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A Framework for the Nonlinear Control of Dual-Stage Systems ⋆

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Abstract: This paper presents a framework for the nonlinear control of dual-stage actuators (DSA). Motivated by various nonlinear controllers that make use of sector bounded and $\mathcal{L}_\infty$ nonlinearities for the control of saturated linear systems, a methodology for integrating such nonlinear functions in order to improve the performance of DSA is presented. The stability of the closed-loop system is assessed by casting the nonlinearities in a mixed sector-bounded plus *quasi*-Linear Parameter Varying (LPV) framework, leading to a set of linear matrix inequalities (LMIs) to be satisfied by the controller parameters. Taking advantage of the developed framework, a new $\mathcal{L}_\infty$ function is proposed to avoid the saturation of the secondary actuator. Simulation results illustrate the validity of the proposed framework and its potential for the performance improvement of DSA.

Keywords: DSA, Nonlinear Feedback, Time-Optimal Performance, Macro-Micro Manipulator.

1. INTRODUCTION

Dual-Stage Actuators (DSA) are comprised of two actuators connected in series with complementary characteristics: a primary (or coarse) actuator with a long travel range but slow response time, and a secondary (fine) actuator of faster response but limited travel range. An improved performance can be achieved by combining both actuators and making use of an appropriate control strategy so that the defects of one are compensated by the merits of the other. This class of systems became popular within high precision motion applications such as the Hard Disk Drives [Mori et al., 1991] and advanced microlithography [Lee and Kim, 1997], providing the motion systems with a significant increase in servo bandwidth and disturbance rejection. Recently other applications have been employing some form of dual stage actution, such as X-Y systems Fung et al. [2009] and nanopositioning scanners Tuma et al. [2013].

Since DSA possess dual-input and single-output configurations, any given trajectory may be generated by different combinations of inputs. This extra freedom makes the control design of such systems a rather challenging process leading to control strategies that vary from advanced nonlinear preview control to coprime factorization, among others (see [Salton et al., 2013] and references therein). While the majority of DSA are linear systems subject to input saturation, they benefit from a number of nonlinear control strategies designed for the performance improvement of linear systems. Maybe the most well known examples are the so-called Proximate-Time Optimal Servomechanism (PTOS), where saturation is neatly taken into account, and the Composite Nonlinear Control,where a form of nonlinear damping is achieved. While the former control method uses a sector bounded nonlinearity [Flores et al., 2013], the latter applies an $\mathcal{L}_\infty$-bounded nonlinear function that may be addressed by *quasi*-Linear Parameter Varying (LPV) methods. This paper proposes to provide a framework for the nonlinear control of DSA by considering both sector and $\mathcal{L}_\infty$-bounded nonlinear control functions subject to saturated inputs.

The stability analysis of linear systems subject to control saturation with, for instance, $\mathcal{L}_\infty$-bounded nonlinear functions that can be cast as polytopic uncertainties was thoroughly addressed by Henrion et al. [1999], Wu et al. [2007] – and references therein – using an LPV approach. Also, Henrion et al. [1999] addressed the problem of robust control design for uncertain linear systems subject to control saturation where a saturating linear output feedback law and a safe set of initial conditions are determined using iterative LMI procedures. The work by Wu et al. [2007] proposes the design of a nonlinear output

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feedback controller in the form of a quasi-LPV system and the saturation nonlinearity is considered by means of a modified polytopic model [Hu and Lin, 2001]. Also note that the stability of the closed-loop system containing sector-bounded nonlinearities can be assessed using the absolute stability framework as shown by Khalil [2002].In this context, the work by Gomes da Silva Jr. et al. [2013] can be considered as a basis for the stability analysis of Lur'e type nonlinear systems with saturating control laws.

In order to provide a framework for the nonlinear control of DSA considering both LPV and sector bounded nonlinearities, this paper proposes to join the ideas presented by Wu et al. [2007] and Gomes da Silva Jr. et al. [2013]. A motivational example will be given justifying the necessity of this general framework for performance improvement of DSA. The paper is organized as follows: Section II will present the system of interest, Section III will derive stability conditions for DSA under nonlinear controllers, Section IV will show how the proposed framework may be used to improve the performance of the system, Section V will show a numerical example and Section VI will conclude the paper.

Notation: v_i denotes the i^{th} element in vector v, N_i denotes the element in position $N(i,i)$ in a diagonal matrix N and $L_{(i)}$ denotes the i^{th} row of matrix L. $Co\{\}$ denotes the convex hull.

