Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Hierachical Fuzzy MPC Concept for Building Heating Control

M.Killian * **B. Mayer** ** **M. Kozek** ***

* *Vienna University of Technology*
Wiedner Hauptstrasse 8-10/325/A5, 1040 Vienna, Austria
(e-mail: michaela.killian@tuwien.ac.at)
** *FH Joanneum Kapfenberg*
Werk-VI-Strasse 46, 8605 Kapfenberg, Austria
(e-mail: barbara.mayer@fh-joanneum.at)
*** *Vienna University of Technology*
Wiedner Hauptstrasse 8-10/325/A5, 1040 Vienna, Austria
(e-mail: martin.kozek@tuwien.ac.at)

Abstract: This paper presents a hierachical model predictive control (MPC) structure with decoupled MPCs for building heating control using weather forcasts and occupancy information. The two level control structure embeds a fuzzy MPC (FMPC) for user comfort optimization and a mixed-integer MPC (MI-MPC) for energy optimization at minimal costs. As FMPC uses a set of local linear models classical linear MPC theory is applicable, though the underlying system dynamics is non-linear. The supply level in a large modern office building always features switching states of aggregates, hence an MI-MPC is used for energy supply optimization. Additionally, both FMPC and MI-MPC consider all relevant constraints. The innovation in this study, beside the usage of FMPC for building control, is the decoupling of the energy supply level and the user comfort with a single coupling node. Although a global optimum is not guaranteed, a decoupled control system often is more attractive for industrial applications and building operators. The perfomance of the proposed control structure is demonstrated in a simulation with a validated building model, and two different disturbance scenarios are presented.

Keywords: Model predictive control; Advanced process control; Fuzzy MPC; Building climate control.

1. INTRODUCTION

Saving energy has become a political and social issue of concern worldwide. As buildings cause 40% of the total final energy consumption, Energy Ag. (2008), and due to the long lifespan of buildings, an emphasis is put on the development of strategies to operate modern large building systems in an energy efficient way. Optimizing the energy consumption while ensuring a given level of comfort within the building leads to a challenging control problem. Numerous different input variables and disturbances influence the desired outputs, such as room temperature. Model predictive control (MPC) has become one of the most promising techniques in this field. This paper presents a new decoupled MPC structure ensuring high comfort with minimal energy and costs expended. The comfort level controller is a Fuzzy MPC using a set of local linear models guaranteeing the desired comfort with minimal amount of energy while complying with the given thermodynamical input constraints, which enables the use of proven linear MPC theory. The models result from a data-driven black-box algorithm, Nelles (2001), which is straightforward to implement and allows for utilization of existing data records. The supply level MPC focuses on the supply of the energy demanded by the building at the best possible costs based on a simplified thermodynamical model including intermittent power supply units (e.g. heat pump). This leads to a mixed-integer quadratic programming (MIQP) problem to be solved for each predictive step. The interface between the two MPCs is the energy demand of the building which constitutes the output of the comfort level controller and input of the supply level controller. This hierarchic approach opens the opportunity to implement either both MPCs or just a single one.

In the field of building control recent papers have shown that energy savings can be reduced significantly with the MPC technology, e.g. Širokí (2011). Taking the uncertainty of disturbances into account either due to the use of weather predictions, Oldenwurtel (2012), and/or occupancy information, Oldenwurtel (2013), by using stochastic MPC (SMPC) was found to be superior in terms of comfort violations. Another approach dealing with uncertain disturbances is a Randomized MPC (RMPC) applicable for non-additive uncertainties, Zhang (2013). However, the common concept is the use of one global model for building Heat Ventilation and Air Condition (HVAC) dynamics, Air Handling units (AHUs), and components for chilled or hot water generation. In Ma (2012) a two level distributed MPC approach is presented distinguishing energy conversion and energy distribution without explicit interface between the controllers. The main difference presented in this work is a decoupled MPC structure with a single coupling parameter, the building's energy demand. One model is used for the building's HVAC dynamics and another is formulated for a simplified thermodynamic behavior of the energy supply components. This hierarchic approach allows to seperate control targets with different time constants to reduce complexity in operation.

The building's HVAC dynamics are generally nonlinear and non-convex. In order to avoid non-convex optimization, a set of local linear models can be extracted from a Takagi-Sugeno (TS) fuzzy model, Takagi (1985), which are then utilized by the MPC. The usage of the resulting Fuzzy MPC, Fischer (1998); Abonyi (2003), is new in the field of building control. TS fuzzy models approximate the nonlinear HVAC dynamics very well and allow to extract a set of models valid for a whole year, since the additional models are identified for varying the ambient temperature, Killian (2013).

Mixed-integer quadratic programming (MIQP) problems are well known as *NP*-hard problems. For predicitve control this implies a demanding and computationally expensive task as an MIQP problem has to be solved online for each prediction step to compute the control input. Therefore, several works report how the optimization problem can be devided into off-line and on-line tasks, e.g. Bemporad (2002). Nevertheless, as long as the system is linear and constraints are linear or second order cone, free powerful solvers, Löfberg (2004), allow straight forward on-line computation if the sampling time is large enough.

The remainder of the paper is structured as follows: In Sec.2 the system models used by the MPCs are introduced. The hierarchic MPC structure and the MPCs' formulations are presented in Sec.3. The simulation results of the hierarchically connected but decoupled MPCs are shown on a demonstration building in Sec.4. The paper is concluded by a discussion of the simulated application and an outlook to further research.

