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Compensation in Repetitive Control System for Aperiodic Disturbances and Input Dead Zone

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Abstract: This paper extends the equivalent-input-disturbance (EID) approach to deal with the problem of aperiodic disturbance rejection for a plant with an input dead zone in a repetitive-control system so as to improve control performance for the tracking of a periodic signal. Since a dead zone greatly degrades control performance, we apply the EID approach to design a compensator for the nonlinearity by treating it as an input-dependent disturbance. An EID estimator is constructed by making the best use of a full-order generalized state observer (GSO). And a method of designing the GSO is explained. The EID estimate, which exhibits the synthetic effect of the nonlinearity and the aperiodic disturbance, is incorporated into a repetitive control law to compensate for the nonlinearity and the aperiodic disturbance. This method does not require any information about the dead zone. It guarantees perfect tracking for periodic reference input and satisfactory compensation of input dead zone and aperiodic disturbance at the same time. Simulation results demonstrate the effectiveness of this method.

Keywords: Dead zone, disturbance rejection, equivalent input disturbance (EID), generalized state observer (GSO), pole placement, repetitive control.

1. INTRODUCTION

Repetitive control (RC) [Inoue et al., 1981] is a widely used control approach that enables the perfect tracking of a periodic reference input and/or the perfect rejection of a periodic disturbance. The core of RC is the use of an internal model of a periodic signal that simulates human behavior of repetitive training with learning.

A dead zone is a common nonlinearity existed in an actuator. It may seriously degrade control performance. We can use an inverse model to directly compensate for it [Recker et al., 1991]. However, it is not a easy task to build a precise model. [Wang et al., 2004] used a sliding mode to deal with a dead zone, but the chatter may damage the plant or cause other problems. Intelligent control methods, such as neural networks and adaptive fuzzy control, have also been used to solve this problem [Selmic & Lewis, 2000, Nishikawa & Yoneyama, 2010], but they are computationally expensive.

The equivalent-input-disturbance (EID) approach is an active disturbance rejection method [She et al., 2008, 2011]. It was extended to compensate for an unknown input dead zone by treating the effect of a dead zone as a state-dependent disturbance [Ouyang et al., 2012]. This study extends the above result to a repetitive-control system (RCS) and presents a method of designing an EID-based RCS that actively compensates for an input dead zone and effectively rejects aperiodic disturbances. The design of the system is divided into two parts: a conventional state-feedback RCS and an EID compensator. A generalized state observer (GSO) is employed in the design of the EID estimator, and a pole-assignment method is used to find the appropriate parameters of the GSO. The introduction of a full-order GSO in the construction of the EID estimator [Liu et al., 2013] increases the flexibility of the EID-based RCS. The main advantages of this method are

- No precise information on the structure or parameters of the dead zone is needed.
- The structure of control system is simple and easy to implement.
- The flexibility of the GSO provides us a potential to achieve good control performance.
- The system compensates satisfactorily for the influence of a dead zone.

2. DESCRIPTION OF PLANT

Consider a continuous-time plant with an input dead zone that is subjected to an aperiodic disturbance

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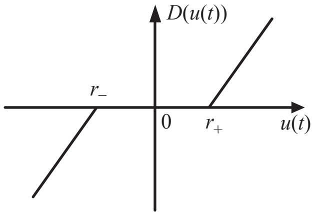

Fig. 1. Model of dead zone.

$$\begin{cases} \dot{x}(t) = Ax(t) + BD(u(t)) + B_d d_{ap}(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the plant; $y(t) \in \mathbb{R}^q$ is the control output; $D(u(t))$ is the input dead zone (Fig. 1); $d_{ap}(t)$ is an aperiodic disturbance; and A, B, B_d, and C are constant real matrices with appropriate dimensions.

The output of the dead zone is

$$u_p(t) = D(u(t)) = \begin{cases} u(t) - r_+, \ u(t) > r_+, \\ \quad 0, \qquad r_- \le u(t) \le r_+, \\ u(t) + r_-, \ u(t) < r_-. \end{cases} \tag{2}$$

We decompose it into

$$u_p(t) = u(t) + d(u(t)), \tag{3}$$

where $d(u(t))$ can be viewed as an input-dependent disturbance.

We make the following assumptions for the plant (1).

