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Hierarchical Network Identification of Large-Scale Systems - An Approach Based on Dissipation Equalities ⋆

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Abstract: Recently, dynamical systems in science and engineering problems become increasingly larger and too complex. One of the ways to solve the difficulty is to model the systems with a hierarchically networked interconnection of subsystems. In this paper, we consider a hierarchical network system which is an interconnection of dissipative subsystems and identify both subsystems and network structure, called network system, preserving the dissipativity properties. We first estimate the state sequences of subsystems via the dissipation equalities instead of the matrix input-output equation in ordinary subspace identification methods. As a main result, we show an identification method in open-loop manner for subsystems and network systems based on the least-squares method. We also propose an identification method in a closed-loop manner.

Keywords: Behavioral approach, Dissipativity, Hierarchical network system, Quadratic difference forms, System identification

1. INTRODUCTION

Recently, due to a variety of social and environmental requirements, dynamical systems in engineering and science problems become increasingly larger and too complex. It is not necessarily desirable to deal with such systems in a uniformed manner, since there exists a limitation in the computational amount and an available information about the systems. One of the ways to solve these difficulties is to divide the systems into some hierarchical layers corresponding to the physical scales (Hara et al. (2008)). Hence, hierarchical and network structure can be considered as an efficient solution to the difficulties, which was proposed by Hara et al. (2009a), Hara et al. (2009b), Sandberg and Murray (2009), Shimizu and Hara (2009), Smith (2005) and etc. This point has also been pointed out in a behavioral framework (Willems (1991)). These concepts may enable us to translate a large-scale system into a hierarchically networked interconnection of subsystems [1] and to deal with them easier. Fig. 1 illustrates the notion of a hierarchical network system, where the circles and arrows represent the states and their interactions, respectively. Subsystems represent local dynamics and they are interconnected via network system.

System identification constructs a mathematical model of a dynamical system from its input and output data. In particular, subspace identification method is well-known as a suitable method for a modeling of large-scale systems, since it is efficiently applicable to multi-input multi-output systems (Van Overschee and Moor (1993), Verhaegen and Dewilde (1992a), Verhaegen and Dewilde (1992b), Katayama (2005)). However, it can only determine an external representation and shrink the dynamics of a large-scale system with some appropriate dimension uniformly. It contains a difficulty that it cannot be applied for a hierarchical network structure directly.

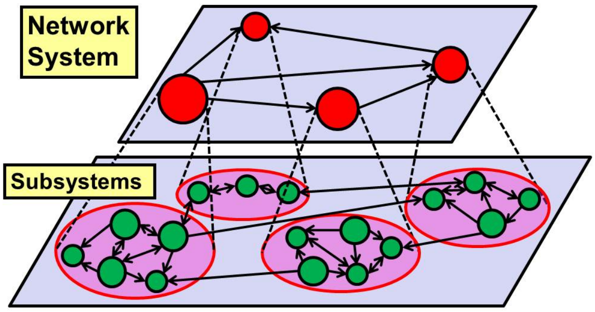

Fig. 1. A hierarchical network dynamical system

On the other hand, dissipativity of a dynamical system is one of the most important property of a system, which considers the system from a view point of energy interactions with its external environment (Willems (1971)). It is important to derive a theory which will not lose conservation law of energy for a physically natural modeling. As an identification method which preserves a dissipativity including passivity, losslessness and etc., Rapisarda and Trentelman (2011) proposed an identification method based on dissipation equalities based on quadratic

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[1] Multi-agent systems are a similar concept to hierarchical network systems, which are considered by Fax and Murray (2004), Olfati-Saber et al. (2007) and etc., for example. In these references, each element of state variables is often regarded as an agent and subsystems are defined as a cluster of the agents.

difference forms (Kaneko and Fujii (2000)) which is a mathematical tool for dissipation theory in a behavioral framework (Willems (1991)). However, this framework has not been extended to aforementioned hierarchical network systems.

