandwidth of the control system, depending on the quality of the current model and current sensor noise level. A smaller λ corresponds to faster closed-loop response for the resonant FCS control system, which on the other hand, it may cause noise amplification and the resulted closed-loop system less robust.

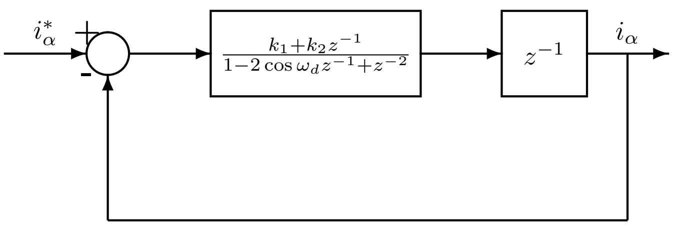


Fig. 3. Outer-loop system with inner-loop approximated by a sample of time delay

If one wishes to use the closed-loop performance specification in continuous-time that closely corresponds to the underlying physical system, then the desired closed-loop polynomial is chosen as $s^2+2\xi w_n s+w_n^2$. For $\xi = 0.707$, the pair of continuous-time complex poles are $s_{1,2} = -\xi w_n \pm j w_n \sqrt{1-\xi^2}$. With a sampling interval Δt, the pair of poles are converted from continuous-time to discrete-time via the following relationships:

$$z_1 = e^{-\xi w_n \Delta t + j w_n \sqrt{1-\xi^2}\Delta t}$$
$$z_2 = e^{-\xi w_n \Delta t - j w_n \sqrt{1-\xi^2}\Delta t}$$

When the desired closed-loop poles are selected this way, the coefficients of the resonant controller are found by comparing the desired closed-loop polynomial with the actual closed-loop polynomial given by the denominator in (18):

$$k_1 = 2\cos\omega_d - 2e^{-\xi w_n \Delta t}\cos(w_n\sqrt{1-\xi^2}\Delta t) \quad (21)$$
$$k_2 = e^{-2\xi w_n \Delta t} - 1 \quad (22)$$

where w_n is the desired bandwidth for the closed-loop current control system specified in the continuous-time.

3.3 Resonant FCS Control System

The control system shown in Figure 2 is the resonant control system without the constraints of using the finite control set in the $\alpha - \beta$ frame. The resonant control algorithm for using the finite control set is an extension of the unconstrained control case. We will first summarize the algorithm, followed by its derivation.

To derive how the control signals are chosen in the presence of constraints, we will resort to the solution via predictive control. Using Euler discretization method, the difference equations of currents i_α and i_β at the sampling time t_i can be described as:

$$i_\alpha(t_i + \Delta t) = (1 - \frac{R_s}{L_s}\Delta t) i_\alpha(t_i) - \frac{\Delta t}{L_s} v_\alpha(t_i) + \frac{1}{L_s}\Delta t\ e_\alpha \quad (23)$$

$$i_\beta(t_i + \Delta t) = (1 - \frac{R_s}{L_s}\Delta t) i_\beta(t_i) - \frac{\Delta t}{L_s} v_\beta(t_i) + \frac{1}{L_s}\Delta t\ e_\beta \quad (24)$$

Because there is no interaction between the variables in the $\alpha - \beta$ reference frame, for simplicity, the prediction for i_α is considered only, and the results will be naturally extended to the variable i_β.

Define the operator $D(z^{-1})$ as

$$D(z^{-1}) = 1 - 2\cos\omega_d z^{-1} + z^{-2}$$

with z^{-1} as the backward shift operator $z^{-1}f(t_i) = f(t_i - \Delta t)$. Applying the operator $D(z^{-1})$ to both sides of (23) yields

$$i_\alpha(t_i + \Delta t)^s = (1 - \frac{R_s}{L_s}\Delta t) i_\alpha(t_i)^s - \frac{\Delta t}{L_s} v_\alpha(t_i)^s \quad (25)$$

where

$$i_\alpha(t_i + \Delta t)^s = D(z^{-1}) i_\alpha(t_i + \Delta t) \quad (26)$$
$$i_\alpha(t_i)^s = D(z^{-1}) i_\alpha(t_i) \quad (27)$$
$$v_\alpha(t_i)^s = D(z^{-1}) v_\alpha(t_i) \quad (28)$$

The variables $i_\alpha(t_i)^s$ and $v_\alpha(t_i)^s$ are the filtered current and voltage signals with the denominator of the resonant controller.