2. SYSTEM MODEL

It is well known, that modeling and identification are the most difficult and time consuming parts of the control design process, particularly for predictive control. Reliable predictions from the identified dynamic building model are absolutely necessary for a good MPC performance. Moreover, predictions of stochastic disturbances like weather forecasts and occupancy are important as well for modeling, Privara (2013); Oldenwurtel (2012, 2013). As already mentioned, the study focuses on two types of model predictive controllers. It is essential to construct two different system models, on the one hand for comfort maximization in the High Level (HiLe) (the building indoor rooms) and on the other hand for energy minimization in the Low Level (LoLe) (supply area of the building). Note that the relevant dynamics in the higher level is comparatively slow, whereas in the lower level it is faster. In the HiLe Takagi-Sugeno models are used, due to the fact that they can represent nonlinearities in a local linear way, see Sec.2.1. For the energy sources in the LoLe a physical white-box model is built, see Sec.2.2.

2.1 HiLe System Model

Data-driven system identification (black-box identification) for modeling nonlinear dynamic systems by local linear model (LLM) networks is an efficient way of model building for complex dynamic systems. One of the main advantages is that existing theory of system identification is extended to globally nonlinear system behaviour. Since the validity of those LLMs is confined to certain regions within the so-called partition space, this model class is also named Takagi-Sugeno Fuzzy models, Takagi (1985). The dynamics in the office building itself is slow, so the sampling time is assumed to be one hour. To set the indoor room temperature ϑ^{in} there exist two main input variables, the supply heat of fan coils (FC) and the supply heat of the thermally activated building system ($TABS$). In addition to the control variable inputs, the linear model tree (LOLIMOT) algorithm also incorporates disturbances, therefore weather forecasts and occupancy information are used. The LOLIMOT algorithm used for this study is described in Nelles (2001), applied for this set of data in Killian (2013) and validated in Mayer (2013).

Takagi-Sugeno (TS) Models

Motivated by results of classical linear MPC theory, e.g. stability theory, Bordons (2004), it is beneficial to use a linear model structure. To fulfill this necessary assumption, TS fuzzy models are used and a fuzzy model predictive controller (FMPC) is presented in Sec.3.2, the results are given in Sec.4.2. Complex dynamical systems can often be represented by a nonlinear autoregressive model structure with exogeneous input (NARX), Abonyi (2003). In general, this structure can be considered as a nonlinear relation between past inputs and ouputs and the predicted ouputs of the sytem:

$$\begin{aligned}\hat{y}(k+1) = f(&y(k), \dots, y(k-n_y+1), u_l(k-n_d), \\ &\dots, u_l(k-n_u-n_d+1)),\end{aligned} \tag{1}$$

where n_y and n_u are the maximum lags considered for the output and input terms, n_d is the discrete dead time, and f represents the nonlinear mapping. TS fuzzy models are proved to be suitable for approximation of such sytems by interpolating between local linear, time-invariant ARX models, Abonyi (2003). The basic element of a fuzzy system is a set of fuzzy inference rules. In general, each inference rule consists of two elements: the IF-part, called the antecedent of a rule, and the THEN-part, called the consequent of the rule. For each rule $\mathbf{R}^j$ the following structure holds:

$$\begin{aligned}\mathbf{R}^j : \text{IF } &\zeta_1 \text{ is } A_1^j \text{ and } \dots \zeta_m \text{is } A_m^j \\ &\text{THEN } y^j(k+1) = \sum_{i=1}^{n_y} a_i^j y(k-i+1) \\ &+ \sum_{i=1}^{n_u} b_i^j u_l(k-i-n_d+1) + c^j.\end{aligned} \tag{2}$$

Here $j = \{TABS, FC\}$, $\zeta = [\zeta_1, \dots, \zeta_m]$ is the vector of input fuzzy variables and $A_1^j, \dots, A_m^j$ are the forgoing fuzzy sets or regions for the j-th rule $\mathbf{R}^j$ with corresponding member ship functions $\mu^j_{A_1}, \ldots, \mu^j_{A_m}$ (note: $\mu_{A_j}(\zeta_j) \mapsto [0,1]$, for $j = 1, \ldots, m$), Nelles (2001); Killian (2013). The elements of the fuzzy vector are usually a subset of the past input and ouputs, Abonyi (2003):

$$\begin{aligned} \zeta \in \{&y(k), \ldots, y(k-n_y+1), u_l(k-n_d), \\ &\ldots, u_l(k-n_u-n_d+1)\}. \end{aligned} \tag{3}$$

The overall output of the TS fuzzy model can be written as

$$y(k+1) = \sum_{j=1}^{r} \omega^j(\zeta) y^j(k+1), \tag{4}$$

where r denotes the number of rules. The degree of fulfillment if the j-th rule can be computed using the product operator:

$$\mu^j(\zeta) = \prod_{i=1}^{m} \mu^j_{A_i}(\zeta_i), \tag{5}$$

furthermore, the normalized degree of fulfillment can be computed as:

$$\omega^j(\zeta) = \frac{\mu^j(\zeta)}{\sum_{i=1}^{r} \mu^i(\zeta)}. \tag{6}$$

If all consequents of the rules have identical structure, the TS model can be expressed as a pseudo-linear model with input-dependent parameters:

$$\begin{aligned} y(k+1) = &\sum_{i=1}^{n_y} a_i(\zeta) y(k-i+1) \\ &+ \sum_{i=1}^{n_u} b_i(\zeta) u_l(k-i-n_d+1) + c(\zeta), \end{aligned} \tag{7}$$

where:

$$\begin{aligned} a_i(\zeta) &= \sum_{j=1}^{r} \omega^j(\zeta) a_i^j, \\ b_i(\zeta) &= \sum_{j=1}^{r} \omega^j(\zeta) b_i^j, \\ c(\zeta) &= \sum_{j=1}^{r} \omega^j(\zeta) c^j. \end{aligned} \tag{8}$$

It is obvious that there are systematic similarities between conditional parametic models and TS fuzzy models. In this context the local neighborhood around each fitting point is dermined by Kernel functions, for fuzzy models the term membership functions is used, Abonyi (2003); Nelles (2001); Takagi (1985).

