

Three-Dimensional Consensus Path-Following for Second-Order Multi-Agent Networks^{*}

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Abstract: In this paper, we address a new consensus problem of coordinately steering a group of multi-agents under directed information flow along a three-dimensional reference path without temporal constraint. The spatial reference path is newly defined by algebraic implicit expressions and the path-following kinematic-error dynamics are then formulated for each agent using two path-following errors and a speed tracking error. Distinct from the stabilizing feedback control design of the path-following problem for a single agent, the proposed new feedback control algorithm augmented with consensus disagreement terms could achieve both the reference goal seeking and consensus during transition. To show effectiveness of the proposed concept, simulation results are included in the end.

Keywords: Consensus, multi-agent systems, nonlinear control, directed graph, path-following.

1. INTRODUCTION

As a fundamental of distributed coordination, network consensus problem (Olfati and Murray, 2004; Yang et al., 2013; Zuo and Tie, 2014a,b), which means that the states of all the agents converge to certain quantities of interest, has been widely investigated in recent years. In some applications including formation flying and coordinated tracking, groups of agents are required to agree upon the state of a dynamic leader with local interaction, i.e., *consensus tracking* problem (Ren, 2010a). However, consensus tracking basically boils down to chasing a time-varying reference dynamics. This means that each agent requires to attain a specific location at a specific pre-assigned instant. To remove the temporal constraint, *consensus path-following* is put forward by us to emphasize the primary geometric task for the groups with flexible dynamic assignment. This concept is developed within the framework of *path-following* control.

Different from the trajectory tracking control, the path-following motion control drives the vehicle to and follow a path with a desired speed profile and without a specific temporal requirement. Aguiar and Hespanha (2007) decomposed the path-following problem into two subproblems: a geometric path following and a speed assignment along the path. In some specific areas, the path-following

motion control is more appealing than the trajectory-tracking control, such as path-following for nonminimum phase systems (Aguiar et al., 2005). The key to the path following problem in a great deal of papers (Soetanto et al., 2003; Breivik and Fossen, 2005; Cunha et al., 2006; Kaminer et al., 2010) is to explicitly control the rate of progression of the virtual target running along the path, and thus provides an extra design degree of freedom to avoid singularities effectively. Within the framework due to Breivik et al. (2005), they further extended the idea to the two-dimensional formation control of multi-agents (Breivik et al., 2006). Based on the work due to Kaminer (2010), Xargay et al. (2012; 2013) investigated the three-dimensional path-following problem of multiple UAVs with stringent spatial and temporal constraints. However, the drawback of the aforementioned path-following control methodologies is that during transition the follower might temporarily deviate from the reference path before it catches up with the virtual target (Nielsen et al., 2010).

This paper builds upon the work due to Zhu and Huo (2013), and the distinctive feature of our work from previous published paper is to address the following key aspects: (i) It presents a new implicit geometric description of reference paths by intersecting two two-dimensional manifolds, and a new singularity-free formulation of the path-following kinematic-error dynamics for each agent in networks using two path-following errors and a speed tracking error. (ii) It puts forward a new concept denoted by *consensus path-following* problem and a corresponding

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design framework of feedback coordination control laws. In contrast to *consensus tracking* problem, it focuses on driving all the agents in networks to a geometric path without stringent temporal constraint and to run along it with a flexible desired speed profile. (iii) A virtual leader staying uniformly on the reference path is incorporated into the directed communication topology between agents and the reference information is only available to a portion of agents in the group. The consensus path-following is achieved for the multi-agents in networks if the extended graph contains a spanning tree.

This paper is organized as follows: Section 2 introduces the preliminaries on graph theory notions. Sections 3 and 4 present the path-following and consensus path-following problems, respectively. In section 5, an illustrative simulation example is discussed. Finally, the paper is ended by concluding remarks in section 6.

2. GRAPH THEORY PRELIMINARIES

Information interchange between agents can be described by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ (Godsil and Royle, 2001) where $\mathcal{V} = \{\pi_1, \pi_2, \dots, \pi_n\}$ is a set of nodes that represent the agents and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges that depict information flow between agents. An edge (π_i, π_j) in \mathcal{G} denotes that the information state of node π_i is available to node π_j , and the node π_i is said to be a neighbour of node π_j . The index set of all neighbours of node π_j is denoted by $\mathcal{N}_j = \{i : (\pi_i, \pi_j) \in \mathcal{E}\}$. In an undirected graph, $(\pi_i, \pi_j) \in \mathcal{E} \Leftrightarrow (\pi_j, \pi_i) \in \mathcal{E}$. Let the adjacent matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ with $a_{ij} = 1$ if $(\pi_j, \pi_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. It is assumed that $a_{ii} = 0$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n \times n}$ is defined as $l_{ii} = \sum_{j=1, j \neq i}^n a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$.

