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Fixed-structure Sparse Control of Interconnected Systems with Polytopic Uncertainty ⋆

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Abstract: This paper deals with the problem of sparse H_∞ dynamic output-feedback control of LTI interconnected systems with polytopic uncertainty. The main objective is to find a controller structure with minimum information exchange between subsystems and local controllers such that stability condition as well as an H_∞ performance criterion are satisfied. To this end, an optimization problem is defined which is the minimization of the cardinality of a special matrix subject to an H_∞ performance constraint. Then, the problem is approximated by a convex optimization. The effectiveness of the proposed approach is evaluated through some simulation results.

1. INTRODUCTION

Control of interconnected systems has attracted considerable attention in recent years because of their numerous applications such as power systems, urban traffic control systems, water distribution, digital communication networks, etc. Conventional methods in the literature for control of such systems are unconstrained control approaches, referred to as the centralized control. In the centralized control approaches, a central controller is designed for the interconnected system and the controller has access to the outputs of all subsystems. However, from a practical point of view, this kind of control structure is costly in terms of required amount of information exchange and communication links between the subsystems and the controller. Moreover, in a number of interconnected systems, there are some restrictions on the accessibility of the outputs of certain subsystems to other subsystems (Lavaei and Aghdam [2008]). In addition, in the centralized control, there is the reliability problem due to the delays, the communication failures, etc. (Siljak [1991]). Therefore, in order to control such systems, some constraints on the control structure should be imposed.

More recently, the constrained control structure has been considered for the control of interconnected systems. The control structure is formulated in a matrix named information flow matrix which includes required information about the existence of communication links between each controller and the subsystems (Lavaei and Aghdam [2008, 2009]). A special class of the constrained control, referred to as the decentralized control, assumes that each local controller uses only the outputs of own subsystems. In this case, the information flow matrix is block-diagonal.

Decentralized control may stabilize the whole system; however, its performance is generally inferior to that of the centralized control approaches. To improve the performance of the whole system, the concept of distributed control is introduced where some local controllers can communicate with each other and also with some subsystems, based on the structure of the information flow matrix.

In most constrained control of interconnected systems, the structure of the controller is fixed a priori (e.g. Lavaei and Aghdam [2008, 2009], Apkarian and Noll [2006], Gahinet and Apkarian [2011]). However, it is possible that the assumed structure is not the best one which can be taken into consideration. In addition, it is difficult to choose the controller structure in advance. Therefore, the main question is how should the controller structure be determined in terms of the information flow matrix with minimum communication links such that the control objectives are satisfied as well as possible?

To answer the above mentioned question, recently, some researchers have focused on the problem of sparse static output (state) feedback control design where the gain between the inputs and outputs (states) of the subsystems is sparsified (e.g. Schuler et al. [2011a], Lin et al. [2013], Schuler et al. [2013], Polyak et al. [2013]). In this way, the communication links between the subsystems and the controllers are reduced. The results of Schuler et al. [2011a] are extended to sparse dynamic output controller design in Schuler et al. [2011b]. However, in Schuler et al. [2011b] some parts of the controller structure have been partially specified in advance (i.e. the structure of controller matrices A_c, B_c, and C_c). In all these approaches, the sparsity is formulated in terms of cardinality of the gain matrix defined as the number of its non-zero elements. Then, the cardinality is relaxed by the (weighted) ℓ_1-norm (see Candes et al. [2008]). Since they have focused on the problem of sparse static output/state feedback, the communication between the local controllers, in the case of dynamic output feedback, has not been considered. Moreover, in these approaches, the parametric uncertainty has not been taken into consideration.

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In this paper, a new LMI-based approach for sparse fixed-structure H_∞ control of interconnected systems with polytopic uncertainty is proposed. The polytopic uncertainty contains a variety of parametric uncertainties such as multiple models and interval uncertainty. In this approach, the controller structure as well as controller parameters are simultaneously designed. To this end, an objective function in terms of cardinality of an information exchange matrix between the subsystems and the local controllers is considered. Then, a weighted ℓ_1-norm is employed as the cardinality relaxation. Finally, the problem is converted into a convex optimization problem which is the minimization of the convex objective function subject to LMI-based H_∞ constraints.

The organization of the paper is as follows: Problem statement and preliminaries are presented in next section. LMI conditions for fixed-structure sparse H_∞ controller design are provided in Sections 3. Two illustrative examples are given in Section 4 to clarify the proposed method. Conclusion remarks are presented in Section 5.

