Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

A Control System Approach to Corrective Maintenance Planning of Building Retrofitted Facilities

Bo Wang, Xiaohua Xia

Center of New Energy Systems, Department of Electrical, Electronic and Computer Engineering, University of Pretoria, 0002 South Africa

Abstract: Traditional building retrofitting planning is a decision making process prior to the building retrofitting investment. Although the long term energy and financial issues of the retrofitting project are considered in some cases, the energy efficiency potentials due to the maintenance and operation of the retrofitted facilities are not sufficiently explored in existing studies. In the broad field of building facilities maintenance, a corrective maintenance planning for energy efficiency refers to the decisions over a sustainability period of a building retrofitting project concerning the replacements or repairs of the failed retrofitted facilities at scheduled maintenance intervals. This paper presents a control system approach to the Building Retrofitted Facilities Corrective Maintenance Planning (BRFCMP) problem. The totality of homogeneous population classes of the retrofitted facilities instead of individual items becomes the main issue of consideration, and a control system framework is proposed, based on which the optimization of BRFCMP is transformed to an optimal control problem. A Model Predictive Control (MPC) approach is then applied to solve the BRFCMP problem with and without consideration of uncertainties and disturbances. An actual building retrofitting project is used as the case study. Simulation results prove the feasibility and the effectiveness of the control system approach, revealing a further energy efficiency potential by optimizing the corrective maintenance of building retrofitted facilities.

Keywords: Energy efficiency, building retrofitting, facilities maintenance, control system framework, optimal control, MPC

1. INTRODUCTION

Buildings are nowadays responsible for a large part of the total energy consumption in the world. In order to reduce the energy demands from the existing buildings, building retrofitting as one of the most feasible and effective methods is widely applied. During building retrofitting, energy inefficient facilities are replaced by perspective facilities, which are applied with alternatives and more energy efficient technologies, namely Energy Conservation Measures (ECMs). Usually, the selection of ECMs must consider a series of energy and non-energy related factors. An optimal building retrofitting plan should not only be energy efficient, but also cost-effective and financially sustainable. The satisfaction of human comfort requirements is also taken into account in some cases. Generally, the task of building retrofitting optimization is to find a solution which strikes a balance between the stakeholders' and occupants' various requirements (Ma et al. (2012)).

Most existing studies on building retrofitting optimization focus on retrofitting planning, i.e., finding a set of optimal ECMs and corresponding implementations to optimize the building energy efficiency (Asadi et al. (2012) and Malatji et al. (2013)). The retrofitting planning is a decision making prior to the building retrofitting investment, in which the long term issues, e.g., the Net Present Value (NPV) of the project, the Life-Cycle Cost Analysis (LCCA), etc., are often involved. However, the optimization and the associated energy efficiency potentials of the maintenance and operation of the retrofitted facilities are not sufficiently and significantly explored. The state of the retrofitted facility population is not constant, but varying over the operating period due to failures, maintenances and the operation of facility items. The maintenance and operation of building facilities is a broad field. Form the perspective of building energy efficiency, maintenance and operation adjust the number and conditions of the available items, so as to maintain the energy efficiency of the project. Given a retrofitting plan, the maintenance plan and operating schedules of the retrofitted facilities can both be optimized to support further improvement of building energy efficiency. A retrofitting plan is usually optimized based on the performance characteristics that are estimated over a long time period e.g., 10 years. The facility population manifests a dynamical change requiring corrective maintenance in 1 or 2 years, while operating schedules concern issues of shorter intervals such as days or hours. In addition, budget limits for maintenance are introduced in practices, which also restrict the implementation of maintenance actions. Therefore, building retrofitting optimization problems can become very complex ones with multiple time scales and substantial magnitude if taking account of maintenance and operation of the retrofitted facilities, due to the complexity and interplay of the retrofitting planning, maintenance and operation.

