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Optimal Control Nodes Selection for Consensus in Multi-agent Systems ⋆

Wen Yang * **Xiaofan Wang** ** **Hongbo Shi** *

* *Key Laboratory of Advanced Control and Optimization for Chemical Processes (East China University of Science and Technology), Ministry of Education, Shanghai, 200237 (Email: weny@ecust.edu.cn, hbshi@ecust.edu.cn).*
** *Department of Automation, Shanghai Jiaotong University, Shanghai, 200240 (Email: xfwang@sjtu.edu.cn).*

Abstract: In a resource limited multi-agent system, it is of practical importance to select a fraction of nodes (agents) to provide control inputs such that consensus can be achieved with optimized performance in terms of network cost and/or convergence speed. In this paper, we investigate the problem of how to select control nodes so as to minimize the network cost, where the control nodes are selected at the beginning and will be fixed all the time. This problem can be transformed to a combinatorial optimization problem, and further relaxed to a convex optimization problem with reweighted l_1 norm. We propose a suboptimal algorithm to solve the convex optimization problem. Finally, we offer several numerical examples to illustrate the efficiency of the proposed strategies, and investigate the relationship how the degrees of control nodes will influence network cost and convergence speed.

Keywords: Node selection; Convex optimization; Multi-agent systems

1. INTRODUCTION

Consensus based dynamic networks have emerged as a flexible framework for multi-agent information sharing when cooperative task is required, such as wireless sensor navigation, spacecraft formation control, mobile robot rendezvous, unmanned aerial vehicle flocking. In the last decade, the consensus problem has been intensively investigated in a lot of the existed works have been focused on the study of system stability under different situations, such as with fixed and switched topology, time delay, and quantization communication, see Fax et al. [2004]-Li et al. [2011]. On the other hand, many efforts have been directed towards the leader-follower system, where the leaders act as interventions which push all the followers reaching consensus on the leaders' state, see Hong et al. [2008], Peng et al. [2009], Ni et al. [2010], Semsar-Kazerooni et al. [2008], Jia et al. [2011].

Recently, to characterize and design useful interaction models systematically, the controllability properties of the underlying interaction network are investigated, where a number of agents are leaders and the remaining agents are followers, see Tanner et al. [2004], Porfiri et al. [2008], Yoon et al. [2011], Jafari et al. [2011], Parlangeli et al. [2011], Lozano et al. [2008], Borsche et al. [2010]. Specifically, Rahmani et al. [2009] have shown how the symmetry structure of the network, characterized in terms of its automorphism group, directly relates to the controllability of the corresponding multi-agent system. Moreover, Egerstedt et al. [2012] systematically discuss the relationship between network structure and controllability properties in single-leader consensus networks and collect some of the key results that have emerged in this area during the last five years.

In most cases, we not only need that all the nodes reach consensus but also hope that the agreement value is a desired value. Since most of real networks consist of a large number of nodes, it is difficult to implement controllers to all the nodes. To save valuable power and cost in a real network, an effective control scheme is adding control input to a fraction of nodes in the network to achieve desired performance. see Li et al. [2004]-Chen et al. [2009].

A natural question that arises is, in a resource constrained network, how to select a fraction of control nodes so as to optimize network performances in terms of lowest energy consumption and/or fastest convergence speed? The optimal selection in real networks are of practical importance, for example, in social networks, the information spreads faster from those influential peoples, and in an animal group, the prey is hunted with higher probability if those animals with much more experiences are selected to be the leaders.

In this work, we reduce the network cost over finite time horizon, we formulate the control nodes selection problem as optimization problems. It is to determine the binary values denoting whether the node is added control input in order to minimize network cost. A related work in (Porfiri et al. [2009]) proposed a node-to-node pinning control strategy to optimize the control performance, where the control input is added on a switched node each time. Clark et al. [2012] presented a framework for selecting leaders based on joint consideration of controllability and performance. In this paper, we discuss the control nodes selection problem with the goal of minimizing the network costs. By replacing the l_0 norm by l_1 norm, the problems are relaxed to a convex optimization problem. In

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this paper, we consider fixed nodes selection problem, where the control inputs are added on fixed nodes over finite time horizon. We formulate the problem as a general combinatorial optimization problem. Through simulations, we explore the relationship between optimality and node properties.