Assumption 1. The plant (A, B, C) is controllable and observable.

Assumption 2. The plant (A, B, C) has no zeros on the imaginary axis.

Assumption 3. The two parameters of the dead zone in (2), r_+ and r_-, have an upper bound and are unknown.

Assumption 1 is standard for designing an observer-based servo system. Assumption 2 is necessary to guarantee the internal stability of a servo system. And Assumption 3 holds in many practical systems. Since it is hard to obtain precise information on a dead zone, an inverse model is difficult to be applied.

Since the control input, $u(t)$, is determined by the state, $x(t)$, $d(u(t))$ in (3) can be viewed as a state-dependent disturbance. According to [She et al., 2010, 2012], there exists an EID whose effect on the output is equivalent to the overall effect of $d(u(t))$ and $d_{ap}(t)$. So, (1) can be represented as a linear plant with an EID (Fig. 2)

$$\begin{cases} \dot{x}(t) = Ax(t) + B[u(t) + d_e(t)], \\ y(t) = Cx(t). \end{cases} \tag{4}$$

Formulating the problem of compensating for the dead zone and rejecting an aperiodic disturbance as the problem of rejecting an EID enables us to design an EID estimator that automatically compensates for the influence of the dead zone, and suppresses the disturbance at the same time.

3. DESIGN OF EID-BASED RCS

This section considers three aspects of the EID-based RCS: the configuration, the stability, and the design algorithm.

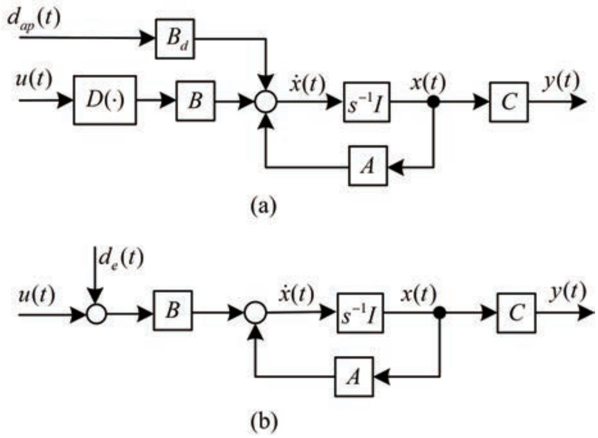

Fig. 2. Equivalent expressions for plant with input dead zone and aperiodic disturbance: (a) original plant and (b) plant with EID.

3.1 System Configuration

An EID-based RCS (Fig. 3) has five parts: the plant, a repetitive controller, a state observer, state feedback, and an EID estimator.

We use a repetitive controller to track a periodic reference input. The repetitive controller contains a time delay, $e^{-\tau s}$, and a low-pass filter, $q_R(s)$. τ is the period of the reference input. And $q_R(s)$ relaxes the stability condition of the system. It is chosen to be

$$q_R(s) = \frac{\omega_r}{s + \omega_r}, \tag{5}$$

where ω_r is the cutoff angular frequency of $q_R(s)$. The state-space representation of the repetitive controller is

$$\dot{x}_f(t) = -\omega_r x_f(t) + \omega_r x_f(t - \tau) + \omega_r e(t). \tag{6}$$

In the EID estimator,

$$B^+ := (B^{\mathrm{T}} B)^{-1} B^{\mathrm{T}}. \tag{7}$$

The EID estimator estimates the overall effect of $d(u(t))$ and $d_{ap}(t)$, and feeds it back to the control input channel to compensate for them. $q_d(s)$ in the EID estimator is a low-pass filter that selects the bandwidth of the angular frequency for estimation. A first-order low-pass filter is used in the study

$$q_d(s) = \frac{1}{T_d s + 1}, \ \omega_d = 1/T_d. \tag{8}$$

The state feedback control law

$$u_f(t) = K_P x(t) + K_R x_R(t) \tag{9}$$

ensures the stability of the system, where

$$x_R(t) = e(t) + x_f(t - \tau). \tag{10}$$

A GSO is

$$\begin{cases} \dot{z}(t) = Fz(t) + Gy(t) + Hu_f(t), \\ \hat{x}(t) = T^{-1} z(t). \end{cases} \tag{11}$$

