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A nonlinear, adaptive observer for gas-lift wells operating under slowly varying reservoir pressure

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Abstract: A nonlinear observer that estimates well flow rates and downhole pressure based on topside measurements only has been presented in the literature. As demonstrated in laboratory experiments the observer is suitable for a coupling with conventional PI control for stabilization of slug flow. The design of the observer is based on a nonlinear model with a linear inflow relation, and demands knowledge of several uncertain well parameters, including the reservoir pressure and production index. In this paper we present an adaptive extension to the nonlinear observer that eliminates one uncertain parameter by estimating the reservoir pressure. The extended observer is shown to be globally uniformly asymptotically stable and retain the properties of the original observer.

Keywords: Gas-lift, Slugging, Casing-heading instability, Nonlinear observer, Adaptive observer.

1. INTRODUCTION

The papers collected in [2] describe a dynamic, non-linear two-phase model of an oil producing well with gas lift. The model captures the oscillatory flow behaviour (slugging) that can occur in gas-lifted wells. This type of instability, often termed *casing-heading instability*, leads to *severe slugging*. Referring to the sketch in Fig. 1, the phenomena can be described as follows:

(1) Gas from the annulus starts to flow through the injection valve and into the tubing. As gas enters the tubing the pressure in the tubing falls, accelerating the inflow of lift-gas.
(2) The gas pushes the major part of the liquid out of the tubing, while the pressure in the annulus falls significantly.
(3) The annulus is practically emptied, and the gas flow into the tubing is blocked by liquid accumulating in the tubing. Due to the blockage, the tubing is filled with liquid and the annulus with gas.
(4) Eventually, the pressure in the annulus becomes high enough for gas to penetrate into the tubing, and a new cycle starts.

Various configurations of PI-controllers have been shown to stabilize slugging in gas lifted wells. A theoretically appealing approach is to control the production choke opening using a downhole pressure measurement. In practice, downhole pressure sensors operate in a harsh environment and have a shorter life expectancy and lower signal-to-noise ratio than surface installed sensors. Because of this they are considered unsuitable for closed-loop control. Adding to the argument is the fact that many wells do not

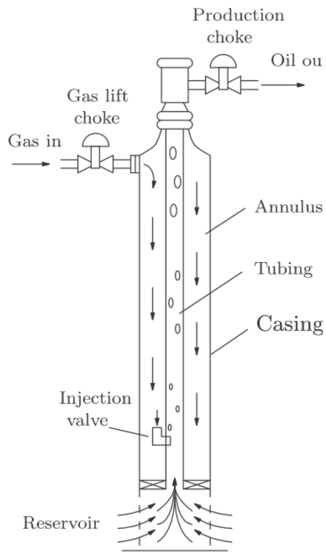

Fig. 1. Production well with gas lift.

have downhole sensors installed. Thus, a realistic control strategy should rely on topside measurements.

The previous argument motivated the design and analysis of the observer in [1]. The observer uses topside measurements to estimate the mass of oil and gas in the system. The downhole pressure can easily be extracted from these states and used in a control scheme. A drawback of the observer is that it demands knowledge of the reservoir pressure and production index (inflow performance). In this paper we aim to eliminate the need of knowing the reservoir pressure by extending the original observer with an adaptation law.

2. MATHEMATICAL MODEL

We follow the notation in [1] and summarize the analytical model of the two-phase production well with gas lift.

The well is modeled using three states: x_1 is the mass of gas in the annulus; x_2 is the mass of gas in the tubing; x_3 is the mass of oil in the tubing. These are defined by the flow of mass in the annulus and tubing (control volumes). The state equations are:

$$\dot{x}_1 = w_{gc} - w_{iv}, \tag{1}$$
$$\dot{x}_2 = w_{iv} - w_{pg}, \tag{2}$$
$$\dot{x}_3 = w_r - w_{po}, \tag{3}$$

where w_{gc} is a constant mass flow rate of lift gas into the annulus, w_{iv} is the mass flow rate of lift gas from the annulus into the tubing, w_{pg} is the mass flow rate of gas through the production choke, w_r is the oil mass flow rate from the reservoir into the tubing, and w_{po} is the mass flow rate of produced oil through the production choke. The flows are modeled by

$$w_{gc} = \text{constant flow rate of lift gas}, \tag{4}$$
$$w_{iv} = C_{iv}\sqrt{\rho_{a,i}\max\{0, p_{a,i} - p_{t,i}\}}, \tag{5}$$
$$w_{pc} = C_{pc}\sqrt{\rho_m \max\{0, p_t - p_s\}}u, \tag{6}$$
$$w_{pg} = \frac{x_2}{x_2 + x_3} w_{pc}, \tag{7}$$
$$w_{po} = \frac{x_3}{x_2 + x_3} w_{pc}, \tag{8}$$
$$w_r = C_r(p_r - p_{t,b}). \tag{9}$$