2.2 Simplified LoLe System Model

By modeling the LoLe the objective is to develop a simplified yet descriptive model which can be used for optimization in an MPC. Modern buildings are supplied by numerous energy sources as district heating, geothermal energy, free cooling, and equipped with electrically powered units supplying different HVAC systems of the building. In terms of an optimization problem these physical energy circuits are decision trees starting from the source and ending at the transfer node to the building. A straight forward modeling approach based on thermodynamical principles is pursued, Ma (2012). Additionally, the units have the possibility to switch their state between on and off or even between several modes. In order not to change modes arbitrarily minimum up-times and minimum down-times are additional constraints to the units' power limitations. The extension with integer or binary control inputs change the model from non-linear to mixed-integer nonlinear programming problem, Ma (2009).

For this work, as the control objective of the LoLe MPC lies on supplying the demanded amount of energy at minimal costs, the interactions of the energy supply units as pumps, heating pumps, or chillers up to the supply connection point to the building are further simplified and modeled in the form of energy balances. Therefore, the following relationship exists for the whole supply system as well as for each circuit of pumps and units:

$$\dot{Q}_{output} = \dot{Q}_{env} + \dot{Q}_{electric} - \dot{Q}_{losses}$$

The output energy $\dot{Q}_{output}$ is gained by the energy drawn from the environment $\dot{Q}_{env}$ and the total electric energy invested $\dot{Q}_{electric}$ minus the sum of all losses $\dot{Q}_{losses}$. Some further simplifying assumptions are made:

(1) As in Vrettos (2013) the heating pump is modeled by a linear Coefficient of Performance (COP), i.e., the ratio of the output thermal power to the input electric power.

$$P_{electric,HP} = \frac{\dot{m}_i \cdot \Delta\vartheta_i \cdot cp}{COP} \tag{9}$$

$$COP = c_0 + c_1 \cdot T_{amb} + c_2 \cdot T_s \tag{10}$$

with $c_0 = 5.593$, $c_1 = 0.0569\ K^{-1}$ and $c_2 = 0.0661\ K^{-1}$. T_{amb} is the ambient temperature of the supply basement which is assumed to be constant. As in Vrettos (2013) the temperature of the supply water on the hot side T_s is a state variable and is also kept constant with the start value.

(2) Pump characteristic curves are linearized, so that the electric power for pump i depends on the corresponding mass flow $\dot{m}_i$.

$$P_{electric,i} = c_i \cdot \dot{m}_i$$

3. MODEL PREDICTIVE CONTROL STRUCTURE

Hierachical MPC structures in building climate control are a possible way to make the implementation and optimization problem easier, even if the global optimum is not guaranteed, Picasso (2010); Scattolini (2007); Skogestad (2000). One motivation for decoupled MPCs in this study was, that industrial partners and building operators wanted to have two products, on the one hand an MPC for comfort maximization and on the other hand an MPC for energy cost minimization. In Sec.3.1 the hierachical control concept is presented. Sec.3.2 shows a High Level MPC (HiLe-MPC) in form of a fuzzy MPC (FMPC), which is used for performance optimization, guaranteeing maximal user comfort. The energy supply for the HiLe-MPC comes from the underlying Low Level mixed integer MPC (LoLe-MPC, MI-MPC), see Sec.3.3, which handles the energy supply in an optimal way.

3.1 Control Concept

The innovation in this paper is the hierachical MPC concept for large office buildings. First of all, a new FMPC method is described to maximize user comfort in the HiLe. In the LoLe energy supply is optimized with MI-MPC concept in a cost minimal way. In Ma (2009) a similar concept for optimizing the supply level was choosen, however with an nonlinear MI-MPC. This study focuses on decoupling of two different types of MPC problems. Their extremly diverse dynamic behavior supports this structure. Communication between the two types auf MPCs is based on one coupling variable, the heating energy $\dot{Q}$. In Fig.1 the hierachical scheme is presented, Picasso (2010); Scattolini (2007); Skogestad (2000). $\dot{Q}$ for the HiLe-MPC

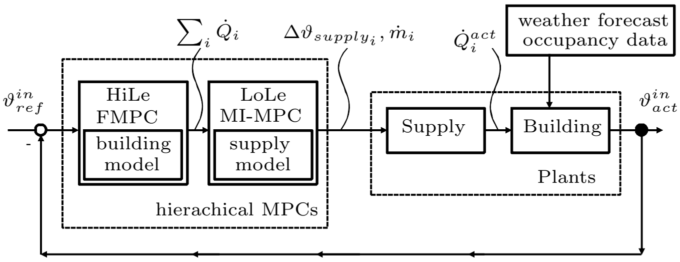

Fig. 1. Concept of hierachical decoupled MPCs

represents the energy demand, which constitutes the control variables of the HiLe-MPC. The sum of these control variables $\sum_i \dot{Q}_i$ is the energy supply of the LoLe-MPC, respectively the reference value. Thus, the LoLe control variables are given by $\Delta\vartheta_{supply_i}$, which is the temperature difference between heat supply and heat return, and massflow $\dot{m}_i$, for $i = \{TABS, FC\}$. ϑ_{ref}^{in} an ϑ_{act}^{in} give the reference and actual value for indoor room temperature. However, the important fact is the decoupling of the optimization problems. Stochastical disturbances like weather and occupancy information only affect the HiLe-MPC, Oldenwurtel (2013); Zhang (2013). Due to the fact that dynamical behavior is highly different in HiLe-MPC and LoLe-MPC, decoupling is useful. Dynamics arise because of different dead and sampling times in the individual MPCs. Latency periods are the reason for dynamics in LoLe. Global optimal solutions may not be reached with a decoupled method, but two local optimal solutions are easier to optimize and implement.