2. SYSTEM OF INTEREST

The system under consideration is comprised of a primary actuator described by a (possibly damped) double integrator, and a mass-spring-damper secondary actuator. This model is depicted in Fig. 1 and describes a general type of DSA where a linear motor is used to drive the primary stage and a piezoelectric actuator to drive the secondary stage. In the figure, M, y_1 and u_1 (m, y_2 and u_2) are the mass, position and input of the primary (secondary) actuator, respectively.

From Lagrangian mechanics one may readily obtain the equations of motion of the system as follows,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_i}\right) - \left(\frac{\partial L}{\partial y_i}\right) = f_i, \quad i = 1,\ 2, \tag{1}$$

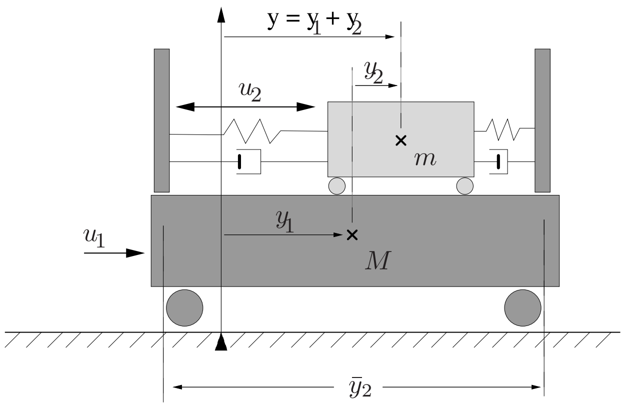

Fig. 1. Schematic representation of a typical dual-stage actuator (DSA).

where $L = K_e - P_e$ is the difference between the kinetic (K_e) and potential (P_e) energies of the system, and f_i describes the external forces acting on each actuator.

Solving (1) for y_1 and y_2, the equations of motion of the DSA are obtained,

$$\begin{bmatrix} M+m & m \\ m & m \end{bmatrix} \ddot{\mathbf{y}} = \begin{bmatrix} 0 & 0 \\ 0 & -k \end{bmatrix} \mathbf{y} - \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \dot{\mathbf{y}} + \begin{bmatrix} \mathrm{sat}(u_1) \\ \mathrm{sat}(u_2) \end{bmatrix} \tag{2}$$

where $\mathbf{y} = [y_1 \quad y_2]'$. Solving the above for $\ddot{\mathbf{y}}$ we achieve the following equations that fully describe the system of interest:

$$\ddot{y}_1 = \frac{u_1 - c_1\dot{y}_1}{M} + \frac{ky_2 - u_2}{M} \tag{3a}$$

$$\ddot{y}_2 = \left(\frac{1}{M} + \frac{1}{m}\right)(u_2 - c_2\dot{y}_2 - ky_2) - \frac{u_1}{M} \tag{3b}$$

Thus, the DSA may be described in its state-space form as follows,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\,\mathrm{sat}(u(t)), \\ y(t) &= C(t)x, \end{aligned} \tag{4}$$

where $x(t) \in \mathbb{R}^4$ are the states

$$x(t) = [y_1(t) - r(t) \ \dot{y}_1(t) \ y_2(t) \ \dot{y}_2(t)]',$$

$u(t) \in \mathbb{R}^2$ are the inputs, $y(t) \in \mathbb{R}$ is the output given by $y(t) = y_1(t) + y_2(t)$, and $r(t) \in \mathbb{R}$. Furthermore, A, B and C are constant matrices with appropriate dimensions. Note also that the input saturation of the secondary actuator implies a maximal displacement for y_2, since this is a mass-spring-damper second order system.

Equation (4) describes all forces acting on a typical dual-stage system, including the coupling forces between the actuators. This coupling between actuators comes from the fact that the forces acting on the primary actuator influence the secondary and vice-versa. Such coupling forces may excite high frequencies and disturb the controller, degrading the system performance. In order to avoid this interaction between the primary and secondary actuators, it is common practice to design dual-stage systems such that the following assumptions are satisfied [Zheng et al., 2009]:

Υ_1: The masses of the actuators are such that $M >> m$.
Υ_2: The bandwidth of the secondary actuator is higher than that of the primary actuator

By carefully picking the actuators such that Υ_1 and Υ_2 are satisfied, equation (3b) may be simplified to:

$$\ddot{y}_2 = \tfrac{u_2 - ky_2 - c\dot{y}_2}{m}, \tag{5}$$

since $1/M$ will be negligible when compared to $1/m$ and the remaining term may be considered a disturbance well inside the bandwidth of the secondary actuator. A similar discussion regarding (3a) leads to,

$$\ddot{y}_1 = \tfrac{u_1 - c_1\dot{y}_1}{M} \tag{6}$$

since $ky_2 - u_2 << u_1$, typically.