We assume that F is Hurwitz, T is nonsingular,

$$TA - FT = GC, \tag{12}$$

and

$$H = TB. \tag{13}$$

For the state observer, we have

$$\dot{\hat{x}}(t) = T^{-1}[Fz(t) + Gy(t) + Hu_f(t)]. \tag{14}$$

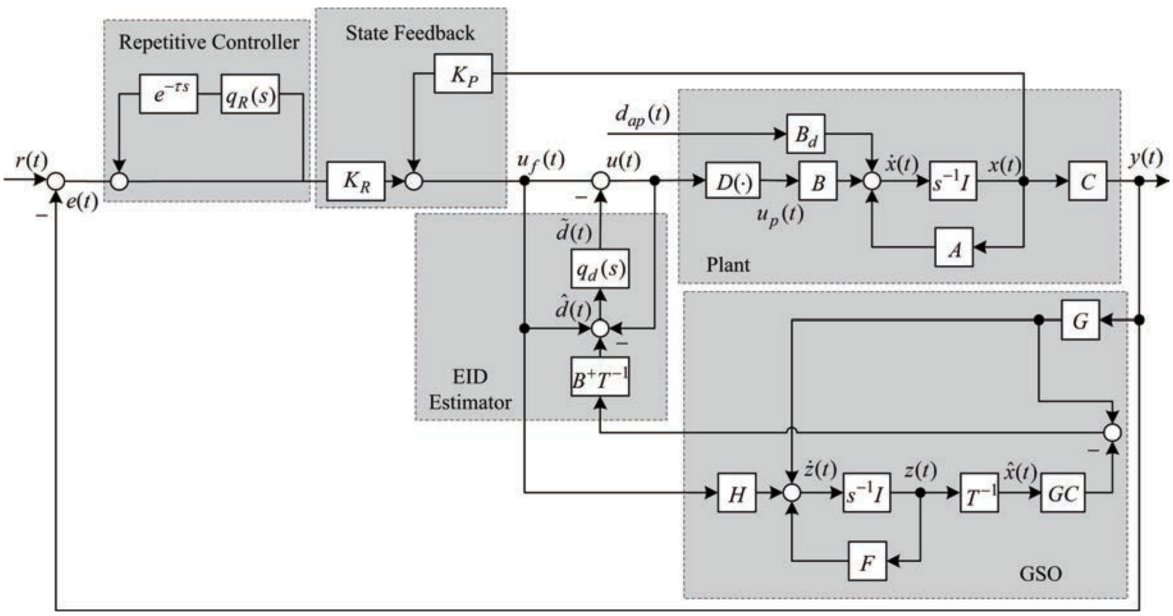

Fig. 3. EID-based RCS.

Letting

$$\delta x(t) = \hat{x}(t) - x(t) \tag{15}$$

and substituting (15) into (4) yield

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + Bd_e(t) + [\delta\dot{x}(t) - A\delta x(t)]. \tag{16}$$

We assume that there exists a $\delta d(t)$ that satisfies

$$B\delta d(t) = \delta\dot{x}(t) - A\delta x(t). \tag{17}$$

Substituting (17) into (16), and defining the estimated value of an EID to be

$$\hat{d}(t) = d_e(t) + \delta d(t) \tag{18}$$

yield

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B[u(t) + \hat{d}(t)]. \tag{19}$$

Combining (14) with (19) gives an estimate of the EID

$$\hat{d}(t) = B^+T^{-1}GC[x(t) - \hat{x}(t)] + u_f(t) - u(t). \tag{20}$$

The filtered EID, $\tilde{d}(t)$, is

$$\tilde{D}(s) = q_d(s)\hat{D}(s), \tag{21}$$

where $\tilde{D}(s)$ and $\hat{D}(s)$ are the Laplace transform of $\tilde{d}(t)$ and $\hat{d}(t)$, respectively.

Combining the EID estimate with the original state-feedback control law yields a new control law (Fig. 3)

$$u(t) = u_f(t) - \tilde{d}(t). \tag{22}$$

3.2 Stability Analysis

To perform the analysis on the stability of the system, we first consider the compensation of the dead zone in the inner loop. Then, we consider the stability issue of the whole system under the assumption that the dead zone is completely compensated for.

In the EID-based RCS, the repetitive controller ensures satisfactory tracking for periodic reference inputs. And the EID estimator actively suppresses the effect of the dead zone and aperiodic disturbances.