Throughout the paper, the matrices I and 0 are the identity matrix and the zero matrix of appropriate dimensions, respectively. The symbol $\star$ indicates symmetric blocks. For symmetric matrices, $P > 0$ and $P < 0$ denote that matrix P is positive-definite and negative-definite, respectively.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Plant Model

Consider an LTI interconnected system consisting of N subsystems. The state space model of the subsystem i is given by:

$$\begin{aligned} x_{g_i}(k+1) &= A_{g_{ii}} x_{g_i}(k) + \sum_{j\neq i}^{N} A_{g_{ij}} x_{g_j}(k) \\ &\quad + \sum_{j=1}^{N} B_{w_{ij}} w_j(k) + B_{g_i} u_i(k) \\ z_i(k) &= \sum_{j=1}^{N} C_{z_{ij}} x_{g_j}(k) + \sum_{j=1}^{N} D_{zw_{ij}} w_j(k) + D_{zu_i} u_i(k) \\ y_i(k) &= C_{g_i} x_{g_i}(k) + \sum_{j=1}^{N} D_{w_{ij}} w_j(k) \end{aligned} \tag{1}$$

where $x_{g_i} \in \mathbb{R}^n$, $u_i \in \mathbb{R}^{n_i}$, $w_i \in \mathbb{R}^r$, $y_i \in \mathbb{R}^{n_o}$, and $z_i \in \mathbb{R}^s$ are the state, the control input, the exogenous input, the measured output, and the controlled output vector of the i^{th} subsystem, respectively. Matrix $A_{g_{ij}} = 0$ if and only if there is no interaction between the subsystems i and j. It is assumed that the matrices either $(A_{g_{ij}}, B_{g_i})$ or $(A_{g_{ij}}, C_{g_i})$ belong to a polytopic region as follows:

$$\{(A_{g_{ij}}(\lambda), B_{g_i}(\lambda), C_{g_i}(\lambda)) = \sum_{l=1}^{q} \lambda_l (A^l_{g_{ij}}, B^l_{g_i}, C^l_{g_i})\} \tag{2}$$

for $i, j = 1, \ldots, N$; where, $\lambda = [\lambda_1 \cdots \lambda_q]^T \in \Lambda$,

$$\Lambda = \left\{ \lambda \,\middle|\, \sum_{l=1}^{q} \lambda_l = 1, \quad \lambda_l \geq 0; \quad l = 1, \ldots, q \right\} \tag{3}$$

In what follows we assume that $(A_{g_{ij}}, B_{g_i})$ belongs to the polytopic uncertainty region. Let us define the following vectors:

$$\begin{aligned} x_g(k) &= [x_{g_1}(k), \ldots, x_{g_N}(k)]^T \\ u(k) &= [u_1(k), \ldots, u_N(k)]^T \\ w(k) &= [w_1(k), \ldots, w_N(k)]^T \\ y(k) &= [y_1(k), \ldots, y_N(k)]^T \\ z(k) &= [z_1(k), \ldots, z_N(k)]^T \end{aligned} \tag{4}$$

Then, the whole network can be presented by the following state space realization:

$$\begin{aligned} x_g(k+1) &= A_g(\lambda) x_g(k) + B_g(\lambda) u(k) + B_w w(k) \\ z(k) &= C_z x_g(k) + D_{zu} u(k) + D_{zw} w(k) \\ y(k) &= C_g x_g(k) + D_w w(k) \end{aligned} \tag{5}$$

where

$$\begin{aligned} A_g(\lambda) &= \begin{bmatrix} A_{g_{11}}(\lambda) & \cdots & A_{g_{1N}}(\lambda) \\ \vdots & \ddots & \vdots \\ A_{g_{N1}}(\lambda) & \cdots & A_{g_{NN}}(\lambda) \end{bmatrix} \\ B_w &= \begin{bmatrix} B_{w_{11}} & \cdots & B_{w_{1N}} \\ \vdots & \ddots & \vdots \\ B_{w_{N1}} & \cdots & B_{w_{NN}} \end{bmatrix} \\ C_z &= \begin{bmatrix} C_{z_{11}} & \cdots & C_{z_{1N}} \\ \vdots & \ddots & \vdots \\ C_{z_{N1}} & \cdots & C_{z_{NN}} \end{bmatrix} \\ D_{zw} &= \begin{bmatrix} D_{zw_{11}} & \cdots & D_{zw_{1N}} \\ \vdots & \ddots & \vdots \\ D_{zw_{N1}} & \cdots & D_{zw_{NN}} \end{bmatrix} \\ D_w &= \begin{bmatrix} D_{w_{11}} & \cdots & D_{w_{1N}} \\ \vdots & \ddots & \vdots \\ D_{w_{N1}} & \cdots & D_{w_{NN}} \end{bmatrix} \end{aligned} \tag{6}$$