Thus, with assumptions Υ_1 and Υ_2 satisfied, the system may be modeled as two independent actuators whose only interaction comes from the system output $y = y_1 + y_2$. The above simplification of the system is a good reason to rely on piezo actuators to drive the second stage, since they match Υ_1 and Υ_2 in most situations.

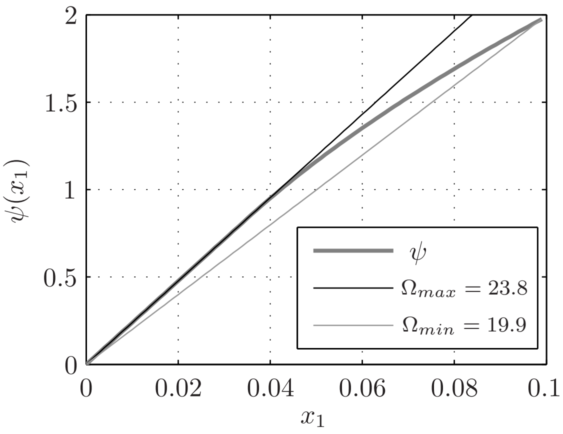

Fig. 2. Nonlinear function $\psi(x_1) = k_2 f(x_1)$ given by (8) with a local sector defined as $(\omega_M,\ \omega_m) = (23.8,\ 19.9)$.

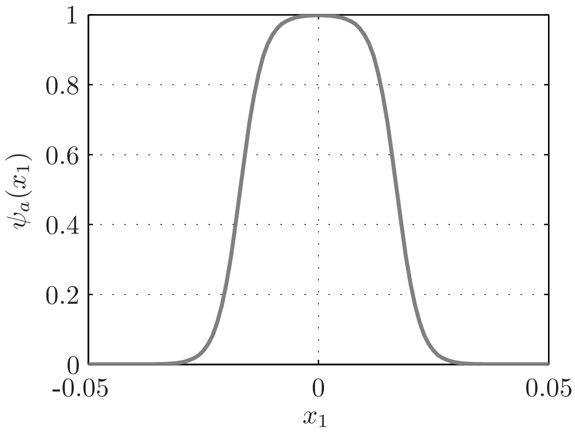

Fig. 3. Nonlinear function $\psi_a(x_1)$ given by (11) with $\beta_1 = 2.0 \times 10^2$ and $\beta_2 = 1.7 \times 10^{-2}$ used to limit the actuation of the secondary actuator.

In the section that follows we will consider the cases where the coupling forces between actuators are *not* negligible. This may occur even when systems are driven by piezo actuators, provided assumptions Υ_1 and Υ_2 are not satisfied in their entirety [Zheng et al., 2009]. We will also consider the cases where the secondary actuator is considerably undamped. In such cases it is crucial that this actuator does not saturate so that the feedback control may restrict the oscillations caused by the mechanical system.

3. IMPROVING THE PERFORMANCE OF DSA

In order to minimize the undesired effects of coupling forces and avoid large oscillations of the secondary actuator, an $\mathcal{L}_\infty$ nonlinear function will be used in the secondary actuator loop. Furthermore, a traditional near time optimal controller will be cast as a sector bounded nonlinearity and applied to the primary actuator, guaranteeing a fast rise time and good performance.

3.1 Primary Controller Design

The traditional Proximate Time-Optimal Servomechanism (PTOS) will be used to control the primary actuator. This is a practical near time-optimal controller commonly used in systems where time-optimal performance is desired. The control law is given by,

$$u_1(t) = -k_2(f(x_1) + x_2), \tag{7}$$

where,

$$f(x_1) = \begin{cases} (k_1/k_2)x_1, & |x_1| \leq y_l, \\ \mathrm{sgn}(x_1)(\sqrt{2\alpha\bar{u}_1|x_1|/M} + \bar{u}_1/k_2), & |x_1| > y_l. \end{cases} \tag{8}$$

where $\bar{u}_1$ is the saturation level of the primary actuator, k_1 is a free parameter and $0 < \alpha < 1$. Further conditions are imposed on $k_2 < 0$ and y_l so that continuity is achieved at the switching instant. It is beyond the scope of this paper to dwell on the particularities of the above controller (see Workman et al. [1987] for details). Here it suffices to say that for $|x_1| > y_l$ a nonlinear function is used to saturate the controller and allow for an aggressive approach to the reference. However, as soon as $|x_1| \leq y_l$, the PTOS becomes a simple proportional-derivative controller, thus settling at the reference without any chattering [Salton et al., 2012].