Let exogenous signals be zero, that is,

$$r(t) = 0, \quad d_{ap}(t) = 0. \tag{23}$$

We redraw Fig. 3 as Fig. 4. The plant is described as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu_p(t), \\ y(t) = Cx(t). \end{cases} \tag{24}$$

Combining (12), (13), (14), (15), (22), and (24) yields

$$\delta\dot{x}(t) = T^{-1}FT\delta x(t) + B[\tilde{d}(t) - d(u(t))] \tag{25}$$

and

$$\hat{d}(t) = B^+(T^{-1}FT - A)\delta x(t) + \tilde{d}(t) \tag{26}$$

Combining (25) and (26) yields

$$G_{dl}(s) = B^+(T^{-1}FT - A)(sI - T^{-1}FT)^{-1}B. \tag{27}$$

Defining

$$G_d(s) = 1 + G_{dl}(s) \tag{28}$$

gives

$$\hat{D}(s) = G_d(s)\tilde{D}(s) - G_{dl}(s)D_u(s) \tag{29}$$

where $D_u(s)$ is the Laplace transform of the signal $d(u(t))$. Note that $G_d(s)$ and $G_{dl}(s)$ in the above equations are shown in Figs. 4 and 5. And $G_{dl}(s)$ is stable if $G_{dl}(s)$ is stable.

The stability condition of the inner loop (the EID estimation and compensation) is derived from the small gain theorem [Zhou et al., 1996] and is given as follows.

Theorem 1. For a suitably designed state-feedback gain $[K_p \;\; K_R]$, to guarantee the stability of the EID-based RCS, the following conditions has to be satisfied:

(1) $q_d(s)$ and $G_d(s)$ are stable; and

(2)

$$\|q_dG_d\|_\infty < 1, \tag{30}$$

where $\|q_dG_d\|_\infty := \sup_{0\leq\omega\leq\infty} \sigma_{\max}(q_d(j\omega)G_d(j\omega))$ and $\sigma_{\max}(\cdot)$ denotes the maximum singular value.

Now, we assume that the input dead zone is fully compensated for by the EID estimator. This gives

$$u_p(t) = u_f(t). \tag{31}$$

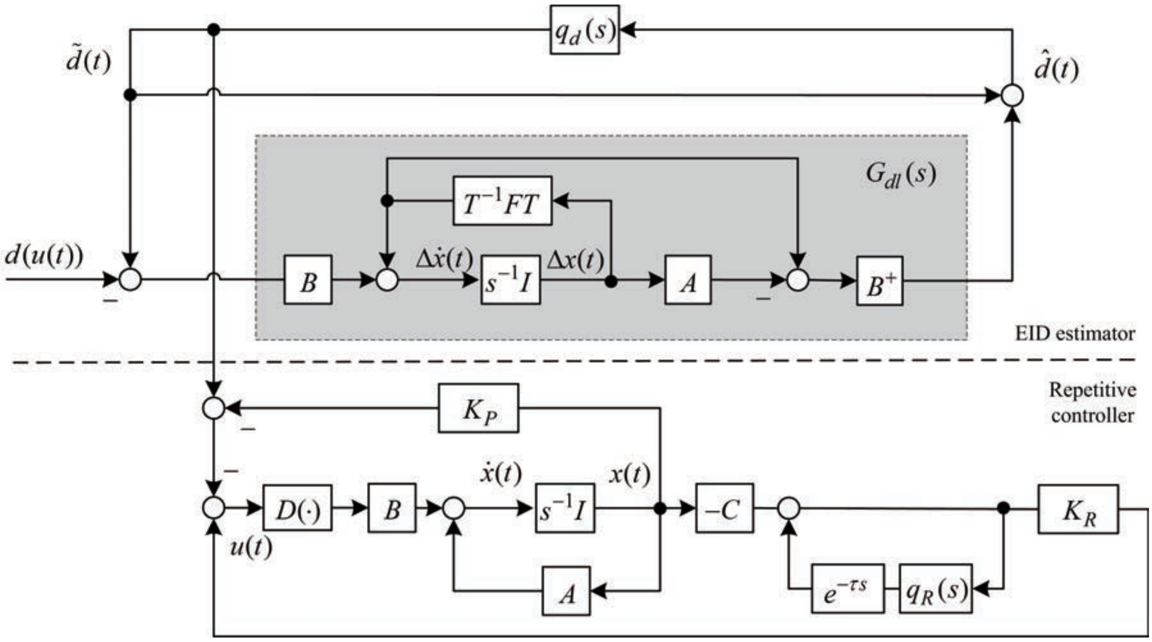

Fig. 4. Block diagram of EID-based RCS for (23).

