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Mixed Integer Optimal Control in Minimum Time Multi-points Traversal Problem of Robotic Manipulators[1]

Qiang Zhang*, **Shurong Li***, **Jianxin Guo****

**College of Information and Control Engineering, China University of Petroleum (East China), Qingdao, China (e-mail:zhangqiangupc@gmail.com, lishuron@upc.edu.cn)*
***National Center for Mathematics and Interdisciplinary Sciences (NCMIS), Chinese Academy of Sciences, Beijing, China (e-mail: guojianxin@amss.ac.cn)*

Abstract: In this paper, the minimum time multi-points traversal problem is studied for robotic manipulators. It is shown that the problem can be formulated as a mixed integer optimal control problem. Cubic Hermite spline is applied to interpolate the desired path points and then a decomposition method is proposed to solve the problem. An outer level iteration and a series of inner level optimization processes are used in the decomposition method. The outer level iteration aims to search optimal permutation of the path points, which is realized by Genetic Algorithm (GA). And the inner level processes aim to cope with a series of path determined minimum time motion planning problems, which are solved by a direct transcription method. Minimum time multi-points traversal task of a 2-DOF robotic manipulator is used to demonstrate the effectiveness of the proposed approach.

1. INTRODUCTION

For the purpose of improving productivity, minimum time motion planning problems are widely studied and several efficient methods are proposed, such as the phase plane analysis approach of Bobrow et al. (1985), the greedy search algorithm of Zhang et al. (2012) and the convex optimization approach of Verscheure et al. (2009). Nevertheless, all these approaches are designed for simple motion planning along given path merely.

In mechanical industry, there exists a class of complex tasks called multi-points traversal problems, such as drilling, spot welding and assembly et al. These tasks have many unordered points, due to which planning strategy is required to design a path to traverse all the given points once and only once and at the same time to satisfy some minimum index such as distance, time or energy. Recently, Petiot et al. (1998) used an elastic net method to study the minimum distance multi-points traversal problem. Dubowsky and Blubaugh (1989) applied a branch and bound method to solve the minimum time multi-points traversal problem for spot welding tasks. Zacharia and Aspragathos (2005) studied the minimum time multi-points traversal problem for non-redundant manipulator by using genetic algorithm. However, all the studies mentioned above need the manipulator to stop completely at each task point.

Different from the works of Petiot et al. (1998), Dubowsky and Blubaugh (1989) and Zacharia and Aspragathos (2005), in this paper we study the minimum time multi-points traversal (MTMPT) problem in which the requirement of full stop at each task point is not mandatory. It is shown that the multi-points traversal problem can also be described as a performance limited traveling salesman problem (TSP): the manipulator effector acts as the salesman, who starts from any path point and passes by each point just by once. So the purpose of this paper is to find an order among the points, through which the salesman can traverse in minimum time under performance limits. Similar works appeared in other applications, Glocker and von Stryk (2002) discussed a minimum time multi-points traversal problem of a motorized car, which is solved by branch-bound combined with the direct collocation methods; Conway et al. (2007) studied a space mission planning problem, which is solved based on an evolutionary principle.

In this paper, a decomposition method is proposed for solving our minimum time multi-points traversal problem for robotic manipulators. As shown in Glocker and von Stryk (2002) and Conway et al. (2007), the MTMPT problem can be formulated as a mixed integer optimal control problem (MIOCP). The dynamics constraints and maximum velocity constraints of manipulator are considered in the problem. By interpolating the path points with unique cubic Hermite spline, we can solve the MIOCP by executing an outer task points sorting process and a series of inner path determined minimum time motion optimization processes.

2. PROBLEM FORMULATION AS A MIXED INTEGER OPTIMAL CONTROL PROBLEM

The dynamics model of a n -DOF robotic manipulator can be formulated as (Constantinescu and Croft (2002)):

$$\boldsymbol{\tau}=\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{G}(\mathbf{q}), \tag{1}$$

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where $\mathbf{q} \in \mathbf{R}^n$ is the vector of joint angular positions, $\boldsymbol{\tau} \in \mathbf{R}^n$ is the vector of joint toques, $\mathbf{M}(\mathbf{q}) \in \mathbf{R}^{n \times n}$ denotes the inertia matrix of the manipulator, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbf{R}^{n \times n}$ contains the information of the centrifugal and Coriolis forces, $\mathbf{G}(\mathbf{q}) \in \mathbf{R}^n$ is the vector of the centrifugal torques.