3.2 Fuzzy MPC

Standard MPC formulations are well known and given in e.g. Bordons (2004). In this paper a local linear model was computed for nonlinear dynamics. To avoid non-convex optimization, a set of local linear models was extracted from a TS fuzzy model which is then utilized by the MPC algorithm, Abonyi (2003). Stochastical disturbances, weather forecast and occupancy information, are noteable in the HiLe of the building, Oldenwurtel (2012, 2013); Zhang (2013). As already mentioned, the FMPC controls the indoor room temperature.
The FMPC optimization problem is formulated as follows:

$$J^\star = \min_U J(U,t) = \alpha \cdot \|\vartheta_{act}^{in}(U,t) - \vartheta_{ref}^{in}\|_2^2 + \beta \cdot \sum_i \dot{Q}_i^2$$

$$\begin{aligned} &\text{s.t.} \\ &\vartheta_{min}^{in} \leq \vartheta^{in}(t) \leq \vartheta_{max}^{in} \\ &u_{i,min} \leq u_i(t) \leq u_{i,max} \end{aligned} \tag{11}$$

for $U = \{u_i\}$ and for $i = \{TABS, FC\}$. Moreover, α and β are weights of the minimization criterium. In this paper α is highly weighted in contrast to β to guarantee high comfort.

Fuzzy MPC formulation
The formulation of the FMPC depends on linear models, which are obtained by interpolating the parameters of the local models in the TS model, see system (8) and equations (7), Takagi (1985); Mollov (2004).
The goal is to locally represent a TS fuzzy model by a linear state-space model

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}_k\mathbf{x}(k) + \mathbf{B}_k\mathbf{u}(k) + \mathbf{E}_k\mathbf{z}(k) \\ \mathbf{y}(k) &= \mathbf{C}_k\mathbf{x}(k) \end{aligned} \tag{12}$$

in which system matrices $\mathbf{A}_k, \mathbf{B}_k, \mathbf{C}_k$ and $\mathbf{E}_k$ are considered to be non-constant. Assume that the MISO TS fuzzy model can be regarded as a multivariable linear parameter-varying system, Abonyi (2003).

$$\begin{aligned} \mathbf{y}(k+1) = &\sum_{i=1}^{n_y} \mathbf{F}_i(\zeta)\mathbf{y}(k-i+1) \\ &+ \sum_{i=1}^{n_u} \mathbf{H}_i(\zeta)\mathbf{u}(k-i+1) + \mathbf{c}(\zeta). \end{aligned} \tag{13}$$

in which parameter matrices frozen at a certain operating point ζ are calculated as

$$\begin{aligned} \mathbf{F}_i(\zeta) &= \sum_{j=1}^{r} \mathbf{W}^j(\zeta)\mathbf{F}_i, \qquad && i = 1,\ldots,n_y \\ \mathbf{H}_i(\zeta) &= \sum_{i=1}^{r} \mathbf{W}^j(\zeta)\mathbf{H}_i^j, && i = 1,\ldots,n_u \\ \mathbf{c}(\zeta) &= \sum_{i=1}^{r} \mathbf{W}^j(\zeta)\mathbf{c}^j, \end{aligned} \tag{14}$$

where $\mathbf{W}^j$ is the diagonal weight matrix which entries are normalized degrees of fulfillment of the j-th rule. In order to *predict the trajectory* of the controlled ouput, system (13) can be used. The linear extracted state-space models can be augmented to provide offset free control. Let $\xi(k)$ denote the augmented state vector at time step k, then future process outputs are computed from the following matrix:

$$\hat{\mathbf{Y}} = \mathbf{F}(k)\xi(k) + \Phi_u(k)\Delta\mathbf{U} + \Phi_z(k)\Delta\mathbf{Z}, \tag{15}$$

the matrices $\mathbf{F}$, Φ_u and Φ_z have to be calculated at each time step and are then used in the following quadratic program for determining optimal future control sequences:

$$J^\star = \min_{\Delta U} J(\Delta U) = (\mathbf{Y}_r - \hat{\mathbf{Y}})^T \mathbf{Q} (\mathbf{Y}_r - \hat{\mathbf{Y}}) + \Delta \mathbf{U}^T \mathbf{R} \Delta \mathbf{U}$$
$$\text{s.t.} \tag{16}$$
$$Y_{min} \leq \hat{Y} \leq Y_{max},$$
$$U_{min} \leq U \leq U_{max},$$
$$\Delta U_{min} \leq \Delta U \leq \Delta U_{max}.$$

$\mathbf{Y}_r$ is the reference trajactory, $U = \{u_i\}$, $i = \{TABS, FC\}$, $\mathbf{Q}$ and $\mathbf{R}$ are positive semidefinite weighting matrices which allow for tuning. The approach can be summarized in the following steps:

(1) Use the obtained linear model (12) at the current operating point $\zeta(k)$ and compute the control signal $\mathbf{u}(k)$ for the whole control horizon.
(2) Simulate the TS fuzzy model over the prediction horizon.
(3) Freeze the TS fuzzy model along each point in the predicted operating point trajectory ζ(k+i) and obtain to parameters of (12), for $i = 1, \dots, N_p$.
(4) Use calculated (12) of step before, $i = 1, \dots, N_p$ to construct MPC matrices $\mathbf{F}$, Φ_u and Φ_z and compute the new control sequence $\mathbf{u}(k)$.