C_{iv}, C_{pc}, and C_r are constants, u is the production choke opening ($u(t) \in [0, 1]$), $\rho_{a,i}$ is the density of gas in the annulus at the injection point, ρ_m is the density of the oil/gas mixture at the top of the tubing. The densities are modeled as follows (using the ideal gas law):

$$\rho_{a,i} = \frac{M}{RT_a} p_{a,i}, \tag{10}$$
$$\rho_m = \frac{x_2 + x_3 - \rho_o L_r A_r}{L_t A_t}. \tag{11}$$

Furthermore, the pressures are described by

$$p_{a,i} = \left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a}\right) x_1, \tag{12}$$
$$p_t = \frac{RT_t}{M} \frac{x_2}{L_t A_t + L_r A_r - \nu_o x_3}, \tag{13}$$
$$p_{t,i} = p_t + \frac{g}{A_t}(x_2 + x_3 - \rho_o L_r A_r), \tag{14}$$
$$p_{t,b} = p_{t,i} + \rho_o g L_r. \tag{15}$$

The separator pressure p_s is assumed to be held constant by a control system. In summary, the model covers the following case:

- Two-phase flow in the tubing, treating oil and water as a single phase;
- No flashing effects;
- Low gas-to-oil ratio (GOR), reflected in the fact that the flow from the reservoir is modeled as pure oil, and;
- Slowly varying components of gas and oil.

Fig. 2 displays a simulation of the well with gas lift and a fully open production choke ($u = 1$). The severe slugging is evident from the spikes in oil production.

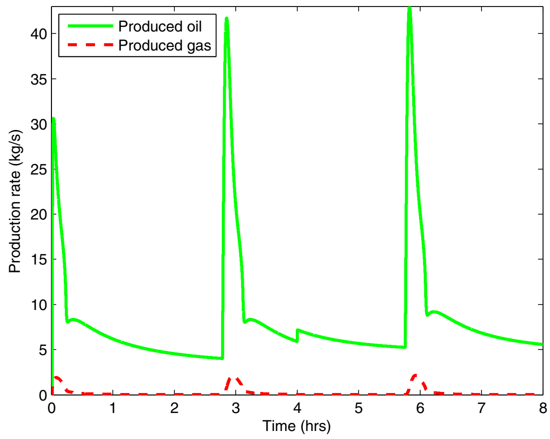

Fig. 2. Well production with fully open production choke and a lift-gas injection rate of $w_{gc} = 0.2$ (kg/s).

3. STATE ESTIMATION

In [1] a reduced order nonlinear observer was derived to estimate x_2 and x_3. The observer was designed by assuming that only topside measurements were available, i.e.

$$\begin{aligned} y_1(t) &= x_1(t), \quad y_2(t) = p_t(t) \\ y_3(t) &= w_{pc}(t), \quad \text{or} \quad y_3(t) = \rho_m(t) \end{aligned} \tag{16}$$

The observer in this paper is derived using the same measurements and similar assumptions as [1]. The slightly modified assumptions are listed below.

Assumption 1. The production choke is not allowed to close completely. That is,

$$u \geq \delta_u > 0, \quad \forall t \geq 0. \tag{17}$$

Assumption 2. The states are bounded away from zero, and the part of the tubing below the gas injection point is filled with oil. More precisely,

$$\begin{aligned} &x_1 \geq \delta_1 > 0, \quad x_2 \geq \delta_2 > 0, \quad \text{and} \\ &x_3 \geq \rho_o L_r A_r + \delta_3 > \rho_o L_r A_r, \quad \forall t \geq 0. \end{aligned} \tag{18}$$

Assumption 3. The gas in the tubing has lower density than the oil. More precisely,

$$L_t A_t + L_r A_r - \nu_o(x_3 + x_2) \geq \delta_g > 0, \quad \forall t \geq 0. \tag{19}$$

Assumption 4. (modified) The pressure drop over the production choke is strictly positive, i.e.

$$p_t - p_s \geq \delta_p > 0, \quad \forall t \geq 0. \tag{20}$$

Assumption 5. The reservoir pressure is slowly varying and can be regarded a constant, that is $p_r(t) = p_r =$ constant.

