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Minimum Variation Guidance Laws for Interceptor Missiles ⋆

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Abstract: This paper introduces a new approach to guidance law design using linear quadratic optimal control theory, minimizing throughout the engagement the variation of the control input as well as the integral control effort. The guidance law is derived for arbitrary order missile dynamics and target maneuvers. Explicit results are provided for a first order missile model and a constant target maneuver. It is shown that in the limiting cases the guidance law degenerates to either Contrell's minimum effort law or to a guidance law that enables mitigating saturation. The performance of the guidance law is analyzed theoretically and demonstrated on a few simulation cases.

1. INTRODUCTION

Guidance law design has been one of the first applications of optimal control theory, as it developed in the late sixties. One of the first results, and a classical one, see e.g. Cottrell [1971] was that Proportional Navigation Guidance (PNG), which was known already thirty years earlier, can be obtained as a solution of a linear quadratic optimal control problem. This has stimulated the discovery of an entire family of guidance laws, that could be called "linear quadratic" guidance laws, that were obtained as solutions of different linear quadratic optimal control problems. The main criticism of the linear quadratic optimal control approach to guidance law design has been that it does not provide a maximum bound for the lateral acceleration command. Thus, some studies have also considered the bounded control variant of the problem (Rusnak and Levy [1991]). In the context of differential games, Gutman [1979], Shima and Shinar [2002] considered explicitly bounds on the lateral acceleration and obtained bang-bang type solutions. However, in the typical case, the strategies are arbitrary on a portion of the game space, which indicates that there is place for further optimization of the guidance laws. Linear quadratic differential games (see Ben-Asher and Yaesh [1998] for a detailed exposition) are not having this disadvantage, but they do not provide any control on the bound of the commanded acceleration. An interesting comparison between the bounded control differential game, and the linear quadratic differential game guidance law can be found in Turetsky and Shinar [2003].

In this work we take an entirely different approach that is based on the following observation. If the target maneuver was known for the entire time interval of guided flight, then the best strategy to avoid saturation, while achieving best performance for the pursuer (even zero miss distance) would be to use constant lateral acceleration, all throughout the flight. Therefore, the solution that we propose here is to derive the guidance law from a quadratic criterion that penalizes the difference between the acceleration command and a constant, in addition to the penalty on the absolute size of the lateral acceleration. The result of solving the optimization problem is shown to be a guidance law of the Augmented PNG type that has the tendency to require an almost constant acceleration of the pursuer, as long as the lateral acceleration of the evader does not change. The proposed methodology gives the guidance loop designer an additional tuning parameter that, if chosen judiciously, may improve guidance performance while keeping the numerical computations for the guidance algorithm relatively simple.

The proposed guidance law design can also be regarded as an alternative to the minimum jerk guidance laws proposed in different versions in recent papers such as Uchiyama et al. [2005], Jeon et al. [2006], Grinfeld and Ben-Asher [2014]. In all these papers, the problem was formulated with the jerk as the input variable. In our approach, the lateral acceleration remains the input variable, but it is constrained to keep close to a constant value. In the extreme case, a constant lateral acceleration imply zero jerk, so the objective of minimum jerk is attained trivially in this case. However, in general, the relation between the approach in this paper and the minimum jerk guidance laws proposed in the cited papers is not so clear and will be a matter of future research.

The structure of the paper is as follows. The model used for deriving the guidance law is presented in Section 2. The proposed guidance law that we call Minimum Variation Guidance (MVG) is derived in Section 3. The performance of the guidance law is analyzed in Section 4. Conclusions and way ahead are formulated in Section 5.