The remainder of this paper is organized as follows. In Section 2, we introduce a general controlled consensus problem for the optimization of network cost involving the constraints. In Section 3, we introduce the nodes selection problem and provide the main results of this paper. Simulation examples are given in Section 4 and some concluding remarks are given in the end.

Notations: $\mathbb{R}^n$ is the $n-$dimensional Euclidean space. $\mathbb{S}^n$ represents symmetric $n \times n$ matrices. $\mathbb{S}^n_+$ and $\mathbb{S}^n_{++}$ represents symmetric positive semidefinite and positive definite matrices, respectively. When X is positive semi-definite matrix and positive definite matrix, it is written as $X \succeq 0$ and $X \succ 0$, respectively. Moreover, $X \succeq Y$ if $X - Y$ is positive semi-definite matrix. $\mathrm{tr}(\cdot)$ is the trace of a matrix. I_n is the n identity matrix. $\mathbf{1}_n$ is the vector with all components one. $\rho(\cdot)$ is the spectral radius of a matrix.

2. PROBLEM FORMULATION

Consider a network described as an undirected graph $G = (V, E)$ with $V = (1, 2, \ldots, n)$ being the set of n nodes, and the edges $E \subset V \times V$ representing the communication links. Denote the set of neighbors of node i by $N_i = \{j : (i, j) \in E\}$. Each node can exchange information with its neighbors. The interconnection topology of the network is described by a weighted matrix $W = [w_{ij}]$, where $w_{ii} = 1 - \sum_{j \in N_i} w_{ij}$ and $0 < w_{ij} \leq 1$ if $(i, j) \in E$; otherwise, $w_{ij} = 0$. Here, we assume that the network G is connected. Thus, W is nonnegative and stochastic.

In this paper, we consider reaching desired agreement by adding control inputs on a fraction of nodes. Each node updates its state as

$$x_i(k+1) = \sum_{j=1}^{n} w_{ij} \cdot x_j(k) + l \cdot \gamma_i \cdot u_i(k) \tag{1}$$

where

$$u_i(k) = c - x_i(k).$$

Here, $x_i(k) \in \mathbb{R}^m$ is the state of ith node at time step k, $c \in \mathbb{R}^m$ is the desired state. Note that $0 < l < \min\{w_{ii}\}$ is a constant gain. If node i is added control input at time step k, then $\gamma_i = 1$; otherwise, $\gamma_i = 0$. Assume that the number of control nodes is less than or equal to q. Let

$$\Gamma \triangleq \mathrm{diag}(\gamma_1, \ldots, \gamma_n),$$

$$u(k) \triangleq [u_1(k)^T, \ldots, u_n(k)^T]^T.$$

Here, we take $m = 1$ for simplicity. However, all the results can be extended to the case $m > 1$ using Kronecker product.

In this paper, we are interested into one problem: how to select a fraction of nodes as the control nodes to minimize network costs while satisfying specific constraints? The nodes selection problem can be formulated as the following optimization problem:

$$\begin{aligned}(P_0):\ & \min_{\Gamma} \sum_{k=0}^{T} x(k)^T x(k) + u(k)^T u(k) \\ & \text{s.t. } \mathrm{tr}(\Gamma) \leq q \\ & \qquad \gamma_i = 0 \text{ or } 1,\ i = 1, \ldots, n,\end{aligned} \tag{2}$$

Before solving the above optimization problem, we analyze the stability of the protocol (1). By concatenating the states of all the nodes, the discrete-time network dynamics is,

$$x(k+1) = W_f x(k) + cl \cdot \Gamma \mathbf{1}_n, \tag{3}$$

where $x(k) = [x_1(k)^T, x_2(k)^T, \ldots, x_n(k)^T]^T$ denotes the aggregated state vector of the network system, and $W_f = W - l\Gamma$. The choice of l guarantees that W_f is nonnegative.

We need the following lemmas in (Horn et al. [1985]) to obtain the stability results.

Lemma 1. The matrix A is irreducible if and only if its corresponding graph is connected.

Lemma 2. If an irreducible matrix is weakly diagonally dominant, but in at least one row (or column) is strictly diagonally dominant, then the matrix is irreducibly diagonally dominant.

Lemma 3. Let A be irreducibly diagonally dominant. Then A is invertible.