Steps 3 and 4 are repeated until $\mathbf{u}$ converges, Abonyi (2003); Fischer (1998); Mollov (2004); Takagi (1985).

3.3 Mixed Integer MPC

Intermittent units at the building's energy supply level require binary variables representing the on/off state as well as continuous variables. In literature such systems are denoted as hybrid systems. Hybrid systems are modeled in discrete-time within the mixed logical dynamical (MLD) framework, Bemporad (1999). First approaches to control dynamic systems with online mixed-integer programming subject to logical conditions have appeared in Tyler (1999). As the traditional MPC objective function is quadratic the optimization routine is a mixed-integer quadratic programming (MIQP) problem. The realization of the MPC of hybrid systems implies that a MIQP program must be solved on-line each prediction step to obtain the control input, which is a computationally expensive task as MIQP are known as *NP*-hard problems. As long as the system is linear, the dynamics are time-invariant, and the control frequency is in the order of minutes, the mixed-integer quadratic programming (MIQP) problem can be solved by readily available off the-shelf optimizers using efficient branch and bound algorithms. The free Matlab toolbox YALMIP, Löfberg (2004) was used for the MPC formulation and gurobi as MIQP solver, Gurobi (2013). Nonlinear systems can be treated by approximation with piecewise affine systems (PWA) which are equivalent to MLD, Bemporad (2000a). Bemporad (2000b) shows criteria for stability of optimal controllers of PWA systems.

As the low level model is a simplified thermodynamical model, namely a static energy balance as introduced in Sec.2.2, only linear constraints are considered. In addition to limits on all control input variables, minimal switch-on/switch-off times expand the set of constraints. The discrete time MI-MPC problem is formulated as follows:

$$J^\star = \min_{U} J(U,t) = \|\sum_i \dot{Q}_i(U,t) - \dot{Q}_{ref}(t)\|_2^2$$
$$+ c_j \cdot E_j^2(U,t)$$
$$\text{s.t.} \tag{17}$$
$$\dot{Q}_i(t) = \dot{m}_i(t) \cdot \Delta\vartheta_i(t) \cdot cp$$
$$E_j(t) = \sum_i \sum_k P_{i,k}^j(t) \Delta t$$
$$u_{i,min} \leq u_i(t) \leq u_{i,max}$$
$$P_{i,k,min} \leq P_{i,k}(t) \leq P_{i,k,max}$$
$$\text{onoff}_k(t) \in \{0,1\}$$
$$\text{minup}_{i,k} \in \mathbb{N}$$
$$\text{mindown}_{i,k} \in \mathbb{N}$$

for $i = \{TABS, FC\}$, $j = \{\text{electric, district heat}\}$ and $U = \{\dot{m}_i, \Delta\vartheta_i\}$. c_j denotes the price for one kWh and E_j the amount of energy demanded from the j^{th} energy source whereas onoff_k are the on/off state of each supply unit k. $P_{i,k}^j$ is the power of unit k run by source j and decision tree i. Minup and mindown are the time constants for minimal up and minimal down periods for unit k and decision tree i.

4. RESULTS

4.1 Demonstration Building

The 27.000 m^2 university building in the center of Salzburg, Austria, has five floors above ground containing several large and numerous smaller meeting rooms, offices and lecture rooms. There are six atriums within the modern building complex. For this study, the north-east quarter of the building is considered, comprised of 400 rooms almost all used as offices. The heat demand for this part is about 300 to 350 kW in winter time. The considered simulation period was February 2012. The samples for this period are drawn from historic databases for the model identification introduced in Sec.2.1, whereas basis for the LoLe Model (see Sec.2.2) were plan data from supply units as pumps or the heat pump. The system disturbances are the outside temperature and the occupancy profile. The historic outside temperature was provided by the ZAMG [1] Austria. As the control simulation is run for a winter month where outside temperature does not exceed 5°C only heating is relevant for the simulation. The office rooms are heated with FC and $TABS$, i.e. fast and slow heating dynamics characterize the HiLe model. To identify the HiLe black-box model different types of sensores were used, to be specific for supply heat and return run from FC or rather $TABS$, indoor room temperatures and ambient temperature. For the LoLe model two decision trees are important for this work. The first delivers hot water for the TABS which is fed by geothermal energy and a heat pump whereas the second is responsible for the supply of hot water for the FC system which is run by district heating. In order to optimize the maintainance costs for the heat pump it is only switched on if necessary. Both trees have one pump in the circuit. The following power limits are

[1] Zentralanstalt für Meteorologie und Geodynamik - The central institute for meteorology and geodynamics

effective for the constraint formulation in Sec.3.3:

$$\begin{aligned} 10\,\frac{\text{kg}}{\text{min}} &\le \dot{m}_i \le\; 40\,\frac{\text{kg}}{\text{min}} \text{ for } i \in \{1,2\} \\ 10\text{ kW} &\le P_{1,HP} \le\; 250\text{ kW} \\ 10\text{ kW} &\le P_{2,DH} \le\; 500\text{ kW} \\ \text{minup}_{1,HP} &= 6\text{ min} \\ \text{mindown}_{1,HP} &= 3\text{ min} \end{aligned} \tag{18}$$