The authors of [1] argue that Assumptions 1-4 are not restrictive and do not limit the application of the observer. Note that in [1] δ_p was assumed to be non-negative ($\delta_p \geq$

0), while we assume it to be strictly positive ($\delta_p > 0$). This modification is reasonable for production wells as it just implies a positive production at all times (coinciding with Assumption 1 since non-producing wells usually are closed). Negative flow through the production choke is not feasible as seen from (6). Using Assumptions 1, 2, and 4 we have that

$$w_{pc} = C_{pc}\sqrt{\rho_m \max\{0, p_t - p_s\}}u > 0, \quad \forall t \geq 0, \tag{21}$$

since $\max\{0, p_t - p_s\} = \max\{\delta_p, p_t - p_s\} \geq \delta_p > 0$ under Assumption 4, $u > 0$ under Assumption 1, and $\rho_m > 0$ under Assumption 2. As explained later, the modification of Assumption 4 helps us select an parameter update law, and it can be interpreted as a persistent excitation (PE) requirement. That is, we would not be able to adapt using measurements from a closed well. Assumption 5, that the reservoir is slowly varying, is reasonable for oil reservoirs.

Lemma 1. Solutions of system (1)-(3) are bounded in the sense that there exist a constant B, depending on the initial state, such that

$$x_i \leq B(x(t_0)), \quad i = 1, 2, 3, \quad \forall t \geq t_0. \tag{22}$$

In particular,

$$x_3 \leq \rho_o(L_t A_t + L_r A_r), \quad \forall t \geq t_0. \tag{23}$$

The bounds in Lemma 1 follow from the derivative of the Lyapunov function candidate $V = 2x_1 + x_2 + x_3 > 0$, which is strictly negative for sufficiently large V.

With Lemma 1 and Assumption 2 we have established upper and lower bounds on the states. Thus, we expect the model outputs y_1, y_2, and y_3, given by Eqs. (16), (13), and (6), respectively, to be upper and lower bounded. That is,

$$0 \leq y_i(t) \leq B^y(B(x(t_0))), \quad i = 1, 2, 3 \tag{24}$$

In practice, we assume the measurements to be non-negative and upper bounded.

The following theorem states the extended nonlinear observer with adaptation of reservoir pressure, when $y_3(t) = w_{pc}(t)$.

Theorem 2. Solutions $\hat{x}(t) = (\hat{x}_2(t), \hat{x}_3(t), \hat{p}_r(t))$ of the observer

$$\dot{\hat{z}}_1 = w_{gc} - \frac{\hat{z}_1 - y_1}{\hat{z}_2 - y_1}y_3 + k_1(\hat{z}_1, \hat{z}_2, y_1, y_2), \tag{25}$$

$$\begin{aligned}\dot{\hat{z}}_2 &= w_{gc} \\ &+ C_r\left(\hat{p}_r - \rho_o g L_r + \frac{A_r}{A_t}\rho_o g L_r + \frac{g}{A_t}y_1 - y_2 - \frac{g}{A_t}\hat{z}_2\right) \\ &- y_3 + k_2(\hat{z}_2, u, y_1, y_2, y_3),\end{aligned} \tag{26}$$

$$\begin{aligned}\dot{\hat{p}}_r = k_3 C_r\Bigg(&\frac{L_t A_t}{\max\{\delta_p, y_2 - p_s\}}\left(\frac{y_3}{C_{pc}u}\right)^2 \\ &- \hat{z}_2 + y_1 + \rho_o L_r A_r\Bigg),\end{aligned} \tag{27}$$

$$\hat{z}_1 \geq \delta_2 + y_1, \quad \text{and} \quad \hat{z}_2 \geq \rho_o L_r A_r + \delta_3 + \hat{z}_1 \tag{28}$$

$$\hat{x}_2 = \hat{z}_1 - y_1 \tag{29}$$

$$\hat{x}_3 = \hat{z}_2 - \hat{z}_1 \tag{30}$$

where $k_3 > 0$ is the adaptation gain, and the output injections, $k_1(\cdot)$ and $k_2(\cdot)$, are given by

$$\begin{aligned}&k_1(\hat{z}_1, \hat{z}_2, y_1, y_2) = \\ &c_1\left(\frac{M}{RT_t}(L_t A_t + L_r A_r - \nu_o(\hat{z}_2 - \hat{z}_1))y_2 - (\hat{z}_1 - y_1)\right),\end{aligned} \tag{31}$$

$$\begin{aligned}&k_2(\hat{z}_2, u, y_1, y_2, y_3) = \\ &c_2\left(\left(\frac{y_3}{C_{pc}u}\right)^2 - \frac{\hat{z}_2 - y_1 - \rho_o L_r A_r}{L_t A_t}\max\{\delta_p, y_2 - p_s\}\right),\end{aligned} \tag{32}$$

converge to the actual state $x(t) = (x_2(t), x_3(t), p_r(t))$, and the dynamics of $e(t) = x(t) - \hat{x}(t)$ is globally uniformly asymptotically stable with Assumptions 1-5.