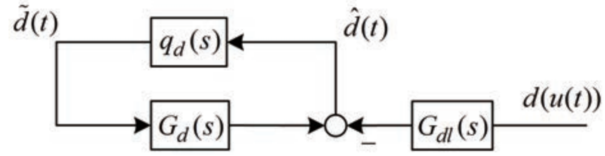

Fig. 5. Simplified block diagram of Fig. 4.

So, we can write

$$u_p(t) = K_P x(t) + K_R x_R(t). \tag{32}$$

Note that the period of the repetitive controller is τ. We define a difference operator, Δ, for any continuous vector-valued function $\xi(t)$

$$\Delta\xi(t) = \xi(t) - \xi(t-\tau). \tag{33}$$

This gives a model for the RC

$$\begin{cases} \Delta\dot{x}(t) = \bar{A}\Delta x(t) + \bar{A}_d\Delta x(t-\tau) + \bar{B}\Delta u_p(t), \\ \Delta e(t) = \bar{C}x(t), \end{cases} \tag{34}$$

and the control law

$$\Delta u_p(t) = [F_P \quad 0]\Delta x(t) + [0 \quad F_R]\Delta x(t-\tau), \tag{35}$$

where

$$\Delta x(t) = [\Delta x^T(t) \quad \Delta x_f{}^T(t)]^T,$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -\omega_r C & -\omega_r \end{bmatrix}, \quad \bar{A}_d = \begin{bmatrix} 0 & 0 \\ 0 & \omega_r \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = [\,-C \;\; 0\,],$$

and

$$F_R = K_R, \; F_P = K_P - K_R C. \tag{36}$$

Substituting (35) into (34) yields the closed-loop system

$$\begin{cases} \Delta\dot{x}(t) = \bar{A}_l\Delta x(t) + \bar{A}_{dl}\Delta x(t-\tau), \\ \Delta e(t) = \bar{C}x(t), \end{cases} \tag{37}$$

where

$$\bar{A}_l = \begin{bmatrix} A + BF_P & 0 \\ -\omega_r C & -\omega_r \end{bmatrix}, \quad \bar{A}_{dl} = \begin{bmatrix} 0 & BF_R \\ 0 & \omega_r \end{bmatrix}.$$

Since RC is a continuous control process, for the RCS in Fig. 3, if there exist a continuous energy functional $V(t)$ that monotonically decreases with time along the closed-loop system, then the whole system is asymptotically stable.

We apply the analysis result of a modified RCS in the two dimensions [Zhou et al., 2013] and obtain the following theorem regarding the stability of the closed-loop system.

Theorem 2. For given ω_r and positive scalars, α and β, if there exist symmetrical positive-definite matrices X_1, X_2, Y_1, and Y_2, together with arbitrary matrices W_1 and W_2, the following LMI holds

$$\begin{bmatrix} L_{11} & -\alpha\omega_r X_1 C^T & 0 & \beta B W_2 & \alpha X_1 & 0 \\ \star & -2\omega_r X_2 & 0 & \omega_r\beta Y_2 & 0 & X_2 \\ \star & \star & -Y_1 & 0 & 0 & 0 \\ \star & \star & \star & -\beta Y_2 & 0 & 0 \\ \star & \star & \star & \star & -Y_1 & 0 \\ \star & \star & \star & \star & \star & -\beta Y_2 \end{bmatrix} < 0, \tag{38}$$

where

$$L_{11} = \alpha X_1 A^T + \alpha A X_1 + \alpha W_1 B^T + \alpha B W_1,$$

then the closed-loop system (37) is asymptotically stable, and the control gains in (35) are

$$F_P = W_1 X_1^{-1}, \quad F_R = W_2 Y_2^{-1}. \tag{39}$$

3.3 System Design

The first algorithm is for the design of the GSO in the EID estimator. The parameters of the GSO are determined by the following steps.