4.2 Control Simulation

This section presents simulation results of the FMPC and MI-MPC communicated by $\dot{Q}_i$, as described in Sec.3.1. The simulations in both examples were run over a heating period. For weather and occupancy, historical data were used. The first example (see Fig.2 and Fig.3) shows a reference value step and a disturance step. The reference step increases by $2°C$ ($t = 50h$), stays at the new value and goes back to the old reference ($t = 80h$). The ambient temperature step increases from $-5°C$ to $0°C$ ($t = 20h$) and decreases to $-2°C$ ($t = 90h$). In the first plot of Fig.2 the reference value step, blue dash-dotted line, and the FMPC, red line, are presented. The green dashed lines represent the constraints on ϑ^{in}. In the second plot of Fig.2 the disturbance steps are shown. The control variables $u_{FMPC_i} = \{TABS, FC\}$ are presented in the plot 3 and 4 of Fig.2. The FMPC is able to enforce constraints in the controlled variable as well as in control variables, shown in plot 3 and 4 of Fig.2. The constraints are also shown in green dashed lines. Fig.3 presents the results

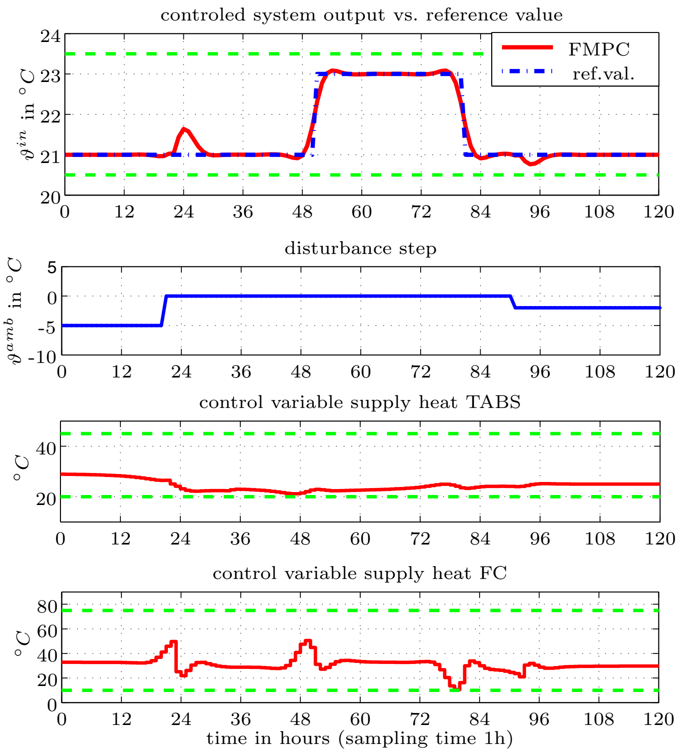

Fig. 2. Fuzzy MPC; reference and disturbance steps

of the underlying MI-MPC, which got as reference value $\dot{Q}_{ref} = \dot{Q}_{TABS} + \dot{Q}_{FC}$ from Fig.2. In the first plot of Fig.3 the different $\dot{Q}$s are shown. $\dot{Q}_{HP}$ and $\dot{Q}_{DH}$ represent the energy taken from different sources, in which HP stands for heat pump and DH for district heating. Also the controlled $\dot{Q}_{act}$, black line, and the reference value from the FMPC control variables $\dot{Q}_{ref}$ is shown, red line. There exists a perfect accordance between these two lines. The green line shows the energy coming from the district heating and the blue line energy from heat pump, binary variable (on/off). It is obvious that an optimizer for minimal costs always takes energy from the cheaper resource (DH), except the upper bound is reached. Then the heating pump is needed for the required $\dot{Q}$. However, the problem switching HP ist not trivial since latency times have to be considered, see eq.(18). In plot 2 and 3 of Fig.3 the control variables $u_{MI-MPC_i} = \Delta\vartheta, \dot{m}$ are shown, green line for district heating and blue line for heat pump.

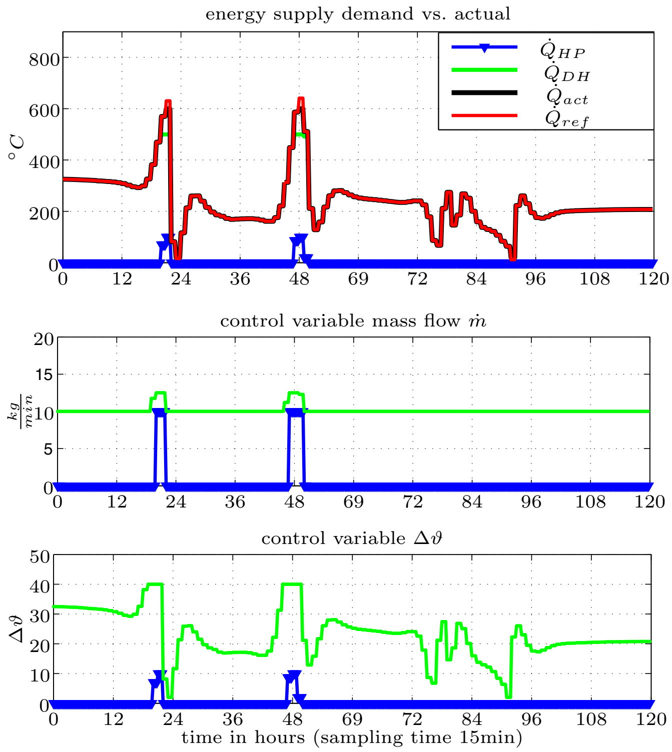

Fig. 3. MI-MPC results; reference and disturbance steps

The second example shows a measured ambient temperature and a constant reference value for the controlled variable. Fig.4 presents a simulation cutout. This cut was chosen, because of the natural stochastic disturbance change from $-20°C$ up to $5°C$, plot 2 of Fig.4. The results of the FMPC are presented in Fig.4, the appropriate results of the MI-MPC shown in Fig.5. The line colors and modes are the same as in the first example.