Algorithm 1

The resonant FCS control signals in the $\alpha-\beta$ reference frame at sampling time t_i, $v_\alpha(t_i)$ and $v_\beta(t_i)$, are found by finding the minimum of the objective function J with respect to the index k,

$$J = \frac{\Delta t^2}{L_s^2}(v_\alpha(t_i)^k - v_\alpha(t_i)^{opt})^2 + \frac{\Delta t^2}{L_s^2}(v_\beta(t_i)^k - v_\beta(t_i)^{opt})^2$$

where the values of $v_\alpha(t_i)^k$ and $v_\beta(t_i)^k$ $(k = 0, 1, 2, \dots, 6)$ are given by the finite control set,

$$\begin{bmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & -1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \frac{2}{3} V_{dc}$$

and the signals $v_\alpha(t_i)^{opt}$ and $v_\beta(t_i)^{opt}$ are computed iteratively using the following equations:

$$\begin{aligned} v_\alpha(t_i)^{opt} &= 2\cos\omega_d v_\alpha(t_i - \Delta t)^{opt} \\ &\quad - v_\alpha(t_i - 2\Delta t)^{opt} + v_\alpha^{sopt}(t_i) \\ v_\alpha^{sopt}(t_i) &= k_{fcs}^\alpha[k_1(i_\alpha(t_i)^* - i_\alpha(t_i)) \\ &\quad + k_2(i_\alpha(t_i - \Delta t)^* - i_\alpha(t_i - \Delta t)) \\ &\quad - (i_\alpha(t_i) - 2\cos\omega_d i_\alpha(t_i - \Delta t) + i_\alpha(t_i - 2\Delta t))] \\ v_\beta(t_i)^{opt} &= 2\cos\omega_d v_\beta(t_i - \Delta t)^{opt} \\ &\quad - v_\beta(t_i - 2\Delta t)^{opt} + v_\beta^{sopt}(t_i) \\ v_\beta^{sopt}(t_i) &= k_{fcs}^\beta[k_1(i_\beta(t_i)^* - i_\beta(t_i)) \\ &\quad + k_2(i_\beta(t_i - \Delta t)^* - i_\beta(t_i - \Delta t)) \\ &\quad - (i_\beta(t_i) - 2\cos\omega_d i_\beta(t_i - \Delta t) + i_\beta(t_i - 2\Delta t))] \end{aligned}$$

The feedback control gains used in the computation are defined as:

$$\begin{aligned} k_{fcs}^\alpha = k_{fcs}^\beta &= \frac{L_s}{\Delta t}(1 - \frac{R_s}{L_s}\Delta t) \\ k_1 &= 2\cos\omega_d - 2\lambda \\ k_2 &= \lambda^2 - 1 \end{aligned}$$

$0 \le \lambda < 1$ is the desired closed-loop pole location for the current control system.

When the operator $D(z^{-1})$ is applied to $\sin(\omega t(t_i))$, we obtain the result that

$$D(z^{-1})\sin(\omega t(t_i)) = 0 \quad (29)$$

By assuming that the grid frequency ω as a constant (say in prefect grid condition), the last term of (23) vanishes when the operator $D(z^{-1})$ is applied to it.

To include the resonant action into the controller, the weighted current errors $e_\alpha^e(t_i) = k_1(i_\alpha(t_i)^* - i_\alpha(t_i)) + k_2(i_\alpha(t_i - \Delta t)^* - i_\alpha(t_i - \Delta t))$ is chosen as the steady-state of the $i_\alpha(t_i)^s$, where k_1 and k_2 are calculated as in Algorithm 1. The steady-state of $v_\alpha(t_i)^s$ is chosen to be zero. Subtracting the steady-state from the model (25) gives:

$$\begin{aligned} & i_\alpha(t_i + \Delta t)^s - e_\alpha^e(t_i) \\ &= (1 - \frac{R_s}{L_s}\Delta t)(i_\alpha(t_i)^s - e_\alpha^e(t_i)) - \frac{\Delta t}{L_s} v_\alpha(t_i)^s \end{aligned} \quad (30)$$

After applying the same procedure to the β-axis current, we obtain the formulation for the i_β variable as

$$\begin{aligned} & i_\beta(t_i + \Delta t)^s - e_\beta^e(t_i) \\ &= (1 - \frac{R_s}{L_s}\Delta t)(i_\beta(t_i)^s - e_\beta^e(t_i)) - \frac{\Delta t}{L_s} v_\beta(t_i)^s \end{aligned} \quad (31)$$

where $e_\beta^e(t_i) = k_1(i_\beta(t_i)^* - i_\beta(t_i)) + k_2(i_\beta(t_i - \Delta t)^* - i_\beta(t_i - \Delta t))$.