GSO design algorithm:

Step 1) Choose expected stable poles for the GSO, $\lambda_1, \lambda_2, \cdots, \lambda_n$, that satisfy $\lambda_i \neq \lambda_j,\; i,j = 1,\cdots,n$.

Step 2) Calculate the characteristic polynomial given by the desirable poles, $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$:

$$\prod_{i=1}^{n}(s-\lambda_i) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0.$$

Step 3) Construct an $n \times n$ matrix $F_0 = \begin{bmatrix}[0 \;\; I]^T & \alpha^T\end{bmatrix}^T$, where $\alpha = [-\alpha_0 \;\; -\alpha_1 \;\; \cdots \;\; -\alpha_{n-1}]$.
Step 4) Choose an arbitrary nonsingular matrix $S \in \mathbb{R}^{n\times n}$, calculate its inverse, and let $F = SF_0S^{-1}$.
Step 5) Choose a matrix G that enables the pair $(F,\ G)$ is controllable.
Step 6) Solving (12) for T.
Step 7) Check whether or not T is nonsingular. If it is, go to the next step; otherwise, go to Step 3.
Step 8) Calculate H using (13).

The EID-based RCS is designed using the following algorithm.

EID-based RCS design algorithm:

Step 1) Select the cutoff frequency ω_r in (5), and positive scalars α and β, and solve the LMI (38).
Step 2) Calculate K_p and K_R using (36) and (39).
Step 3) Select the cutoff angular frequency ω_d in (8).
Step 4) Calculate matrices H, G, F, and T^{-1} in the GSO using the design algorithm of GSO.
Step 5) Check whether or not the conditions in Theorem 1 hold. If not, go back to Step 3; otherwise, finish.

4. SIMULATION VERIFICATION

A numerical example illustrates the design procedure and the validity of our method. The parameters of the plant (1) are

$$A = \begin{bmatrix} -31.31 & 0 & -2.833\times 10^4 \\ 0 & -10.25 & 8001 \\ 1 & -1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 28.06 \\ 0 \\ 0 \end{bmatrix},$$

$$C = [\,1\ 0\ 0\,].$$

The parameters in the dead zone are

$$r_- = -0.8, \quad r_+ = 0.7.$$

Let the reference input be

$$r(t) = \sin \pi t. \tag{40}$$

and the aperiodic disturbance be

$$d_{ap}(t) = \begin{cases} \sin 0.5\pi t + \cos 0.5\pi t + \sin t & \\ +\cos 2\pi t + \sin 4\pi t, & 10 \le t \le 20, \\ 0, & \text{otherwise.} \end{cases} \tag{41}$$

The delay time in the repetitive controller is

$$\tau = 2 \text{ s}.$$

The cutoff frequency of the low-pass filter was chosen to be

$$\omega_r = 100 \text{ rad/s}.$$

The performance index was chosen to be

$$J = \sum_{k=0}^{20} \int_{k\tau}^{(k+1)\tau} e^2(t)dt. \tag{42}$$

The two parameters in the LMI (38) were selected based on the evaluation result of the performance index and were

$$\alpha = 1.57, \quad \beta = 1. \tag{43}$$

The corresponding gains in the feedback controller are

$$K_R = 9.6607, K_P = [1.4 \;\; 0 \;\; 1009.0]\,. \tag{44}$$

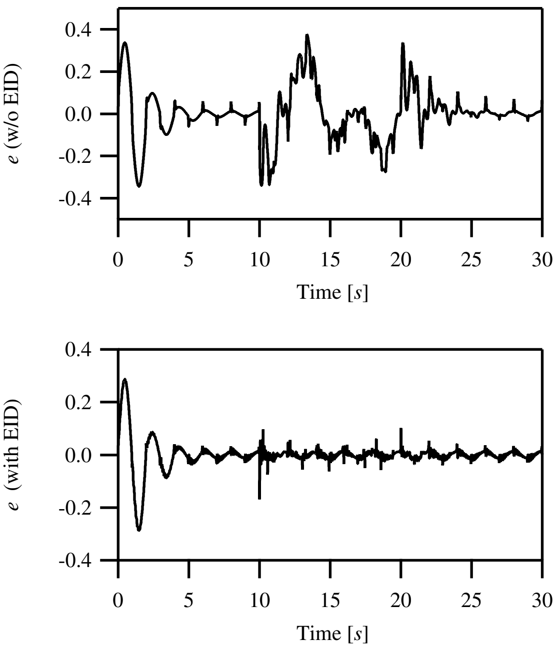

Fig. 6. Tracking error: (a) without EID compensation and (b) with EID compensation.

