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Stability in small signals investigation of nonlinear dynamic power systems

Stanislav N. Vassilyev*, Igor B. Yadikin**, Nataliya N. Bakhtadze***, Vladimir A. Lototsky****

*V.A. Trapeznikov Institute of Control Sciences,
65, Profsoyuznaya, Moscow, 117997, Russia
(e-mail: jadikin1@mail.ru)

**V.A. Trapeznikov Institute of Control Sciences,
65, Profsoyuznaya, Moscow, 117997, Russia
(e-mail: jadikin1@mail.ru)

***V.A. Trapeznikov Institute of Control Sciences,
65, Profsoyuznaya, Moscow, 117997, Russia
(e-mail: sung7@yndex.ru)

****V.A. Trapeznikov Institute of Control Sciences,
65, Profsoyuznaya, Moscow, 117997, Russia
(e-mail: valot@yandex.ru)

Abstracts. The new technique of Frobenius norm calculation of Gramians, sub-Gramians and transfer functions of electric power systems (EPS) is suggested. It is based on spectral expansion of gramian by combinational modes of dynamic matrix of EPS predicting model. It is more effective comparing to modal analysis as it allows to prescribe the criterion of EPS stability loss risk, and also to evaluate the interaction of ill-stable system modes. The suggested approach has a transparent physical interpretation, i.e., stability loss risk corresponds to the energy of ill-stable modes group. The predicting model is formed basing on associative search technique for nonlinear dynamic plants.

Keywords: stability control, Gramian method, system identification, virtual model, associative search, knowledge base

1. INTRODUCTION

The problem of the degree of stability determining and power system monitoring of local or district oscillations is known in the world of science for over 50 years and still has not lost its relevance (Kundur, 2005). Research in this field were concentrated mainly around the direct Lyapunov method, modal analysis and research of the eigenvalues of the characteristic equation of the EPS mathematical model's matrix (Barquin, et al, 2012). Many powerful mathematical methods and their combinations were used .

However, the practical value of these studies are largely limited, primarily due to the fact that the mathematical models of electric power systems are complex: they are non-linear, non-stationary and distributed, include a variety of descriptions for slow and fast processes. Each method has a limited application and is effective only for certain modes of the system (Preiss and Wegmann, 2001). Major problems arise due to the high dimension of the mathematical models of EPS and the need to solve problems of high dimensionality in real time (Ahmetzyanov, 2012).

Under methods of modal analysis is Prony analysis, robust recursive least squares method, the Yule-Walker algorithms, wavelet analysis, neural networks and genetic algorithms.

To determine the static stability of a power grid the paper presents a Gramian method based on a new mathematical technique of solving Lyapunov and Silvester differential and algebraic equations that was developed in the Institute of Control Sciences for analysis of stability degree of linear dynamic systems. The method operates by decomposing the Gramian matrix that is a solution of the Lyapunov or Silvester equation into a spectrum of matrices that make up these equations. It is more effective comparing to modal analysis as it allows to prescribe the criterion of EPS stability loss risk (Sukhanov, 2012), and also to evaluate the interaction of ill-stable system modes (Grobovoi, et al., 2013).

To develop immune system it is necessary to create virtual analyzer (soft sensor) of the stability loss threat. There are several approach to design such analyzer:

- Ill-stable oscillation power measurement (Gaglioti, et al., 2011; CIEE, 2010),
- Selective modal analysis (Barquin, 2012),
- Modified Arnoldi method,
- Parametric identification methods (Bakhtadze, 2008)
- Kalman filter,
- Neural networks.

For soft sensor design one can use direct Lyapunov method, based on analysis of solution of the Lyapunov differential or algebraic equation in frequency domain.