As mentioned in the introduction section, the goal of this paper is to plan a reasonable path which passes through all the given points in minimum time under the dynamics limits of the manipulator.

Let n_c denote the number of the given points. The joint angular positions vector of the manipulator corresponding to each given task point is denoted by $\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_{n_c}$ with $\mathbf{p}_i \in \mathbf{R}^n$. Joint torque constraints are considered in our MTMPT problem and are written as

$$-\boldsymbol{\tau}_{\mathrm{B}} \le \boldsymbol{\tau} \le \boldsymbol{\tau}_{\mathrm{B}}. \tag{2}$$

The joint velocity constraints are

$$-\dot{\mathbf{q}}_{\mathrm{B}} \le \dot{\mathbf{q}} \le \dot{\mathbf{q}}_{\mathrm{B}}, \tag{3}$$

Then the minimum time multi-points traversal problem of robotic manipulator can be formulated as the following mixed integer optimal control problem:

$$\min_{\{\mathbf{q}(t), \mathbf{W}\}} T_{\mathrm{f}}$$
$$\text{s.t.} \begin{cases} \mathbf{q}(t_i) - \mathbf{Q}\mathbf{W}_i = 0, \\ \dot{\mathbf{q}}(t_i) = \dot{\mathbf{q}}_i, i = 1, 2, \cdots, n_c, \\ \sum_{i=1}^{n_c} \mathbf{W}_i = [1, 1, \cdots, 1]_{n_c}^{\mathrm{T}}, \\ \boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}), \\ -\boldsymbol{\tau}_{\mathrm{B}} \le \boldsymbol{\tau} \le \boldsymbol{\tau}_{\mathrm{B}}, -\dot{\mathbf{q}}_{\mathrm{B}} \le \dot{\mathbf{q}} \le \dot{\mathbf{q}}_{\mathrm{B}}, \end{cases} \tag{4}$$

where t_i denotes the time when manipulator reaches the ordered i th path point, $\mathbf{Q} = [\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_{n_c}]_{n \times n_c}$ contains all joint positions of the path points, $\dot{\mathbf{q}}_i$ denotes the desired joint velocity at ordered point i, $\mathbf{W}_i \in \{0,1\}^{n_c}$ is the i th unit column vector of matrix $\mathbf{W}$, $\mathbf{W} \in \mathbf{Z}^{n_c \times n_c}$,

$$0 = t_1 < t_2 < \cdots < t_{n_c} = T_{\mathrm{f}}.$$

Problem (4) is an optimal control problem with continuous and discrete decision variables. The common solution methods include branch and bound (von Stryk and Glocker (2000)), bender decomposition (Sager (2005)), et al. In this paper, we use a GA with embedded direct transcription method to solve this problem.

3. PROBLEM DECOMPOSITION USING GENETIC ALGORITHM AND DIRECT TRANSCRIPTION METHOD

The common multi-points traversal problem is described as a minimum distance TSP. Hence the planned path is a series of straight lines with sharp corners at the path points, which means the motion of the manipulator must stop or slow down at the corners, which may fly away from the path otherwise. Thus, it is reasonable to plan a smooth path to travel by the points.

In this section, cubic Hermite spline is applied to interpolate the desired path points, because of which the generated path has C^2 continuity.

Assuming the travel order of the points is known, denoted by $\mathbf{L} \in \mathbf{Z}^{n_c}$ with each element of $\mathbf{L}$ be the point label, then the sequence of the point positions is $\{\mathbf{Q}_{\mathbf{L}(1)}, \mathbf{Q}_{\mathbf{L}(2)}, \cdots, \mathbf{Q}_{\mathbf{L}(n_c)}\}$.

Define path parameter $s \in [0,1]$ with $s_i = \dfrac{i-1}{n_c - 1}, i = 1, 2, \cdots, n_c$.