2. MODELS DERIVATION

As usual, a two dimensional intercept model will be used for the derivation of the guidance law, under the assump-

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tion that motion of the missile can be separated into two orthogonal channels. Figure 1 presents a schematic view of the planar endgame geometry, where $X_I - O_I - Z_I$ is a Cartesian inertial reference frame. The missile and target are denoted by the subscripts M and T, respectively. The speed, normal acceleration, and flight path angles are denoted by V, a, and γ, respectively, the relative range between the two vehicles is r, and θ is the angle between the LOS and the X_I axis. The X-axis, aligned with the LOS used for linearization, is denoted as LOS_0. z is the relative displacement between the target and the missile normal to this direction. The target and missile accelerations normal to LOS_0 are denoted by a_{TN} and a_{MN}, respectively; and satisfy $a_{TN} = a_T \cos(\gamma_{T0} + \theta_0)$, $a_{MN} = a_M \cos(\gamma_{M0} - \theta_0)$.

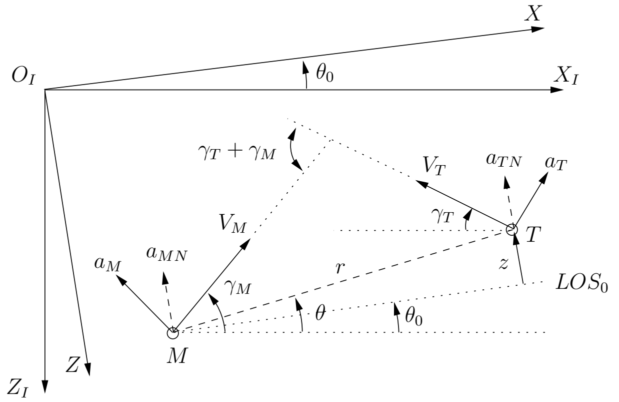

Fig. 1. Planar engagement geometry.

Neglecting the gravitational force, the engagement kinematics, expressed in a polar coordinate system (r, θ) attached to the missile, is $\dot{r} = V_r$ and $\dot{\theta} = V_\theta / r$, where the speed V_r is

$$V_r = -\left[V_M \cos(\gamma_M - \theta) + V_T \cos(\gamma_T + \theta)\right] \tag{1}$$

and the speed perpendicular to the LOS is

$$V_\theta = -V_M \sin(\gamma_M - \theta) + V_T \sin(\gamma_T + \theta) \tag{2}$$

During the endgame, the target and missile are assumed to move at a constant speed. The lateral maneuver dynamics of the target is assumed to be ideal.

The lateral maneuver dynamic of the missile is assumed to be represented by arbitrary order linear equations

$$\dot{\boldsymbol{x}}_M = \mathbf{A}_M \boldsymbol{x}_M + \mathbf{B}_M u_M \tag{3}$$

$$\dot{\gamma}_M = a_M / V_M \tag{4}$$

where

$$a_M = \mathbf{C}_M \boldsymbol{x}_M + d_M u_M \tag{5}$$

and $\boldsymbol{x}_M$ is the state vector of the interceptor's internal state variables with $dim(\boldsymbol{x}_M) = n$.

The derivation of the guidance laws in this paper will be performed based on a linearized model around collision triangle trajectories.

The state vector of the linearized problem is

$$\boldsymbol{x} = \left[\, z \;\; \dot{z} \;\; \boldsymbol{x}_M^T \,\right]^T \tag{6}$$

where z is the relative displacement perpendicular to the initial line-of-sight. The equations of motion are

$$\dot{\boldsymbol{x}} = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_T \cos(\gamma_{T0} + \theta_0) - a_M \cos(\gamma_{M0} - \theta_0) \\ \dot{\boldsymbol{x}}_M = \mathbf{A}_M \boldsymbol{x}_M + \mathbf{B}_M u_M \end{cases} \tag{7}$$