2. THE EVALUATION OF EPS STABILITY LOSS RISK BY GRAMIAN H2- NORM TECHNIQUE

Let us suppose that power system mathematical model is defined as nonlinear algebraic-differential equations system

$$\begin{aligned}&\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x},\mathbf{u},t), \quad \mathbf{x}(t_0)=0,\\ &\mathbf{M}(\mathbf{x},t)\mathbf{x}(t) = \mathbf{N}(\mathbf{x},t)\mathbf{u}(t).\end{aligned} \tag{1}$$

Linearized model of the power system fixed mode is defined as the linear algebraic-differential equations system

$$\begin{aligned}&\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad \mathbf{x}(t_0)=0,\\ &\mathbf{M}\mathbf{x}(t) = \mathbf{N}\mathbf{u}(t).\end{aligned} \tag{2}$$

Suppose that matrix M is nonsingular one. Then equations system (2) may be transformed to

$$\begin{aligned}&\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad \mathbf{x}(t_0)=0,\\ &\mathbf{A} = \mathbf{A}_1 + \mathbf{M}^{-1}\mathbf{N},\end{aligned} \tag{3}$$

Consider linear stationary difference MISO system

$$\begin{aligned}&x(k+1) = \mathbf{F}x(k) + \mathbf{G}u(k), \quad x(0)=0,\\ &y(k) = \mathbf{H}x(k),\end{aligned}$$

where $x(k)$ is state vector, $u(k)$ is scalar control, and $y(k)$ is scalar output, $\mathbf{F}$ is $[n\text{x}n]$ dynamics matrix, $\mathbf{G}$ and $\mathbf{H}$ are $[n\text{x}1]$ and $[1\text{x}\ n]$ matrices respectively. Applying to (1) bilinear transform

$$s = \frac{z-1}{z+1}, \tag{4}$$

which maps unit circle interiority onto the left complex half-plane, one obtains equivalent stationary linear dynamic SISO system

$$\begin{aligned}&\frac{dx(t)}{dt} = \mathbf{A}x(t) + \mathbf{B}u(t), \quad x(0)=0,\\ &y(t) = \mathbf{C}x(t),\end{aligned} \tag{5}$$

where $x(t)$ is state vector, $u(t)$ is scalar control, $y(t)$ is scalar output, $\mathbf{A}$ - dynamics $[n\text{x}n]$-matrix, $\mathbf{B}$ and $\mathbf{C}$ are matrices of size $[n\text{x}1]$ and $[1\text{x}n]$ respectively.

The matrices of continuous system are related to matrices of discrete-time system by relations

$$\mathbf{A} = (\mathbf{F}+\mathbf{I})^{-1}(\mathbf{F}-\mathbf{I}), \quad \mathbf{B} = \sqrt{2}(\mathbf{F}+\mathbf{I})^{-1}\mathbf{G}, \quad \mathbf{C} = \sqrt{2}\mathbf{H}(\mathbf{F}+\mathbf{I})^{-1}.$$

The nice property of bilinear transform is that infinite Gramians of controllability, observability, and cross-Gramians coincide. This fact implies that when eigenvalues of discrete system approximate the unit circle, and respective eigenvalues of continuous system approximate imagine axis from the left asymptotic models of Gramians of controllability, observability, and cross-Gramians will coincide too. So stability analysis of electric power systems is identical both for continuous and discrete models.

Let us consider following continuous direct differential and algebraic Lyapunov (Lyapunov, 1934) equations

$$\begin{aligned}&\frac{d\mathbf{P}(t)}{dt} = \mathbf{A}\mathbf{P}(t) + \mathbf{P}(t)\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{B}^{\mathrm{T}}, \quad \mathbf{P}(0) = 0_n,\\ &\mathbf{A}\mathbf{P}(\infty) + \mathbf{P}(\infty)\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{B}^{\mathrm{T}} = 0.\end{aligned} \tag{6}$$

By substitution of matrix Lyapunov integral $\mathbf{P}(t) = \int_0^t e^{\mathbf{A}\tau}\mathbf{R}e^{\mathbf{A}^{\mathrm{T}}\tau}d\tau$, called finite controllability gramian, in first of Eq. (6) one can find that this integral is the solution of relevant Lyapunov equations.