and

$$\begin{aligned} B_g(\lambda) &= diag(B_{g_1}(\lambda), \ldots, B_{g_N}(\lambda)) \\ C_g &= diag(C_{g_1}, \ldots, C_{g_N}) \\ D_{zu} &= diag(D_{zu_1}, \ldots, D_{zu_N}) \end{aligned} \tag{7}$$

2.2 Controller Dynamic

It is assumed that there is one local controller for each subsystem described by:

$$\begin{aligned} x_{c_i}(k+1) &= \sum_{j=1}^{N} A_{c_{ij}} x_{c_j}(k) + \sum_{j=1}^{N} B_{c_{ij}} y_j(k) \\ u_i(k) &= \sum_{j=1}^{N} C_{c_{ij}} x_{c_j}(k) + \sum_{j=1}^{N} D_{c_{ij}} y_j(k) \end{aligned} \tag{8}$$

for $i = 1, \ldots, N$; where, $x_{c_i} \in \mathbb{R}^m$ is the state vector of the i^{th} local controller. In this structure, each sub-controller uses the outputs of its own subsystem as well as the outputs of other subsystems and the states of other sub-controllers. The centralized controller K with this structure is given by:

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c y(k) \\ u(k) &= C_c x_c(k) + D_c y(k) \end{aligned} \tag{9}$$

where $x_c(k) = [x_{c_1}(k), \ldots, x_{c_N}(k)]^T$. The controller matrices A_c, B_c, C_c, and D_c are of appropriate dimensions.

The main objective is to design a controller such that each local controller uses a minimum amount of information exchanges between the subsystems and the local controllers. In order to design such controller, first, the following matrix is defined.

$$\mathcal{Z}(K) = \begin{bmatrix} card\left(\begin{bmatrix} A_{c_{11}} & B_{c_{11}} \\ C_{c_{11}} & D_{c_{11}} \end{bmatrix}\right) & \dots & card\left(\begin{bmatrix} A_{c_{1N}} & B_{c_{1N}} \\ C_{c_{1N}} & D_{c_{1N}} \end{bmatrix}\right) \\ \vdots & \ddots & \vdots \\ card\left(\begin{bmatrix} A_{c_{N1}} & B_{c_{N1}} \\ C_{c_{N1}} & D_{c_{N1}} \end{bmatrix}\right) & \dots & card\left(\begin{bmatrix} A_{c_{NN}} & B_{c_{NN}} \\ C_{c_{NN}} & D_{c_{NN}} \end{bmatrix}\right) \end{bmatrix} \quad (10)$$

where $card(.)$ is the cardinality operator. Element z_{ij} of $\mathcal{Z}(K)$ represents the communication links between the local controller i and the subsystem j. The number of the non-zero elements of $\mathcal{Z}(K)$ is defined as the number of the communication links of the controller. Note that $z_{ij} = 0$ if and only if $\begin{bmatrix} A_{c_{ij}} & B_{c_{ij}} \\ C_{c_{ij}} & D_{c_{ij}} \end{bmatrix} = 0$. The relation between the information flow matrix $\mathcal{I}(K) = [I_{ij}]$ and $\mathcal{Z}(K)$ is as follows:

$$I_{ij} = sgn(z_{ij}) \quad (11)$$

where sgn is the signum function. Therefore, in order to find a controller structure with minimum communication links between the subsystems and the local controllers, matrix $\mathcal{Z}(K)$ should be as sparse as possible. The sparsity of this matrix can be presented by its cardinality which is equal to the number of its non-zero elements.