For an edge (π_i, π_j) in a directed graph, π_i is the parent node and π_j is the child node. A directed path from node π_j to π_i in \mathcal{G} is a sequence of edges $(\pi_j, \pi_{i_1}), (\pi_{i_1}, \pi_{i_2}), \dots, (\pi_{i_{l-1}}, \pi_i)$ in \mathcal{G} with distinct nodes $\pi_{i_k}, k = 1, 2, \dots, l$. A directed tree (Ren and Beard, 2005) is a directed graph, in which every node has exactly one parent except for the node, called root, which has no parent, and the root has a directed path to every other node. A spanning tree of a directed graph is a directed tree formed by graph edges which connect all the nodes in \mathcal{G} . We say that a graph has (or contains) a spanning tree if a subset of the edges forms a spanning tree.

3. PATH-FOLLOWING CONTROL FOR A SINGLE AGENT

In this section, we briefly review the implicit geometric path following control methodology (Zhu and Huo, 2013) for an agent system at the dynamic level as

$$\begin{cases} \dot{p} = v \\ \dot{v} = u \end{cases} \quad (1)$$

where $p = [x, y, z]^T \in \mathbb{R}^3$ denotes the position vector, $v = [v_x, v_y, v_z]^T \in \mathbb{R}^3$ the velocity vector, and $u \in \mathbb{R}^3$ the control input.

The reference path to be followed in this paper is described by two algebraic equations, i.e., an implicit geometric description, defined by

$$\mathcal{P}_r = \{[x, y, z] \in \mathbb{R}^3 | f_1(x, y, z) = 0, f_2(x, y, z) = 0\} \quad (2)$$

where $f_1(x, y, z)$ and $f_2(x, y, z)$ are both C^∞ functions with respect to x, y and z . To guarantee the regularity of the reference path in (2), we assume that the tangent vector of $p_r \in \mathcal{P}_r$ satisfies

$$\left. \frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right|_{p=p_r} \neq 0 \quad (3)$$

where \times denotes the cross product of two vectors. For a regular curve, $\frac{\partial f_1}{\partial p}$ and $\frac{\partial f_2}{\partial p}$ are unparallel, implying that (3) holds.

Remark 1. The C^∞ property of functions $f_1(x, y, z)$ and $f_2(x, y, z)$ ensures that $\frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p}$ is C^∞ . Thus, the assumption (3) holds in the near region of \mathcal{P}_r .

Define the path-following errors as

$$\begin{cases} \varepsilon_1 \triangleq f_1(x(t), y(t), z(t)) \\ \varepsilon_2 \triangleq f_2(x(t), y(t), z(t)) \end{cases} \quad (4)$$

Given the desired speed profile $\bar{v}(t)$ which is not identically equal to zero, we define the following speed tracking error as

$$\varepsilon_3 \triangleq \left(\frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right)^T v - \left\| \frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right\| \bar{v} \quad (5)$$

Remark 2. Instead of intuitively defining the speed error as $\varepsilon_3 \triangleq \|v\| - \bar{v}$, the modified one in (5) removes local singularities in the path following control design. Actually, this error definition (5) assigns the desired velocity along the tangent vector of the reference path.

The path-following kinematic-error dynamics can be obtained by differentiating (4) and (5), yielding

$$\begin{aligned} [\dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3]^T &= H(x, y, z, \dot{x}, \dot{y}, \dot{z}) \\ &\quad + G(x, y, z)u \end{aligned} \quad (6)$$

where $H = [h_1, h_2, h_3]^T \in \mathbb{R}^3$ and $G = [g_1^T, g_2^T, g_3^T]^T \in \mathbb{R}^{3 \times 3}$ with the corresponding elements defined by