The above nonlinear controller may be cast as the interconnection of a linear feedback with a sector bounded nonlinearity as proposed by Flores et al. [2013], matching the proposed framework. In order to do so, it suffices to define $\psi(x_1) = k_2 f(x_1)$ and,

$$u_1 = -k_e x_1 - k_2 x_2 - \psi(x_1), \tag{9}$$

where $k_e > 0$ is added so that the closed loop system matrix $A + BK$ is Hurwitz, a characteristic that shall be explored when searching for stability results. We now focus on the design of the secondary actuator control law in order to reduce the system coupling forces and oscillations.

3.2 Secondary Controller Design

When a large reference step is given to a DSA [1] the secondary actuator saturates and the system will be limited only by the bandwidth of the primary actuator. For coupled systems an extra problem occurs due to the (unproductive) acceleration of the secondary actuator as the reference step arrives. This acceleration will push the primary actuator back, delaying the output transition and demanding unnecessary effort from both actuators. Furthermore, if the second stage is considerably undamped, the saturation of the input will lead to oscillations that could be avoided by feedback. It is, therefore, desirable that the secondary actuator,

i only moves when the reference is within its reach;
ii does not saturate so oscillations are avoided;

To satisfy the above, we propose an $\mathcal{L}_\infty$ function so the secondary actuator's acceleration profile is smooth and only active when the reference is within its reach. In other words, we propose a nonlinear function that minimizes the secondary actuator effort when it is far from the reference, but allows for its full action as it approaches the desired set point. This may be achieved by setting,

$$u_2 = -k_3(x_3 + x_1\psi_a(x_1)) - k_4 x_4 \tag{10}$$

[1] By "large" we mean a reference that is beyond the range of the secondary (fine) actuator.

for some $k_3, k_4 > 0$ and,

$$\psi_a(x_1) = \frac{1 - \text{htan}(\beta_1(|x_1| - \beta_2))}{2} \tag{11}$$

Nonlinear function (11) is depicted in Fig. 3. Notice that for large values of x_1, $\psi_a(x_1) \approx 0$, however, as the system approaches the reference point due to the primary actuator, x_1 decreases and $\psi_a(x_1) \to 1$. Recalling that $x_1 := y_1 - r$, it is clear that the secondary actuator attempts to reduce the tracking error only when the system approaches the desired set point, thus avoiding its saturation and reducing oscillations. With appropriate choices of β_1 and β_2 it is straightforward to achieve a smooth transition between $u_2 \approx -k_3x_3 - k_4x_4$ and

$$u_2 \approx -k_3(x_3 + x_1) - k_4x_4.$$

Given the above, the multivariable control signal may be expressed as follows,

$$\begin{aligned}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} &= \begin{bmatrix} -k_e & -k_2 & 0 & 0 \\ 0 & 0 & -k_3 & -k_4 \end{bmatrix} x \\ &+ \begin{bmatrix} 0 \\ -k_3 \end{bmatrix} \psi_a(x_1) - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \psi(x_1),\end{aligned} \tag{12}$$

where $\psi(x_1)$ and $\psi_a(x_1)$ are used to improve the performance of the primary and secondary actuator, respectively.

We now explore the fact that ψ_a is an $\mathcal{L}_\infty$ nonlinear function and ψ is a sector bounded nonlinear function and develop stability results for system (4) under control law (12).

4. STABILITY OF DSA UNDER NONLINEAR CONTROLLERS

The controllers presented in the previous section may be cast in the following form,

$$u(t) = Kx(t) + K_a\psi_a(L_ax(t)) - \psi(L_cx(t)) \tag{13}$$

where $K \in \mathbb{R}^{2\times 4}$ is such that $A + BK$ is Hurwitz, $\psi_a(L_ax(t)) \in \mathbb{R}^2$ is an $\mathcal{L}_\infty$ nonlinear function and $\psi(L_cx(t)) \in \mathbb{R}^2$ a sector bounded nonlinearity. Both $\psi_a(\cdot)$ and $\psi(\cdot)$ may be used in order to improve the system performance.

4.1 $\mathcal{L}_\infty$ nonlinear function

Stability conditions will be derived in the sequel assuming an LPV approach [Wu et al., 2007] so that the $\mathcal{L}_\infty$ nonlinear function can be rewritten as

$$\psi_a(L_ax(t)) = \Delta(t)x(t) \tag{14}$$

with Δ being a diagonal matrix such that $\underline{\delta}_i \leq \Delta_i(t) \leq \bar{\delta}_i$. Under this assumption it follows from (13) and (14) that

$$u(t) = K_\Delta x(t) - \psi(L_cx(t)) \tag{15}$$

where $K_\Delta = K + K_a\Delta(t)$. To describe K_Δ we adopt a polytopic description [Geromel et al., 1998] considering $K_\Delta \in Co\{K_j,\ j = 1, \cdots, 2^h\}$ with h representing the number of time varying parameters, i.e., the value of K_Δ can be determined by a convex combination of vectors $K_j,\ j = 1, \cdots, 2^2$ such that

$$K_\Delta = \sum_{j=1}^{2^h} q_jK_j,\ 0 \leq q_j \leq 1,\ \sum_{j=1}^{2^h} q_j = 1. \tag{16}$$