2.3 Closed-loop System Structure

The state space representation of the closed-loop system $H_{zw}(z)$, transfer matrix from w to z, can be written as:

$$\begin{aligned} x(k+1) &= A(\lambda)x(k) + B(\lambda)w(k) \\ z(k) &= Cx(k) + Dw(k) \end{aligned} \quad (12)$$

where $x(k) = [x_g(k) \quad x_c(k)]^T$ and

$$\begin{aligned} A(\lambda) &= \begin{bmatrix} A_g(\lambda) + B_g(\lambda)D_cC_g & B_g(\lambda)C_c \\ B_cC_g & A_c \end{bmatrix} \\ B(\lambda) &= \begin{bmatrix} B_w + B_g(\lambda)D_cD_w \\ B_cD_w \end{bmatrix} \\ C &= [C_z + D_{zu}D_cC_g \quad D_{zu}C_c] \\ D &= D_{zw} + D_{zu}D_cD_w \end{aligned} \quad (13)$$

It is known that the closed-loop state matrix $A(\lambda)$ is called robustly stable if all its eigenvalues are located inside the unit circle for all $\lambda \in \Lambda$.

3. LMI REPRESENTATION OF FIXED-STRUCTURE SPARSE H_∞ CONTROL DESIGN

The problem addressed in this paper is to design fixed-structure controllers for the interconnected systems with polytopic uncertainty described by (5) such that:

(1) The closed loop system in (13) is robustly stable and $\|H_{zw}(\lambda)\|_\infty^2 < \mu$.
(2) The cardinality of $\mathcal{Z}(K)$ is minimized.

The aforementioned conditions can be formulated as the following optimization problem:

$$\begin{aligned} &\min_K && card(Z(K)) \\ &\text{subject to} && \|H_{zw}(\lambda)\|_\infty^2 < \mu \end{aligned} \quad (14)$$

The mentioned problem is non-convex because of the non-covexity of the cardinality operator and the non-convex H_∞ constraint. In the next subsections, a convex relaxation of the cardinality and an inner convex approximation of the H_∞ constraint are presented.

3.1 Convex Relaxation of Cardinality

To reduce the required amount of information exchange between subsystems and sub-controllers in an interconnected system, matrix $\mathcal{Z}(K)$ in (10) should be sparse. The sparsity requirements are expressed in terms of the cardinality which is non-convex. It has been shown that the non-convex cardinality minimization can be relaxed by the convex one-norm (ℓ_1) minimization (Candes et al. [2008]). In fact, one-norm is the convex envelope of the cardinality (see Fazel [2002]).

To better approximate the cardinality, the weighted ℓ_1 norm is used. In Candes et al. [2008], an iterative algorithm for choosing the weights has been given. Therefore, the objective function in (14) can be written as:

$$J = \|W * \mathcal{Z}(K)\|_1 \quad (15)$$

where W is the matrix of weights. Matrix $\|W * \mathcal{Z}(K)\|_1$ is defined as follows:

$$\|W * \mathcal{Z}(K)\|_1 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} \left\| \begin{bmatrix} A_{c_{ij}} & B_{c_{ij}} \\ C_{c_{ij}} & D_{c_{ij}} \end{bmatrix} \right\|_1 \quad (16)$$

where w_{ij} is the ij^{th} entry of W.

3.2 Convex Set of Fixed-structure H_∞ Controllers

In Sadabadi and Karimi [2013], a set of LMI conditions for fixed-structure H_∞ control of the polytopic systems in (5) and (2) has been proposed and the results are given in the following theorem.

Theorem 1. Suppose that two auxiliary matrices M and T are given. Then, the fixed-structure controller in (9) guarantees the stability of the closed-loop system given in (13) with $\|H_{zw}(\lambda)\|_\infty^2 < \mu$, if there exist matrices $P^l > 0$ such that:

$$\begin{bmatrix} P^l - M^TP^lM & \star & \star & \star \\ P^lM - M + T^{-1}A^lT & 2I - P^l & \star & \star \\ 0 & (T^{-1}B^l)^T & I & \star \\ CT & 0 & D & \mu I \end{bmatrix} > 0 \quad (17)$$

for $l = 1, 2, \dots, q$.