$$\begin{aligned} h_1 &= \frac{\partial^2 f_1}{\partial x^2} v_x^2 + \frac{\partial^2 f_1}{\partial y^2} v_y^2 + \frac{\partial^2 f_1}{\partial z^2} v_z^2 \\ &\quad + 2 \frac{\partial^2 f_1}{\partial x \partial y} v_x v_y + 2 \frac{\partial^2 f_1}{\partial y \partial z} v_y v_z + 2 \frac{\partial^2 f_1}{\partial z \partial x} v_z v_x, \\ h_2 &= \frac{\partial^2 f_2}{\partial x^2} v_x^2 + \frac{\partial^2 f_2}{\partial y^2} v_y^2 + \frac{\partial^2 f_2}{\partial z^2} v_z^2 \\ &\quad + 2 \frac{\partial^2 f_2}{\partial x \partial y} v_x v_y + 2 \frac{\partial^2 f_2}{\partial y \partial z} v_y v_z + 2 \frac{\partial^2 f_2}{\partial z \partial x} v_z v_x, \\ h_3 &= \left[\frac{d}{dt} \left(\frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right)^T \right] v - \left[\frac{d}{dt} \left\| \frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right\| \right] \bar{v} \\ &\quad - \left\| \frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right\| \dot{\bar{v}}, \\ g_1 &= \frac{\partial f_1}{\partial p}, \quad g_2 = \frac{\partial f_2}{\partial p}, \quad g_3 = \left(\frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right)^T \end{aligned}$$

It can be verified that $\det(G) = \left\| \frac{\partial f_1}{\partial p} \times \frac{\partial f_2}{\partial p} \right\|^2 > 0$ holds locally around the reference path.

At this point, the path-following generalized error vector can be formally defined as $\varepsilon_{pf} \triangleq [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T$. The statement of path-following for a single agent can now be described.

Definition 3. (Path-Following Problem). For a given reference path $p_r \in \mathcal{P}_r$ defined in (2) and a reference speed profile $\bar{v}(t)$, design feedback control law u for the agent system in (1) such that the path-following generalized error ε_{pf} converges to the origin as long as the initial position is sufficiently close to the reference path.

Lemma 4. Given a reference path $p_r \in \mathcal{P}_r$ and a desired speed profile $\bar{v}(t)$, feedback control law

$$u = G^{-1}(-H + \mu) \quad (7)$$

solves the path-following problem locally, where $\mu = [-k_{11}\dot{\varepsilon}_1 - k_{12}\varepsilon_1, -k_{21}\dot{\varepsilon}_2 - k_{22}\varepsilon_2, -k_{31}\varepsilon_3]^T$ with positive k_{ij} .

Proof. Substituting (7) into (6) yields the closed-loop path-following kinematic-error system:

$$[\dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3]^T = \mu \quad (8)$$

It is straightforward to verify that ε_{pf} is locally exponentially stable, since the closed-loop dynamics in (8) are Hurwitz.

Remark 5. The local property in Lemma 4 stems from the regularity requirement for the reference path, as discussed in Remark 1.

4. CONSENSUS PATH-FOLLOWING FOR MULTI-AGENTS

The previous section provided a solution to the path-following problem for a single agent. In this section, we move on to the consensus path-following problem for multi-agents in networks with directed interaction topology.

Consider a group of n continuous-time agents, indexed by $i \in \mathcal{I}_n = \{1, 2, \dots, n\}$, with dynamics in the form of

$$\begin{cases} \dot{p}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \quad (9)$$

where $p_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3$ and $v_i = [v_{x_i}, v_{y_i}, v_{z_i}]^T \in \mathbb{R}^3$ denote the position vector and velocity vector of the i -th agent, respectively, and $u_i \in \mathbb{R}^3$ the control input of the i -th agent.

Employing the path-following formulation presented in section 3, we transfer the dynamics of n agents into the path-following kinematic-error dynamic forms

$$\begin{aligned} [\ddot{\varepsilon}_{1_i}, \ddot{\varepsilon}_{2_i}, \ddot{\varepsilon}_{3_i}]^T &= H_i(x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i) \\ &+ G_i(x_i, y_i, z_i)u_i \end{aligned} \quad (10)$$

where $\varepsilon_{1_i} = f_1(x_i, y_i, z_i)$ and $\varepsilon_{2_i} = f_2(x_i, y_i, z_i)$ represent the path-following errors of the i -th agent, $\varepsilon_{3_i} = (\frac{\partial f_1}{\partial p_i} \times \frac{\partial f_2}{\partial p_i})^T v - \|\frac{\partial f_1}{\partial p_i} \times \frac{\partial f_2}{\partial p_i}\| \bar{v}$ the speed tracking error of the i -th agent, H_i and G_i are defined in a similar way as in (6) with respect to the states of the i -th agent. Information state to be interchanged between agents for coordination is the path-following generalized error vectors defined similarly as $\varepsilon_{pf,i} \triangleq [\varepsilon_{1_i}, \varepsilon_{2_i}, \varepsilon_{3_i}]^T$. Toward this end, we can now define the consensus path-following problem for a group of multi-agents.