Given the above, the closed loop system (4) may be described as,

$$\begin{aligned}\dot{x}(t) &= (A + BK_\Delta)x(t) - B\phi(u(t)) - B\psi(L_cx(t)), \\ y(t) &= Cx(t)\end{aligned} \tag{17}$$

where the saturation function is rewritten by means of a decentralized deadzone nonlinearity

$$\phi(u(t)) = u(t) - \text{sat}(u(t)). \tag{18}$$

4.2 Sector-bounded nonlinear functions

The change to the deadzone nonlinearity (18) is considered in order to apply the modified sector approach in (Tarbouriech et al. [2011]). Note that if $\psi(L_cx(t))$ belongs to the sector $[\Omega_{min}, \Omega_{max}]$ (see Fig. 2), then there exists a diagonal positive definite matrix Θ that verifies

$$(\psi(t) - \Omega_{min}L_cx(t))'\Theta(\psi(t) - \Omega_{max}L_cx(t)) \leq 0, \tag{19}$$

for all $x(t) \in \mathcal{S}_1 \subseteq \mathbb{R}^2$. Sector condition (19) is satisfied globally if $\mathcal{S}_1 = \mathbb{R}^2$, otherwise it is locally verified in $\mathcal{S}_1$.

Applying a loop transformation, it follows from Khalil [2002] that (17) with $\psi(L_cx(t))$ bounded by sector $[\Omega_{min}, \Omega_{max}]$ is equivalent to

$$\dot{x}(t) = (A_c + BK_\Delta)x(t) - B\phi(u(t)) - B\psi_c(L_cx(t)) \tag{20}$$

with $A_c = A - B\Omega_{min}L_c$ and $\psi_c(L_cx(t)) = \psi(L_cx(t)) - \Omega_{min}L_cx(t)$ bounded by sector $[0, \Omega_c]$ with $\Omega_c = \Omega_{max} - \Omega_{min}$. In this case, the sector condition

$$\psi_c(t)'\Theta(\psi_c(t) - \Omega_cL_cx(t)) \leq 0,\ \forall x(t) \in \mathcal{S}_c \subseteq \mathbb{R}^2 \tag{21}$$

is satisfied locally in $\mathcal{S}_c$ with

$$\mathcal{S}_c = \{x \in \mathbb{R}^4; |L_{c(i)}x| \leq \rho_i,\ \rho_i > 0,\ i = 1, 2\} \tag{22}$$

From the results in (Gomes da Silva Jr. et al. [2013]), if $x(t) \in \mathcal{S}$ with

$$\mathcal{S} = \{x \in \mathbb{R}^4; |(K_{\Delta(i)} - H_{(i)})x| + (1 - N_i)\psi_{ci} \leq \bar{u}_i, i = 1, 2\} \tag{23}$$

then the modified sector condition

$$\phi(t)'T(\phi(t) - Hx(t) - N\psi_c(t)) \leq 0 \tag{24}$$

is satisfied for some diagonal definite positive matrix T. In this case $H \in \mathbb{R}^{2\times 4}$ and $N := \text{diag}\{n_1,\ n_2\}$ are free variables to be determined.

4.3 Stability Result

We are now ready to state the stability conditions for system (20) under control law (15).

Theorem 1. Consider $\Omega_c, K_j,\ j = 1, \cdots, 2^2$ and ρ *a priori* fixed such that $\mathbb{A}_j = A_c + BK_j$ is Hurwitz. If there exist symmetric positive definite matrices $W \in \mathbb{R}^{4\times 4}$, $S \in \mathbb{R}^{2\times 2}$, a diagonal positive matrix $\nabla \in \mathbb{R}^2$, and matrices $X \in \mathbb{R}^{2\times 4}$ and $\Gamma \in \mathbb{R}^{2\times 1}$ such that the following LMIs are verified,

$$\begin{bmatrix} \mathbb{A}_jW + W\mathbb{A}_j' & -B\nabla + WL_c'\Omega_c' & -BS + X \\ * & -2\nabla & \Gamma \\ * & * & -2S \end{bmatrix} < 0, \quad j = 1, \cdots, 2^2 \tag{25}$$