Proof. Define $z_2 = x_1 + x_2 + x_3$, which is the total amount of mass in the system. From (1)-(3), its time derivative is

$$\begin{aligned}\dot{z}_2 &= w_{gc} \\ &+ C_r\left(p_r - \rho_o g L_r + \frac{A_r}{A_t}\rho_o g L_r + \frac{g}{A_t}y_1 - y_2 - \frac{g}{A_t}z_2\right) \\ &- y_3\end{aligned} \tag{33}$$

We estimate z_2 by $\hat{z}_2$, which is governed by

$$\begin{aligned}\dot{\hat{z}}_2 &= w_{gc} \\ &+ C_r\left(\hat{p}_r - \rho_o g L_r + \frac{A_r}{A_t}\rho_o g L_r + \frac{g}{A_t}y_1 - y_2 - \frac{g}{A_t}\hat{z}_2\right) \\ &- y_3 + k_2(\cdot),\end{aligned} \tag{34}$$

where $\hat{p}_r$ is an estimate of the reservoir pressure p_r, and $k_2(\cdot)$ is an output injection term to be determined. The observer error, $e_2 \triangleq z_2 - \hat{z}_2$, is governed by

$$\dot{e}_2 = -\frac{C_r g}{A_t}e_2 + C_r\tilde{p}_r - k_2(\cdot), \tag{35}$$

where we have defined the parameter error $\tilde{p}_r \triangleq p_r - \hat{p}_r$. Take the Lyapunov function candidate $V_2 = \frac{1}{2}e_2^2$. Its time derivative along solutions of (35) is

$$\dot{V}_2 = e_2\left(-\frac{C_r g}{A_t}e_2 + C_r\tilde{p}_r - k_2(\cdot)\right). \tag{36}$$

Utilizing (6) and the fact that $y_3(t) = w_{pc}(t)$ we get that

$$\begin{aligned}&e_2\frac{\max\{\delta_p, y_2 - p_s\}}{L_t A_t} \\ &= \left(\frac{y_3}{C_{pc}u}\right)^2 - \frac{\hat{z}_2 - y_1 - \rho_o L_r A_r}{L_t A_t}\max\{\delta_p, y_2 - p_s\}.\end{aligned} \tag{37}$$

Selecting

$$\begin{aligned}&k_2(\hat{z}_2, u, y_1, y_2, y_3) \\ &= c_2\left(\left(\frac{y_3}{C_{pc}u}\right)^2 - \frac{\hat{z}_2 - y_1 - \rho_o L_r A_r}{L_t A_t}\max\{\delta_p, y_2 - p_s\}\right),\end{aligned} \tag{38}$$

where $c_2 > 0$, and inserting (38) into (36), we get

$$\dot{V}_2 = -\left(\frac{C_r g}{A_t} + c_2\frac{\max\{\delta_p, y_2 - p_s}{L_t A_t}\right)e_2^2 + C_r\tilde{p}_r e_2. \tag{39}$$

Due to the last term, and the fact that $\tilde{p}_r$ is unknown, we cannot establish negative definiteness. To mend this problem, we augment the Lyapunov function candidate with $\tilde{p}_r$, i.e $V_3 = V_2 + \frac{1}{2k_3}\tilde{p}_r^2$. The time derivative is

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + \frac{1}{k_3}\tilde{p}_r\dot{\tilde{p}}_r \\ &= -\left(\frac{C_r g}{A_t} + c_2\frac{\max\{\delta_p, y_2 - p_s\}}{L_t A_t}\right)e_2^2 \\ &\quad + \tilde{p}_r(C_r e_2 + \frac{1}{k_3}\dot{\tilde{p}}_r).\end{aligned} \tag{40}$$

Due to Assumption 5, we now have the freedom to cancel the unknown error $\tilde{p}_r$ by selecting the update law $\dot{\tilde{p}}_r = -\dot{\hat{p}}_r = -k_3 C_r e_2$. We reuse the above trick of expressing e_2 in terms of known variables and measurements and end up with the parameter error update law

$$\begin{aligned}\dot{\hat{p}}_r &= k_3 C_r e_2 \\ &= k_3 C_r \Bigg(\frac{L_t A_t}{\max\{\delta_p, y_2 - p_s\}}\left(\frac{y_3}{C_{pc}u}\right)^2 \\ &\quad - \hat{z}_2 + y_1 + \rho_o L_r A_r\Bigg).\end{aligned} \tag{41}$$

Note that we have expressed e_2, which is unknown, in terms of known measurements. We could not have done this without the modification of Assumption 4, letting us divide by the max function. Inserting (41) into (40) gives

$$\dot{V}_3 = -\left(\frac{C_r g}{A_t} + c_2\frac{\max\{\delta_p, y_2 - p_s\}}{L_t A_t}\right)e_2^2, \tag{42}$$

which is negative semi-definite. Applying [4, Theorem 8.4], we get that for any initial state, e_2 and $\tilde{p}_r$ are bounded, and $\lim_{t\to\infty} e_2(t) = 0$. The theorem is inconclusive on convergence of the parameter error $\tilde{p}_r$, which is a desired property of the observer.