In an arbitrary interval $[s_i, s_{i+1}]$, cubic Hermite spline path has the following formulation:

$$\begin{aligned} \mathbf{q}(s) = {} & h_{00}(u)\mathbf{Q}_{\mathbf{L}(i)} + h_{10}(u)(s_{i+1} - s_i)\mathbf{r}_i + \\ & h_{01}(u)\mathbf{Q}_{\mathbf{L}(i+1)} + h_{11}(u)(s_{i+1} - s_i)\mathbf{r}_{i+1}, \end{aligned} \tag{5}$$

where $u = \dfrac{s - s_i}{s_{i+1} - s_i}$, $u \in [0,1]$. h refers to the basis function and can be defined as below.

$$\begin{aligned} h_{00} &= 2u^3 - 3u^2 + 1, \\ h_{01} &= -2u^3 + 3u^2, \\ h_{10} &= u^3 - 2u^2 + u, \\ h_{11} &= u^3 - u^2. \end{aligned} \tag{6}$$

In (5), $\mathbf{r}_i$ denotes the tangent value at point $\mathbf{L}(i)$ which is defined as the following three-point difference form:

$$\mathbf{r}_i = \frac{\mathbf{Q}_{\mathbf{L}(i+1)} \text{-} \mathbf{Q}_{\mathbf{L}(i)}}{2(s_{i+1} - s_i)} + \frac{\mathbf{Q}_{\mathbf{L}(i)} \text{-} \mathbf{Q}_{\mathbf{L}(i-1)}}{2(s_i - s_{i-1})} \tag{7}$$

for $i = 2, 3, \cdots, n_c - 1$ and one-sided difference at the end points of the path. According to the property of Hermite spline, for the given travel order of the points, the interpolated smooth path is unique.

As mentioned in Bobrow et al. (1985) and Verscheure et al. (2009), if there exists optimal motion along a given path, the minimum travel time (written as $T_{\min}$) is unique, accordingly the manipulator performance in which satisfying the corresponding limits. Therefore the minimum time objective can be treated as an implicit function of point traversal order $\mathbf{L}$. Hence the mixed integer optimal control problem (4) can be solved as a two-level nested optimization problem. The outer level iteration is a combinational optimization problem aiming to obtain optimal traversal order and corresponding interpolated path, while, at each outer iteration, an inner optimization process is performed to calculate a minimum travel time for the current interpolated path.

In this section, a GA is used to solve the combinational optimization problem, in which each minimum travel time for current path is obtained through a direct transcription method.

3.1 Planning Minimum Time Motion using Direct Transcription Method

According to (5), the gradient informations of path $\mathbf{q}(s)$ w.r.t parameter s are

$$\begin{aligned}\mathbf{q}'(s) &= \frac{\mathrm{d}\mathbf{q}}{\mathrm{d}s} \\ &= h_{00}^{'}(u)\mathbf{Q}_{\mathrm{L}(i)} + h_{10}^{'}(u)(s_{i+1}-s_i)\mathbf{r}_i + \\ &\quad h_{01}^{'}(u)\mathbf{Q}_{\mathrm{L}(i+1)} + h_{11}^{'}(u)(s_{i+1}-s_i)\mathbf{r}_{i+1},\end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{q}''(s) &= \frac{\mathrm{d}^2\mathbf{q}}{\mathrm{d}s^2} \\ &= h_{00}^{''}(u)\mathbf{Q}_{\mathrm{L}(i)} + h_{10}^{''}(u)(s_{i+1}-s_i)\mathbf{r}_i + \\ &\quad h_{01}^{''}(u)\mathbf{Q}_{\mathrm{L}(i+1)} + h_{11}^{''}(u)(s_{i+1}-s_i)\mathbf{r}_{i+1}.\end{aligned} \tag{9}$$