The above inequalities are LMIs with respect to the controller parameters (A_c, B_c, C_c, D_c), μ, and the matrices $P^l > 0$ for $l = 1, \dots, q$. The instrumental matrices M and T are determined based on a set of initial controllers designed for each vertex of the polytope and the following lemma:

Lemma 1. The following set of inequalities are equivalent with (17):

$$\begin{bmatrix} P_T^l - A^{l^T}P_T^lA^l & \star & \star & \star \\ P_T^lA^l + M_T - XA^l & 2X - P_T^l & \star & \star \\ B^{l^T}M_T - B^{l^T}XA^l & B^{l^T}X & I & \star \\ C & 0 & D & \mu I \end{bmatrix} > 0 \quad (18)$$

for $l = 1, \ldots, q$, where

$$\begin{aligned} M_T &= T^{-T} M T^{-1} \\ {P_T}^l &= T^{-T} P^l T^{-1} \\ X &= T^{-T} T^{-1} \end{aligned} \tag{19}$$

Now, consider a set of initial fixed-structure H_∞ controllers independently designed for each vertex of the polytopic system. Then, compute $(\bar{A}^l, \bar{B}^l, \bar{C}^l, \bar{D}^l)$ from (13) by replacing (A_k, B_k, C_k, D_k) with the initial controllers. Then, the auxiliary matrices M_T and X can be obtained through an optimization problem which is minimizing μ subject to LMIs in (18), by simply replacing (A^l, B^l, C^l, D^l) with ($\bar{A}^l$, $\bar{B}^l$, $\bar{C}^l$, $\bar{D}^l$).

Finally, the auxiliary matrices M and T can be chosen as follows:

$$\begin{aligned} M &= T^T M_T T \\ T &= (chol(X))^{-1} \end{aligned} \tag{20}$$

where $chol$ is Cholesky factorization. The results can be further improved if the resulting controller is used as the initial controller to update the instrumental matrices iteratively (Sadabadi and Karimi [2013]).

3.3 Convex Set of Sparse H_∞ Controllers

In this subsection, an iterative algorithm for the problem of fixed-structure sparse H_∞ controller design is presented. The iterative procedure can be summarized by the following steps. To ease the presentation, the inequalities in (17) and (18) are respectively defined as follows:

$$\mathcal{F}_1^l(P^l, K, \mu \mid M, T) > 0 \tag{21}$$

$$\mathcal{F}_2^l({P_T}^l, M_T, X, \mu \mid K) > 0 \tag{22}$$

for $l = 1, \ldots, q$. The sign $\mid$ in the arguments of $\mathcal{F}_1^l$ and $\mathcal{F}_2^l$ separates the decision variables and the known parameters in the related LMIs. Therefore, LMIs in (21) are used to find the controller parameters, $K = (A_c, B_c, C_c, D_c)$, for a given pair of (M, T). In the same way, LMIs in (22) are used to find M_T and X for a given controller K.

Step 1: Design some initial controllers for each vertex of the polytope (${K^l}^{[0]}$). Put the iteration number $h = 1$, a small tolerance for $\epsilon > 0$, and $w_{ij}^{[1]} = 1$, $i, j = 1, ..., N$.

Step 2: Determine $M_T^{[h]}$ and $X^{[h]}$ from the following optimization problem:

$$\begin{aligned} \mu_2^{[h]} = \min \quad & \mu \\ \text{subject to} \quad & \mathcal{F}_2^l({P_T}^l, M_T^{[h]}, X^{[h]}, \mu \mid {K^l}^{[h-1]}) > 0; \\ & l = 1, \ldots, q \end{aligned} \tag{23}$$

Compute the auxiliary matrices $M^{[h]}$ and $T^{[h]}$ using $(M_T^{[h]}, X^{[h]})$ and (20).

Step 3: Solve the following optimization problem to obtain a fixed-structure sparse H_∞ controller $K^{[h]}$:

$$\begin{aligned} \mu_1^{[h]} = \min \quad & \mu + \beta \| W^{[h]} * \mathcal{Z}(K) \|_1 \\ \text{subject to} \quad & \mathcal{F}_1^l(P^l, K^{[h]}, \mu \mid M^{[h]}, T^{[h]}) > 0; \\ & l = 1, \ldots, q \end{aligned} \tag{24}$$

where β is a trade-off between the H_∞ performance and the sparsity of the controller structure.

Step 4: Find $\mathcal{Z}^{[h]}(K)$ based on the current controller $K^{[h]}$ and (10).

Step 5: Update the values of ϵ:

$$\epsilon^{[h+1]} = \alpha \epsilon^{[h]} \tag{25}$$

where $0 < \alpha \leq 1$.

Step 6: Update the ij^{th} elements of the weighting matrix $W^{[h+1]}$:

$$w_{ij}^{[h+1]} = \begin{cases} \frac{1}{z_{ij}^{[h]} + \epsilon^{[h]}}, & i \neq j \\ 0, & i = j \end{cases} \tag{26}$$

for $i, j = 1, ..., N$.