The following studies are considered for identification of a system with a network structure. Massioni and Verhaegen considered a subspace identification for circulant systems (Massioni and Verhaegen (2008)) and distributed decomposable systems (Massioni and Verhaegen (2010)). Note that they assumed that the complete information of the network structure are available. In biological networks, identifications of protein and gene network structures are considered based on the least square method (Takahashi et al. (2010)) and the ℓ_1-norm minimization (Julius et al. (2009)), respectively. A partial information may be available for us both in subsystems and network systems.

Based on the above observations, we focus on a hierarchical network structure and system identification method for an efficient modeling of large-scale systems as interconnections of dissipative subsystems. In particular, hierarchical network structure and dissipativity properties are the properties which are equipped in not only artificial systems, e.g. large-scale plant, but also natural systems such as meteorological phenomena and biological systems as a gift. It may be reasonable that we consider system identification with focus on such a priori information. Kojima et al. (2012) proposed a subspace identification method for hierarchical network systems consisting of homogeneous subsystems. However, this framework has not been generalized to the heterogeneous and dissipative case.

In this paper, we discuss how to identify subsystems and network structure under assumption on the dissipativity of subsystems. For this purpose, we propose an identification method for hierarchical network systems based on dissipation equalities. The result leads us to an efficient theoretical and numerical framework of mathematical modeling, analysis and control of large-scale systems.

The organization of the paper is as follows. In Section 2, we review some basic definitions and results on quadratic difference forms and dissipativity. The problem formulation is provided in Section 3. In Section 4, we show an approach for an identification method of hierarchical network systems based on dissipation equalities. as a main result. Two different approaches, so called open-loop and closed-loop approaches, are proposed for the identification.

We use the following notations throughout this paper. The set of $m\times n$ real and $m\times m$ symmetric matrices are denoted by $\mathbb{R}^{m\times n}$ and $\mathbb{R}_{\mathrm{s}}^{m\times m}$, respectively. The set of $m\times n$ rational matrices is denoted by $\mathbb{R}^{m\times n}(z)$. The set of $m \times n$ real and $m \times m$ symmetric two-variable polynomial matrices are denoted by $\mathbb{R}^{m\times n}[\zeta,\eta]$ and $\mathbb{R}_{\mathrm{s}}^{m\times m}[\zeta,\eta]$, respectively. The matrix $A^{\dagger}$ denotes the pseudo inverse matrix of A. We denote $\mathbb{W}^{\mathbb{T}}$ as the set of maps from $\mathbb{T}$ to $\mathbb{W}$. We define the signal space l_2^q by

$$l_2^q := \left\{ w \in (\mathbb{R}^q)^{\mathbb{Z}} \,\middle|\, \sum_{t=-\infty}^{\infty} \|w(t)\|^2 < \infty \right\}.$$

2. QUADRATIC DIFFERENCE FORMS AND DISSIPATIVITY

In this section, we will review the basic definitions and results of quadratic difference forms and dissipativity (Willems and Trentelman (1998), Kaneko and Fujii (2000), Kaneko and Fujii (2003)).

We consider a linear discrete-time system Σ described by the state-space equation

$$x(t+1) = Ax(t) + Bu(t), \tag{1}$$
$$y(t) = Cx(t) + Du(t) \tag{2}$$

throughout this section, where $x \in (\mathbb{R}^n)^{\mathbb{Z}}$, $u \in (\mathbb{R}^m)^{\mathbb{Z}}$ and $y \in (\mathbb{R}^p)^{\mathbb{Z}}$ are the state, input and output variables of Σ, respectively, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$ are constant matrices. We assume that the pairs (A, B) and (C, A) are reachable and observable, respectively, throughout this section. We define the variable $w \in (\mathbb{R}^q)^{\mathbb{Z}}$ of Σ by $w := \begin{bmatrix} u \\ y \end{bmatrix}$, $q := p + m$, which is mentioned as a manifest variable in behavioral approach (Willems (1991)).