The joint velocities can be rewritten as

$$\dot{\mathbf{q}}(t)=\mathbf{q}'(s(t))\dot{s}(t). \tag{10}$$

The joint torques are

$$\boldsymbol{\tau} = \mathbf{m}(s)\ddot{s}+\mathbf{c}(s)\dot{s}^2+\mathbf{g}(s), \tag{11}$$

where

$$\begin{aligned}&\mathbf{m}(s)=\mathbf{M}(\mathbf{q}(s))\mathbf{q}'(s)\in\mathbf{R}^n, \\ &\mathbf{c}(s)=\mathbf{M}(\mathbf{q}(s))\mathbf{q}''(s)+\mathbf{C}(\mathbf{q}(s),\mathbf{q}'(s))\mathbf{q}'(s)\in\mathbf{R}^n, \\ &\mathbf{g}(s)=\mathbf{G}(\mathbf{q}(s))\in\mathbf{R}^n.\end{aligned}$$

Then according to Verscheure et al. (2009), the inner minimum time travel problem for certain path can be formulated as the following convex optimal control problem:

$$\min_{b(s)} \; T_{\mathrm{f}} = \int_0^1 \frac{1}{\sqrt{a(s)}}\mathrm{d}s$$

$$\text{s.t.}\begin{cases} a'(s)=2b, a(s)>0, s\in[0,1], \\ \boldsymbol{\tau}(s)=\mathbf{m}(s)b(s)+\mathbf{c}(s)a(s)+\mathbf{g}(s), \\ -\dot{\mathbf{q}}_{\mathrm{B}} \le \mathbf{q}'(s)\sqrt{a(s)} \le \dot{\mathbf{q}}_{\mathrm{B}}, \\ -\boldsymbol{\tau}_{\mathrm{B}} \le \boldsymbol{\tau}(s) \le \boldsymbol{\tau}_{\mathrm{B}}, \\ \mathbf{q}'(s_i)\sqrt{a(s_i)} = \dot{\mathbf{q}}_i, i=1,2,\cdots,n_c, \end{cases} \tag{12}$$

where $a=\dot{s}^2$, $b=\ddot{s}$.

Then a direct transcription method is used to parameterize the problem(12). Define knot vector as

$$\mathbf{s}_{\mathbf{n}} = [\bar{s}_1,\cdots,\bar{s}_k,\cdots,\bar{s}_N], \tag{13}$$

where $0<\bar{s}_1<\bar{s}_2<\cdots<\bar{s}_N<1$, and $N>>n_c$. The knot intervals are

$$\Delta s_j = \begin{cases}\bar{s}_1, j=1 \\ \bar{s}_j-\bar{s}_{j\text{-}1}, j=2,3,\cdots,N \\ 1-\bar{s}_N, j=N+1\end{cases}. \tag{14}$$

A radial basis function network is applied to approximate the desired optimal trajectory, formulated as

$$a(s)=\sum_{k=1}^{N}\omega_k\phi_k(s),$$

where $\phi_k(s)=\exp\left(-\frac{(s-\bar{s}_k)^2}{2\delta^2}\right)>0$ denotes the $k-th$ radial basis function as shown in Fig. 1, δ denotes the width of the basis function.

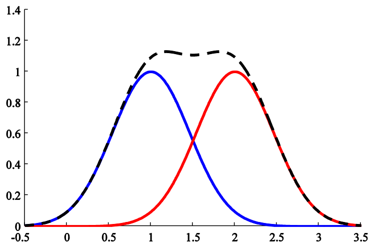

Fig.1. Typical radial basis function

Since $a(s)=\dot{s}^2(s)\ge 0$ for all $s\in[0,1]$, we expect that the weight coefficient $\omega_k\ge 0$ for all $k=1,2,\cdots,N$. Here we make a little offset of trajectory $a(s)$ as

$$a(s)=\sum_{k=1}^{N}\omega_k\phi_k(s)-\varepsilon=\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}(s)-\varepsilon \tag{15}$$

with ε is a small positive. And

$$\boldsymbol{\omega}=[\omega_1,\omega_2,\cdots,\omega_N]^{\mathrm{T}}, \tag{16}$$

$$\boldsymbol{\Phi}(s)=[\phi_1(s),\phi_2(s),\cdots,\phi_N(s)]^{\mathrm{T}}. \tag{17}$$