The matrix form of the equation set is therefore

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}u_M + \mathbf{C}a_T \tag{8}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_k & \mathbf{A}_{12} \\ [0]_{n\times 2} & \mathbf{A}_M \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ -d_M \cos(\gamma_{M0} - \theta_0) \\ \mathbf{B}_M \end{bmatrix},$$
$$\mathbf{C} = \begin{bmatrix} 0 \\ \cos(\gamma_{T0} + \theta_0) \\ [0]_{n\times 1} \end{bmatrix}, \tag{9}$$

and

$$\mathbf{A}_k = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{A}_{12} = \begin{bmatrix} [0]_{1\times n} \\ -\mathbf{C}_M \cos(\gamma_{M0} - \theta_0) \end{bmatrix}, \tag{10}$$

with $[0]$ denoting a matrix of zeros with appropriate dimensions. In the neighbourhood of a collision triangle the closing speed V_r is approximately constant and the interception time, given by $t_f = -r_0/V_r$ can be assumed fixed. For the guidance law implementation we approximate time-to-go by

$$t_{go} = -r/V_r \tag{11}$$

3. DERIVATION OF THE MINIMUM VARIATION GUIDANCE LAW

Based on the linear intercept model derived in the previous section, the guidance law design problem is to determine the input u_M in such a way that the miss distance is zero

$$z(t_f) = 0, \tag{12}$$

where t_f is the time duration of the intercept. We are assuming here full information, so that the target acceleration and the initial condition of the intercept are both known. Also, we assume in the sequel that the target acceleration a_T is constant.

A popular way to derive a guidance law is to optimize the quadratic criterion

$$J_s(\mathbf{x}(0), u_M(\cdot)) = \frac{1}{2}\int_0^{t_f} u_M^2(t)dt, \tag{13}$$

such as e.g. Ben-Asher and Yaesh [1998], leading to the Minimum Effort Guidance Law. In fact, it is well-known that for a missile with ideal dynamics, the optimal guidance law according to this criterion is the Augmented Proportional Navigation (Garber [1968])

$$u_M = 3V_c\dot{\lambda} + \frac{3}{2}a_T.$$

The Minimum Variation Guidance law, that we propose here, is obtained by changing the quadratic criterion (13) to become

$$J(\mathbf{x}(0), u_M(\cdot), \xi) = \frac{1}{2}\int_0^{t_f} [(u_M(t) - \xi)^2 + \rho u_M^2(t)]dt. \tag{14}$$

Here $\rho \geq 0$ is a fixed constant, whereas the minimum is taken with respect to $u_M(\cdot)$ and $\xi \in \mathbb{R}$. In fact, the approach can be extended without any complication to the case that ρ is time varying and there might be advantage in doing so, however we will not follow this extension here.

To solve this optimal control problem it is advantageous to perform in (8) the well-known Zero Effort Miss distance transformation

$$Z(t) = \left[\, 1 \;\; 0 \;\; [0] \,\right] \left(\Phi(t_f, t)\mathbf{x}(t) + \int_t^{t_f} \Phi(t_f, \tau)d\tau \mathbf{C}a_T \right),$$

where $\Phi(\cdot,\cdot)$ is the transition matrix

$$\Phi(t, \tau) = \exp(\mathbf{A}(t - \tau)). \tag{15}$$

In fact, Z can be written more explicitly as

$$Z(t) = z(t) + (t_f - t)\dot{z}(t) - \left(\int_t^{t_f} (t_f - \sigma)\mathbf{C}_M e^{\mathbf{A}_M \sigma} d\sigma\right) \mathbf{x}_M(t) + \frac{(t_f - t)^2}{2} a_T \cos(\gamma_{T0} + \theta_0) \quad (16)$$

With this change of coordinates

$$\dot{Z} = b_M(t) u_M, \quad (17)$$

where

$$b_M(t) = \left[\, 1 \;\; 0 \;\; [0] \,\right] \Phi(t_f, t)\mathbf{B}, \quad (18)$$

with the boundary conditions

$$Z(0) = z(0) + t_f \dot{z}(0) - \left(\int_0^{t_f} (t_f - \sigma)\mathbf{C}_M e^{\mathbf{A}_M \sigma} d\sigma\right) \boldsymbol{x}_M(0) + \frac{t_f^2}{2} a_T \cos(\gamma_{T0} + \theta_0), \quad (19)$$
$$Z(t_f) = 0.$$