We pursue convergence of $\tilde{p}_r$ by observing that the error dynamics for e_2 and $\tilde{p}_r$, with the selected update law and output injection term, can be expressed as a linear, time-varying system (since the states are bounded)

$$\begin{bmatrix}\dot{e}_2 \\ \dot{\tilde{p}}_r\end{bmatrix} = \underbrace{\begin{bmatrix}-a(t) & C_r \\ -k_3 C_r & 0\end{bmatrix}}_{A(t)}\begin{bmatrix}e_2 \\ \tilde{p}_r\end{bmatrix}, \tag{43}$$

where we have assigned $a(t) = \left(\frac{C_r g}{A_t} + c_2\frac{\max\{\delta_p, y_2(t) - p_s\}}{L_t A_t}\right)$. Note that $||A(t)|| < \infty, \forall t$, due to the upper bound B^y on y_2. Also, the system has a single equilibrium point in the origin. Global exponential stability of (43) follows from [4, Theorem 8.5] if we can show that

$$V_3(t+\delta, \phi(t+\delta; t, x)) - V_3(t, x) \le -\lambda V_3(t, x), \tag{44}$$

where $0 \le \lambda \le 1$, $x = [e_2 \;\; \tilde{p}_r]^\mathsf{T}$, and $\phi(\tau; t, x) = \Phi(\tau, t)x$ is the solution of (43) starting at (t, x).

We define $C(t) = [\sqrt{a(t)} \quad 0]$ so that $\dot{V}_3 = -x^\mathsf{T} C^\mathsf{T}(t)C(t)x$. The left hand side of (44) can then be written as

$$V_3(t+\delta, \phi(t+\delta; t, x)) - V_3(t, x) = -x^\mathsf{T} W(t, t+\delta)x, \tag{45}$$

where $W(t, t+\delta)$ is the observability Gramian of the pair $(A(t), C(t))$, i.e.

$$W(t, t+\delta) = \int_t^{t+\delta} \Phi^\mathsf{T}(\tau, t)C^\mathsf{T}(\tau)C(\tau)\Phi(\tau, t)d\tau. \tag{46}$$

The requirement $x^\mathsf{T} W(t, t+\delta)x \ge \lambda V_3(t, x)$ from (44) is implied by uniform observability of $(A(t), C(t))$ (see [4]). Uniform observability of $(A(t), C(t))$ is equivalent to uniform observability of $(A(t) - K(t)C(t), C(t))$, for any piecewise continuous, bounded matrix $K(t)$ (see [3]). By selecting $K(t) = [-\sqrt{a(t)} \quad -k_3 C_r/\sqrt{a(t)}]^\mathsf{T}$ the pair simplifies to

$$A(t) - K(t)C(t) = \begin{bmatrix}0 & C_r \\ 0 & 0\end{bmatrix}, \quad C(t) = [\sqrt{a(t)} \;\; 0]. \tag{47}$$

The observability Gramian of the above pair can be shown to be

$$W(t, t+\delta) = \int_t^{t+\delta}\begin{bmatrix}a(\tau) & a(\tau)C_r(\tau - t) \\ a(\tau)C_r(\tau - t) & a(\tau)C_r^2(\tau - t)^2\end{bmatrix}d\tau.$$

To verify uniform observability we need to show that the Gramian W is nonsingular [1]. From $\det W$ we get that W is singular when

$$\int_t^{t+\delta} a(\tau)d\tau \int_t^{t+\delta} a(\tau)\tau^2 d\tau = \left(\int_t^{t+\delta} a(\tau)\tau d\tau\right)^2. \tag{48}$$

Using Hölder's inequality we have that

$$\left(\int_t^{t+\delta} \underbrace{\sqrt{a(\tau)}}_{f}\,\underbrace{\sqrt{a(\tau)}\tau}_{g}\,d\tau\right)^2 \tag{49}$$

$$\le \int_t^{t+\delta} a(\tau)d\tau \int_t^{t+\delta} a(\tau)\tau^2 d\tau, \tag{50}$$

with equality iff $|f| = \alpha|g|$, for a constant α. Since $|f| = \sqrt{a(t)}$ and $|g| = \sqrt{a(t)}\tau$ we have a strict inequality in (50), and by contradiction W is not singular. Thus, the pair $(A(t) - K(t)C(t), C(t))$ is uniformly observable and the error dynamics of $(e_2, \tilde{p}_r)$, described by (43), is globally exponentially stable. [2]