Consider a two-variable polynomial matrix in $\mathbb{R}^{q_1\times q_2}[\zeta,\eta]$ described by

$$\Phi(\zeta,\eta) = \sum_{i=0}^{N_1}\sum_{j=0}^{N_2} \Phi_{ij}\zeta^i\eta^j, \tag{3}$$

where $\Phi_{ij} \in \mathbb{R}^{q_1\times q_2}$ and $N \geq 0$. This $\Phi(\zeta,\eta)$ induces a bilinear difference form (BDF)

$$\mathsf{L}_\Phi : (\mathbb{R}^{q_1})^{\mathbb{Z}} \times (\mathbb{R}^{q_2})^{\mathbb{Z}} \to \mathbb{R}^{\mathbb{Z}},$$
$$\mathsf{L}_\Phi(\ell_1,\ell_2)(t) := \sum_{i=0}^{N_1}\sum_{j=0}^{N_2} \ell_1(t+i)^\top \Phi_{ij}\ell_2(t+j).$$

This means that ζ and η correspond to the shift operations on $\ell_1(t)$ and $\ell_2(t)$, respectively.

We call $\Phi(\zeta,\eta)$ symmetric when $q_1 = q_2 =: q$, $N_1 = N_2 =: N$ and $\Phi(\zeta,\eta)^\top = \Phi(\eta,\zeta)$. In this case, $\Phi(\zeta,\eta)$ induces a *quadratic difference form* (QDF)

$$\mathsf{Q}_\Phi : (\mathbb{R}^q)^{\mathbb{Z}} \to \mathbb{R}^{\mathbb{Z}},\ \mathsf{Q}_\Phi(\ell)(t) := \mathsf{L}_\Phi(\ell,\ell)(t).$$

A QDF $\mathsf{Q}_\Phi(\ell)$ is called the *rate of change* of $\mathsf{Q}_\Psi(\ell)$ if $\mathsf{Q}_\Psi(\ell)(t+1) - \mathsf{Q}_\Psi(\ell)(t) = \mathsf{Q}_\Phi(\ell)(t)$.

Here, we introduce the notion of dissipativity for linear discrete-time systems due to Kaneko and Fujii (2003).

Let $\Phi \in \mathbb{R}_{\mathrm{s}}^{q\times q}[\zeta,\eta]$. A system Σ is called dissipative with respect to the supply rate $\mathsf{Q}_\Phi(w)$ if $\sum_{t=-\infty}^{\infty} \mathsf{Q}_\Phi(w)(t) \geq 0$ holds for all $w \in l_2^q$ satisfying (1) and (2). Moreover, Σ is called half-line dissipative with respect to the supply rate if $\sum_{t=-\infty}^{0} \mathsf{Q}_\Phi(w)(t) \geq 0$ holds for all $w \in l_2^q$ satisfying (1) and (2). We can think of $\mathsf{Q}_\Phi(w)$ as the power delivered to the system Σ. The dissipativity implies that the net flow of energy into the system is non-negative, i.e. the system dissipates energy. Hence, the rate of increase of the energy stored inside of the system does not exceed the power supplied to it.

We give definitions of a storage function and a dissipation rate. The QDF $\mathsf{Q}_\Psi(w)$ induced by $\Psi \in \mathbb{R}_s^{q\times q}[\zeta,\eta]$ is a storage function for $\mathsf{Q}_\Phi(w)$ if

$$\mathsf{Q}_\Psi(w)(t+1) - \mathsf{Q}_\Psi(w)(t) \le \mathsf{Q}_\Phi(w)(t) \quad (4)$$

holds for all $t \in \mathbb{Z}$ and $w \in (\mathbb{R}^q)^{\mathbb{Z}}$ satisfying (1) and (2). We call (4) the dissipation inequality. The QDF $\mathsf{Q}_\Delta(w)$ induced by $\Delta \in \mathbb{R}_s^{q\times q}[\zeta,\eta]$ is a dissipation rate for $\mathsf{Q}_\Phi(w)$ if

$$\sum_{t=-\infty}^{\infty} \mathsf{Q}_\Phi(w)(t) = \sum_{t=-\infty}^{\infty} \mathsf{Q}_\Delta(w)(t) \quad (5)$$

and $\mathsf{Q}_\Delta(w)(t) \ge 0$ hold for all $t \in \mathbb{Z}$ and $w \in (\mathbb{R}^q)^{\mathbb{Z}}$ satisfying (1) and (2). Moreover, there is a one-to-one relation between a storage function $\mathsf{Q}_\Psi(w)$ and a dissipation rate $\mathsf{Q}_\Delta(w)$ defined by

$$\mathsf{Q}_\Psi(w)(t+1) - \mathsf{Q}_\Psi(w)(t) = \mathsf{Q}_\Phi(w)(t) - \mathsf{Q}_\Delta(w)(t). \quad (6)$$

The equation (6) is called the dissipation equality.