Proposition 1. If $W = (w_{ij})^n_{i,j=1}$ is an irreducible non-negative matrix and satisfying $w_{ij} = w_{ji}$, if $i \neq j$, and $W\mathbf{1}_n = \mathbf{1}_n$. Then, if $\varepsilon < \min_i(w_{ii})$, all the eigenvalues of the matrix

$$\tilde{W} = W - \varepsilon e_i e_i^T,\ 1 \leq i \leq n$$

are within $(-1, 1)$. Here, e_l denotes the vector with a 1 in the ith coordinate and 0's elsewhere.

Proof: Note that $\sum_{j=1}^n \tilde{w}_{ij} \leq 1$ for $i = 1, 2, \ldots, n$. According to Gershgorin circle theorem, all the eigenvalues of $\tilde{W}$ are within $(-1, 1]$ because each diagonal element $\tilde{w}_{ii} > 0$. Suppose that $\lambda = 1$ is an eigenvalue of $\tilde{W}$ with corresponding eigenvector $v = [v_1, \ldots, v_n]^T$, that is, $(I_n - \tilde{W})v = 0$.

Without loss of generality, we assume that $i = 1$.

$$\det(\tilde{W} - I_n) = \begin{vmatrix} \tilde{w}_{11} - \varepsilon - 1 & \tilde{w}_{12} & \ldots & \tilde{w}_{1n} \\ \tilde{w}_{21} & \tilde{w}_{22} - 1 & \ldots & \tilde{w}_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ \tilde{w}_{n1} & \tilde{w}_{n2} & \ldots & \tilde{w}_{nn} - 1 \end{vmatrix}$$

$$= \begin{vmatrix} -\varepsilon & \tilde{w}_{12} & \ldots & \tilde{w}_{1n} \\ 0 & \tilde{w}_{22} - 1 & \ldots & \tilde{w}_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ 0 & \tilde{w}_{n2} & \ldots & \tilde{w}_{nn} - 1 \end{vmatrix}$$

Denote

$$B := \begin{bmatrix} \tilde{w}_{22} - 1 & \ldots & \tilde{w}_{2n} \\ \vdots & \ldots & \vdots \\ \tilde{w}_{n2} & \ldots & \tilde{w}_{nn} - 1 \end{bmatrix},$$

and let $E = (\tilde{w}_{12}, \ldots, \tilde{w}_{1n})$. It is easy to verify that $-B$ is weakly diagonally dominant matrix. Further, we can rearrange the indices of $n-1$ nodes such that B can be rewritten as a block diagonal matrix

$$B = \begin{bmatrix} B_1 & 0 & \ldots & 0 \\ 0 & B_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & B_h \end{bmatrix},$$

where each block matrix B_i, $i = 1,2,\dots,h$ corresponds to a connected subgraph. By Lemma 1, each B_i is irreducible. Again since W is irreducible, there must exist a node in each block B_i which connects with node 1, otherwise, the graph associated with W is not connected. It implies that at least one row is strictly diagonally dominant in each B_i because $\tilde{w}_{j1} \neq 0$, and the jth row is one row in B_i. By Lemma 2, because each $-B_i$ is weakly diagonally dominant matrix, thus $-B_i$ is irreducibly diagonally dominant. By Lemma 3, $-B_i$ is invertible. Thus, $\det(B) \neq 0$. Further,

$$\det(\tilde{W} - I_n) = \begin{vmatrix} -\varepsilon & E \\ 0 & B \end{vmatrix} = -\varepsilon \cdot \det(B) \neq 0.$$

Thus, $\lambda \neq 1$. Further, $|\lambda| < 1$.■

Let

$$z(k) = x(k) - c\mathbf{1}_n.$$

The state error system is,

$$\begin{aligned} z(k+1) &= x(k+1) - c\mathbf{1}_n \\ &= W_f x(k) + cl\Gamma\mathbf{1}_n - c\mathbf{1}_n \\ &= W_f(x(k) - c\mathbf{1}_n) + cW_f\mathbf{1}_n + cl\Gamma\mathbf{1}_n - c\mathbf{1}_n \\ &= W_f z(k). \end{aligned} \tag{4}$$

The last equality holds because

$$(W_f + l\Gamma)\mathbf{1}_n = W\mathbf{1}_n = \mathbf{1}_n.$$

It is easy to find that all the nodes reach agreement on the desired state c if $z(k) \to 0$ as $k \to \infty$. By Proposition 1, we directly give the following result.