The Hamiltonian function for the problem (17),(19),(14) is

$$H(z, \lambda, u_M, t) = p b_M(t) u_M - \frac{1}{2}[(u_M(t) - \xi)^2 + \rho u_M^2(t)], \quad (20)$$

where

$$-\dot{p} = \frac{\partial H}{\partial z} = 0,$$

thus p is constant. $u_M(t)$ can be uniquely determined to maximize the Hamiltonian (20) as

$$u_M(t) = \frac{b_M(t)}{\rho + 1} p + \frac{\xi}{\rho + 1}. \quad (21)$$

Introducing this expresion into (17) and integrating between 0 and t_f, we obtain

$$0 = Z(t_f) = Z(0) + \frac{\int_0^{t_f} b_M^2(\sigma) d\sigma}{\rho + 1} p + \frac{\int_0^{t_f} b_M(\sigma) d\sigma}{\rho + 1} \xi$$

From this, we determine p to be

$$p = -\frac{\rho + 1}{\int_0^{t_f} b_M^2(\sigma) d\sigma} Z(0) - \frac{\int_0^{t_f} b_M(\sigma) d\sigma}{\int_0^{t_f} b_M^2(\sigma) d\sigma} \xi.$$

Using (18) and substituting this expression into (21), we obtain the optimal guidance command as a function of ξ in the form

$$u_M(t) = a_\xi(t) Z(0) + b_\xi(t)\xi, \quad (22)$$

where

$$a_\xi(t) = -\frac{b_M(t)}{\int_0^{t_f} b_M^2(\sigma) d\sigma}, \quad (23)$$

$$b_\xi(t) = \frac{1}{\rho + 1}\left(1 - \frac{b_M(t) \int_0^{t_f} b_M(\sigma) d\sigma}{\int_0^{t_f} b_M^2(\sigma) d\sigma}\right). \quad (24)$$

Introducing the expression (22) into (14), the quadratic cost function can be written as an algebraic quadratic function of ξ with a positive leading coefficient and its minimum with respect to ξ can readily be found to be attained for

$$\xi_{opt} = - \frac{(\rho + 1) \int_o^{t_f} a_\xi(s) b_\xi(s) ds - \int_o^{t_f} a_\xi(s) ds}{(\rho + 1) \int_o^{t_f} b_\xi^2(s) ds - 2 \int_o^{t_f} b_\xi(s) ds + t_f} Z(0) \quad (25)$$

which can be substituted back in (22) to obtain the optimal acceleration command as

$$u_M(t) = \left[a_\xi(t) - b_\xi(t) \times \frac{(\rho + 1) \int_o^{t_f} a_\xi(s) b_\xi(s) ds - \int_o^{t_f} a_\xi(s) ds}{(\rho + 1) \int_o^{t_f} b_\xi^2(s) ds - 2 \int_o^{t_f} b_\xi(s) ds + t_f} \right] Z(0) \quad (26)$$

Here, $Z(0)$ can be substituted from equation (19). This optimal command can be implemented as a guidance law by writing this acceleration command for the interval $[t, t_f]$ and as a function of $t_{go} = t_f - t$:

$$u_M(t) = \left[\bar{a}(t_{go}) - \bar{b}(t_{go}) \times \frac{(\rho + 1) \int_0^{t_{go}} \bar{a}(s)\bar{b}(s) ds - \int_0^{t_{go}} \bar{a}(s) ds}{(\rho + 1) \int_0^{t_{go}} \bar{b}^2(s) ds - 2 \int_0^{t_{go}} \bar{b}(s) ds + t_{go}} \right] \times \left(z + t_{go}\dot{z} - \left(\int_0^{t_{go}} (t_{go} - \sigma)\mathbf{C}_M e^{\mathbf{A}_M \sigma} d\sigma\right) \mathbf{x}_M + \frac{t_{go}^2}{2} a_T\right), \quad (27)$$

where, using the expressions (23), (24), and (18),

$$\bar{a}(t) = -\frac{\bar{b}_M(t)}{\int_0^t \bar{b}_M^2(\sigma) d\sigma}, \quad (28)$$

$$\bar{b}(t) = \frac{1}{\rho + 1}\left(1 - \frac{\bar{b}_M(t) \int_0^t \bar{b}_M(\sigma) d\sigma}{\int_0^t \bar{b}_M^2(\sigma) d\sigma}\right), \quad (29)$$

with

$$\bar{b}_M(t) = \left[\, 1 \;\; 0 \;\; 0 \,\right] e^{\mathbf{A}t}\mathbf{B}. \quad (30)$$