Kaneko and Fujii (2003) proved that the dissipativity of Σ is characterized in terms of a storage function and a dissipation rate, i.e. the following conditions (i), (ii), and (iii) are equivalent.

(i) The system Σ is dissipative with respect to the supply rate $\mathsf{Q}_\Phi(w)$.
(ii) The QDF $\mathsf{Q}_\Phi(w)$ admits a storage function.
(iii) The QDF $\mathsf{Q}_\Phi(w)$ admits a dissipation rate.

3. PROBLEM FORMULATION

In this section, we first give a mathematical formulation of a hierarchical network system. After the formulation, we declare the hierarchical network identification problem based on dissipation inequalities.

3.1 Hierarchical Network Systems

We give mathematical formulation of a hierarchical network system in this section. Hierarchical network system consists of L-tuple of subsystems in lower layer and network system in higher layer. Each subsystem represents local dynamics of the hierarchical network system. On the other hand, the network system plays a role for communication of the internal information between subsystems.

We give indices $i = 1, 2, \cdots, L$ for each subsystem and define ith subsystem Σ_i. The subsystem Σ_i represented by the state-space equation

$$x_i(t+1) = A_i x_i(t) + B_i\left(u_i(t) + r_i(t)\right), \quad (7)$$
$$y_i(t) = C_i x_i(t) + D_i\left(u_i(t) + r_i(t)\right), \quad (8)$$

where $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$, $C_i \in \mathbb{R}^{p_i\times n_i}$, $D_i \in \mathbb{R}^{p_i\times m_i}$ are constant matrices and $r_i \in \left(\mathbb{R}^{m_i}\right)^{\mathbb{Z}}$, $u_i \in \left(\mathbb{R}^{m_i}\right)^{\mathbb{Z}}$, $x_i \in \left(\mathbb{R}^{n_i}\right)^{\mathbb{Z}}$, $y_i \in \left(\mathbb{R}^{p_i}\right)^{\mathbb{Z}}$ are the input variable for identification, the input variable from network system, the state variable, and the output variable of Σ_i, respectively. We define the manifest variable $w_i \in \left(\mathbb{R}^{q_i}\right)^{\mathbb{Z}}$ of Σ_i by $w_i := \begin{bmatrix} u_i + r_i \\ y_i \end{bmatrix}$, $q_i := m_i + p_i$.

Assume that (A_i, B_i) is reachable and (C_i, A_i) is observable for all $i = 1, 2, \cdots, L$ throughout this paper. The transfer function $G_i \in \mathbb{R}^{p_i\times m_i}(z)$ of Σ_i is expressed as $G_i(z) := C_i\left(zI_i - A_i\right)^{-1} B_i + D_i$.

Each subsystem interconnects with a network structure in the upper layer via a contraction of state informations. We call the structure as *network system*. Suppose that the network system Σ_0 is represented by a static equation

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_L(t) \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1L} \\ K_{21} & K_{22} & \cdots & K_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ K_{L1} & K_{L2} & \cdots & K_{LL} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_L(t) \end{bmatrix}, \quad (9)$$

where $K_{ij} \in \mathbb{R}^{m_i\times n_i}$ $(i,j = 1,2,\cdots,L)$. In (9), the constant matrix

$$K := \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1L} \\ K_{21} & K_{22} & \cdots & K_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ K_{L1} & K_{L2} & \cdots & K_{LL} \end{bmatrix} \in \mathbb{R}^{(m_1+\cdots+m_L)\times(n_1+\cdots+n_L)}$$

is called the *network matrix*.