Fig.6 shows the MPC trajectory of the second example in the fuzzy space decomposition constructed by LOLIMOT. The starting point is the red diamond, black points are operating points. The trajectory changes between LLM 2 and 3 and after the stochastical disturbance change goes to LLM 1. The green circle is the end, which is the area of highest ambient temperature. The stochastic disturbance change switches from LLM 2 to LLM 1. As shown in this plot, FMPC is able to handle frequent transitions in the local linear models without any problems.

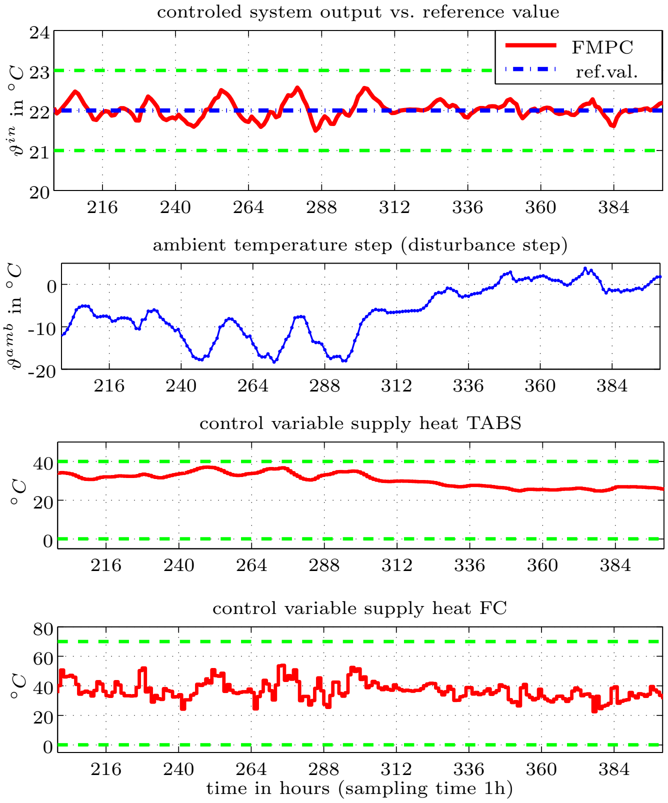

Fig. 4. Fuzzy MPC; disturbance step

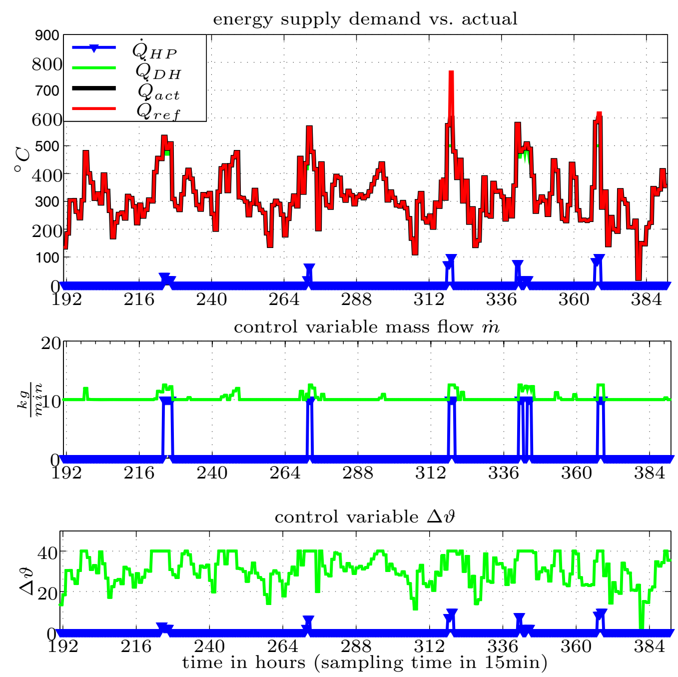

Fig. 5. MI-MPC results; disturbance step

5. CONCLUSION AND FUTURE RESEARCH

The paper introduced a hierachical MPC concept for efficient decoupled building heating control. For the optimization of the user comfort on the high level a Fuzzy MPC is presented which is a new technology to this field. Whereas the Mixed-Integer-MPC solves the problem of minimizing

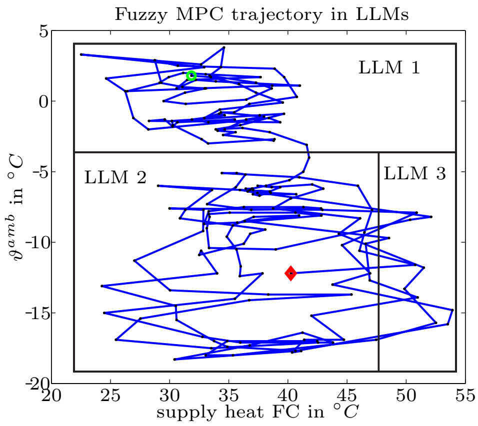

Fig. 6. Fuzzy MPC trajectory; disturbance step

energy at minimal costs seperatly in the lower supply level of the building. Due to this hierarchic concept, two optimization problems of different dynamics and sampling frequency can be optimized decoupled but still connected in a single coupling node within one control structure. Although the hierachical MPC will only find local optima, the implementation effort and formulation of the different optimization problems can be significantly reduced. There exists no global optimal solution to the given problem, therefore a sub-optimal solution was presented. This sub-optimal solution is difficult to measure or compare with other control strategies as far as the real implementation in the demonstration building is concerned.
Future work will deal with the extension to control cooling cuircuits and heating circuits over a longer period of the year and with the implementation in the real building.

ACKNOWLEDGEMENTS

This work was supported by the project "SMART MSR" (FFG, No. 832103) in cooperation with evon GmbH.