Next, we define $z_1 = x_1 + x_2$, which is the total mass of gas in the system. From (1) and (2), its time derivative is

$$\dot{z}_1 = w_{gc} - \frac{z_1 - y_1}{z_2 - y_1}y_3. \tag{51}$$

We estimate z_1 by $\hat{z}_1$, which is governed by

$$\dot{\hat{z}}_1 = w_{gc} - \frac{\hat{z}_1 - y_1}{\hat{z}_2 - y_1}y_3 + k_1(\cdot). \tag{52}$$

where $k_1(\cdot)$ is an output injection term to be determined. The observer error, $e_1 = z_1 - \hat{z}_1$, is governed by

$$\dot{e}_1 = -\frac{z_1 - y_1}{z_2 - y_1}y_3 + \frac{\hat{z}_1 - y_1}{\hat{z}_2 - y_1}y_3 - k_1(\cdot). \tag{53}$$

Notice that the observer error dynamics is in cascade form, where the dynamics of $(e_2, \tilde{p}_r)$ is independent of e_1. Thus, we aim to show that the dynamics of e_1 is input-to-state stable with e_2 as input. Then, we can apply [4, Lemma 4.7] to prove global uniform asymptotic stability of the cascade system.

[1] In the simple case when $y_2(t)$ is a constant signal we can easily confirm that the pair (A, C) is observable, and y_2 is persistently exciting. Thus, we are able to identify a single parameter with a constant signal, as expected.

[2] The requirements $||A(t)|| < \infty$ and $\lim_{t\to\infty} e_2(t) = 0$ are similar to what we would expect from an output injection lemma or integral lemma (see [6]), or a lemma based on limiting functions [5].

We investigate the e_1-subsystem using the Lyapunov function candidate $V_1 = \frac{1}{2}e_1^2$. Its time derivative along the solutions of (53) is

$$\begin{aligned}\dot{V}_1 = &-\frac{y_3}{z_2 - y_1}e_1^2 + y_3\frac{\hat{z}_1 - y_1}{(z_2 - y_1)(\hat{z}_2 - y_1)}e_1 e_2 \\ &- e_1 k(\cdot).\end{aligned} \tag{54}$$

As in [1], we select

$$\begin{aligned}&k_1(\hat{z}_1, \hat{z}_2, y_1, y_2) = \\ &c_1\left(\frac{M_g}{RT_t}(L_t A_t + L_r A_r - \nu_o(\hat{z}_2 - \hat{z}_1))y_2 - \hat{z}_1 + y_1\right),\end{aligned} \tag{55}$$

where $c_1 > 0$, and obtain

$$\begin{aligned}\dot{V}_1 = &-\left(\frac{y_3}{z_2 - y_1} + c_1\frac{L_t A_t + L_r A_r - \nu_o(z_2 - y_1)}{L_t A_t + L_r A_r - \nu_o(z_2 - z_1)}\right)e_1^2 \\ &+ \left(y_3\frac{\hat{z}_1 - y_1}{(z_2 - y_1)(\hat{z}_2 - y_1)}\right. \\ &\left. - c_1\frac{\nu_o(z_1 - y_1)}{L_t A_t + L_r A_r - \nu_o(z_2 - z_1)}\right)e_1 e_2\end{aligned} \tag{56}$$

Using Lemma 1, Assumptions 2 and 3, and noticing that $(\hat{z}_1 - y_1)/(\hat{z}_2 - y_1) < 1$, we obtain

$$\begin{aligned}\dot{V}_1 \leq &-\left(\frac{y_3}{2B} + \underbrace{c_1\frac{\delta_g}{L_t A_t + L_r A_r}}_{d_1 > 0}\right)e_1^2 \\ &+ \left(\frac{y_3}{\delta_2 + \delta_3} + c_1\frac{B\nu_o}{\delta_g}\right)||e_1|| \cdot ||e_2||\end{aligned} \tag{57}$$

Remembering that $y_3(t) = w_{pc}(t) > 0$, we get

$$\dot{V}_1 \leq -d_1 e_1^2 + d_2||e_1|| \cdot ||e_2||, \tag{58}$$

where we have defined the constants

$$d_1 = c_1\frac{B\nu_o}{\delta_g} \quad \text{and} \quad d_2 = \left(\frac{B^y}{\delta_2 + \delta_3} + c_1\frac{B\nu_o}{\delta_g}\right). \tag{59}$$

We use the term $-d_1 e_1^2$ to dominate $d_2||e_1|| \cdot ||e_2||$ for large $||e_1||$ by writing

$$\dot{V}_1 \leq -d_1(1 - \theta)e_1^2 - \theta e_1^2 + d_2||e_1|| \cdot ||e_2||, \tag{60}$$

where $0 < \theta < 1$, and finally arrive at the desired result

$$\dot{V}_1 \leq -d_1(1 - \theta)e_1^2, \quad ||e_1|| \geq \frac{d_2||e_2||}{d_1\theta}. \tag{61}$$

From (61) we have that the subsystem (53) is ISS with $\gamma(r) = (d_2/d_1\theta)r$ when e_2 is regarded as input. We now satisfy all requirements of [4, Lemma 4.7], and we can conclude the proof by stating that the error dynamics of the observer in Theorem 2 is globally uniformly asymptotically stable under the given assumptions.