In this paper, Corrective Maintenance, a subcategory of facilities maintenance which mainly refers to the replacements and repairs of the failed items, is investigated. A subproblem of the optimization of the corrective maintenance of retrofitted facilities is accordingly introduced, namely the Building Retrofitted Facilities Corrective Maintenance Planning (BRFCMP) problem, as the main issue discussed in this paper. The BRFCMP problem only considers corrective maintenances for failures that prevent retrofitted facilities from working and contributing to the energy saving. For simplicity, the BRFCMP issue concerned is a maintenance planning at the population level rather than at level of individual items, and at predefined maintenance schedules. Some important facilities, whose failures must be immediately repaired, are not included in the BRFCMP. In this problem, homogeneous classes of facilities are grouped, so as to obtain the managed facility populations. The classification of facilities must take account of three kinds of characteristics: the inherent energy and reliability performance, the operational environment of the items and the corresponding operating schedules. The energy and financial performance with respect to the classes is thus a key issue of the BRFCMP. From the perspective of control systems, the totality of the facility population of retrofitted facilities can be considered as the control plant, and the quantities of available items in each homogeneous class can be considered as the state variables of the system, which are also the key parameters to decide the energy and financial performance of the retrofitting project. The corrective maintenance actions, i.e., the numbers of maintained items with respect to the classes, are taken as the control inputs. The measurements of the state variables are taken as the system output. Accordingly, the internal dynamics of the state variables is the facility population dynamics due to failures. In this way, the control system approach also brings in the studies of reliability engineering, where deterministic or stochastic models of facility reliability are vigorously built. A series of common failure distributions, reliability and hazard rate functions for facilities with various reliability characteristics are provided by O'Connor and Kleyner (2011), according to which the facility population dynamics can be characterized. It is expected that the research progress in reliability engineering will facilitate the advance of the control system approaches for building energy efficiency optimization, and vice versa. The characterization of facility population dynamics can also be found from some existing studies in Clean Development Mechanism (CDM) environment, which consider the facility population dynamics either by a simplified linear assumption (UNFCCC (2010)) or an experimental data fitting (Carstens et al. (2013)). Finally, the uncertainties must be taken into account, as well as disturbances of the system.

The purpose of this paper is to cast the BRFCMP problem into a control system framework, so that it can be solved with the control system techniques. As introduced above, the key issues of the BRFCMP problem can be reasonably modeled by a control system framework, and the optimal control techniques can be employed to solve the corresponding optimization problem. The optimization objectives are transformed into control objectives. Similar to the retrofitting planning, the objectives are maximizing the energy savings and economic viability of the retrofitted facilities over the sustainability period of the retrofitting project. The optimal maintenance plan is obtained to further improve the long term energy efficiency and cost-effectiveness of the given retrofitting plan. An optimal control model is accordingly defined for the BRFCMP problem. For simplicity, two further assumptions are made: the disturbances of the system are generally considered as a random noise on state variables; the sampling errors are also simplified as a random noise on measured output. Secondly, the optimization of maintenance scheduling or the timing of corrective maintenance actions are not considered in the current BRFCMP problem. Rather, it is assumed that corrective maintenances are done at known, pre-defined intervals. In practice, corrective maintenances do not continuously take place. Usually, for convenience of management, such maintenances can happen once every two or three years. A Model Predictive Control (MPC) based approach is applied to solve the problem without or taking into account the control system disturbances. As a case study, a practical building retrofitting project is used to test and verify the feasibility of the presented approach.

The remainder of the paper consists of four sections. Section 2 gives the formulation of the control system framework and the optimal control problem for BRFCMP. Section 3 introduces the MPC based approach. Section 4 provides the simulation and analysis. Section 5 draws conclusion and discusses future research.

2. OPTIMAL BUILDING MAINTENANCE PROBLEM FORMULATION

2.1 A control system framework for BRFCMP problem

Assuming there are I groups of retrofitted facilities with respect to the homogeneous classes in the building, where items from the same class are considered as one group. Let $t_k = kS, k = 0, 1, 2, ...$ denote the sampling instants over the control period, where S is the sampling interval. Let x_i denote the quantity of the initially installed items corresponding to each group of retrofitted facilities, $i = 1, 2, ..., I$. Let $\mathbf{x}(t_k) = [x_1(t_k), x_2(t_k), ..., x_I(t_k)]^\tau$ denote the state variable vector, which represents the quantities of the items working over sampling period $[t_k, t_{k+1})$, with respect to the I groups; $\mathbf{u}(t_k) = [u_1(t_k), u_2(t_k), ..., u_I(t_k)]^\tau$ denote the control input vector, which represents the corrective maintenance actions with respect to the facility groups; $\mathbf{w}(t_k) = [w_1(t_k), w_2(t_k), ..., w_I(t_k)]^\tau$ as well as $\mathbf{d}(t_k) = [d_1(t_k), d_2(t_k), ..., d_I(t_k)]^\tau$ denote the disturbances on the state variables and measured output, respectively. Given $\mathbf{D}(\cdot) = [D_1(\cdot), D_2(\cdot), ..., D_I(\cdot)]^\tau$ denoting the decline of the facility populations. $D_i(x_i(t_k))$ represents the decline of population x_i over $[t_k, t_{k+1})$, and $x_i(t_{k+1})$ can be calculated by the following equation:

$$x_i(t_{k+1}) = D_i(x_i(t_k)) + u_i(t_k). \qquad (1)$$

The measured output $\mathbf{y}(t_k)$ is the measurement of $\mathbf{x}(t_k)$. Generally, $\mathbf{y}(t_k)$ is obtained by sampling the group $x_i(t_k)$ at time t_k. The accuracy and confidential level of sampling is determined by the sampling size (Ye et al. (2013)). The corrective maintenance of retrofitted facilities over a sustainability period $[0, TS)$ of the retrofitting project is thus formulated as follows:

$$\begin{cases} \mathbf{x}(t_{k+1}) = \mathbf{D}(\mathbf{x}(t_k)) + \mathbf{u}(t_k) + \mathbf{w}(t_k), \\ \mathbf{y}(t_k) = \mathbf{x}(t_k) + \mathbf{d}(t_k). \end{cases} \tag{2}$$

Given the initial state $x_i(0) = x_i$, $\mathbf{x}(0) = (x_1, x_2, ..., x_I)$. Let $\mathbf{x} = \{\mathbf{x}(1), \mathbf{x}(2), ..., \mathbf{x}(T)\}$ denote the time series of the system state and $\mathbf{u} = \{\mathbf{u}(1), \mathbf{u}(2), ..., \mathbf{u}(T)\}$ denote the time series of the control input, respectively. The formulation of the optimal control problem for the BRFCMP is then introduced as follows.

2.2 Optimal control problem formulation

As mentioned in the previous section, the nature of the BRFCMP problem is a multi-objective optimization problem. In order to make it easier to solve, a weighted sum method is implemented to establish the objective function, which is defined as:

$$J = -\lambda_1 f_e(\mathbf{x}, \mathbf{u}) - \lambda_2 f_n(\mathbf{x}, \mathbf{u}), \tag{3}$$

where λ_1 and λ_2 are the weights. $f_e(\mathbf{x}, \mathbf{u})$ represents the total energy saving and $f_n(\mathbf{x}, \mathbf{u})$ represents the NPV during the sustainability period, which indicates the financial benefit of the project, respectively. $f_e(\mathbf{x}, \mathbf{u})$ and $f_n(\mathbf{x}, \mathbf{u})$ are calculated by the following equations:

$$f_e(\mathbf{x}, \mathbf{u}) = \sum_{k=0}^{T-1} \sum_{i=1}^{I} a_i(t_k) x_i(t_k), \tag{4}$$

$$f_n(\mathbf{x}, \mathbf{u}) = \sum_{k=0}^{T-1} \frac{B(t_k) - C_m(t_k)}{(1+d)^k}, \tag{5}$$

where

$$B(t_k) = \sum_{i=1}^{I} a_i(t_k) x_i(t_k) p(t_k), \tag{6}$$

$$C_m(t_k) = \sum_{i=1}^{I} u_i(t_k) m_i(t_k). \tag{7}$$

In equation (4) - (7), $a_i(t_k)$ represents the energy saving of an item from the *i-th* group of retrofitted facilities over the sampling period $[t_k, t_{k+1})$. $B(t_k)$ is the cash inflow over the same sampling period, which comes from the profit of saved energy, and $C_m(t_k)$ is the cash outflow which comes from the cost of maintenances. $p(t_k)$ denotes the energy price over $[t_k, t_{k+1})$ and $m_i(t_k)$ denotes the maintenance price for an item from the *i-th* group. d is the discount rate of NPV calculation, by which the cash flow in the future are translated into the present value of money.