REFERENCES

J. Abonyi. *Fuzzy model identification for control.* Birkäuser Boston, 2003.

A. Bemporad and M. Morari. Control of systems integrating logic, dynamics and constraints. In *Automatica*, volume 35(3), pages 407–427, 1999.

A. Bemporad, G. Ferrari-Trecate, and M. Morari. Observability and controllability of piecewise affine and hybrid systems. In Proc. *Conference on Decision and Control*, 1999.

A. Bemporad, F. Borrelli, and M. Morari. Optimal controllers for hybrid systems: Stability and piecewise linear explicit form. In Proc. *IEEE Conf. on Decision and Control*, 2000.

A. Bemporad, M. Morari, V. Dua, and E.N. Pistikopoulos. The explicit linear quadratic regulator for constrained systems. In *Automatica*, volume 38(1), pages 3-20, 2002.

C. Bordons, and E.F. Camacho *Model Predictive Control (Advanced Textbooks in Control and Signal Processing).* Springer Verlag, London, 2004.

International Energy Agency. Energy Efficiency Requirements in Building Codes - Energy Efficiency Policies for New Buildings, 2008.

M. Fischer, and O. Nelles. Predictive control based on local linear fuzzy models. In *International Journal of System Science*, volume 29, pages 661–675, 1998.

Gurobi Optimization, Inc. Gurobi Optimizer Reference Manual. http://www.gurobi.com, retrieved on 29th of October,2013.

M. Killian, S. Großwindhager, M. Kozek, and B. Mayer. Pre-processing of Partition Data for Enhancement of LOLIMOT. In Proc. *EUROSIM Congress on Modelling and Simulation* Cardiff, Wales, United Kingdom, 2013.

J. Löfberg. YALMIP : A Toolbox for Modeling and Optimization in MATLAB. In Proc. *CACSD Conference*, Taipei, Taiwan, 2004.

Y. Ma, F. Borrelli, B. Hencey, A. Packard, and S. Bortoff. Model predictive control of thermal energy storage in building cooling systems. In Proc. *IEEE Conference on Decision and Control*, 2009.

Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves. Model Predictive Control for the Operation of Building Cooling Systems. In *IEEE Transactions on Control Systems Technology*, volume 20(3), pages 796–803, 2013.

B. Mayer, M. Killian, and, M. Kozek. Modeling workflow for industrial building model for control purposes. In Proc. *IEEE International Conf. on Systems and Control*, Algiers, Algeria, 2013.

S. Mollov, R. Babuška, J. Abonyi, and H.B. Verbruggen. Effective Optimization for Fuzzy Model Predictive Control. In *IEEE Transactions on Fuzzy Systems*, volume 12(5), pages 661–675, 2004.

O. Nelles. *Nonlinear System Identification: From Classical Approaches, to Neural Networks and Fuzzy Models* Springer Verlag, Berlin Heidelberg, 2001.

F. Oldewurtel, A. Parisio, C.N. Jones, M. Morari, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari. Use of model predictive control and weather forecasts for energy efficient building climate control. In *Energy and Buildings*, volume 45, pages 15–27, 2012.

F. Oldewurtel, D. Sturzenegger, and M. Morari. Importance of occupancy information for building climate control. In *Applied Energy*, volume 101, pages 521–532, 2013.

B. Picasso, D. De Vito, R. Scattolini, and P. Colaneri. An MPC approach to the design of two-layer hierachical control systems. In *Automatica*, volume 46, pages 823–832, 2010.

S. Prívara, J. Cigler, Z. Váňa, F. Oldenwurtel, C. Sagerschnig, and E. Žáčeková. Building modeling as a crucial part for building predictive control. In *Energy and Buildings*, volume 56, pages 8–22, 2013.

R. Scattolini, and P. Colaneri. Hierachical Model Predicitive Control. In Proc. *IEEE Conference on Decision and Control*, New Orleans, LA, USA, 2007.

J. Širokí, F. Oldewurtel, J. Cigler, and S. Prívara. Experimental analysis of model predictive control for an energy efficient building heating system. In *Applied Energy*, volume 88, pages 3079–3087, 2011.

S. Skogestad. Plantwide control: the search for the self-optimizing control structure. In *Journal of Process Control*, volume 10(5), pages 487–507, 2000.

T. Takagi, and M. Sugeno. Fuzzy identification of systems and its application to modeling and control. In *IEEE Trans. Systems, Man and Cybernetics*, volume 15(1), pages 116–132, 1985.

M.L. Tyler and M. Morari. Propositional Logic in Control and Monitoring problems. *Automatica*, volume 35, pages 565–582, 1999.

E. Vrettos, K. Lai, F. Oldewurtel, and G. Andersson. Predictive control of buildings for demand response with dynamic day-ahead and real-time prices. In *European Control Conference Conference*,Zürich, Switzerland, 2013.

X. Zhang, G. Schildbach, D. Sturzenegger, and M. Morari. Scenario-Based MPC for Energy-Efficient Building Climate Control under Weather and Occupancy Uncertainty. In Proc. *European Control Conference*, Zürich, Switzerland, 2013.

Appendix A. ACRONYMS

Notation	**Description**
MPC	model predictive control
FMPC	fuzzy model predictive control
MI-MPC	mixed-integer model predictive control
SMPC	stochastic model predictive control
RMPC	randomized model predictive control
MIQP	mixed-integer quadratic program
LLM	local linear model network
LOLIMOT	local linear model tree
TS	Takagi-Sugeno
MISO	multi-input single-output
ARX	autoregressive model structure with exogeneous input
NARX	nonlinear autoregressive model structure with exogeneous input
HiLe	high level
LoLe	low level
AHU	air handling unit
HVAC	heat ventilation and air condition
FC	fan coil
TABS	thermally actived building system
HP	heat pump
DH	district heating