If y_3 measures density, i.e. $y_3(t) = \rho_m(t)$, we can simply replace y_3 with

$$C_{pc}\sqrt{y_3 \max\{\delta_p, y_2 - p_s\}} \tag{62}$$

in (25), (26), (27), and (32). $\square$

4. SIMULATIONS

The well model is simulated using parameters from an offshore oil well in the Petrobras operated Marlim field. Two cases are considered: Case A where all measurements are noise-free; and Case B, where y_3 is noisy.

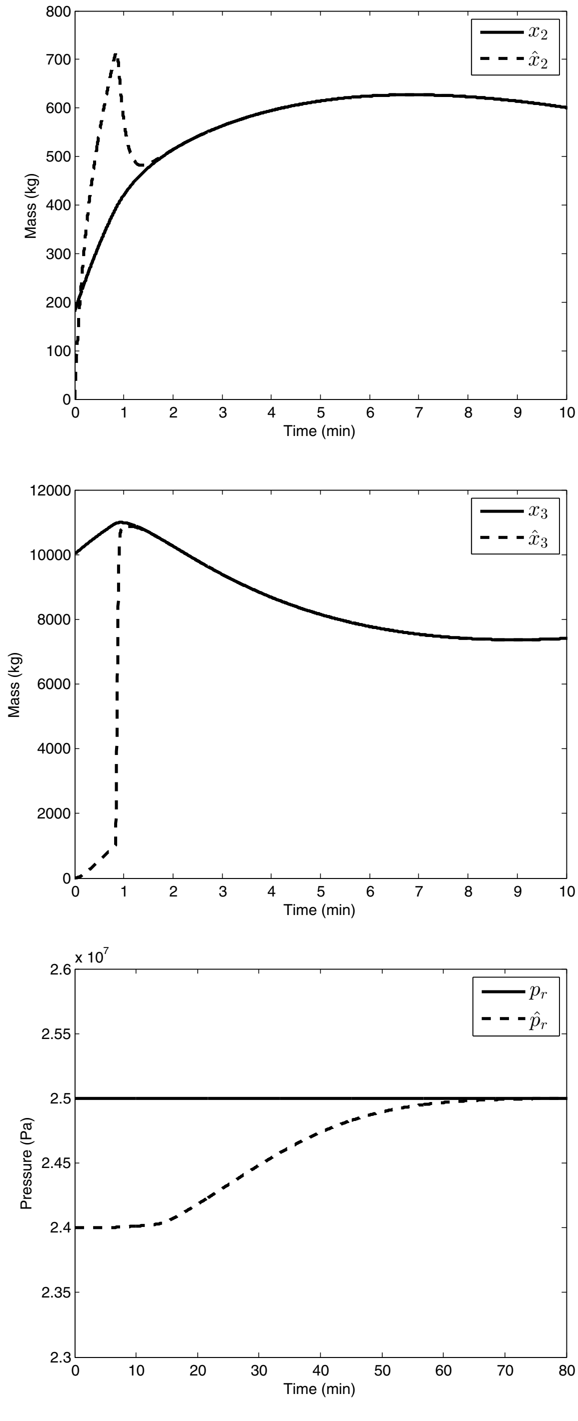

Fig. 3. Case A. Convergence of estimated states with observer gains $c_1 = 0.1$, $c_2 = 0.0001$, and $c_3 = 1\times 10^8$. Note the different time-scale in the last plot.

In both cases the reservoir pressure is $p_r = 250$ bar $= 2.5\times 10^7$ Pa, while the initial guess on the reservoir pressure is $\hat{p}_r(t) = 240$ bar $= 2.4 \times 10^7$ Pa. The observer gains are set to $c_1 = 0.1$, $c_2 = 0.0001$, and $c_3 = 1 \times 10^8$. The high value on c_3 is due to poor scaling ($C_r \approx 10^{-6}$ in (27)).

A common approach to avoid initial kicks in the parameter estimation (due to large initial errors in the estimated states) is to start the adaptation after the estimated states have converged. In both cases the adaptation is started after 5 minutes.

Case A. We can see from Fig. 3 that the states $\hat{x}_2$ and $\hat{x}_3$ converge before the 5 minute mark, before the adaptation starts. This tells us that the estimated states are robust against error in the reservoir pressure. At 5 minutes the adaptation starts and the reservoir pressure estimate tends towards the correct reservoir pressure without affecting the state estimation.

Case B. White noise with a signal-to-noise ratio of 20 is added to the measurement $y_3 = w_{pc}$. This is a high, but reasonable noise signal for a topside flow meter. From Fig. 4, we see that the estimated states are fairly unaffected by the noise. On the other hand, we observe that the parameter estimation converges, albeit deteriorated by noise. This is expected from (27), where y_3 is squared. The only way to decrease sensitivity to noise is to lower c_3 and accept a slower convergence rate. The unrealistic reservoir pressure step change in Fig. 4 is included to illuminate the adaptation performance.