$$\begin{bmatrix} W & -WL_{c(i)}'\Omega_{ci}' & WK_{j(i)}' - X_{(i)}' \\ * & 2\nabla_i & \nabla_i - \Gamma_i \\ * & * & \bar{u}_i^2 \end{bmatrix} > 0 \quad \begin{matrix} i = 1, 2 \\ j = 1, \cdots, 2^2 \end{matrix} \tag{26}$$

$$\begin{bmatrix} W & WL'_{c(i)} \\ * & \rho_i^2 \end{bmatrix} > 0 \quad i = 1, 2 \tag{27}$$

then the trajectories starting on the ellipsoidal set

$$\mathcal{E}(P, 1) = \{x \in \mathbb{R}^4; x'Px \leq 1\},$$

for $P := W^{-1}$, remain bounded to this set and converge asymptotically to the origin.

Proof: Let $V(x) = x'Px$ be a quadratic Lyapunov candidate function and $\dot{V}(x) = 2x'P\dot{x}$. Assuming $x \in \mathcal{S} \cap \mathcal{S}_c$ such that (24) and (21) are satisfied locally, if

$$\dot{V}(x) - 2\phi' T(\phi - Hx - N\psi_c) - 2\psi_c' \Theta(\psi_c - \Omega_c L_c x) < 0 \tag{28}$$

then, $\forall T$ and $\Theta > 0$, it follows that $\dot{V}(x) < 0$. From (17) we have that (28) can be written in the form $\xi' \Sigma \xi < 0$ with $\xi' = [x' \ \psi_c' \ \phi']'$ and

$$\Sigma = \begin{bmatrix} P\mathbb{A}_\Delta + \mathbb{A}_\Delta' P & -PB + L_c'\Omega_c'\Theta & -PB + H'T' \\ * & -2\Theta & NT \\ * & * & -2T \end{bmatrix} \tag{29}$$

with $\mathbb{A}_\Delta = A_c + BK_\Delta$. Clearly, $\Sigma < 0$ implies (28) and, hence, $\dot{V}(x) < 0$.

Consider now $W = P^{-1}$, $\nabla = \Theta^{-1}$, $S = T^{-1}$, $X = HW$ and $\Gamma = WM'$. From convexity arguments (Tarbouriech and Garcia [1997]), condition (29) holds for all $K_\Delta \in Co\{K_j, \ j = 1, \cdots, 2^2\}$, if (29) is jointly satisfied for $K_\Delta = K_j, j = 1, \cdots, 2^2$. Right and left multiplying (29) by diag$\{W, \nabla, S\}$ it follows that (25) is equivalent to $\Sigma < 0$ and therefore we can conclude that (25) implies that (28) is verified. Then, the satisfaction of (25) implies that, $\dot{V}(x) < 0 \Rightarrow V(x(t)) \leq V(x(0))$, $\forall x(0) \in \mathcal{E}(P, 1) \subset \mathcal{S} \cap \mathcal{S}_c$, which ensures that $\mathcal{E}(P, 1)$ is an invariant set and that $x(t) \to 0$ as $t \to \infty$, i.e. that $\mathcal{E}(P, 1)$ is included in the region of attraction of the origin of the closed loop system (17).

On the other hand, (26) ensures that $\mathcal{E}(P, 1) \subset \mathcal{S}$. The proof this fact follows the same steps as presented in Flores et al. [2013] assuming convexity arguments. Condition (27) is a standard condition to ensure $\mathcal{E}(P, 1) \subset \mathcal{S}_c$, which concludes the proof. □

5. NUMERICAL EXAMPLE

In order to demonstrate the effectiveness of the proposed framework, let us consider a DSA with parameters given by the table below under controllers (9) and (10). We

Parameter	Value	Unit
M	0.1	kg
m	0.05	kg
c_1	0.1	m/s^2
c_2	0.1	m/s^2
k	10^3	N/m

also let $\bar{u}_1 = 5$ and $\bar{u}_2 = 1$ be the saturation levels and consider constant gains $k_{DC_1} = 1$ and $k_{DC_2} = 10$ that relate the control signal to the actual force applied by the actuator. Furthermore, the control gain K was heuristically computed, and is given by,

$$K = \begin{bmatrix} -71.1 & -6.0 & 0 & 0 \\ 0 & 0 & -10^3 & -1 \end{bmatrix}. \tag{30}$$