The global system Σ can be regarded as a feedback interconnection of subsystems and network system. This is illustrated in Fig. 2.

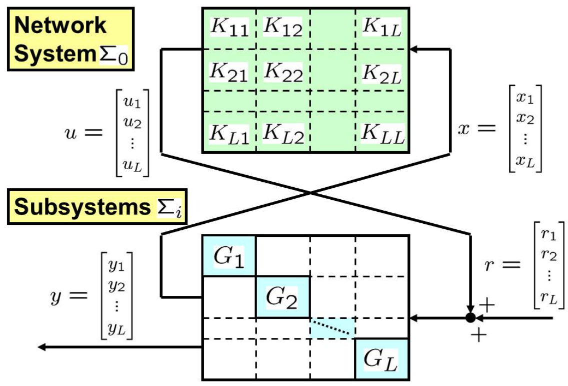

Fig. 2. Global system as a feedback interconnection of subsystems and network system

3.2 Problem Formulation

In an actual modeling, there can be many situations where a preliminary partial information on network structure and signals are available, for example circulant network, orthogonality of input sequences, dissipativity and etc. It may be desirable to use such information for identifications actively. We propose an identification method which preserves dissipativity properties in this paper.

We formulate the hierarchical network identification problem based on dissipation equalities in the following.

Problem 1. Suppose that input-output data

$$\{r_i(t), u_i(t), y_i(t); t = 0, 1, \cdots, N-1\} \quad (10)$$

of Σ_i are available for $i = 1, \cdots, L$, where N is a sufficiently large number. Moreover, Σ_i is half-line dissipative with respect to the supply rate $\mathsf{Q}_{\Phi_i}(w_i)$, and the supply rate $\mathsf{Q}_{\Phi_i}(w_i)$ and the dissipation rate $\mathsf{Q}_{\Phi_i}(w_i)$ are available. The problem is to identify the system matrices $\{A_i, B_i, C_i, D_i\}$ of each subsystem Σ_i and the matrices K of the network system Σ_0 up to similarity transformations.

Remark 1. We can also regard $y_i(t)$ and $x_i(t)$ as observable and unobservable outputs of the subsystem Σ_i, respectively. The setting of Problem 1 means that the data of the representative values such unobservable variables are available for the identification. Thus, we actively make use of the signals of different layers, namely the subsystems and network system.

We set the following assumption on the interconnection with subsystems and network system.

Assumption 1.

(i) The global system is well-posed in the sense that u_i and y_i are uniquely determined by the state of Σ_i ($i = 0, 1, \cdots, L$) and by the input r_i for identification. This implies that $I_{p_1+\cdots+p_L} - H(\infty)K$ and $I_{m_1+\cdots+m_L} - KH(\infty)$ are nonsingular, where $H(\infty) \in \mathbb{R}^{(p_1+\cdots+p_L)\times(m_1+\cdots+m_L)}$ is defined by $H(\infty) := \lim_{z\to\infty} H(z)$, $H(z) := \mathrm{diag}\,(H_1(z), , \cdots, H_L(z))$, $H_i(z) := (zI_{n_i} - A_i)^{-1} B_i$ $(i = 1, \cdots, L)$.

(ii) The global system Σ is internally stable.

(iii) The input variables $r_0, r_1, \cdots, r_L$ are uncorrelated each other.

Assumption 1 (i) is the condition to avoid the occurrence of algebraic loops in the global system Σ.

Remark 2. As we have pointed out in Section 3.1, the global system Σ can be regarded as a feedback interconnection of plants and a controller if we consider subsystems and network system as a plant and a controller, respectively. Hence, we need to take care of the following points for avoiding specific difficulties related to an identification of feedback systems. However, the conventional approach cannot be applied to our identification problem.

Firstly, we reduce the identification problem to an identification of feedback systems via dissipation equalities. However, it is not desirable to use the input-output pair of global system from the view points of a clearness of system description and a computational amount. Hence, the identification should be completed by each subsystem in a distributed manner. Two-approaches, so-called open-loop approach and closed-loop approach, are considered for the identification in this manner. These are proposed in Sections 4.3 and 4.4.