Definition 6. (Consensus Path-Following Problem). Given a team of n second-order agents under directed interaction topology, and the reference path $p_r \in \mathcal{P}_r$ and reference speed profile $\bar{v}(t)$, design feedback control laws u_i such that, for $i \in \mathcal{I}_n$, the path-following generalized error vector $\varepsilon_{pf,i}$ converge to the origin.

Let $\varepsilon_{pf,0} = [\varepsilon_{1_0}, \varepsilon_{2_0}, \varepsilon_{3_0}]^T \in \mathbb{R}^3$ denote the information state of a virtual leader π_0 for the multi-agent system in (9). It is worth noticing, however, that $\varepsilon_{pf,0}$ is available not to all agents but to only a portion of agents. Here, we define a nonnegative diagonal matrix $\Omega = \text{diag}\{\omega_1, \omega_2, \dots, \omega_n\}$ to indicate the accessibility of $\varepsilon_{pf,0}$ by the agents, where $\omega_i = 1$ if $\varepsilon_{pf,0}$ is accessible by the i -th agent, and $\omega_i = 0$ otherwise. Without loss of generality, we assume that $\varepsilon_{pf,0} \equiv 0$ represents the reference path. The directed graph incorporating π_0 into \mathcal{G} is denoted by \mathcal{G}^e . We moreover make the following assumption on \mathcal{G}^e :

Assumption 7. \mathcal{G}^e has a spanning tree with π_0 being its root vertex.

Remark 8. Actually, the virtual leader π_0 can be viewed as a particle running along the reference path all the time, and thus we have $\varepsilon_{pf,0} \equiv 0$.

To streamline the technical proof of the main result, a lemma is presented before moving on.

Lemma 9. All the eigenvalues of matrix $\mathcal{L} + \Omega$ have positive real parts if and only if the assumption 7 holds.

Proof. Note that the Laplacian matrix of \mathcal{G}^e can be described in the form of

$$\mathcal{L}(\mathcal{G}^e) = \begin{bmatrix} 0 & 0 \\ -\Omega \mathbf{1}_n & \mathcal{L} + \Omega \end{bmatrix}. \quad (11)$$

where $\mathbf{1}_n = [1, 1, \dots, 1]^T \in \mathbb{R}^n$.

Necessity: If all the eigenvalues of $\mathcal{L} + \Omega$ have real positive parts, it is obvious from (11) that $\mathcal{L}(\mathcal{G}^e)$ has exactly one zero eigenvalue, and its other eigenvalues have positive real parts. This guarantees that the directed graph \mathcal{G}^e associated with the Laplacian matrix of (11) has a spanning tree (Ren and Beard, 2005). Since π_0 can not get information from all agents in \mathcal{G} , the root vertex of the spanning tree in \mathcal{G}^e can only be π_0 .

Sufficiency: If assumption 7 holds, then $\mathcal{L}(\mathcal{G}^e)$ has a simple zero eigenvalue and the other eigenvalues have positive real parts (Ren and Beard, 2005). This together with (11) guarantees that the eigenvalues of $\mathcal{L} + \Omega$ all have positive real parts. This completes the proof.

Theorem 10. For a given group of multi-agents (9) in networks under directed information flow with assumption 7, the feedback control laws

$$u_i = G_i^{-1}(-H_i + \mu_i), \quad i \in \mathcal{I}_n \quad (12)$$

solve the consensus path-following problem locally, where μ_i denote the consensus disagreement terms with their elements defined by

$$\begin{aligned} \mu_{1_i} &= - \sum_{j \in \mathcal{N}_i} a_{ij} [(\varepsilon_{1_i} - \varepsilon_{1_j}) + \gamma_1 (\dot{\varepsilon}_{1_i} - \dot{\varepsilon}_{1_j})] \\ &\quad - \omega_i [\varepsilon_{1_i} + \gamma_1 \dot{\varepsilon}_{1_i}] \end{aligned} \quad (13)$$

$$\begin{aligned} \mu_{2_i} &= - \sum_{j \in \mathcal{N}_i} a_{ij} [(\varepsilon_{2_i} - \varepsilon_{2_j}) + \gamma_2 (\dot{\varepsilon}_{2_i} - \dot{\varepsilon}_{2_j})] \\ &\quad - \omega_i [\varepsilon_{2_i} + \gamma_2 \dot{\varepsilon}_{2_i}] \end{aligned} \quad (14)$$

$$\mu_{3_i} = - \sum_{j \in \mathcal{N}_i} a_{ij} (\varepsilon_{3_i} - \varepsilon_{3_j}) - \omega_i \varepsilon_{3_i} \quad (15)$$

where γ_1 and γ_2 are positive damping constants.