Theorem 1. Consider an undirected network consisting of n nodes with dynamics (1) . All the nodes' state converge to the desired state if any one node is controlled.

Proof: By the assumption that G is connected, W is nonnegative and irreducible. It is easy to show that the state error system (4) is asymptotically stable by Proposition 1.■

3. MAIN RESULTS

As discussed above, the whole network reaches agreement on a desired state if any one node is controlled. This section is devoted to relaxing the optimization problem (P_0) to make it explicit and convex with respect to the optimization variables.

3.1 Fixed nodes selection problem

In this section, we investigate the fixed control nodes selection problem.

Let

$$u(k) = c\mathbf{1}_n - x(k).$$

One obtains

$$u(k+1) = (W - l\Gamma)u(k). \tag{5}$$

By iteration, one has

$$u(k) = (W - l\Gamma)^k u(0).$$

Let $\hat{W} = W - l\Gamma$. Then,

$$\begin{aligned} \sum_{k=0}^{\infty} u(k)^T u(k) &= u(0)^T \sum_{k=0}^{\infty} \hat{W}^{kT}\hat{W}^k u(0) \\ &= u(0)^T (I_n - \hat{W}^2)^{-1} u(0) \end{aligned}$$

Because

$$(I_n - \hat{W}^2)^{-1} = \frac{1}{2}(I_n + \hat{W})^{-1} + \frac{1}{2}(I_n - \hat{W})^{-1},$$

we have

$$\sum_{k=0}^{\infty} u(k)^T u(k) = \frac{1}{2}u(0)^T((I + \hat{W})^{-1} + (I_n - \hat{W})^{-1})u(0).$$

By Proposition 1, we know that $-I \prec \hat{W} \prec I$. By recursion, we have

$$\begin{aligned} x(k) &= \hat{W}^k x(0) + l\sum_{i=0}^{k-1} \hat{W}^i \Gamma c\mathbf{1}_n, \\ &= \hat{W}^k x(0) + l[I_n - \hat{W}^k](I_n - \hat{W})^{-1}\Gamma c\mathbf{1}_n, \\ &= \hat{W}^k x(0) + c(I_n - \hat{W}^k)\mathbf{1}_n, \end{aligned}$$

The last equality follows from the fact that

$$l\Gamma\mathbf{1}_n = (W - \hat{W})\mathbf{1}_n = (I_n - \hat{W})\mathbf{1}_n.$$

Further, we have

$$\begin{aligned} &\sum_{k=0}^{T} x(k)^T x(k) + u(k)^T u(k) \\ &\quad < \sum_{k=0}^{T} ||\hat{W}^k x(0) + c(I_n - \hat{W}^k)\mathbf{1}_n||_2^2 \\ &\quad + \frac{1}{2}u(0)^T((I + \hat{W})^{-1} + (I - \hat{W})^{-1})u(0). \end{aligned} \tag{6}$$

Since the inverse of a positive definite matrix is a convex function of the matrix (See Exercise 3.18, Boyd [2009]), the second term of (6) is a convex function. Next, we are going to investigate the convexity of the first term of (6).

Lemma 4. Consider the minimization problem

$$\min_{\Gamma} \sum_{k=0}^{T} ||\hat{W}^k x(0) + c(I_n - \hat{W}^k)\mathbf{1}_n||_2^2, \tag{7}$$

with $\gamma_i \in \{0,1\}$, $i = 1,\dots,n$. The suboptimal solution of (7) can be derived by solving the following problem,

$$\begin{aligned} \min_{\Gamma} \quad & x(0)^T (I_n - \hat{W}^2)^{-1} x(0) + 2cx(0)^T (I_n - \hat{W})^{-1}\mathbf{1}_n \\ & + c^2\mathbf{1}_n^T(TI_n - \hat{W} - 2(T-2)\hat{W}^2 + (T-2)\hat{W}^4)\mathbf{1}_n \\ & - 2cx(0)^T(I_n - \hat{W}^2)^{-1}\mathbf{1}_n \end{aligned} \tag{8}$$

where the function in (8) is the upper bound of the function in (7).