By using the small deviation from the collision triangle assumption, the displacement z, normal to the initial line-of-sight, can be approximated by

$$z \approx (\theta - \theta_0)\, r \quad (31)$$

Differentiating Eq. (31) with respect to time yields

$$\dot{\theta} = \frac{z(t) + t_{go}\dot{z}}{-V_r t_{go}^2}$$

in the expression (27), the optimal guidance law can be written in a form reminiscent of the Augmented Proportional Navigation (APN) guidance law:

$$u_M(t) = N(t_{go})\left(-V_r \dot{\theta} - \frac{1}{t_{go}^2} \left(\int_0^{t_{go}} (t_{go} - \sigma)\mathbf{C}_M e^{\mathbf{A}_M \sigma} d\sigma\right) x + \frac{1}{2} a_T\right), \quad (32)$$

where

$$N(t_{go}) = \left[\bar{a}(t_{go}) - \bar{b}(t_{go}) \frac{(\rho + 1) \int_0^{t_{go}} \bar{a}(s)\bar{b}(s) ds - \int_0^{t_{go}} \bar{a}(s) ds}{(\rho + 1) \int_0^{t_{go}} \bar{b}^2(s) ds - 2 \int_0^{t_{go}} \bar{b}(s) ds + t_{go}} \right] t_{go}^2. \quad (33)$$

Two particular cases are of great interest. For a first order model of the missile dynamics,

$$\mathbf{A}_M = -\frac{1}{\tau_M},\ \mathbf{B}_M = \frac{1}{\tau_M},\ \mathbf{C}_M = 1,\ \mathbf{D}_M = 0, \quad (34)$$

and the variable $\boldsymbol{x}$ coincides with the missile acceleration a_M that should be available from accelerometer measurements. The navigation constant (33) becomes

$$\begin{aligned} N(t_{go}) &= \frac{A(t_{go})}{B(t_{go})}, \quad (35) \\ A(t_{go}) &= t_{\mathrm{go}}^2 e^{\frac{t_{\mathrm{go}}}{\tau_M}} \Bigg[-2\tau_M t_{\mathrm{go}} \left((\rho+1) e^{\frac{t_{\mathrm{go}}}{\tau_M}} - \rho \right) \\ &\quad + (2\rho+1) t_{\mathrm{go}}^2 e^{\frac{t_{\mathrm{go}}}{\tau_M}} + 2\tau_M^2 \left(e^{\frac{t_{\mathrm{go}}}{\tau_M}} - 1 \right) \Bigg] \\ B(t_{go}) &= \tau_M^3 t_{\mathrm{go}} \left(e^{\frac{t_{\mathrm{go}}}{\tau_M}} - 1 \right) \left((3+4\rho) e^{\frac{t_{\mathrm{go}}}{\tau_M}} + \rho \right) \\ &\quad + 2\tau_M^2 t_{\mathrm{go}}^2 e^{\frac{t_{\mathrm{go}}}{\tau_M}} \left((\rho+2) e^{\frac{t_{\mathrm{go}}}{\tau_M}} - 2\rho - 1 \right) \\ &\quad - 2(\rho+1)\tau_M t_{\mathrm{go}}^3 e^{\frac{2t_{\mathrm{go}}}{\tau_M}} + \frac{4\rho+3}{6} t_{\mathrm{go}}^4 e^{\frac{2t_{\mathrm{go}}}{\tau_M}} \\ &\quad + 2\tau_M^4 \left(e^{\frac{t_{\mathrm{go}}}{\tau_M}} - 1 \right)^2 \end{aligned}$$

and the guidance law can be written as

$$\begin{aligned} u_M(t) = N(t_{go})[-V_r\dot{\theta} - \frac{\tau_M}{t_{go}} \left(1 - \frac{\tau_M}{t_{go}} (1 - e^{-\frac{\tau_M}{t_{go}}}) \right) a_M \\ + \frac{1}{2} a_T], \end{aligned} \quad (36)$$