Proof. Since the virtual leader π_0 stays on and moves along the reference path all the time, the information state of π_0 for consensus satisfies $\varepsilon_{pf,0} \equiv 0$ and $\dot{\varepsilon}_{1_0} = \dot{\varepsilon}_{2_0} \equiv 0$. Thus, the consensus laws (13)–(15) can be rewritten as

$$\begin{aligned}\mu_{1_i} &= - \sum_{j \in \mathcal{N}_i} a_{ij} [(\varepsilon_{1_i} - \varepsilon_{1_j}) + \gamma_1 (\dot{\varepsilon}_{1_i} - \dot{\varepsilon}_{1_j})] \\ &\quad - \omega_i [(\varepsilon_{1_i} - \varepsilon_{1_0}) + \gamma_1 (\dot{\varepsilon}_{1_i} - \dot{\varepsilon}_{1_0})] \\ \mu_{2_i} &= - \sum_{j \in \mathcal{N}_i} a_{ij} [(\varepsilon_{2_i} - \varepsilon_{2_j}) + \gamma_2 (\dot{\varepsilon}_{2_i} - \dot{\varepsilon}_{2_j})] \\ &\quad - \omega_i [(\varepsilon_{2_i} - \varepsilon_{2_0}) + \gamma_2 (\dot{\varepsilon}_{2_i} - \dot{\varepsilon}_{2_0})] \\ \mu_{3_i} &= - \sum_{j \in \mathcal{N}_i} a_{ij} (\varepsilon_{3_i} - \varepsilon_{3_j}) - \omega_i (\varepsilon_{3_i} - \varepsilon_{3_0})\end{aligned}$$

The assumption 7 ensures that at least one $\omega_i = 1$ for $i \in \mathcal{I}_n$, which implies that all the eigenvalues of $(L + \Omega)$ have positive real parts by Lemma 9. Stack the consensus information states of n agents as three vectors $\varepsilon_1 = [\varepsilon_{1_1}, \varepsilon_{1_2}, \dots, \varepsilon_{1_n}]^T$, $\varepsilon_2 = [\varepsilon_{2_1}, \varepsilon_{2_2}, \dots, \varepsilon_{2_n}]^T$ and $\varepsilon_3 = [\varepsilon_{3_1}, \varepsilon_{3_2}, \dots, \varepsilon_{3_n}]^T$. Substituting (12) into (10) obtains the path-following closed-loop dynamic equations

$$\ddot{\varepsilon}_1 = -(\mathcal{L} + \Omega)\dot{\varepsilon}_1 - \gamma_1(\mathcal{L} + \Omega)\varepsilon_1 \quad (16)$$

$$\ddot{\varepsilon}_2 = -(\mathcal{L} + \Omega)\dot{\varepsilon}_2 - \gamma_2(\mathcal{L} + \Omega)\varepsilon_2 \quad (17)$$

$$\dot{\varepsilon}_3 = -(\mathcal{L} + \Omega)\varepsilon_3 \quad (18)$$

It can be straightforwardly verified that $\varepsilon_3(t) = \exp(-(\mathcal{L} + \Omega)t)\varepsilon_3(0)$, which implies that the speed tracking error $\varepsilon_3(t)$ converges to the origin exponentially. As for the path-following errors, we rewrite (16) and (17) into the state space forms:

$$\begin{bmatrix} \dot{\varepsilon}_k \\ \ddot{\varepsilon}_k \end{bmatrix} = \begin{bmatrix} 0 & \mathbb{I}_n \\ -(\mathcal{L} + \Omega) & -\gamma_k(\mathcal{L} + \Omega) \end{bmatrix} \begin{bmatrix} \varepsilon_k \\ \dot{\varepsilon}_k \end{bmatrix} \quad (19)$$

where \mathbb{I}_n denotes the $n \times n$ identity matrix and $k = 1, 2$. Let $\mu_i (i \in \mathcal{I}_n)$ be the eigenvalues of $(\mathcal{L} + \Omega)$ and all $\mu_i > 0$ follows from Lemma 9. From (19) we have

$$\begin{aligned}\det \left(\lambda \mathbb{I}_{2n} - \begin{bmatrix} 0 & \mathbb{I}_n \\ -(\mathcal{L} + \Omega) & -\gamma_k(\mathcal{L} + \Omega) \end{bmatrix} \right) \\ = \det(\lambda^2 \mathbb{I}_n + (1 + \gamma_k \lambda)(\mathcal{L} + \Omega)) \\ = \prod_{i=1}^n (\lambda^2 + (1 + \gamma_k \lambda)\mu_i)\end{aligned}$$