Proof: Again since $\rho(\hat{W}) < 1$, it is easy to obtain

$$\begin{aligned} \sum_{k=0}^{T} x(0)^T \hat{W}^{kT}\hat{W}^k x(0) &< \sum_{k=0}^{\infty} x(0)^T \hat{W}^{kT}\hat{W}^k x(0) \\ &= x(0)^T (I_n - \hat{W}^2)^{-1} x(0). \end{aligned} \tag{9}$$

Further, one obtains

$$\begin{aligned} \sum_{k=0}^{T} x(0)^T \hat{W}^k (I_n - \hat{W}^k)\mathbf{1}_n &< \sum_{k=0}^{\infty} x(0)^T \hat{W}^k (I_n - \hat{W}^k)\mathbf{1}_n \\ &= \sum_{k=0}^{\infty} x(0)^T (\hat{W}^k - \hat{W}^{2k})\mathbf{1}_n \\ &= x(0)^T (I_n - \hat{W})^{-1}\mathbf{1}_n - x(0)^T (I_n - \hat{W}^2)^{-1}\mathbf{1}_n. \end{aligned} \tag{10}$$

Note that

$$\sum_{k=0}^{T}(I_n-\hat{W}^k)^2 < \sum_{k=0}^{T}(2I_n-\hat{W})+(T-2)(I_n-\hat{W}^2)^2$$
$$= TI_n-\hat{W}-2(T-2)\hat{W}^2+(T-2)\hat{W}^4 \quad (11)$$

Following from (9), (10) and (11), it is directly to have, the objective function of (8) is the upper bound of the objective function 7. Thus, we can obtain a suboptimal solution of (7) by solving the minimization problem (8). ■

Using all the previous arguments, and denoting the ith column of the matrix $\hat{W}$ as $(\hat{W})_i$, we can reformulate the optimization problem P_0 as the following problem.

Theorem 2. Consider the linear system (1) and let Γ be the node selection matrix. The minimization problem over Γ of the cost function

$$\sum_{k=0}^{T} x(k)^T x(k)+u(k)^T u(k)$$

can be formulated as

$$\begin{aligned}(P_1): \min_{\Gamma}\quad & x(0)^T(I_n-\hat{W}^2)^{-1}x(0) \qquad (12)\\ & +\frac{1}{2}(x(0)+\mathbf{1}_n)^T(I_n-\hat{W})^{-1}(x(0)+\mathbf{1}_n)\\ & -\frac{1}{2}x(0)^T(I_n-\hat{W})^{-1}x(0)-\frac{1}{2}\mathbf{1}_n^T(I_n-\hat{W})^{-1}\mathbf{1}_n\\ & -\frac{1}{2}(x(0)+\mathbf{1}_n)^T(I_n+\hat{W})^{-1}(x(0)+\mathbf{1}_n)\\ & +\frac{1}{2}x(0)^T(I_n+\hat{W})^{-1}x(0)+\frac{1}{2}\mathbf{1}_n^T(I_n+\hat{W})^{-1}\mathbf{1}_n\\ & +c^2\mathbf{1}_n^T[TI_n-\hat{W}-2(T-2)\hat{W}^2+(T-2)\hat{W}^4]\mathbf{1}_n\\ & +u(0)^T(I_n-\hat{W}^2)^{-1}u(0)\\ s.t.\quad & \Gamma\mathbf{1}\le q,\\ & \gamma_i \ge \left\|1-||(\hat{W})_i||_1\right\|_0,\ i=1,\cdots,n,\end{aligned}$$

where $||\cdot||_0$ (called the l_0 norm) of a scalar is 0 if the scalar if 0 (it is 1 otherwise) and $\bar{k}$ is the smallest number which satisfies the third constraint.

Proof: Again since

$$(I_n-\hat{W}^2)^{-1}=\frac{1}{2}(I_n+\hat{W})^{-1}+\frac{1}{2}(I_n-\hat{W})^{-1},$$

we have

$$\begin{aligned}& 2x(0)^T[(I_n-\hat{W})^{-1}-(I_n-\hat{W}^2)^{-1}]\mathbf{1}_n\\ &= x(0)^T[(I_n-\hat{W})^{-1}-(I_n+\hat{W})^{-1}]\mathbf{1}_n\\ &= \frac{1}{2}(x(0)+\mathbf{1}_n)^T(I_n-\hat{W})^{-1}(x(0)+\mathbf{1}_n)\\ &-\frac{1}{2}(x(0)+\mathbf{1}_n)^T(I_n+\hat{W})^{-1}(x(0)+\mathbf{1}_n)\\ &-\frac{1}{2}x(0)^T(I_n-\hat{W})^{-1}x(0)-\frac{1}{2}\mathbf{1}_n^T(I_n-\hat{W})^{-1}\mathbf{1}_n\\ &+\frac{1}{2}x(0)^T(I_n+\hat{W})^{-1}x(0)+\frac{1}{2}\mathbf{1}_n^T(I_n+\hat{W})^{-1}\mathbf{1}_n.\end{aligned}$$