Then we can have

$$b(s)=\frac{1}{2}a'(s)=\frac{1}{2}\sum_{k=1}^{N}\omega_k\phi_k'(s)=\frac{1}{2}\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}'(s), \tag{18}$$

where

$$\phi_k'(s)=-\frac{(s-\bar{s}_k)}{\delta^2}\exp\left(-\frac{(s-\bar{s}_k)^2}{2\delta^2}\right),$$

$$\boldsymbol{\Phi}'(s)=[\phi_1'(s),\phi_2'(s),\cdots,\phi_N'(s)]^{\mathrm{T}}.$$

At each knot point, the torque function can be

$$\boldsymbol{\tau}(\bar{s}_k)=\frac{1}{2}\mathbf{m}(\bar{s}_k)\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}'(\bar{s}_k)+\mathbf{c}(\bar{s}_k)\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}(\bar{s}_k)+\mathbf{g}(\bar{s}_k)-\varepsilon\mathbf{c}(\bar{s}_k). \tag{19}$$

By using the rectangle rule, the objective function can be calculated by

$$T_{\mathrm{f}} = \int_0^1 \frac{1}{\sqrt{a(s)}} \mathrm{d}s \approx \sum_{j=1}^{N+1} \frac{\Delta \bar{s}_j}{\sqrt{\left(\boldsymbol{\omega}^{\mathrm{T}} \boldsymbol{\Phi}\left(\frac{\bar{s}_{j-1}+\bar{s}_j}{2}\right)\right) - \varepsilon}}. \tag{20}$$

Hence the optimal control problem (12) can be parameterized as the following convex optimization problem:

$$\min_{b(s)} T_{\mathrm{f}} = \sum_{j=1}^{N+1} \frac{\Delta \bar{s}_j}{\sqrt{\left(\boldsymbol{\omega}^{\mathrm{T}} \boldsymbol{\Phi}\left(\frac{\bar{s}_{j-1}+\bar{s}_j}{2}\right)\right) - \varepsilon}}$$

$$\text{s.t.} \begin{cases} \boldsymbol{\tau}(\bar{s}_k) = \frac{1}{2}\mathbf{m}(\bar{s}_k)\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}'(\bar{s}_k) + \mathbf{c}(\bar{s}_k)\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}(\bar{s}_k) + \\ \qquad \mathbf{g}(\bar{s}_k) - \varepsilon\mathbf{c}(\bar{s}_k), \\ -\dot{\mathbf{q}}_{\mathrm{B}} \le \mathbf{q}'(\bar{s}_k)\sqrt{\left(\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}(\bar{s}_k)\right) - \varepsilon} \le \dot{\mathbf{q}}_{\mathrm{B}}, \\ -\boldsymbol{\tau}_{\mathrm{B}} \le \boldsymbol{\tau}(\bar{s}_k) \le \boldsymbol{\tau}_{\mathrm{B}}, k = 1, 2, \cdots, N, \\ \mathbf{q}'(s_i)\sqrt{\left(\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Phi}(s_i)\right) - \varepsilon} = \dot{\mathbf{q}}_i, i = 1, 2, \cdots, n_c. \end{cases} \tag{21}$$

The efficient solution of problem (21) can be achieved by using the SQP routine *fmincon* in Matlab environment.

3.2 Path Selection using GA

In problem (4), assuming the traversal order $\mathbf{L}$ is known, then the traversal order matrix $\mathbf{W}$ is determined. The problem will be reduced to problem (12) which can be solved by the method in Section 3.1. Since problem (12) is convex, the optimal travel time $T_{\min}$ of problem (12) is unique. So for each traversal order $\mathbf{L}$, there is a unique minimum time $T_{\min}$ (if it exists), which means the solution of problem (4) can be obtained by solving the following combinational optimization problem:

$$\min T_{\mathrm{f}}(\mathbf{L})$$
$$\text{s.t.} \begin{cases} \mathbf{L} \in \{1, 2, \cdots, n_c\}^{n_c}, \\ \mathbf{L}(i) \ne \mathbf{L}(j) \ for\ i \ne j, \\ i, j = 1, 2, \cdots, n_c. \end{cases} \tag{22}$$

In this paper, a genetic algorithm is used to search the optimal traversal order $\mathbf{L}$. The algorithm procedure is programmed as follows:

Table 1. GA based path selection procedure (reference to Moon et al. (2002)).