Pre-scheduled corrective maintenance is represented by an additional constraint for BRFCMP. Let $Q = \{k_1, k_2, ...\}$ denote the set of instants that the maintenance takes place during $[0, TS)$. If $k \notin Q$, $u_i(t_k) = 0$.

The BRFCMP problem is then defined as follows:

Given λ_1, λ_2, $a_i(t_k)$, $p(t_k)$, $m_i(t_k)$, d and the decay models $D_i(\cdot)$, $x_i(0) = x_i$, $u_i(0) = 0$, $0 \le k < T$, $1 \le i \le I$, let

$$x_i(t_{k+1}) = D_i(x_i(t_k)) + u_i(t_k) + w_i(t_k),$$

and solve the minimization problem:

$$min\ J, \tag{8}$$

subject to

$$\begin{cases} \sum_{k=0}^{T-1} \sum_{i=1}^{I} a_i(t_k) x_i(t_k) \ge \sum_{k=1}^{T} \alpha(t_k), \\ \sum_{i=1}^{I} u_i(t_k) m_i(t_k) \le \beta(t_k), \\ u_i(t_k) = 0,\ k \notin Q, \end{cases} \quad zhang2009jade \tag{9}$$

where $\alpha(t_k)$ is the targeted energy saving value and $\beta(t_k)$ is the maintenance budget limit over $[t_k, t_{k+1})$. The adopted constraints come from the practical building retrofitting project requirements. In practical projects, the actual energy saving amounts must be guaranteed larger than the targeted values. The maintenance costs are also regulated by the budget limits. Thus $\alpha(t_k)$ and $\beta(t_k)$ are pre-decided by the decision maker.

2.3 Population decay model

In the current BRFCMP problem, two types of decay models are employed to characterize the facility population dynamics. One of them corresponds to non-repairable items such as the lighting facilities and motion sensors, and the other one corresponds to repairable items such as air conditioners, chillers and heat pumps. Equations (10) and (11) describe both decay models respectively:

$$D_i(x_i(t_k)) = b_i c_i x_i(t_k)^2 / x_i - b_i x_i(t_k) + x_i(t_k), \tag{10}$$

$$D_i(x_i(t_k)) = x_i(t_k) e^{-\zeta_i}, \tag{11}$$

where the coefficients b, c, ζ are estimated by the Mean Time To Failure (MTTF) of the non-repairable items and Mean Time Between Failures (MTBF) of the repairable items. The decay model for non-repairable products as described in equation (10) is taken from Carstens et al. (2013). Let L_i denote the MTTF, i.e., the rated lifetime of the item from population x_i. The general form of time-domain decay model $P_i(t) = (c_i + e^{b_i t - L_i})^{-1}$ can be found in Carstens et al. (2013), where $P_i(t)$ is the proportion of surviving items in the whole population. For this decay model, given L_i is known , b_i and c_i can be obtained by solving out the following equations:

$$\begin{cases} P_i(0) = 1, \\ P_i(L_i) = 0.5. \end{cases} \tag{12}$$

b_i and c_i can also be identified from the experimental data. Equation (11) describes the decay model for repairable products. As the length of the rated life time is usually several times longer than the MTBF for such items, according to the reliability bathtub curve, the failure rate of the population is an approximately low constant before the end of the lifetime. Therefore an exponential decay model is adopted from O'Connor and Kleyner (2011) in equation (11). Let θ_i denote the MTBF of the facility, then ζ_i is obtained from the following equation:

$$\zeta_i = (\theta_i)^{-1}. \tag{13}$$

Both equations (10) and (11) are actually statistical models, which are considered as first-order Markov processes in the current BRFCMP problem. Yet again for simplicity of discussion, another important assumption is made: the replacement or repaired items of the failed items are from the same respective homogeneous classes, they thus share the same decay rates. L_i and θ_i are a necessary priori knowledge according to the model. They can be obtained from the facilities producers or the historical performances of the items.

3. MPC APPROACH TO OPTIMAL BUILDING MAINTENANCE

This section will propose MPC based approaches to solve the BRFCMP problem. The MPC approaches for BRFCMP Problem without or taking into account the control system disturbances are discussed respectively.