4. HIERARCHICAL NETWORK IDENTIFICATION BASED ON DISSIPATION EQUALITIES

We give an identification method for hierarchical network system based on dissipation equalities as a main result in this section. We first introduce the data matrices of the variables of each subsystem and set some assumptions on the matrices, or equivalently the variables, in Section 4.1. In Section 4.2, we show an estimate procedure of the data matrices of the state variables of each subsystem using dissipation equalities. Next, we show an open-loop approach, which can be regarded as the simplest method, for the identification of subsystem and network system in Section 4.3. Finally, we give a closed-loop approach which excludes a biased estimate in Section 4.4.

4.1 Data Matrices

We introduce the data matrices of the variables of the subsystem Σ_i and set some assumptions on the matrices before we move to the main part of this paper.

We define $U_i \in \mathbb{R}^{m_i\times N}$, $R_i \in \mathbb{R}^{m_i\times N}$ and $Y_i \in \mathbb{R}^{p_i\times N}$ as the data matrices of u_i, r_i and y_i satisfying (10).

$$U_i := [u_i(0)\ u_i(1)\ \cdots\ u_i(N-1)], \tag{11}$$

$$R_i := [r_i(0)\ r_i(1)\ \cdots\ r_i(N-1)], \tag{12}$$

$$Y_i := [y_i(0)\ y_i(1)\ \cdots\ y_i(N-1)] \tag{13}$$

We also construct $X_{i,j} \in \mathbb{R}^{n_i\times N}$ as the data matrix of the state variable of Σ_i.

$$X_{i,j} := [x_i(j)\ x_i(j+1)\ \cdots x_i(j+N-1)] \tag{14}$$

We set the following assumptions on these matrices.

Assumption 2. The following conditions (i) and (ii) hold.

(i) $\mathrm{rank}\begin{bmatrix} R_{1,0|0} \\ \vdots \\ R_{L,0|0} \end{bmatrix} = m_1 + \cdots + m_L$

(ii) $\mathrm{span}(X_0) \cap \mathrm{span}(R_{i,0|0}) = \{0\}$

Assumption 2 (i) implies that the input $r(t)$ satisfies the PE (persistently exciting) condition of order $m_1+\cdots+m_L$. The condition (ii) is a feedback free condition.

4.2 Estimation of the State Sequence

In this subsection, we estimate the state sequence of Σ_i by using a method proposed by Rapisarda and Trentelman (2011).

The dissipation equality of Σ_i is described by

$$\mathsf{Q}_{\Psi_i}(w_i)(t+1) - \mathsf{Q}_{\Psi_i}(w_i)(t) = \mathsf{Q}_{\Phi_i}(w_i)(t) - \mathsf{Q}_{\Delta_i}(w_i)(t),$$

where $\Psi_i \in \mathbb{R}_s^{q_i\times q_i}[\zeta, \eta]$ and $\Delta_i \in \mathbb{R}_s^{q_i\times q_i}[\zeta, \eta]$ are the two-variable polynomial matrices which induces the storage function and the dissipation rate with respect to the supply rate $\mathsf{Q}_{\Phi_i}(w_i)$. Define the S-matrix $\mathsf{S}_i \in \mathbb{R}_s^{N\times N}$ of Σ_i by

$$\mathsf{S}_i := \begin{bmatrix} \mathsf{S}_i(w)(0,0) & \mathsf{S}_i(w)(0,1) & \cdots & \mathsf{S}_i(w)(0,N-1) \\ \mathsf{S}_i(w)(1,0) & \mathsf{S}_i(w)(1,1) & \cdots & \mathsf{S}_i(w)(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathsf{S}_i(w)(N-1,0) & \mathsf{S}_i(w)(N-1,1) & \cdots & \mathsf{S}_i(w)(N-1,N-1) \end{bmatrix},$$

where $\mathsf{S}_i(w)(t_1, t_2)$ is the (t_1, t_2) element of S_i given by

$$\mathsf{S}_i(w)(t_1,t_2) = \sum_{k=0}^{\infty} \mathsf{L}_{\Phi_i}(w_i(t_1+k), w_i(t_2+k)) - \sum_{k=0}^{\infty} \mathsf{L}_{\Delta_i}(w_i(t_1+k), w_i(t_2+k)).$$