Input:	The number of points n_c, the location of each path point $\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_{n_c}$, the desired joint velocity $\dot{\mathbf{q}}_i$ at each ordered point i.
Output:	Optimized traversal path $\mathbf{q}(s)$ and corresponding minimum time trajectory $\mathbf{q}(t)$.
Initialization	Generation index $i = 0$; Population size pop_size; Number of generation max_gen; Initialize the traversal order population $Pa\_pop(i)$=Randperm(pop_size, n_c); Using the method in section 3.1 to evaluate the corresponding minimum time of each individual of the population $Pa\_pop(i)$ and select the best solution $T_{\min}^*$.
Loop:	*While* ($i < max\_gen$), *Step 1.* Regenerate offspring population $Ch\_pop(i)$ from $Pa\_pop(i)$ by applying the crossover and mutation operations; *Step 2.* Use the method in section 3.1 to evaluate the corresponding minimum time of each individual of the population $Ch\_pop(i)$ and select the best solution $T_{\min}$; *Step 3.* Update the best solution $T_{\min}^* = T_{\min}$, if the current $T_{\min}^* > T_{\min}$; *Step 4.* Select new population $Pa\_pop(i+1)$ from $Pa\_pop(i)$ and $Ch\_pop(i)$; *Step 5.* $i = i + 1$. *End while.*

4. NUMERICAL EXAMPLE

In this section, a minimum time multi-points traversal example for a 2-DOF robotic manipulator is presented to verify the effectiveness of the proposed approach.

The dynamic equations of the manipulator model in Fig. 2 is given as (Feng et al. (2002))

$$M(q) = \begin{bmatrix} a_{11}(q_1) & a_{12}(q_1) \\ a_{21}(q_1) & a_{22}(q_1) \end{bmatrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} -\beta(q_2)\dot{q}_1^2 - 2\beta(q_2)\dot{q}_1\dot{q}_2 \\ \beta(q_2)\dot{q}_2^2 \end{bmatrix}, \ G(q) = \begin{bmatrix} \gamma_1(q_1, q_2)g \\ \gamma_2(q_1, q_2)g \end{bmatrix},$$

where

$$a_{11}(q_1) = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2),$$

$$a_{12}(q_1) = m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2),$$

$$a_{22}(q_1) = m_2 l_2^2,$$

$$\beta(q_2) = m_2 l_1 l_2 \sin(q_2),$$

$$\gamma_1(q_1, q_2) = (m_1 + m_2) l_1 \cos(q_2) + m_2 l_2 \cos(q_1 + q_2),$$

$$\gamma_2(q_1, q_2) = m_2 l_2 \cos(q_1 + q_2).$$

The model parameters are

$$l_1 = 1m \,,\; l_2 = 0.8m \,,\; m_1 = 1.5kg \,,\; m_2 = 2.5kg \,.$$

The path points used in this example are obtained by solving the following equations.

$$\begin{cases} x = sin(n\pi s)cos(\pi s), \\ y = asin(n\pi s)sin(\pi s), \\ n = 5, a = 1.5, \\ s = 0.18 : 0.08 : 1. \end{cases}$$

The distribution of the path points is shown in Fig.3. There are total 11 points, that is $n_c = 11$. Torque limits of the two joints are set as [140; 140;]N.m. Initial and termination feedrate of the end effecter are assumed to be zero. The desired feedrate of the end effecter at each intermediate point is set as 2m/s.

The typical GA set is shown in Table 2. The parameterization number of problem (12) is set as $N = 200$. Since problem (21) is convex, the solution process can be very fast. Fig. 4 presents the best solution history of the GA based path point permutation process. The best solution is obtained at 38th generation with the mean computation time about 12min on a laptop, Matlab environment, 32-bit system, 2.5 GHz Core i3 processor, 2GB RAM memory.

Fig.5 presents the optimal traversal path for the given path points with the path length 6.087m. Along this traversal path, the minimum motion time is 1.3316s. Fig. 6 shows the corresponding minimum time feedrate of the end effecter for the optimal path. In Fig. 7, the optimal structure of the joint torques is shown to be bang-bang, which is an indication of time optimality for the desired path.