3.1 MPC approach to undisturbed BRFCMP problem

In MPC approaches, an open loop optimal control problem is repeatedly solved over a finite horizon according to the plant model prediction. The obtained optimal open loop control is then used to generate the optimal control input for the problem to be solved, with which the state variables executed over the next finite horizon are obtained. As the optimal controller over the next finite horizon is actually a function of the system state from the previous control step, a closed-loop feedback is thus obtained. Consider a horizon with length N, a mathematical transformation of the BRFCMP problem is applied, and the open loop optimal control problem over $[t_m, t_{m+N})$ is accordingly defined as the following minimization problem:

$$min\ J' = -\lambda_1 f'_e(\mathbf{x}, \mathbf{u}) - \lambda_2 f'_n(\mathbf{x}, \mathbf{u}), \tag{14}$$

subject to

$$\begin{cases} x_i(t_{m+1}) = D_i(x_i(t_m)) + u_i(t_m),\ initialized\ at\ \mathbf{x}(t_m), \\ f'_e(\mathbf{x}, \mathbf{u}) \geq \sum_{k=m}^{m+N-1} \alpha(t_k), \\ \sum_{i=1}^{I} u_i(t_k) m_i(t_k) \leq \beta(t_k), \\ u_i(t_k) = 0,\ k \notin Q, ... \end{cases} \tag{15}$$

where

$$f'_e(\mathbf{x}, \mathbf{u}) = \sum_{k=m}^{m+N-1} \sum_{i=1}^{I} a_i(t_k) x_i(t_k), \tag{16}$$

$$f'_n(\mathbf{x}, \mathbf{u}) = \sum_{k=m}^{m+N-1} \frac{B(t_k) - C_m(t_k)}{(1+d)^{t_k}}. \tag{17}$$

This problem is solved over the interval $[t_m, t_{m+N})$ when $m \in Q$, and a series of optimal control inputs are obtained, represented by $\mathbf{u}'|_m = \{u'_i|_m(t_k) : i = 1, 2, ..., I, k = m, m+1, ..., m+N-1\}$. Given the decay model in equation (10), the minimization problem in equation (14) is nonlinear. A DE based approach is thus applied to solve problem (14) (Zhang and Sanderson (2009)). Only the optimal solution in the first sampling period $[t_m, t_{m+1})$ is applied, represented by $\bar{\mathbf{u}}|_m = \{u'_i|_m(t_m)\} = \{\bar{u}|_m(\mathbf{x}(t_m))\}$, where the last equation is to emphasize the functional dependence of the optimal control on the initial state $\mathbf{x}(t_m)$ of the MPC formulation in equations (14)-(17). According to equation (1), $\bar{\mathbf{u}}|_m$ is applied, and $\mathbf{x}(t_{m+1})$ is obtained. $\mathbf{x}(t_{m+1})$ then becomes the initial condition of the MPC formulation over the time horizon $[t_{m+1}, t_{m+N+1})$. When $m \notin Q$, the control $\mathbf{u}(t_m) = 0$ is implemented as a solution. These are taking place consecutively over the control period to obtain the optimal control inputs $\bar{\mathbf{u}}$. For an undisturbed control system model, where the disturbances $\mathbf{w}(t_k)$ and $\mathbf{d}(t_k)$ are not taking into account, the system output $\mathbf{y}(t_k)$ equals the predicted state variable $\mathbf{x}(t_k)$. $\mathbf{x}(t_k)$ is also applied as the initial state for the open loop optimal control problem over the next finite horizon. In summary, the following MPC algorithm can thus be formulated (Xia et al. (2011)):

MPC Algorithm Initialization: Let initial state $\mathbf{x}(0) = [x_1, x_2, ..., x_I]^\tau$ and $m = 0$.
(i) Compute the open loop optimal solution $\{u'_i|_m(t_k)\}$ of the problem formulation (14)-(17), where $i = 1, 2, ..., I$, $k = m, m+1, ..., m+N-1$.
(ii) The MPC controller $\bar{\mathbf{u}}|_m = \{u'_i|_m(t_m) : i = 1, 2, ..., I\}$ is applied to the plant in the sampling interval $[m, m+1)$. The remains of the open loop optimal solution $\{u'_i|_m(t_k) : i = 1, 2, ..., I, k = m+1, ..., m+N-1\}$ are discarded. $\mathbf{x}(t_{m+1})$ is then obtained according to:

$$x_i(t_{m+1}) = D_i(x_i(t_m)) + u_i(t_m)$$

and executed over the period $[t_m, t_{m+1})$.
(iii) Let $m := m+1$ and go back to step (i).