It is straightforward to see that the system eigenvalues of (19) are given by

$$\lambda_{i\pm} = \frac{-\gamma_k \mu_i \pm \sqrt{\gamma_k^2 \mu_i^2 - 4\mu_i}}{2} \quad (20)$$

Thus, all system eigenvalues $\lambda_{i\pm}$ in (20) have negative real parts and the exponential convergence of ε_k and $\dot{\varepsilon}_k$ follows immediately for $k = 1, 2$.

With the above preparation, $\lim_{t \rightarrow \infty} \varepsilon_{pf,i} = 0$ follows immediately. Thus, the feedback control laws (12) solve the consensus path-following problem.

Remark 11. In this control design, we can view the reference path $p_r \in \mathcal{P}_r$ is generated by a virtual leader π_0 . The introduction of a virtual leader π_0 in the directed interaction topology under assumption 7 implies that only a portion of agents can access the reference path information.

Remark 12. From the *pinning control* point of view (Chen et al., 2009), the feedback control laws in (13)–(15) imply that the i -th agent is actually a pinned node for $\forall i \in \{i \in \mathcal{I}_n | \omega_i \neq 0\}$.

Remark 13. Distinct from the stabilizing law (7) for a single agent, the disagreement term μ_i in the feedback control law (12) introduces mutual synchronization between neighboring agents. Gains a_{ij} and ω_i , $i, j \in \mathcal{I}_n$, determine the relative weight for consensus ($\varepsilon_{pf,i} \rightarrow \varepsilon_{pf,j}$) during transition and goal seeking ($\varepsilon_{pf,i} \rightarrow 0$) (Ren, 2010b). In contrast, consensus during transition is not guaranteed if the stabilization laws are used for each agent in a group.

5. SIMULATION

Consider a group of $n = 4$ agents in a network with fixed directed information flow as shown in Fig. 1 from which we have $\Omega = \text{diag}\{1, 0, 0, 1\}$ and

$$\mathcal{L} + \Omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The design parameters $\gamma_1 = \gamma_2 = 2$ are set for each feedback control law. Let the initial conditions of the agents be $p_1(0) = [0, 2, -2]^T$, $p_2(0) = [-2, -1, 2]^T$, $p_3(0) = [-3, 3, -3]^T$, $p_4(0) = [3, -3, 3]^T$ and $v_1(0) = v_2(0) = v_3(0) = v_4(0) = 0$. The desired speed profile and reference path are chosen respectively as $\bar{v} = 2\text{m/s}$ and

$$\begin{cases} f_1(x, y, z) = x^2 + y^2 + z^2 - 25 = 0 \\ f_2(x, y, z) = x + y + z = 0 \end{cases}$$

which is a circular curve formed by the intersection of a ball and a plane, as shown in Fig. 2. After direct computations, we have

$$\begin{aligned}h_{1_i} &= 2(v_{x_i}^2 + v_{y_i}^2 + v_{z_i}^2) \\ h_{2_i} &= 0 \\ h_{3_i} &= 2(v_{y_i} - v_{z_i})v_{x_i} + 2(v_{z_i} - v_{x_i})v_{y_i} + 2(v_{x_i} - v_{y_i})v_{z_i} \\ &\quad - 2\bar{v} \frac{(y_i - z_i)(v_{y_i} - v_{z_i}) + (z_i - x_i)(v_{z_i} - v_{x_i})}{\sqrt{(y_i - z_i)^2 + (z_i - x_i)^2 + (x_i - y_i)^2}} \\ &\quad - 2\bar{v} \frac{(x_i - y_i)(v_{x_i} - v_{y_i})}{\sqrt{(y_i - z_i)^2 + (z_i - x_i)^2 + (x_i - y_i)^2}}\end{aligned}$$

$$G_i = \begin{bmatrix} 2x_i & 2y_i & 2z_i \\ 1 & 1 & 1 \\ 2(y_i - z_i) & 2(z_i - x_i) & 2(x_i - y_i) \end{bmatrix}$$

$$\dot{\varepsilon}_{1_i} = 2x_i v_{x_i} + 2y_i v_{y_i} + 2z_i v_{z_i}$$

$$\dot{\varepsilon}_{2_i} = v_{x_i} + v_{y_i} + v_{z_i}$$

Fig. 3 demonstrates that all four agents in the group converge to the reference path coordinately and thus achieve the path-following consensus. The agents' position states and control inputs are shown in Figs. 4 and 5, respectively. Since the communication topology in Fig. 1 has a spanning tree, the simulation results validate the correctness of the Theorem 10.