Step 6: Terminate on convergence or when maximum number of iterations h_{max} reaches. Otherwise, use the obtained controller in Step 3 as an initial controller (${K^l}^{[h+1]} \leftarrow K^{[h]}; l = 1, \ldots, q$) and go to Step 2 with $h \leftarrow h + 1$.

4. SIMULATION EXAMPLES

In this section, two simulation examples are provided in order to evaluate the effectiveness of the proposed method. It should be noted that LMI optimization problems are solved by YALMIP (Löfberg [2004]) and SDPT3 (Toh et al. [1999])/SeDuMi (Sturm [1999]) as the interface and the solver, respectively.

In the examples, $\epsilon = 10^{-5}$, $\alpha = 1$, and $\beta = 0.5$ are considered.

Example 1: Consider a network of three interconnected second-order subsystems given in Schuler et al. [2011b] with the following state space matrices:

$$\begin{aligned} A_g &= \left[\begin{array}{cc|cc|cc} a_{11} & 0.1 & a_{13} & 0 & -0.3 & 0 \\ 0.1 & 0.1 & 0 & 0 & -0.3 & 0.2 \\ \hline 0.3 & 0.1 & 0.6 & 0.1 & 0 & 0 \\ 0.2 & 0.5 & 0.1 & a_{44} & 0 & 0 \\ \hline 0 & 0 & -0.2 & 0 & 0.4 & 0 \\ 0 & 0 & 0.4 & -0.1 & 0.2 & 0.3 \end{array}\right] \\ B_g &= diag\left(\begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix} \right) \\ B_w &= diag\left(\begin{bmatrix} -0.2 \\ 1.0 \end{bmatrix}, \begin{bmatrix} 0.4 \\ -0.2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \right) \\ C_g &= diag\left(\begin{bmatrix} 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix} \right) \\ C_z &= diag\left(\begin{bmatrix} 1.3 & 0.4 \end{bmatrix}, \begin{bmatrix} 0 & -2 \end{bmatrix}, \begin{bmatrix} -0.5 & 0 \end{bmatrix} \right) \\ D_{zu} &= diag(-0.3, 0.1, 0.5) \\ D_{zw} &= diag(0.1, 0.5, 0) \\ D_w &= diag(-0.3, 0, 0.5) \end{aligned} \tag{27}$$

where $a_{11} = 0.2$, $a_{13} = 0$, and $a_{44} = 0.4$.

It is assumed that there exists some uncertainty in the parameters a_{11} and a_{44} of the subsystems up to $\pm 100\%$ of their nominal values and in the parameter a_{13} of the interaction terms such that $-1 \leq a_{13} \leq 1$. The objective of this example is to design a first-order sparse H_∞ controller for the polytope of eight vertices.

Based on the control design procedure in Subsection 3.3, at the first step, eight initial first-order centralized controllers are designed by using the command *hinfstruct* in MATLAB for each vertex of the polytope. The initial controllers are transformed to discrete-time ones by using

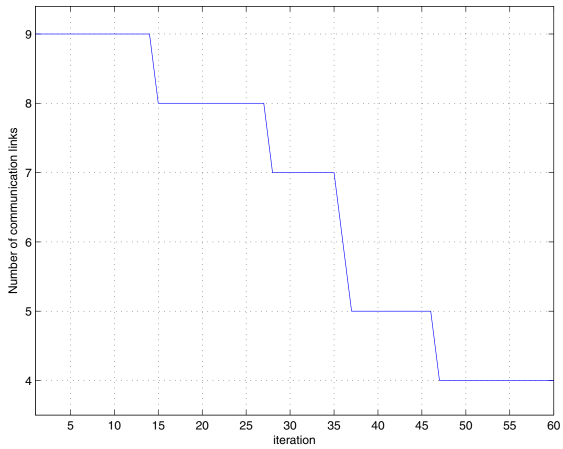

Fig. 1. Candidates for the control structure in Example 1

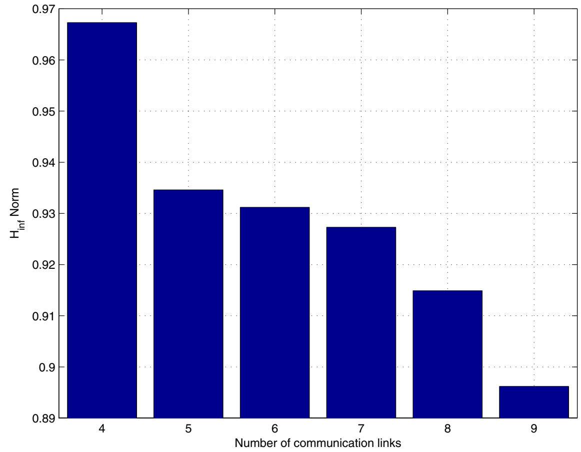

Fig. 2. Upper bound of $\|H_{zw}(\lambda)\|_\infty$ versus the number of communication links in Example 1