If the missile dynamics is assumed ideal, that is $\tau_M \to 0$, the navigation constant becomes

$$N = \frac{6(2\rho+1)}{4\rho+3}. \quad (37)$$

and does not depend on the time-to-go. The optimal guidance command is in this case

$$u_M = N(-V_r\dot{\theta} + \frac{1}{2} a_T). \quad (38)$$

Notice that taking the limit $\rho \to \infty$ in (37), the navigation constant for the ideal missile dynamic is $N = 3$, and corresponds to the Minimum Effort guidance, whereas for $\rho = 0$, the optimal navigation constant is $N = 2$, leading to a constant maneuver and a circular path.

4. PERFORMANCE EVALUATION STUDIES

There are many performance evaluation issues that can be raised about the proposed class of guidance laws. We will only limit ourselves in this paper to a few of them, leaving many interesting questions for future work.

4.1 Linear analysis using the Method of Adjoints

The first aspect that we consider in this section is the effect of the design parameter ρ. For this, we consider only the linear case, without acceleration saturation, and we investigate the performance of the guidance law (38), when applied to a guidance loop of a missile of first order dynamics described by (3) with (34). For this case, we use the Method of Adjoints (Zarchan [2002]) to determine the contribution of the heading error, and of the target acceleration to the miss distance. Notice that it makes no sense to analyse the miss distance performance for the case of the guidance law (36) applied to a first order missile dynamics model, or the guidance law (38) to a missile model with ideal dynamics, since these guidance laws were designed to provide zero miss distance.

The results of the adjoint simulation for the case $V_M = 1000$, and $\tau_M = 0.2$ are represented in Figures 2a and 2b (different values of these parameters lead to different absolute values, but the relative aspect of the plots is unchanged). The parameter ρ was given four values 10^{-2}, 10^{-1}, 1 and 10.

As it is apparent from these figures, there is no uniform tendency of the performance as ρ is decreased. There are values of the time of flight for each of the chosen value of ρ to achieve best performance with respect to the target maneuver. This indicates that the choice of ρ may not be trivial, and that presumably a variable ρ over the time of flight may provide performance improvement. However, this question will remain for future research.

4.2 Performance analysis with bounded missile acceleration

As the stated motivation for proposing these guidance laws was to avoid the loss of performance due to limitations on the missile lateral acceleration, it is natural to examine how these guidance law perform when the lateral acceleration of the missile is not unlimited. In this case, we use the guidance law (36) that compensates for the first order missile dynamic. The effect of varying the parameter ρ is clearly visible in Figure 3. For $\rho = 0$, the missile acceleration is approximately constant along the flight, as expected, and it is only slightly larger than the target acceleration as can be seen in Figure 3a. As ρ is increased to 10, the missile acceleration peaks (and saturates) early in the flight as visible in Figure 3b. Although in the last case, the miss distance is larger, especially for the slower missile dynamics $\tau_M = 0.3$, it is still relatively small despite the saturation.

An entire different picture is revealed in Figure 4. In this case, the acceleration was allowed to switch sign in the middle of the flight (an S-maneuver). The maximum missile acceleration was allowed in this case to be $700 m/s^2$. Even so, for $\rho = 10$, the miss distance is very large for the larger values of the missile time constant. However, for the case $\rho = 0$, the missile achieves good performance and the lateral acceleration does not saturate.

5. CONCLUSIONS AND FUTURE WORK

We introduced a new class of guidance laws for homing missiles that does not only attempt to minimize the total maneuvering effort, but also to reduce the variation of the lateral acceleration during the intercept. The motivating idea behind this approach was to obtain high intercept accuracy even in the case that the maneuverability advantage of the interceptor with respect to the evader is minimal. We have shown that the proposed guidance law design succeeds indeed to deliver better performance in this respect.