From the half-line dissipativity and Propositions 2 and 3 in Rapisarda and Trentelman (2011), we can prove the existence of $K_i \in \mathbb{R}_s^{n_i\times n_i}$ satisfying a rank-revealing factorization

$$\mathsf{S}_i = X_{i,0}^\top K_i X_{i,0} \tag{15}$$

with $n_i := \mathrm{rank} S_i(w)$. Thus, we can estimate the data matrix of the state variable of Σ_i, which will be used for the identification of the subsystems and the network system in the next subsections.

4.3 Open-Loop Approach

In this subsection, we propose an identification method in an open-loop manner. The method is mentioned as the open-loop approach. This approach chooses $u_i + r_i$ and y_i as an input and output variables in each subsystem.

This can be the simplest method in identification of a feedback system. Of course, it can also be the simplest for our problem and we can identify subsystems separately via the approach. Since there exists a feedback from the state variable of Σ_i to the input variables $u_i + r_i$, the feedback-free condition, which is necessary to system identification, is violated (Katayama (2005)). Hence, it may give a biased estimate if there exists a system noise to each subsystem.

(a) Identification of Subsystems We first consider a least-squares identification of subsystems based on the state estimation in Section 4.2.

From the state-space equation (7) and (8), we see that the data matrices in (11)-(14) satisfy the matrix equation

$$\begin{bmatrix} X_{i,1} \\ Y_i \end{bmatrix} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \begin{bmatrix} X_{i,0} \\ U_i + R_i \end{bmatrix}.$$

Then, we have the following theorem.

Theorem 1. Assume that Assumption 2 holds. Let $U_i \in \mathbb{R}^{m_i \times N}$, $R_i \in \mathbb{R}^{m_i \times N}$ and $Y_i \in \mathbb{R}^{p_i \times N}$ $(i = 1, \cdots, L)$ be given by (11), (12) and (13), respectively. Suppose that $X_{i|0} \in \mathbb{R}^{n \times N}$ $(i = 1, \cdots, L)$ is estimated by the factorization (15). Then, the system matrices $\{A_i, B_i, C_i, D_i\}$ are estimated by the formula

$$\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \left(\begin{bmatrix} X_{i,1} \\ Y_i \end{bmatrix} \begin{bmatrix} X_{i,0} \\ U_i + R_i \end{bmatrix}^\top \right) \cdot \left(\begin{bmatrix} X_{i,0} \\ U_i + R_i \end{bmatrix} \begin{bmatrix} X_{i,0} \\ U_i + R_i \end{bmatrix}^\top \right)^{-1} \tag{16}$$

for $i = 1, \cdots, L$.

Remark 3. As we have pointed out at the front of this subsection, due to the existence of a feedback from x_i to u_i, the proposed method gives biased estimates. Moreover, the full row rank condition of $\begin{bmatrix} X_{i,0} \\ U_i + R_i \end{bmatrix}$ is not guaranteed at the current problem setting. This implies that the matrix $\begin{bmatrix} X_{i,0} \\ U_i + R_i \end{bmatrix} \begin{bmatrix} X_{i,0} \\ U_i + R_i \end{bmatrix}^\top$ may not be nonsingular. We of course recognize such difficulties in this approach, however we proceed the identification method anyway.

(b) Identification of Network System As we have identified subsystems, we consider an identification of the network system Σ_0 based on the least-squares method.

We see that the equation (9) is rewritten as

$$\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_L \end{bmatrix} = K \begin{bmatrix} X_{1,0} \\ X_{2,0} \\ \vdots \\ X_{L,0} \end{bmatrix}.$$

Then, we can estimate the network matrix K by the following proposition derived in Kojima et al. (2012).