the bilinear (Tustin) approximation with the sampling time $T_s = 0.1sec$. Then, these controllers are utilized to obtain the instrumental matrices M and T using LMIs in (23). The next step is to determine the sparse controller by solving the convex optimization problem in (24). These steps iteratively repeated and finally after 60 iterations, some control structures are obtained. The computational time is about $187sec$.

Figure 1 shows the number of communication links versus the iteration numbers. Then, for each obtained control structure, an H_∞ controller is iteratively designed where the structure of controller is fixed a priori. For example, in the case of 4 communication links, a distributed H_∞ controller with the upper bound 0.9673 is obtained as follows:

$$
\begin{aligned}
A_c &= \begin{bmatrix} 0.0074 & 0 & 0 \\ -0.3107 & 0.6538 & 0 \\ 0 & 0 & 0.1519 \end{bmatrix} \\
B_c &= \begin{bmatrix} 0.0945 & 0 & 0 \\ 0.0430 & 0.0013 & 0 \\ 0 & 0 & 0.0662 \end{bmatrix} \\
C_c &= \begin{bmatrix} 2.6502 & 0 & 0 \\ -3.8635 & 0.4953 & 0 \\ 0 & 0 & -3.2312 \end{bmatrix} \\
D_c &= \begin{bmatrix} 1.2327 & 0 & 0 \\ -1.4132 & -0.8750 & 0 \\ 0 & 0 & 0.4798 \end{bmatrix}
\end{aligned} \quad (28)
$$

This sparse controller guarantees the stability as well as the H_∞ performance of the whole polytope.

Figure 2 shows the upper bound of $\|H_{zw}(\lambda)\|_\infty$ versus the number of communication links. It is observed that by increasing the sparsity of the controller structure, a decrease in the H_∞ performance is achieved. For example, the closed-loop system with a centralized controller (with 9 communication links) has an H_∞ upper bound of $\|H_{zw}(\lambda)\|_\infty < 0.8962$ whereas the distributed controller given in (28) with 4 communication links leads to $\|H_{zw}(\lambda)\|_\infty < 0.9673$.

Example 2: Let G be an interconnected system of five SISO subsystems, borrowed form Schuler et al. [2011a], with the state space matrices given in (29).

Assume that there is some parameter uncertainty in the parameters of the 5^{th} subsystems and its interaction with the 4^{th} subsystems such that $0.378 \le a_1 \le 0.702$, $-0.182 \le a_2 \le -0.098$, and $-0.52 \le b_1 \le -0.28$. The goal here is to design a sparse static output feedback controller which minimizes the H_∞ norm of the closed-loop system H_{zw} for the whole polytope.

To this end, the iterative procedure in 3.3 is used. Since a static output feedback is sought, eight static output feedbacks are designed by *hinfstruct* as initial controllers for the vertices. After 30 iterations, some candidates for the control structure are obtained.

Now $\|H_{zw}(\lambda)\|_\infty$ is iteratively minimized using LMI conditions in (17) and subject to structural constraints determined by the candidates. The upper bound of $\|H_{zw}(\lambda)\|_\infty$ versus the number of communication links is plotted in Figure 3. The static output-feedback controller with 8 communication links is as follows:

$$
D_c = \begin{bmatrix} 0.1113 & 0 & 0 & 0.7823 & 0 \\ 0 & -3.7119 & 0 & 0 & 0 \\ 0.5719 & 0 & 0.5807 & 0 & 0 \\ 0 & 0 & 0.2012 & -0.0380 & 0 \\ 0 & 0 & 0 & 0 & 6.3590 \end{bmatrix} \quad (30)
$$

with $\mu = 1.7447^2$.