Yadykin (2010) obtained spectral expansion both finite and infinite controllability Gramians with respect to dynamics matrix spectrum supposing that all characteristic roots of matrix are simple and non symmetrical w.r.t. zero:

$$\begin{aligned}&\mathbf{P}(\infty) = \sum_{k=1}^{n}\sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1}\frac{s_k^j(-s_k)^\eta}{N'(s_k)N(-s_k)}\mathbf{A}_j\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{A}_\eta^{\mathrm{T}},\\ &P(t) = -\sum_{k=1}^{n}\sum_{\lambda=1}^{r}\sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1}\frac{s_k^j s_\lambda^\eta}{(s_\lambda+s_k)N'(s_k)N'(s_\lambda)}\mathbf{A}_j\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{A}_\eta^{\mathrm{T}}(1\text{-}e^{(s_k+s_\lambda)t}).\end{aligned} \tag{7}$$

In (7) $N(s) = s^n + a_{n-1}s^{n-1}...+as+a_0$ is characteristic polynomial of matrix $\mathbf{A}$, s_k - k-th root of characteristic equation, $\mathbf{A}_j$ - Faddeev matrix, which can be calculated as follows (Faddeev and Faddeyeva, 1963; Kwakernaak and Sivan, 1972):

$$\mathbf{A}_j = \sum_{i=j+1}^{n} a_i \mathbf{A}^{i-j-1}. \tag{8}$$

According to the second Lyapunov technique, system stability analysis is connected to the investigation of quadratic form $x^{\mathrm{T}}\mathbf{P}^c(\infty)x = \mathbf{E}$, which in fact is the energy of perturbed motion.

The advantage of Gramian spectral expansions of form (7) is possibility to separate in it additive components which are matrix quadratic forms, respecting to certain modes of dynamics matrix and their different combinations. Among different matrix modes, let separate those which correspond to left half-plane roots and have small absolute value of real part. Let call this modes *ill-stable,* and dynamic systems the dynamics matrix of which contains such modes let call *ill-stable systems.* Let call *infinite sub-Gramian of certain mode* following matrix quadratic form

$$\mathbf{P}_k^c(\infty) = \sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1} \frac{s_k^j(-s_k)^\eta}{N'(s_k)N(-s_k)} \mathbf{A}_j \mathbf{B}\mathbf{B}^{\mathrm{T}} \mathbf{A}_\eta^{\mathrm{T}}, \tag{9}$$

and *infinite sub-Gramian of combining mode* - matrix quadratic form

$$\mathbf{P}_{k,\lambda}^c = -\sum_{k=1}^{n}\sum_{\lambda=1}^{r}\sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1} \frac{s_k^j s_\lambda^\eta}{(s_\lambda + s_k)N'(s_k)N'(s_\lambda)} \mathbf{A}_j \mathbf{B}\mathbf{B}^{\mathrm{T}} \mathbf{A}_\eta^{\mathrm{T}}. \tag{10}$$

The most important property of infinite subgramian of certain mode is that the square of subgramian Frobenius norm tends to infinity when the root or pair of conjugated roots approach imagine axis of complex plate $n \le 30$ (Ahmetzyanov *et al*, 2012).

First applications of Gramian technique for investigation of EPS static stability relate to stability analysis of Kundur test model of two district power system (Ahmetzyanov *et al*., 2012) and the model of Russky Island power system (macrogrid) (Grobovoi *et al*., 2013). This investigation confirmed mentioned earlier properties of gramians and subgramians, and detected a number of peculiarities of gramian technique numerical realization, in particular numerical instability of large dimension Faddeev matrices calculation. It was found that for Kundur scheme the maximal order of dynamics matrix is limited: $n \le 30$, and for EPS of Russky Island - $n \le 70$

To overcome this difficulty one can use order diminishing techniques for EPS mathematical model, which widely use calculations of infinite observability and controllability gramians (Sorensen and Antoulas, 2005). For SISO systems, more simple solution exists, based on using in gramian spectral expansion instead of matrix quadratic forms numeric ones, that is the forms produced by coefficients of numerator of scalar transfer function of EPS.