For which,

$$J = 0.0007.$$

Then we set $T_d = 0.001$ s. Selecting the poles of the GSO to be $\{-40 \pm 65j, -310\}$ yielded the GSO

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1805750 & -30625 & -390 \end{bmatrix},\ G = \begin{bmatrix} 972.31334 \\ -11.57738 \\ -0.11083 \end{bmatrix},$$

$$T = \begin{bmatrix} 3.868 & 13.781 & 937.105\times 10^4 \\ -156.318 & -1078.355 & 675.193 \\ 5581.091 & 10377.948 & -4199426.897 \end{bmatrix},$$

$$H = \begin{bmatrix} 108.53837 \\ -4386.28692 \\ 156605.40984 \end{bmatrix}.$$

As a result,

$$\|q_d G_d\|_\infty = 0.927 < 1.$$

Fig. 6 shows the tracking errors of the RCS without EID compensation and the EID-based RCS. It is clear that, since the EID estimator compensated for the effect of dead zone and aperiodic disturbance, the largest steady-state peak-to-peak tracking error decreased from 0.105 to 0.067, and the transient tracking error was also greatly reduced. The aperiodic disturbance, the control input, and the output of the EID-based RCS are shown in Fig. 7.

5. CONCLUSION

In this study, we treated an input dead zone as an input-independent disturbance and inserted an EID estimator in an RCS to estimate and compensate for the synthetic effect caused by the dead zone and the aperiodic disturbance. A GSO was introduced in the EID estimator so as to increase the flexibility in the design of the system. Under the assumption that the dead zone was fully compensated

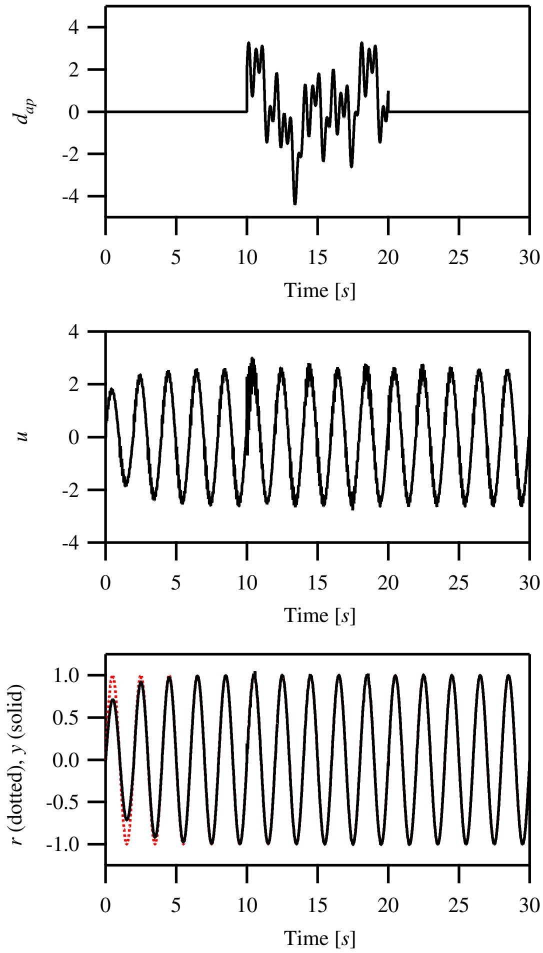

Fig. 7. Simulation results of EID-based RCS.

for, the design of the system was divided into two parts: the EID compensator and the conventional state-feedback-based RCS. We carried out an analysis on the stability of the system, and presented the design algorithms for the EID estimator and the EID-based RCS.

The advantage of this method is that we do not need any information on a dead zone or construct an inverse model to compensate for it. The simulation results show that the designed system is stable, and tracking performance was improved by the incorporation of the EID estimator.

This paper present only with constant input dead zone. Variable input dead zone and relationship between the dead zone and control performance will be considered in the near future.

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