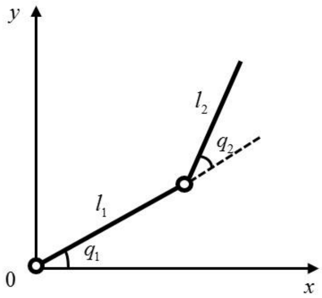

Fig. 2. Two link robot manipulator model

Table 2. Typical Genetic Algorithm Set.

pop_size	40
max_gen	100
Total mutation rate	75%
Mutation operations	Flip 25% Swap 25% Slide 25%

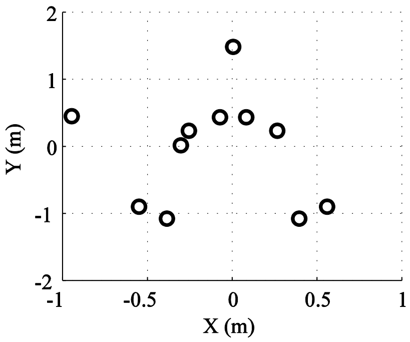

Fig. 3. Distribution of the path points

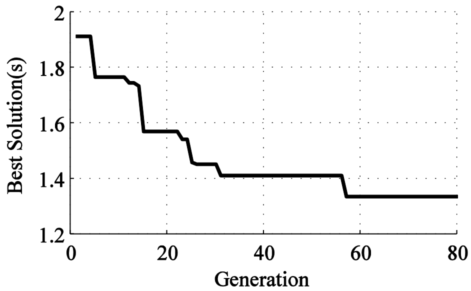

Fig. 4. Best solution history of the GA based path point permutation process

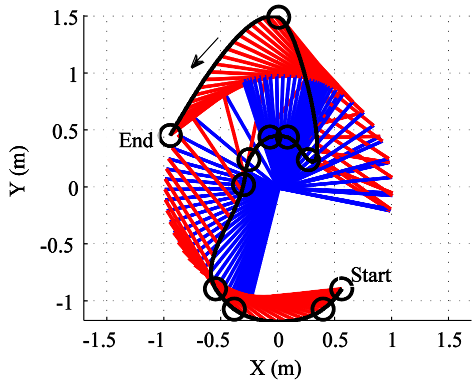

Fig.5. Optimal traversal path for the given path points

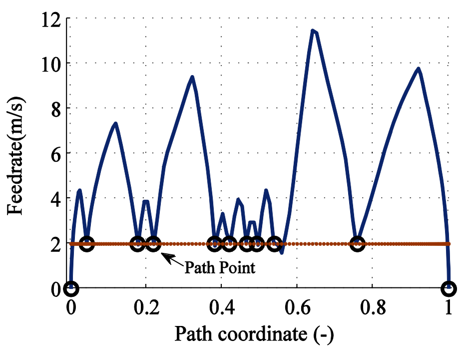

Fig. 6. Minimum time feedrate of the end effecter for the optimal path

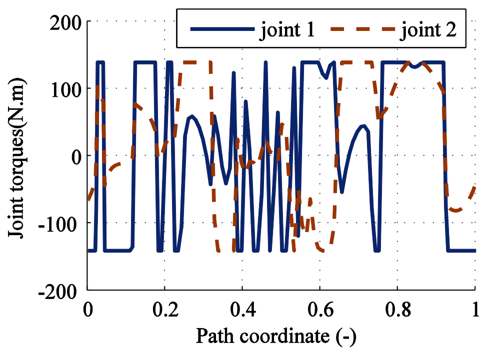

Fig. 7. Minimum time joint torque trajectory for the optimal path

5. CONCLUSIONS

In this paper, a decomposition method is proposed to solve the minimum time multi-points traversal problem for robotic manipulators. By applying cubic Hermite spline to interpolate the desired path points, the minimum time multi-points traversal problem is decomposed into an outer level iteration which aims to search the optimal permutation of the path points and a series of inner level optimization processes which aim to obtain the minimum motion time along the interpolated spline paths. The numerical efficiency of the proposed method has been demonstrated through the 2-DOF robotic manipulator example.

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