5. CONCLUSION

In this paper, the problem of sparse H_∞ dynamic output-feedback control of interconnected systems with polytopic uncertainty has been studied. The objective of this paper is to simultaneously design the controller structure with minimum communication links between the subsystems and the local controllers as well as the controller parameters. For this purpose, the minimization of the sparsity degree of a special matrix has been considered as an objective function subject to some H_∞ performance constraints. The sparsity can be achieved by the cardinality minimization problem which is relaxed by a weighted ℓ_1-norm. The

$$
\begin{aligned}
A_g &= \left[\begin{array}{cc|cc|cc|cc|cc}
0.30 & -0.29 & 0 & 0 & -0.24 & 0.21 & -0.16 & 0.03 & 0 & 0 \\
-0.29 & 0.32 & 0 & 0 & -0.18 & 0 & -0.28 & -0.32 & 0 & 0 \\
\hline
0 & 0 & 0.50 & 0.04 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.18 & 0.59 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & -0.08 & -0.51 & 0.36 & -0.10 & 0 & 0 & -0.28 & 0.15 \\
0 & 0 & -0.13 & 0.04 & -0.24 & 0.69 & 0 & 0 & -0.29 & -0.20 \\
\hline
-0.38 & -0.20 & 0 & 0 & 0.47 & 0.07 & -0.10 & 0.11 & 0 & 0 \\
0.06 & -0.34 & 0 & 0 & -0.01 & -0.22 & -0.09 & 0.18 & 0 & 0 \\
\hline
0 & 0 & 0.04 & -0.01 & 0 & 0 & a_1 & -0.01 & a_2 & 0.15 \\
0 & 0 & 0.03 & 0.13 & 0 & 0 & 0.05 & 0.34 & 0.15 & 0.25
\end{array}\right] \\
B_g &= diag\left(\begin{bmatrix}-0.2\\-1\end{bmatrix}, \begin{bmatrix}0\\-0.8\end{bmatrix}, \begin{bmatrix}0\\-0.4\end{bmatrix}, \begin{bmatrix}1.9\\1.6\end{bmatrix}, \begin{bmatrix}-0.4\\b_1\end{bmatrix}\right) \\
B_w &= diag\left(\begin{bmatrix}-0.5\\0\end{bmatrix}, \begin{bmatrix}0\\0.8\end{bmatrix}, \begin{bmatrix}0\\1.3\end{bmatrix}, \begin{bmatrix}-1.1\\0\end{bmatrix}, \begin{bmatrix}-0.6\\-1\end{bmatrix}\right) \\
C_g &= diag\left(\begin{bmatrix}1.8 & 0\end{bmatrix}, \begin{bmatrix}0 & -0.2\end{bmatrix}, \begin{bmatrix}-1.5 & 0\end{bmatrix}, \begin{bmatrix}-0.3 & 0\end{bmatrix}, \begin{bmatrix}0 & 0.2\end{bmatrix}\right) \\
C_z &= diag\left(\begin{bmatrix}1.3 & 0.4\end{bmatrix}, \begin{bmatrix}0 & -2\end{bmatrix}, \begin{bmatrix}-0.5 & 0\end{bmatrix}, \begin{bmatrix}-1.3 & -0.2\end{bmatrix}, \begin{bmatrix}0 & 0.2\end{bmatrix}\right) \\
D_{zu} &= 0, \quad D_{zw} = 0, \quad D_w = 0
\end{aligned} \tag{29}
$$

where $a_1 = 0.54$, $a_2 = -0.14$, and $b_1 = 0$.

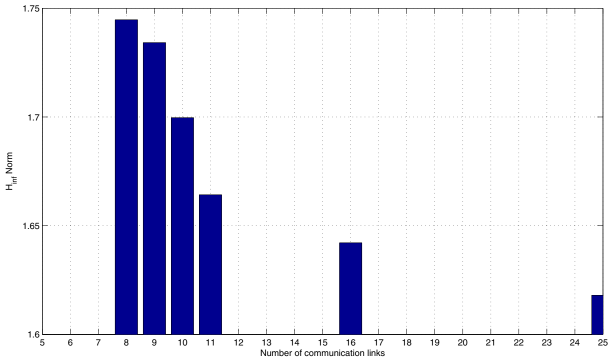

Fig. 3. Upper bound of $\|H_{zw}(\lambda)\|_\infty$ versus the number of communication links in Example 2

non-convex H_∞ constraints have been also convexified by using some instrumental matrices. The weights as well as the instrumental matrices have been computed from an iterative algorithm and based on the values of the previous solution.

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