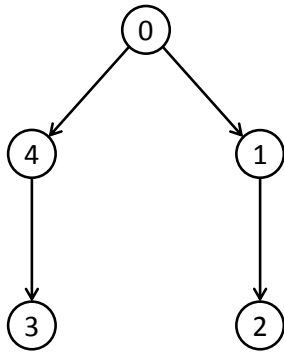


Fig. 1. Directed Communication Topology

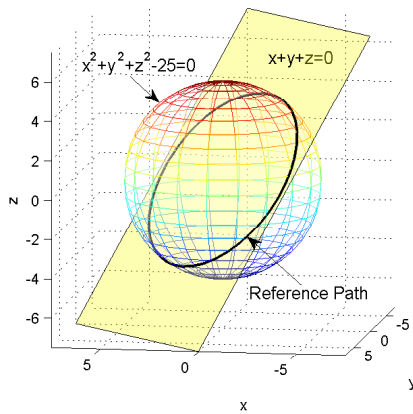


Fig. 2. Three-Dimensional Reference Path

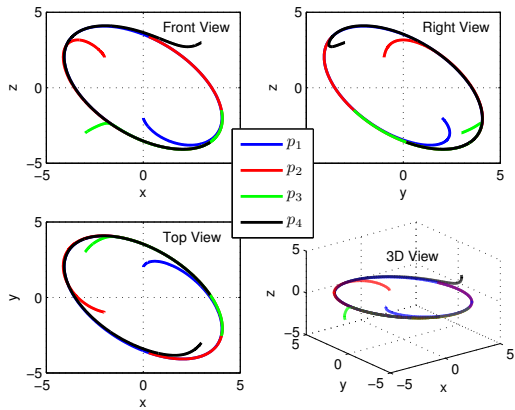


Fig. 3. Three-Dimensional Consensus Path-Following

6. CONCLUSION

In this paper, the implicit algebraic path-following control methodology is expanded to address the consensus problem of multi-agent system. A new concept of consensus path-following is proposed and a corresponding design framework of feedback consensus control laws is developed. The future work includes the extensions of the proposed framework to the coordinated path-following and formation controls of multi-agent systems or UAVs with various uncertainties.

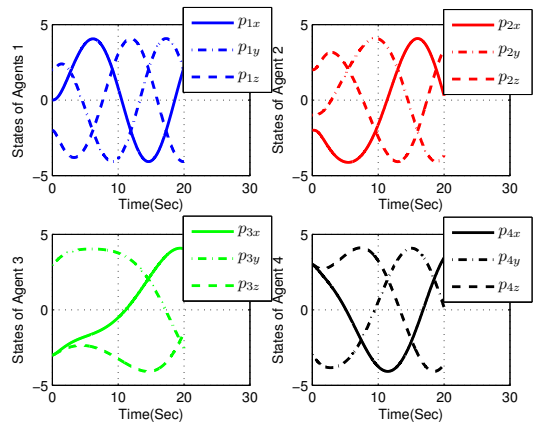


Fig. 4. Positions of the Four Agents

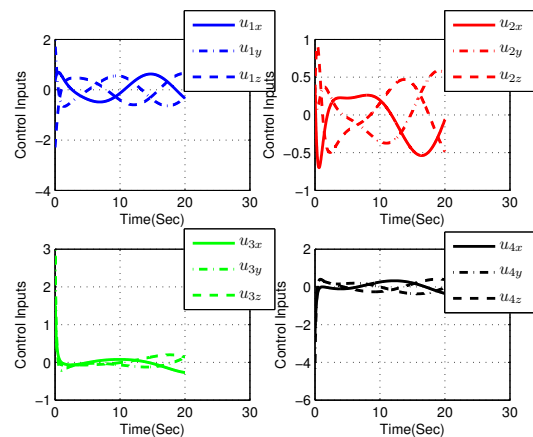


Fig. 5. Feedback Control Inputs

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REFERENCES

Aguiar, A.P., Hespanha, J.P., and Kokotović, P.V. Path-Following for Nonminimum Phase Systems Removes Performance Limitations. *IEEE Transactions on Automatic Control*, 50:234–239, 2005.