The control objective is to minimize the error function J, where

$$J = \begin{bmatrix} i_\alpha(t_i + \Delta t)^s - e_\alpha^e(t_i) \\ i_\beta(t_i + \Delta t)^s - e_\beta^e(t_i) \end{bmatrix}^T \begin{bmatrix} i_\alpha(t_i + \Delta t)^s - e_\alpha^e(t_i) \\ i_\beta(t_i + \Delta t)^s - e_\beta^e(t_i) \end{bmatrix} \quad (32)$$

which is to regulate the filtered current signals $i_\alpha(t_i + \Delta t)^s$, $i_\beta(t_i + \Delta t)^s$ to be as close as possible to $e_\alpha^e(t_i)$ and $e_\beta^e(t_i)$.

By substituting (30) and (31) into (32), it can be shown that the optimal solutions of $v_\alpha(t_i)^s$ and $v_\beta(t_i)^s$ that will minimize the objective function (32) are given by

$$v_\alpha(t_i)^{sopt} = k_{fcs}^\alpha(e_\alpha^e(t_i) - i_\alpha(t_i)^s) \quad (33)$$

$$v_\beta(t_i)^{sopt} = k_{fcs}^\beta(e_\beta^e(t_i) - i_\beta(t_i)^s) \quad (34)$$

where the feedback controller gains are defined as

$$k_{fcs}^\alpha = k_{fcs}^\beta = \frac{L_s}{\Delta t}(1 - \frac{R_s}{L_s}\Delta t) \quad (35)$$

Furthermore, the objective function J can be expressed via completing squares as

$$\begin{aligned} J &= \frac{\Delta t^2}{L_s^2}(v_\alpha(t_i)^s - v_\alpha(t_i)^{sopt})^2 \\ &\quad + \frac{\Delta t^2}{L_s^2}(v_\beta(t_i)^s - v_\beta(t_i)^{sopt})^2 \end{aligned} \quad (36)$$

Now, note that $v_\alpha(t_i)^s$, $v_\alpha(t_i)^{sopt}$, $v_\beta(t_i)^s$, $v_\beta(t_i)^{sopt}$ are filtered voltage variables. Thus, by definition of the filtered control signals, the following relationships are true:

$$\begin{aligned} v_\alpha(t_i)^{opt} &= \frac{v_\alpha(t_i)^{sopt}}{1 - 2\cos\omega_d z^{-1} + z^{-2}} \\ v_\beta(t_i)^{opt} &= \frac{v_\beta(t_i)^{sopt}}{1 - 2\cos\omega_d z^{-1} + z^{-2}} \end{aligned}$$

which leads to the expressions of $v_\alpha(t_i)^{sopt}$ and $v_\beta(t_i)^{sopt}$ in an iterative manner:

$$\begin{aligned} v_\alpha(t_i)^{sopt} &= v_\alpha(t_i)^{opt} - 2\cos\omega_d v_\alpha(t_i - \Delta t)^{opt} \\ &\quad + v_\alpha(t_i - 2\Delta t)^{opt} \end{aligned} \quad (37)$$

$$\begin{aligned} v_\beta(t_i)^{sopt} &= v_\beta(t_i)^{opt} - 2\cos\omega_d v_\beta(t_i - \Delta t)^{opt} \\ &\quad + v_\beta(t_i - 2\Delta t)^{opt} \end{aligned} \quad (38)$$

By calculating the actual filtered control signals using the same past optimal control signal states, we obtain

$$v_\alpha(t_i)^s = v_\alpha(t_i) - 2\cos\omega_d v_\alpha(t_i - \Delta t)^{opt} + v_\alpha(t_i - 2\Delta t)^{opt} \quad (39)$$

$$v_\beta(t_i)^s = v_\beta(t_i) - 2\cos\omega_d v_\beta(t_i - \Delta t)^{opt} + v_\beta(t_i - 2\Delta t)^{opt} \quad (40)$$

By substituting the filtered variables (37)-(40) into the objective function (36), it becomes

$$J = \frac{\Delta t^2}{L_s^2}(v_\alpha(t_i) - v_\alpha(t_i)^{opt})^2 + \frac{\Delta t^2}{L_s^2}(v_\beta(t_i) - v_\beta(t_i)^{opt})^2 \quad (41)$$

With the finite control set, the derived objective function here is identical to the one used in the Algorithm 1. This completes the derivation of the resonant FCS control algorithm.