Appearing of WAMS technologies for EPS analysis makes reality the calculation of EPS transfer functions from measurement data for systems of large enough dimension (CIEE report, 2010). Let the system transfer function be

$$\mathbf{W}(s) = \frac{y(s)}{u(s)} = \frac{\mathbf{C}\mathbf{A}_{n-1}\mathbf{B}s^{n-1} + \ldots + \mathbf{C}\mathbf{A}_1\mathbf{B}s + \mathbf{C}\mathbf{A}_0\mathbf{B}}{N(s)} = \\ = \frac{b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0}. \tag{11}$$

Multiplying Eq.(9) by $\mathbf{C}$ from the left, and then by $\mathbf{C}^{\mathrm{T}}$ from the right one obtains

$$\|\mathbf{W}(\mathrm{s})\|_2^2 = \mathrm{tr}\mathbf{C}\mathbf{P}^c(\infty)\mathbf{C}_{\mathrm{T}} = \sum_{k=1}^{n}\sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1} \frac{s_k^j(-s_k)^\eta}{N'(s_k)N(-s_k)} b_j b_\eta = \\ = \tilde{\mathbf{b}}^{\mathrm{T}}\mathbf{G}\tilde{\mathbf{b}}, \quad \tilde{\mathbf{b}}^{\mathrm{T}} = \begin{bmatrix} b_{n-1} & \ldots & b_1 & b_0 \end{bmatrix}, \\ \mathbf{G}_k = \sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1} \frac{s_k^j(-s_k)^\eta}{N'(s_k)N(-s_k)}, \mathbf{G} = \sum_{k=1}^{n}\mathbf{G}_k. \tag{12}$$

$$\|\mathbf{W}(\mathrm{s})\|_2^2 = -\sum_{\lambda=1}^{\mathrm{n}}\sum_{k=1}^{n}\sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1} \frac{s_k^j s_\lambda{}^\eta}{(s_k + s_\lambda)N'(s_k)N'(s_\lambda)} b_j b_\eta = \\ = \tilde{\mathbf{b}}^{\mathrm{T}}\tilde{\mathbf{G}}\tilde{\mathbf{b}}, \quad \tilde{\mathbf{b}}^{\mathrm{T}} = \begin{bmatrix} b_{n-1} & \ldots & b_1 & b_0 \end{bmatrix}, \\ \tilde{\mathbf{G}}_{k,\lambda} = -\sum_{j=0}^{n-1}\sum_{\eta=0}^{r-1} \frac{s_k^j s_\lambda{}^\eta}{(s_k + s_\lambda)N'(s_k)N'(s_\lambda)}, \tilde{\mathbf{G}} = \sum_{\lambda=1}^{n}\sum_{k=1}^{n}\tilde{\mathbf{G}}_{k,\lambda}. \tag{13}$$

Eqs. (12) – (13) give the spectral expansion of Frobenius norm square of EPS transfer function. They are much more simple comparing to analogue spectral expansion of Frobenius norm square of controllability Gramian.

3. ASYMPTOTIC MODELS FOR FROBENIUS NORM CALCULATION FOR ANALYSIS OF POWER SYSTEM STABILITY IN SMALL SYGNALS

Consider process of transition the power system from stable to unstable state while left-side roots on complex plane are moving to imagine axis so that part of roots have fixed position, and others are moving parallel to real axis. The investigation of power system stability in small signals by modal analysis shows that for large scale power systems the risk of stability loss is determined by only several ill-stable roots (Kundur, 1994; Martins, 1997; Gagliotti et al., 2011). It gives the opportunity to suppose that in analysis of stability by gramian techniques the leading role pertains to subgramians of ill-stable modes, or relative real quadratic forms in Eqs.(9)-(10).

Let call the quadratic form $\tilde{\mathbf{b}}^{\mathrm{T}}\tilde{\mathbf{G}}_{k,\lambda}\tilde{\mathbf{b}}$ for ill-stable combining mode $s_k + s_\lambda$ in Eq.(10) *partial component* of the square of Frobenius norm of EPS transfer function.

Generally speaking Eq.(10) implies that matrix $\tilde{\mathbf{G}}_{k,\lambda}$ has complex range, so such matrices have to be considered in spectral expansion of square of Frobenius norm of EPS transfer function complex conjugate pairs. The matrix $\left[s_k^j s_\lambda^\eta\right]$ have unique rank. In the case of ill-stable real combining modes, respective partial component of transfer function Frobenius norm square becomes a real positive number. In case of pair of complex conjugate combining modes respective pair of partial components of transfer function Frobenius norm square is real positive number.