Additionally, the nonlinear function $\psi_a(x_1)$ is as depicted in Fig. 3 and the PTOS parameters are defined with $\alpha = 0.8$ and $k_1 = -142$. This results in $y_l = 0.035$ and the nonlinearity $\psi(x_1)$ depicted in Fig. 2, locally bounded by $[\Omega_{min}, \Omega_{max}] = [19.9, \ 23.8]$ in the set $\mathcal{S}_1$,

$$\mathcal{S}_1 = \{x_1 \in \mathbb{R}; |x_1(t)| < 0.1, \forall t > 0\}.$$

Two simulation results are presented in Figures 4 and 5, where the top plots depict the system outputs y, y_1 and y_2, and the bottom plots the inputs u_1 and u_2. Fig. 4 shows the system response to a step reference of $r = 0.05$ when the primary actuator is controlled by (9) and the secondary actuator is controlled by a proportional-derivative controller with gains k_3 and k_4, respectively. Note that as soon as the step change occurs the secondary actuator rushes toward the reference pushing the primary back, due to coupling forces. Furthermore, it soon saturates on its displacement limit causing a large oscillation on the system that is also reflected to the primary actuator. While the system response is still faster than that of the primary actuator alone, a significant amount of energy is wasted due to the saturation of secondary actuator, along with the effort of the primary in reducing the oscillations caused by this saturation (bottom plot of Fig. 4).

The results presented in Fig. 5 show the system response under controllers (9) and (10). The top plot clearly shows the benefits of using the nonlinearity $\psi_a(x_1)$ in order to limit the excursion of the secondary actuator. When the system is far from the reference $\psi_a(x_1) \approx 0$ and the secondary actuator rests at the origin. However, as the system output approaches the reference $\psi_a(x_1)$ smoothly increases allowing the secondary actuator to reach the reference ahead of time, improving the system performance. From the bottom plot of Fig. 5 it is clear that the system oscillations are considerably smaller than those of Fig. 4 and that energy may be saved without compromising the performance. While the resulting controller reduces the mechanical loads on the system prolonging its durability, it also reduces the input energy at no performance cost.

6. CONCLUSION

This paper has proposed a framework for the nonlinear control of dual-stage actuators (DSA). Stability results have been provided considering the system of interest under the effects of saturation and nonlinear controllers. The proposed approach gives a great deal of freedom for the design of nonlinear controllers for DSA since it encompasses both sector bounded and $\mathcal{L}_\infty$ nonlinear control functions. In order to illustrate the proposed framework a numerical example was presented where nonlinear functions were explored to improve the system performance. It has been shown that the traditional PTOS fits the framework nicely and a novel nonlinear feedback that avoids the saturation of the secondary actuator – by limiting its excursion – has been introduced. The numerical example illustrated that, by a proper choice of nonlinear control methods, the system performance may be significantly improved, while, at the same time, the input efforts are reduced.

REFERENCES

J.V. Flores, N.B. Neto, A.T. Salton, and J.M. Gomes da Silva Jr. Acceleration enhancement factor for damped

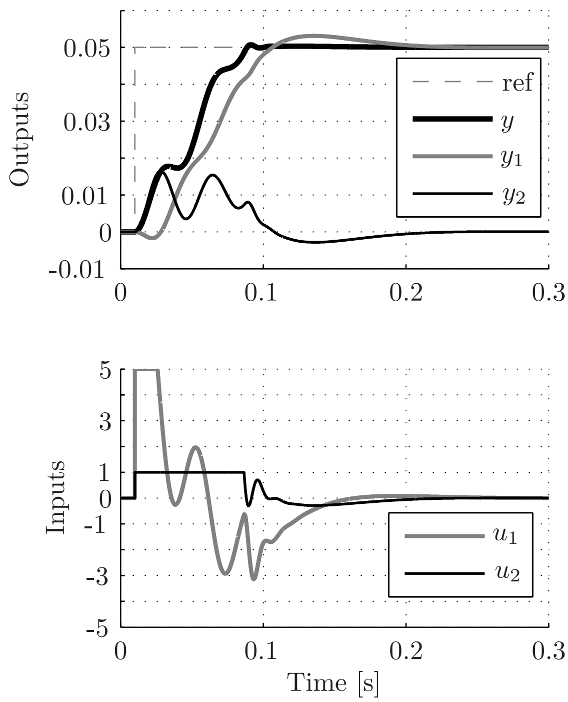

Fig. 4. Simulation results: system response for u_1 given by PTOS and u_2 by linear feedback. The oscillations are a result of the saturation of u_2.