It is emphasized that the resonant FCS control algorithm 1 used the past optimal control signal states ($v_\alpha(t_i - \Delta t)^{opt}$,

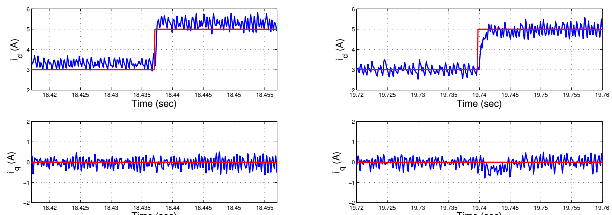
$v_\alpha(t_i - 2\Delta t)^{opt}$) together with the current ($v_\alpha(t_i)$) to predict the filtered $v_\alpha(t_i)^s$. If the past implemented control signal states ($v_\alpha(t_i - \Delta t)$, $v_\alpha(t_i - 2\Delta t)$) were used in the prediction, then it could result in accumulated errors from the finite control set and lead to steady-state errors in the resonant FCS control system.

4. EXPERIMENTAL RESULTS

Laboratory prototype of three-phase boost rectifier is used to validate the control design as discussed in the previous section. In the experimental set-up, the system parameters are $V_{ac} = 30$V, $L_s = 6.3$mH, $C_{dc} = 296\mu$F and $R_{load} = 20\Omega$. The desired closed-loop pole location λ is chosen as 0.95 for resonant FCS.

4.1 Comparison study with and without resonant controller

To illustrate the performance of the resonant FCS predictive control, a comparison study is done between the original FCS and the resonant FCS in response to step change on i_d current from 3A to 5A. The experimental results shown in Fig. 4 are the i_d and i_q currents of the original FCS and the resonant FCS in the synchronous reference frame. It is shown that the original FCS had a clear steady-state error and responded to a step change in 0.9 milliseconds, whereas the resonant FCS did not have a steady-state error. It took about 1.85 milliseconds for i_d current to reach the new reference value and 4 millisecond for i_q current to return to zero. The response time of the resonant FCS can be tuned by changing the λ which is the closed-loop pole value of the controller. The original FCS controller had a mean value of 3.3129A and 5.2666A for the set point 3A and 5A respectively, while resonant FCS controller had a mean value of 3.0008A and 4.9611A.



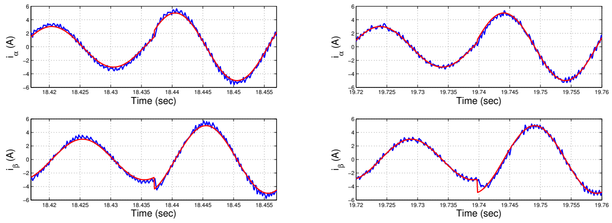
(a) Original FCS (b) Resonant FCS

Fig. 4. i_d and i_q currents of FCS and Resonant FCS in response to step change from 3A to 5A.

Fig. 5 shows the experimental results in stationary reference frame. It is seen from the figure that original FCS had a clearly steady-state error at peak and trough of the sinusoidal waveform, while resonant FCS tracked the reference without steady-state error.

5. CONCLUSION

This paper has investigated the design and implementation of a finite control set predictive control for a two level power converter. Specifically, the proposed approach included a discrete-time resonant controller in the algorithm to eliminate the sinusoidal error. Experimental results verified that the design algorithm and implementation are efficacious.



(a) Original FCS (b) Resonant FCS

Fig. 5. i_α and i_β currents of FCS and Resonant FCS in response to step change from 3A to 5A.

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REFERENCES

Aguilera, R., Lezana, P., and Quevedo, D. (2013). Finite-control-set model predictive control with improved steady-state performance. *Industrial Informatics, IEEE Transactions on*, 9(2), 658–667. doi: 10.1109/TII.2012.2211027.

Kawabata, T., Miyashita, T., and Yamamoto, Y. (1990). Dead beat control of three phase PWM inverter. *Power Electronics, IEEE Transactions on*, 5(1), 21–28. doi: 10.1109/63.45996.

Kukrer, O. (1996). Deadbeat control of a three-phase inverter with an output LC filter. *Power Electronics, IEEE Transactions on*, 11(1), 16–23. doi: 10.1109/63.484412.

Linder, A. and Kennel, R. (2005). Model predictive control for electrical drives. In *Power Electronics Specialists Conference, 2005. PESC '05. IEEE 36th*, 1793–1799. doi:10.1109/PESC.2005.1581874.

Rodriguez, J., Pontt, J., Silva, C., Salgado, M., Rees, S., Ammann, U., Lezana, P., Huerta, R., and Cortes, P. (2004). Predictive control of three-phase inverter. *Electronics Letters*, 40(9), 561–563. doi: 10.1049/el:20040367.

Wang, L. and Gan, L. (2014). Integral FCS predictive current control of induction motor drive. In *IFAC 19th World Congress*.