Sufficient algebraic asymptotic EPS instability condition. Let perturbed EPS movement with small deviations can be described by eqations similar to Eq.(7). Consider the sequence of equations of type (3) where matrices *B, C* are constant, and Hurvitz matrix *A* parameters are changing so that *m* roots are fixed on complex plane, and *n – m* are moving toward imagine axis so that their real parts simultaneously increase by positive value *d* until one or more of roots reach ε neighborhood of imagine axis. Such situation let call asymptotic conditions of EPS instability development.

Under this assumptions, following statements are simultaneously true.

1. If the roots $s_k = \alpha_k + j\beta_k, s_\lambda = \alpha_\lambda + j\beta_\lambda$ are ill-stable, then $\lim\limits_{\alpha_k,\alpha_\lambda \to 0} \mathrm{Re}(s_k + s_\lambda) = 0$
2. $\lim\limits_{\alpha_k,\alpha_\lambda \to 0} \left\|P_{k,\lambda}^c\right\|_2^2 = \infty, \quad \lim\limits_{\alpha_k,\alpha_\lambda \to 0} \left\|P^c\right\|_2^2 = \infty.$
3. $\lim\limits_{\alpha_k,\alpha_\lambda \to 0} \left\|\tilde{b}^{\mathrm{T}}\tilde{G}_{k,\lambda}\tilde{b}\right\|_2^2 = \infty, \ \lim\limits_{\alpha_k,\alpha_\lambda \to 0} \left\|W(s)\right\|_2^2 = \infty$

4. INSTABILITY DEVELOPMENT ANALYSIS OF NONLINEAR ELECTRIC POWER SYSTEMS

Now suppose that in Eq.(1) **f** is nonlinear operator, and system dynamics is characterized by essentially nonlinear properties which in certain points operating range may be discontinuous, ambiguous, or do not exist at all. When one will linearize such system in usual manner it may give wrong results.

The predicting models design may be fulfilled by *associative search technique* for nonlinear dynamic objects (Bakhtadze et al., 2008). With this aim, let transfer from system description in state space (4) to “input – output” description.

The predicting model design is carried out with the help of intellectual algorithms of nonlinear dynamic system identification. Those are based on inductive learning: associative search of analogues by intellectual analysis of both archives of energy system technology parameters (Data Mining) and the knowledge base of technologies.

The construction of predictive model by associative search of dynamic plant on each step is based on technological knowledge. This approach allows to use any available a priori information about the plant. To increase quickness of algorithm and to save computational resources, one is to learn the system. The criterion of input vectors choice from the archive for current virtual model building underlies clustering technique choice.

The linear dynamic model looks as follows:

$$y_t = a_0 + \sum_{i=1}^{r} a_i y_{t-i} + \sum_{j=1}^{s}\sum_{p=1}^{P} b_{jp} x_{t-l_j,p}, \ \ l_j \le t \qquad (14)$$

where y_t is the object’s output forecast at the t-th step, x_t is the input vector, r is the output memory depth, s is the input memory depth, P is the input vector length. The equation (1) differs from the ordinary regression because $t-l_j$ are being selected not in chronological sequence but rather according to a certain criterion, which describes the proximity of input vectors with the index $t-l_j$ to the current input vector x_t. Such criteria are named associative impulses.

The original dynamic algorithm consists in the design of an approximating hypersurface in the input vector space and the related one-dimensional outputs at every time step (see Fig. 1). To build a virtual model for a specific time step, the vectors close in a manner to the current input vector are selected. This selection procedure is called *associative impulse*. The output value at the next step is further calculated using least-squares method (LSM).

To increase the speed of the virtual models-based algorithm, an approach is applied based on employing a *model of process operator’s or operator’s associative thinking* for predicting (Bakhtadze et al., 2011). For modeling the associative search procedure imitating the intuitive prediction of process status by an analyst we assume that the sets of process variable values, which are the components of an input vector, as well as the system outputs at previous time steps altogether create a set of symptoms, making an image *R* of the object output at the next step.