Due to the constraint $u_i(t_m) = 0$, $m \notin Q$, it is not necessary to solve the open loop optimal control problem over $[t_m, t_{m+N})$, and $\mathbf{x}(t_{m+1})$ is obtained by $\mathbf{D}(\mathbf{x}(t_m))$, as $\mathbf{u}(t_m) = 0$. The above MPC algorithm will go over the control period to solve out the optimal control strategy.

3.2 MPC approach to disturbed BRFCMP problem

In practical cases, where the disturbances $\mathbf{w}(t_k)$ and $\mathbf{d}(t_k)$ cannot be ignored, the applied MPC approach is different. In step (ii) of the MPC algorithm, the state variable is updated according to:

$$\hat{x_i}(t_{m+1}) = D_i(x_i(t_m)) + u_i(t_m) + w_i(t_m), \tag{18}$$

and measured output is updated according to:

$$y_i(t_m) = \hat{x_i}(t_m) + d_i(t_m). \tag{19}$$

where $w_i(t_m)$ and $d_i(t_m)$ are random noises.

4. SIMULATION AND VERIFICATION

4.1 Case study

The case study in our previous paper (Wang et al. (2013)) is extended to verify the effectiveness of the present approach to solve the BRFCMP problem. Let the sampling instants be the end of each year during the sustainability period, e.g., the first year is represented by $[t_0, t_1)$, the quantities of facility groups over the first year are represented by $\mathbf{x}(0)$, which is the initial state obtained from the optimal solution of the optimization model in Wang et al. (2013). The control input executed at the end of the first year is thus $\mathbf{u}(0)$. 35 alternative interventions are involved in that case. There are 19 types of facilities employing the decay model in equation (10), including the motion sensors, lighting facilities and shower heads. The rest 16 types of facilities employ the decay model in equation (11), including the chillers, heat pumps and thermal traps. Over 3100 items are involved in this retrofitting project. The sustainability period of the retrofitting project is 10 years, which is an average value among similar projects. The targeted energy saving amount is 10% of the energy baseline, which is 5,870,911 kWh per year. The corrective maintenance schedule and planning were the following:

$$u_i(t_k) = \begin{cases} 0, & k = 0, 2, 4... \\ x_i - x_i(t_k), & k = 1, 3, 5... \end{cases} \qquad (20)$$

which means the maintenances take place at the end of the year 2,4,6... i.e., the sampling periods $[t_1, t_2), [t_3, t_4), [t_5, t_6)...$ All the failed items are replaced or repaired during the maintenance. In the present case study, maintenances are also scheduled at the end of year 2,4,6...and thus $Q = \{1, 3, 5, 7, 9\}$.

There are 4 scenarios in Table 1 optimized at retrofitting plannings in Wang et al. (2013), each aims at optimizing the maintenance plan over 10 years, so as to further improve the energy and financial performances of the given retrofitting solution, which is considered optimal in Wang et al. (2013). Maintenance budget limits are introduced in these scenarios. For Scenario A and B, the budget limit is \$10,000 per year. For Scenario C the limit per year is \$12500 and for Scenario D is \$15000. The performance characteristics with respect to the scenarios are illustrated in Table 1 as the baseline.

4.2 Illustrative results and analysis

Table 2 shows the energy and financial performance characteristics of the present MPC approach to control systems without the disturbances. All results from the 4 scenarios are better than the given retrofitting plans without BRFCMP optimization, compared with Table 1. The energy savings slightly increase, whereas the overall investments become smaller. More importantly, the payback periods are further reduced. Given that the main purpose of the existing optimization model is to maximize the energy saving amount and minimize the payback period, the MPC approach generally outperforms the retrofitting optimization without optimal BRFCMP. Table 3 shows the application of MPC approach to the control systems with disturbances. In the simulations, a White Gaussian Noise (WGN) is used to represent the disturbances $\mathbf{w}(t_k)$ and $\mathbf{d}(t_k)$, and the Signal-to-Noise-Ratio (SNR) is 20dB. As system output feedback is employed in the applied MPC approach, this noise is added as a total on the system output. The illustrated results prove the effectiveness of the MPC approach to the disturbed BRFCMP problem. The performance characteristics are better than given retrofitting plans without BRFCMP optimization. It can be observed that the energy and financial performance of the retrofitting project is maintained by the present control approach under the influence of uncertainties.