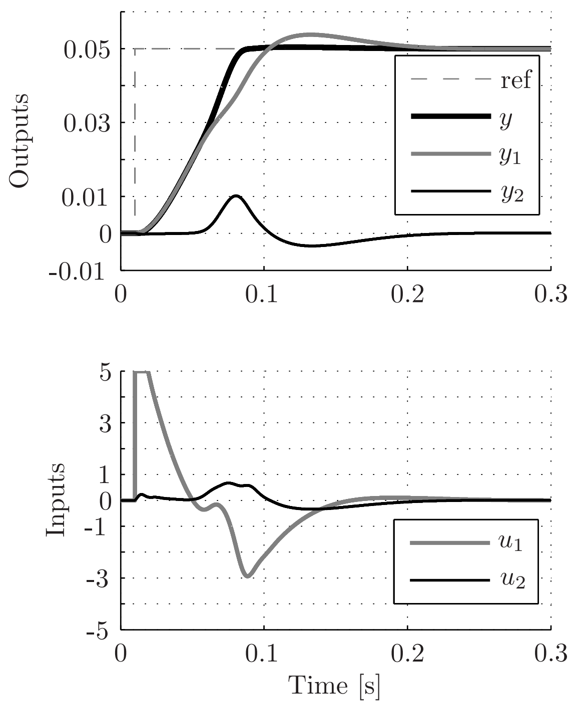

Fig. 5. Simulation results: system response for u_1 given by PTOS and u_2 by (10).

systems subject to the discrete proximate time-optimal servomechanism. In *Decision and Control, 2013 (CDC 2013). Proceedings of the 48th IEEE Conference on*, pages 1 – 6. IEEE, 2013.

R.F. Fung, Y.L. Hsu, and M.S. Huang. System identification of a dual-stage xy precision positioning table. *Precision Engineering*, 33(1):71–80, 2009.

J.C. Geromel, M.C. de Oliveira, and L. Hsu. {LMI} characterization of structural and robust stability. *Linear Algebra and its Applications*, 285(13):69 – 80, 1998.

J.M. Gomes da Silva Jr., E.B. Castelan, J. Corso, and D. Eckhard. Dynamic output feedback stabilization for systems with sector-bounded nonlinearities and saturating actuators. *Journal of the Franklin Institute*, (0):–, 2013.

D. Henrion, S. Tarbouriech, and G. Garcia. Output feedback robust stabilization of uncertain linear systems with saturating controls: an lmi approach. *IEEE Transactions on Automatic Control*, 44(11):2230–2237, 1999.

T. Hu and Z. Lin. *Control systems with actuator saturation: analisys and design.* Birkhauser, Boston, MA, 2001.

H.K. Khalil. *Nonlinear Systems.* Prentice Hall, 3rd edition, December 2002.

C.W. Lee and S.W. Kim. An ultraprecision stage for alignment of wafers in advanced microlithography. *Precision Engineering*, 21(2?3):113 – 122, 1997. ISSN 0141-6359. doi: http://dx.doi.org/10.1016/S0141-6359(97)00080-9.

K. Mori, T. Munemoto, H. Otsuki, Y. Yamaguchi, and K. Akagi. A dual-stage magnetic disk drive actuator using a piezoelectric device for a high track density. *Magnetics, IEEE Transactions on*, 27(6):5298–5300, 1991.

A.T. Salton, Zhiyong Chen, and Minyue Fu. Improved control design methods for proximate time-optimal servomechanisms. *IEEE/ASME Transactions on Mechatronics*, 17(6):1049 –1058, dec. 2012.

A.T. Salton, Z. Chen, J. Zheng, and M. Fu. Optimal trajectories for the preview control of dual-stage actuators. In *American Control Conference (ACC'13)*, pages 5005–5010. IEEE, jun 2013.

S. Tarbouriech and G. Garcia. *Control of uncertain systems with bounded inputs.* Lecture notes in control and information sciences. Springer, 1997.

S. Tarbouriech, G. Garcia, J.M. Gomes da Silva Jr., and I. Queinnec. *Stability and Stabilization of Linear Systems with Saturating Actuators.* Springer, 2011.

T. Tuma, W. Haeberle, H. Rothuizen, J. Lygeros, A. Pantazi, and A. Sebastian. A high-speed electromagnetically-actuated scanner for dual-stage nanopositioning. In *Mechatronic Systems*, number 1, pages 125–130, 2013.

M.L. Workman, R.L. Kosut, and G.F. Franklin. Adaptive proximate time-optimal servomechanisms: Discrete-time case. In *Decision and Control, 1987. 26th IEEE Conference on*, volume 26, pages 1548–1553. IEEE, 1987.

F. Wu, Z. Lin, and Q. Zheng. Output feedback stabilization of linear systems with actuator saturation. *IEEE Transactions on Automatic Control*, 52(1):122–128, 2007.

J. Zheng, A.T. Salton, and M. Fu. A novel rotary dual-stage actuator positioner. In *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pages 5426–5431. IEEE, 2009.