The associative search process consists in the recovery of all symptoms describing the specific object based on its images. Denote the image initiating the associative search by R_0 and the corresponding resulting image of the associative search by R. A pair of images (R_0, R) will be further called association A or $A(R_0, R)$.

The set of all associations over the set of images forms the memory of the intelligent system’s knowledgebase.

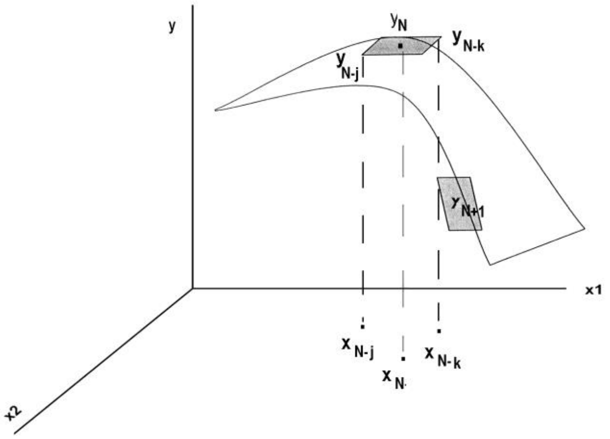

Fig. 1. Approximating hypersurface design

At the system's learning phase, an archive of images is created. In our case, a set of input vectors selected form the process history will be considered as an image. At the prediction stage, the input vector x_t will be considered as an initial image R_0^a of the associative search, while approximating hypersurface formed by the input vectors from the process history will be the final image R^a of the associative search. The selected hypersurface is an image of the current input vector which is used for output prediction. The algorithm implements the process of image R^a recovery based on R_0^a, i.e., the associative search process, and can be described by a predicate $\Xi = \left\{\Xi_i\left(R_{0i}^a, R_i^a, T^a\right)\right\}$ where $R_{0i}^a \subset R_0$, $R_i^a \subset R$, and T^a is the duration of the associative search.

Predictive model design for nonlinear dynamic plants based on associative search permits to forecast the dynamics of investigated plant and detect approaching the stability border. The gramian technique for stability analysis of power systems in small signals permits to estimate the risk of power system stability loss basing on calculation of Frobenius norm of subgramians of ill-stable combining modes including aperiodic modes, ill-stable pairs of complex conjugated modes, and ill-stable wobble modes. Two first groups of mode one can detect by modal analysis.

The Gramian technique allows to estimate the influence of dangerous electro-mechanical oscillations on transmission capacitance of power grid, and localize ill-stable grid cross-sections (Sorensen and Antoulas, 2005). The method disadvantage is the necessity of ill-stable modes calculation. It may be partially abolished by determining of numerical mode values from amplitude frequency characteristics of the system. The predicting model designed by associative search technique for nonlinear dynamic plants, permits to forecast the dynamics off the plant under study and detect the approaching to the stability boarder.

To predict system approaching the unstable state from output forecast made by associate search technique one need investigate transfer function of linear regression virtual model. For this, controllability gramian of respective state space system realization can be used.

The techniques of minimal realization for SISO and MISO systems are well known. For MIMO systems, Gilbert pseudo-canonical realizations can be used.

5. CONCLUSION

Using the proposed mathematical methods to develop and install MAS for early warning of the system losing stability will step up the network throughput by at least 3%; this increase will improve the network company's profit, save capital investments in network construction and modernization and payment for various additional system services. Furthermore, losses of the active power in the electric network will be reduced, the range of admissible modes will be extended by limiting power flows and improving the system controllability and consequently the mode reliability. The development of a real time programmable sensor that would specify the point where the system stability may be lost will make it possible to start the tackling of the following new technological tasks:

- development of an adaptive automatic vector dampening control of dangerous low frequency oscillations in a power grid;
- development of a computation technique and program for real time emergency control devices;
- development of a new generation of multi-agent systems that would counter stability disturbances in
- intelligent adaptive systems which maintain system stability.

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