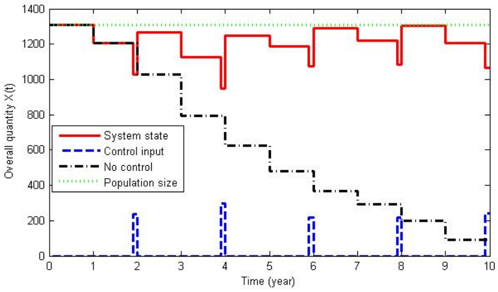

Fig. 1. Optimal control trajectories of the Scenario A

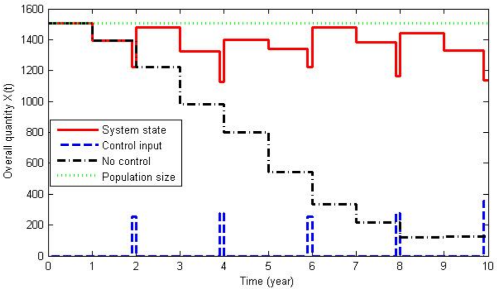

Fig. 2. Optimal control trajectories of the Scenario B

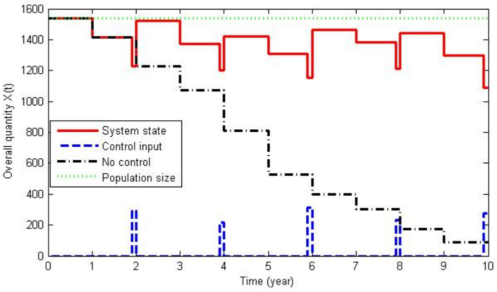

Fig. 3. Optimal control trajectories of the Scenario C

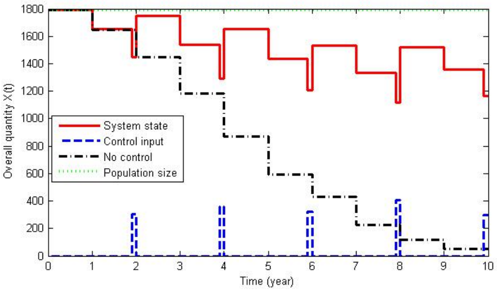

Fig. 4. Optimal control trajectories of the Scenario D

The optimal control actions and the dynamics of the total facility quantities are shown in Figs. 1 - 4. In all four figures, the horizontal axis indicates the sampling instants and the vertical axis indicates the overall quantity of retrofitted facilities. The solid curve illustrates the dynamics of the available items at each year over the sustainability period. The dashed curve illustrates optimal control input, i.e., the corrective maintenance quantities of the failed population. The dashdoted curve shows how the facility population will decay if there are no maintenances. The dotted curve shows the maximum size of the facility population, which is given by the original retrofitting plan. It is observed that in all four scenarios, the optimal corrective maintenance does not replace all failed items as the case in equation (20).

5. CONCLUSION

This paper proposes a control system approach to the Building Retrofitted Facilities Corrective Maintenance Planning problem. A subcategory of the building facilities maintenance, namely corrective maintenance, which refers to the replacements and repairs of the failed retrofitted facilities in the building, is introduced and investigated. The totality of the retrofitted facility population, instead

Table 1. **Four scenarios of optimal retrofitting solutions**

	Payback Period (Month)	Energy Savings (kWh)	Percentage Saved	Investment($)		Overall Profit($)	NPV($)
				Initial	Overall		
Scenario A	15	5875415	10.01%	59999.96	119023	590995.1	336762.8
Scenario B	25	8413665	14.33%	94996.67	163635.1	641663.7	353218.9
Scenario C	30	10709500	18.24%	124992.1	198926.3	640576.5	338938.6
Scenario D	36	13322410	22.69%	165775.1	262621.7	653733.8	319045.3