5. CONCLUSIONS

The nonlinear observer in [1] has been extended with an adaptation law for the reservoir pressure. Without any strict assumptions or extra measurements, the adaptation law eliminates the need of knowing the reservoir pressure. The design of the extended observer is similar to that of the original, and exploits the same structure of the model with respect to internal flows between the annulus and tubing. The extension does however come at the cost of reduced robustness properties as the stability is proved asymptotic, whereas the original observer converged in an exponential manner. As demonstrated with simulations, the observer is able to estimate the model states when measurement noise is present. However, the parameter estimate of the slowly varying reservoir pressure is sensitive to noise. Given that the model accurately describes the well the observer will successfully indicate the unknown reservoir pressure.

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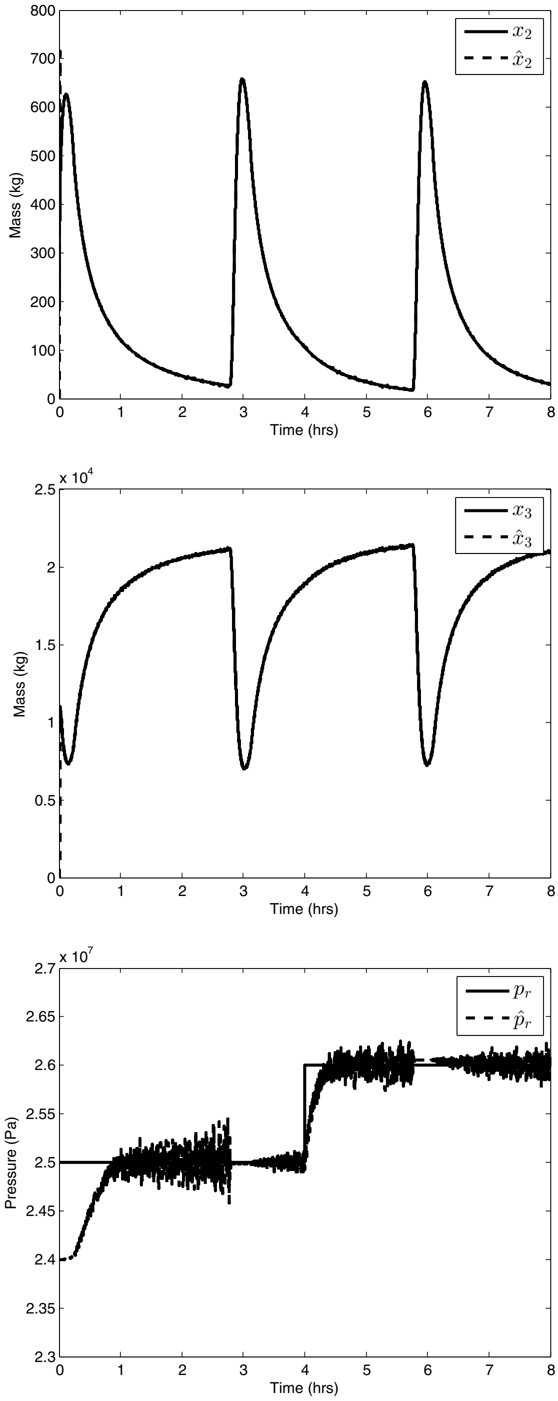

Fig. 4. Case B. Convergence of estimated states with observer gains $c_1 = 0.1$, $c_2 = 0.0001$, and $c_3 = 1 \times 10^8$. After 4 hours the reservoir pressure is stepped from 2.5 to 2.6×10^7 Pa.

REFERENCES

[1] Aamo, O., Eikrem, G., Siahaan, H., and Foss, B. (2004). Observer design for multiphase flow in vertical pipes with gas-lift – theory and experiments. *Journal of process control.*

[2] Eikrem, G. (2006). Stabilization of Gas-Lift Wells by Feedback Control. *Doctoral Thesis, NTNU.*

[3] Ioannou, P.A. and Sun, J. (1995). Robust adaptive control.

[4] Khalil, H.K. (2000). *Nonlinear Systems.* Pearson Education.

[5] Lee, T.C. and Jiang, Z.P. (2005). A generalization of Krasovskii–LaSalle theorem for nonlinear time-varying systems: converse results and applications. *Automatic Control, IEEE Transactions on*, 50(8), 1147–1163.

[6] Loria, A., Panteley, E., Popovic, D., and Teel, A. (2008). Persistency of excitation for uniform convergence in nonlinear control systems. URL `http://arxiv.org/abs/math/0301335`.