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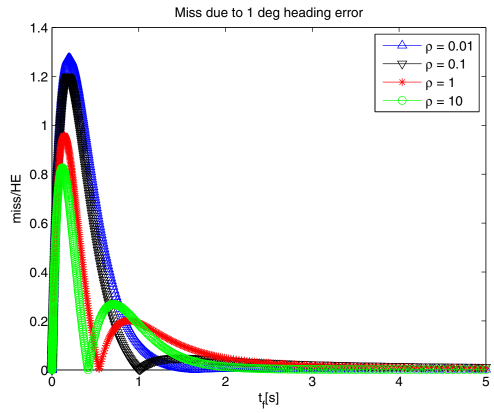
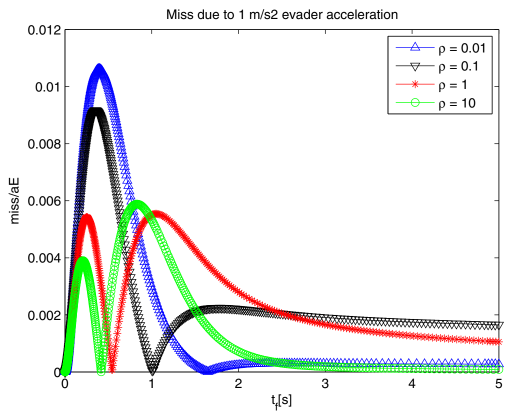
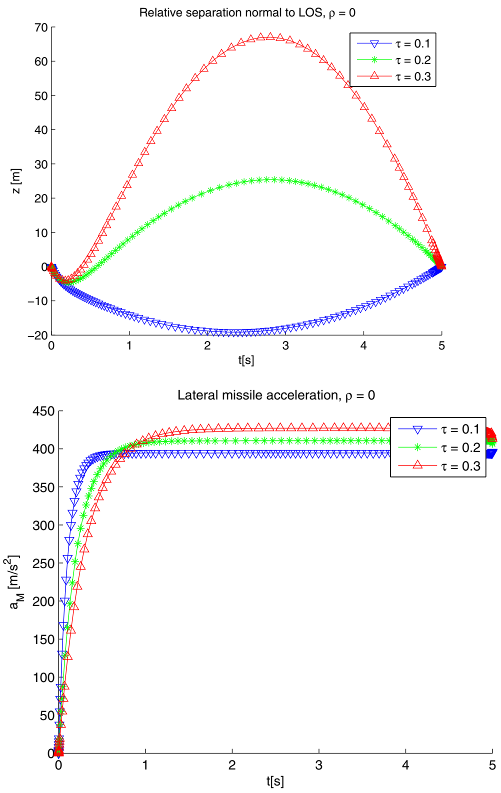


(a) Influence of heading error on the miss distance for different values of ρ.

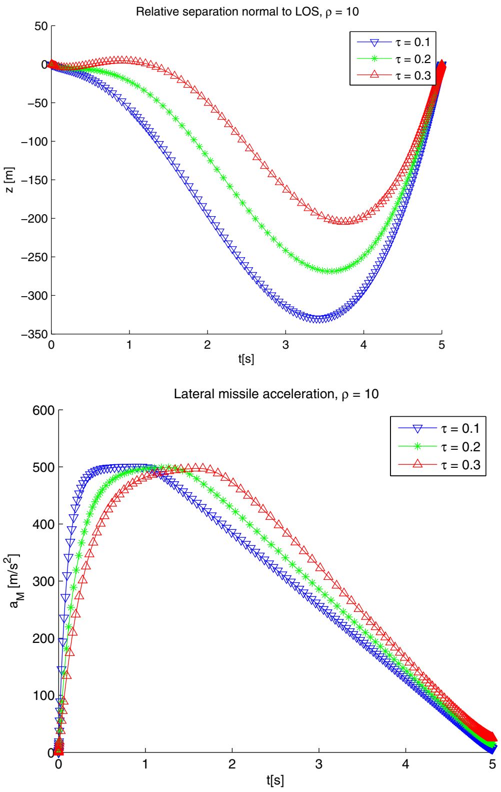


(b) Influence of target acceleration on the miss distance for different values of ρ.