The remainder of the reformulation follows directly from Lemma 4. ■

Remark 1. The main improvement of formulation (P_1) over (P_0) is that instead of

$$\sum_{k=0}^{T} x(k)^T x(k)+u(k)^T u(k),$$

we now have an explicit quadratic objective function which is independence of time horizon, and show an explicit relation between the objective function and the γ_is. Further note that each term of the objective function is a matrix fractional function. For a function $f:\mathbb{R}^n\times\mathbb{S}^n\to\mathbb{R}$, defined as

$$f(x,Y)=x^TY^{-1}x$$

is convex on $\mathbf{dom} f=\mathbb{R}^n\times\mathbb{S}^n_{++}$ (See, Example 3.4, Boyd [2009]). However, the problem P_1 is hard combinatorial problem because the constrain on γ_i contains the l_0 norm. In recent years, some heuristic methods have been proposed to replace l_0 norm to other norm like l_1 norm. Note that $(I_n-\hat{W}^2)^{-1}$, $(I_n+\hat{W}^2)^{-1}$ and $(I_n-\hat{W})^{-1}$ are positive definite matrix. Therefore, we can relax the problem P_1 to a convex optimization problem by using the reweighed l_1 norm, which are discussed in the subsection.

3.2 Sub-optimal nodes selection algorithm

For large scale networks, the combinatorial problem P_1 becomes computationally infeasible since it should check all possible nodes selection strategies satisfying the constraints. In this subsection, we relax the problem P_1 to a convex optimization problem, where the solutions are suboptimal but computationally feasible.

In recent years, some technique on replacing l_0 norm with l_1 norm are proposed, see Candes et al. [2008]. To avoid the main drawback that the larger numbers in l_1 are penalized much more than in l_0, we replace l_0 norm by a weighted l_1 norm which is similar to the work by Mo et al. [2011]. Using all the previous results, we propose this algorithm to solve the suboptimal problem P_1' relaxed by the problem P_1. The algorithm is composed of 4 steps:

1: Initialization: $l=0$, $w_i(0)=1\ for\ i=1,\ldots,n.$
2: Solve the weighted l_1 minimization problem:

$$\begin{aligned}(P_1'): \min_{\Gamma}\quad & x(0)^T(I_n-\hat{W}^2)^{-1}x(0) \qquad (13)\\ & +\frac{1}{2}(x(0)+\mathbf{1}_n)^T(I_n-\hat{W})^{-1}(x(0)+\mathbf{1}_n)\\ & -\frac{1}{2}x(0)^T(I_n-\hat{W})^{-1}x(0)-\frac{1}{2}\mathbf{1}_n^T(I_n-\hat{W})^{-1}\mathbf{1}_n\\ & -\frac{1}{2}(x(0)+\mathbf{1}_n)^T(I_n+\hat{W})^{-1}(x(0)+\mathbf{1}_n)\\ & +\frac{1}{2}x(0)^T(I_n+\hat{W})^{-1}x(0)+\frac{1}{2}\mathbf{1}_n^T(I_n+\hat{W})^{-1}\mathbf{1}_n\\ & +c^2\mathbf{1}_n^T[TI_n-\hat{W}-2(T-2)\hat{W}^2+(T-2)\hat{W}^4]\mathbf{1}_n\\ & +u(0)^T(I_n-\hat{W}^2)^{-1}u(0)\\ s.t.\quad & \Gamma\mathbf{1}\le q,\\ & \gamma_i\ge w_i^l(1-||(\hat{W})_i||_1),\ i=1,\cdots,n.\end{aligned}$$

Let the solution be $\gamma_1(l),\ldots,\gamma_n(l)$.
3: Update the weights:
$w_i(l+1)=\frac{1}{\gamma_i(l)+\delta}$, $\delta>0$.
4: Terminate if either l reaches a specified maximum number of iterations l_{max} or the solution has converged. Otherwise, increase l and return to step 2.