Aguiar, A.P., and Hespanha, J.P. Trajectory-Tracking and Path-Following of Underactuated Autonomous Vehicles with Parametric Modeling Uncertainty. *IEEE Transactions on Automatic Control*, 52:1362–1379, 2007.

Breivik, M., and Fossen, T.I. Principles of Guidance-Based Path Following in 2D and 3D. *Proceedings of 44th IEEE Conference on Decision and Control and the European Control Conference*, Seville, Spain, pages 627–634, 2005.

Breivik, M., Subbotin, M.V., and Fossen, T.I. Kinematic Aspects of Guided Formation Control in 2D. *Group Coordination and Cooperative Control*, Lecture Notes in Control and Information Sciences, Berlin: Springer-Verlag, 336:55–74, 2006.

- Chen, F., Chen, Z., Xiang, L., Liu, Z., and Yuan, Z. Reaching A Consensus via Pinning Control. *Automatica*, 45: 1215–1220, 2009.
- Cunha, R., Antunes, D., Gomes, P., and Silvestre, C. A Path-Following Preview Controller for Autonomous Air Vehicles. *Proceedings of AIAA Guidance, Navigation, and Control Conference and Exhibit*, Keystone, Colorado, USA, 2006.
- Godsil, C., and Royle, G. *Algebraic Graph Theory* New York: Springer-Verlag, 2001.
- Kaminer, I., Pascoal A., Xargay, E., Hovakimyan, N., Cao, C., and Dobrokhodov V. Path Following for Unmanned Aerial Vehicles Using \mathcal{L}_1 Adaptive Augmentation of Commercial Autopilots. *Journal of Guidance, Control, And Dynamics*, 33:550–564, 2010.
- Khalil, H.K. *Nonlinear Sstems (3rd ed.)* Upper Saddle River, NJ: Prentice Hall, 2005.
- Nielsen, C., Fulford, C., and Maggiore, M. Path Following Using Transverse Feedback Linearization: Application to a Maglev Positioning System. *Automatica*, 46:585–590, 2010.
- Olfati-Saber, R., and Murray, R.M. Consensus Problems in Networks of Agents with Switching Topology and Time-Delays. *IEEE Transactions on Automatic Control*, 49:1520–1533, 2004.
- Ren, W., and Beard, R.W. Consensus Seeking in Multi-agent Systems. *IEEE Transactions on Automatic Control*, 50:655–661, 2005.
- Ren, W. Consensus Tracking Under Directed Interaction Topologies: Algorithms and Experiments. *IEEE Transactions on Control System Technology*, 18:230–237, 2010.
- Ren, W. Distributed Cooperative Attitude Synchronization and Tracking for Multiple Rigid Bodies. *IEEE Transactions on Control System Technology*, 18:383–392, 2010.
- Soetanto, D., Lapierre, L., and Pascoal, A.M. Adaptive, Non-Singular Path-Following Control of Dynamic Wheeled Robots. *Proceedings of the International Conference on Advanced Robotics*, Coimbra, Portugal, pages 1387–1392, 2003.
- Xargay, E., Dobrokhodov, V., Kaminer, I., Pasoal, A.M., Hovakimyan, N., and Cao, C. Time-Critical Cooperative Control for Multiple Autonomous Systems. *IEEE Control Systems Magazine*, 32:49–73, 2012.
- Xargay, E., Kaminer, I., Pasoal, A.M., Hovakimyan, N., Dobrokhodov, V., Cichella, V., Aguiar, A.P., and Ghabcheloo, R. Time-Critical Cooperative Path Following of Multiple Unmanned Aerial Vehicles over Time-Varying Networks. *Journal of Guidance, Control, And Dynamics*, 36:499–516, 2013.
- Yang, W., Wang, X., and Shi, H. Fast Consensus Seeking in Multi-Agent Systems with Time Delay *Systems & Control Letters*, 62:269–276, 2013.
- Zhu, B., and Huo, W. 3-D Path-Following Control for a Model-Scaled Autonomous Helicopter. *IEEE Transactions on Control System Technology*, Published online, 2014.
- Zuo, Z., and Tie, L. A new class of finite-time nonlinear consensus protocols for multi-agent systems. *International Journal of Control*, 87:363–370, 2014.
- Zuo, Z., and Tie, L. Distributed Robust Finite-Time Nonlinear Consensus Protocols for Multi-Agent Systems. *International Journal of Systems Science*, Published online, 2014.