Fig. 2. Results of adjoint simulation: influence on the miss distance as function of the time of flight t_f.

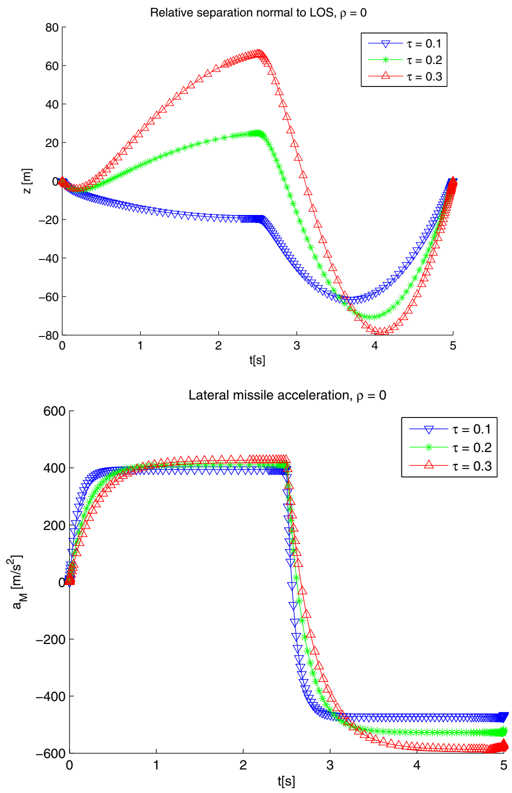


(a) Separation from the line-of-sight and missile acceleration for $\rho = 0$.

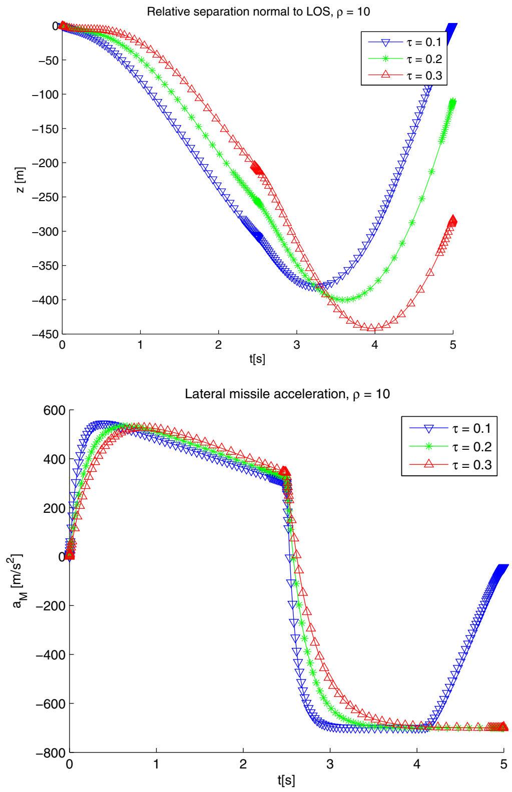


(b) Separation from the line-of-sight and missile acceleration for $\rho = 10$.

Fig. 3. Single flight results for different ρ and τ_M values, for $a_{M,max} = 500m/s^2$ and a constant target maneuver $a_T = 400m/s^2$.



(a) Separation from the line-of-sight and missile acceleration for $\rho = 0$.



(b) Separation from the line-of-sight and missile acceleration for $\rho = 10$.

Fig. 4. Single flight results for different ρ and τ_M values, for $a_{M,max} = 700m/s^2$ and a target maneuver $a_T = 400m/s^2$ switching signs in the middle of the flight.