Table 2. **Optimal performance without the disturbances**

	Payback Period (Month)	Energy Savings (kWh)	Percentage Saved	Investment($)		Overall Profit($)	NPV($)
				Initial	Overall		
Scenario A	14	6295377	10.72%	59999.96	97231.69	658343.4	394473.9
Scenario B	21	8887561	15.14%	94996.67	143864.3	713277	412702.1
Scenario C	28	11178906	19.04%	124992.1	182860.4	710702.9	394627.1
Scenario D	34	13523763	23.04%	165775.1	231607.1	701196.3	372092.6

Table 3. **Optimal performance taking into account the disturbances**

	Payback Period (Month)	Energy Savings (kWh)	Percentage Saved	Investment($)		Overall Profit($)	NPV($)
				Initial	Overall		
Scenario A	14	6107913	10.40%	59999.96	98958.6	649947.9	386938.3
Scenario B	21	8213134	13.99%	94996.67	143735.8	687207.5	394766.8
Scenario C	27	8920008	15.19%	124992.1	184883.9	657913.6	362637.1
Scenario D	34	12944136	22.05%	165775.1	235005.3	674324.5	355860.1

of individual items, becomes the main consideration of the optimization of BRFCMP. Given the dynamics of retrofitted facility population due to the failures among facility items, such optimization problem is difficult to solve because of the complexity and interplay of the retrofitting planning, maintenance and operation. Therefore, a control system approach to BRFCMP problem is proposed. The BRFCMP problem is firstly formulated in a control system framework, the optimization of which is accordingly transformed to an optimal control problem. An application of Model Predictive Control technique is then presented to solve the BRFCMP problem, and a Differential Evolutionary algorithm is adopted to solve the finite horizon optimal control problem for MPC. By the present MPC approach, an optimal building maintenance plan can be derived to further improve the long term energy efficiency and cost-effectiveness of the given retrofitting plan over a sustainability period of the retrofitting project. In the end, a practical building retrofit project as the case study is used to test and verify the feasibility of the present control approach. The simulation results prove the effectiveness of the control system approach to solve building retrofitting optimization problems. The energy efficiency and cost effectiveness of the retrofitting plan is further improved by the obtained optimal solutions, as the energy and financial performances with respect to four different retrofitting scenarios appear better than the original cases without BRFCMP optimization. Furthermore, the application of MPC approach to the disturbed system model in all four scenarios illustrates satisfactory performances.

There are several topics which call for the further studies: the retrofitting planning optimization can be combined with the maintenance planning optimization; the uncertainty factors are possible to be recognized and taken into account; the operating schedule optimization can be introduced to further improve the building energy performance. The relaxing of the assumption that maintenance keeps the homogeneous class of facilities may call for new modelling and control of systems of varying dimensions.

REFERENCES

Asadi, E., da Silva, M.G., Antunes, C.H., and Dias, L. (2012). Multi-objective optimization for building retrofit strategies: A model and an application. *Energy and Buildings*, 44, 81–87.

Carstens, H., Xia, X., Zhang, J., and Ye, X. (2013). Characterising compact fluorescent lamp population decay. In *11th IEEE AFRICON Conference in Africa, Mauritius.*

Ma, Z., Cooper, P., Daly, D., and Ledo, L. (2012). Existing building retrofits: Methodology and state-of-the-art. *Energy and Buildings.*

Malatji, E.M., Zhang, J., and Xia, X. (2013). A multiple objective optimisation model for building energy efficiency investment decision. *Energy and Buildings*, 61, 81–87.

O'Connor, P. and Kleyner, A. (2011). *Practical Reliability Engineering.* Wiley.

UNFCCC (2010). Approved small scale methodology ams ii.j, demand-side activities for efficient lighting technologies. *Technical Report.*

Wang, B., Xia, X., and Zhang, J. (2013). A multi-objective optimization model for the life-cycle cost analysis and retrofit planning of buildings. *Energy and Buildings, submitted.*

Xia, X., Zhang, J., and Elaiw, A. (2011). An application of model predictive control to the dynamic economic dispatch of power generation. *Control Engineering Practice*, 19(6), 638–648.

Ye, X., Xia, X., and Zhang, J. (2013). Optimal sampling plan for clean development mechanism energy efficiency lighting projects. *Applied Energy*, 112, 1006–1015.

Zhang, J. and Sanderson, A.C. (2009). Jade: adaptive differential evolution with optional external archive. *IEEE Transactions on Evolutionary Computation*, 13(5), 945–958.