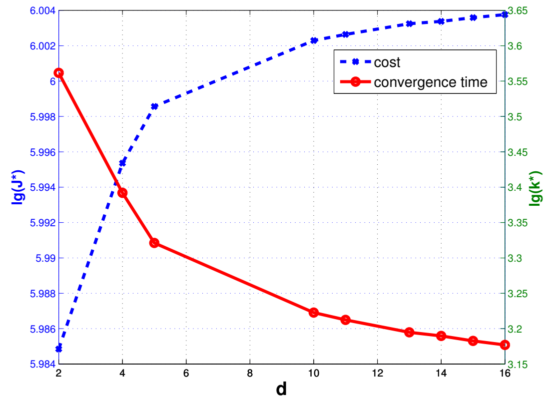

Fig. 1. The network cost $J^\star$ and the settling time $k^\star$ as functions of the degree of control node. Here, $p = 1$.

Remark 2. In real applications, we do not need a very accurate solution because we will threshold γ_i and make it binary. Usually the problem take less number of reweightings for the solution to converge. Since a function is convex if and only if its epigraph is a convex set Boyd [2009], we can represent the problem P_1' as a series of LMIs form, which can be solved by many methods, for example, interior point and steepest descent method.

4. NUMERICAL EXAMPLES

In this section, we investigate the efficiency of the proposed strategy for control nodes selection problem. Let us assume that only $p < n$ nodes are selected to add control at each time step. Throughout this section, we consider an undirected network with $n = 20$ nodes. Its second largest eigenvalue $\lambda_2(W) = 0.9129$. For the simulations, we impose $l = 0.05$, $c = [3,2]^T$.

In the case of $p = 1$, by solving the problem P_1', we find that the control node with lowest degree leads to minimal cost. We arrange nodes in increasing order of degree. In the simulations, we let the settling time be the minimal k satisfying $\frac{1}{n}\sum_{i=1}^{n}|x_i(k) - c|^2 \leq 10^{-5}$. Define $J^\star$ as $\sum_{k=0}^{4000} x(k)^T x(k) + u(k)^T u(k)$. Figure 1 shows the cost varying with node degree over window size $T = 4000$. Obviously, controlling those nodes with lower degree leads to less cost. In contrary, controlling the node with lower degree requires longer convergence time. It implies that a tradeoff exists between the network cost and convergence time. This result provides us an effective method to select control nodes to achieve the goal.

Further, we show the gap between the random selection strategy and the proposed selection strategy. In the simulations, we compute the cost of both strategies over the window size $T = 4000$ for different p. Note that the constraint in the problem P_1' is changed to $\Gamma\mathbf{1} = q$. As Figure 2 shows the cost gap decreases with the increasing of number of control nodes. The heuristic reason is that, the convergence speed increases with the increasing of number of control nodes. After all the nodes reach the target state, the cost equals to $n||c||_2^2$, which means that the gap accumulates during a short time horizon. On the other side, when p increases, the probabilistic of selecting those optimal control nodes increases as well in the random strategy. Thus, when $p = n$, the cost gap decreases to zero.

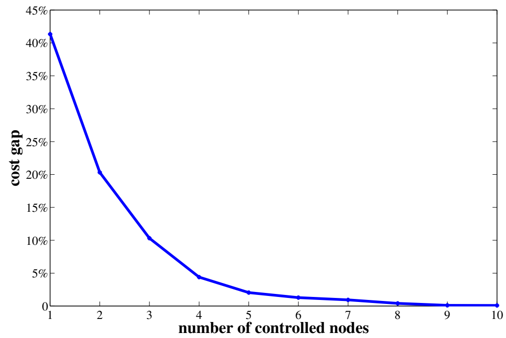

Fig. 2. Average cost gap of the proposed suboptimal strategy and the random strategy.

5. CONCLUSIONS

In this paper, we considered control nodes selection problem with the goal of minimizing network cost in a resource limited multi-agent system. We transformed the problem to a combinatorial optimization problem, and further reformulated it using a convex relaxation based on a reweighed l_1 approximation. The simulation results verify the effectiveness of the proposed suboptimal algorithm, and also show that, the node with lower degree leads to less network cost and longer convergence time, which provides a method to help us select the control nodes in real applications. In a counterpart work, we further discuss the control nodes selection problem in switched nodes case.

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