Proposition 1. Assume that Assumption 2 holds. Let $U_i \in \mathbb{R}^{m_i \times N}$ $(i = 1, \cdots, L)$ be defined by (11). Suppose that $X_{i|0} \in \mathbb{R}^{n_i \times N}$ $(i = 1, \cdots, L)$ is estimated by the factorization (15). Then, the network matrix $K \in \mathbb{R}^{(m_1 + \cdots + m_L) \times (n_1 + \cdots + n_L)}$ is estimated by the formula

$$K = \left(\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_L \end{bmatrix} \begin{bmatrix} X_{1,0} \\ X_{2,0} \\ \vdots \\ X_{L,0} \end{bmatrix}^\top \right) \left(\begin{bmatrix} X_{1,0} \\ X_{2,0} \\ \vdots \\ X_{L,0} \end{bmatrix} \begin{bmatrix} X_{1,0} \\ X_{2,0} \\ \vdots \\ X_{L,0} \end{bmatrix}^\top \right)^{-1}. \tag{17}$$

This matrix is determined uniquely up to similarity transformations.

4.4 Closed-Loop Approach

In this subsection, we propose an identification method in a closed-loop. The method in this subsection is called by the closed-loop approach. This approach identifies a subsystem Σ_i using the input variable r_i and the output variables u_i, y_i based on the joint input-output approach (Katayama (2005)). It needs a rather large computational amount comparing with the open-loop approach, however the feedback-free condition (Katayama (2005)) is guaranteed, which is fundamental for a system identification. This implies that there is no biased estimate in identification results.

From (7), (8) and (9), the subsystem Σ_i can be expressed as the state-space equation in closed-loop form

$$\begin{aligned} x_i(t+1) &= A_{i,\mathrm{cl}} x_i(t) + B_i \sum_{j \neq i} K_{ij} x_j(t) + B_i r_i(t), \\ u_i(t) &= \sum_{j=1}^{L} K_{ij} x_j(t), \\ y_i(t) &= C_{i,\mathrm{cl}} x_i(t) + D_i \sum_{j \neq i} K_{ij} x_j(t) + D_i r_i(t), \end{aligned}$$

where $A_{i,\mathrm{cl}} \in \mathbb{R}^{n_i \times n_i}$ and $C_{i,\mathrm{cl}} \in \mathbb{R}^{p_i \times n_i}$ are the constant matrices defined by $A_{i,\mathrm{cl}} = A_i + B_i K_{ii}$ and $C_{i,\mathrm{cl}} = C_i + D_i K_{ii}$, respectively. In terms of the data matrices, the above equation is rewritten by

$$\begin{bmatrix} X_{i,1} \\ U_i \\ Y_i \end{bmatrix} = \begin{bmatrix} A_{i,\mathrm{cl}} & B_i K_{i1} & \cdots & B_i K_{ii-1} & B_i K_{ii+1} & \cdots & B_i K_{iL} & B_i \\ K_{ii} & K_{i1} & \cdots & K_{ii-1} & K_{ii+1} & \cdots & K_{iL} & 0 \\ C_{i,\mathrm{cl}} & D_i K_{i1} & \cdots & D_i K_{ii-1} & D_i K_{ii+1} & \cdots & D_i K_{iL} & D_i \end{bmatrix} \cdot \begin{bmatrix} X_{i,0} \\ X_{1,0} \\ \vdots \\ X_{i-1,0} \\ X_{i+1,0} \\ \vdots \\ X_{L,0} \\ R_{i,0|0} \end{bmatrix}.$$

From Assumption 2, we see that there holds

$$\begin{aligned} &\begin{bmatrix} A_{i,\mathrm{cl}} & B_i K_{i1} & \cdots & B_i K_{ii-1} & B_i K_{ii+1} & \cdots & B_i K_{iL} & B_i \\ K_{ii} & K_{i1} & \cdots & K_{ii-1} & K_{ii+1} & \cdots & K_{iL} & 0 \\ C_{i,\mathrm{cl}} & D_i K_{i1} & \cdots & D_i K_{ii-1} & D_i K_{ii+1} & \cdots & D_i K_{iL} & D_i \end{bmatrix} \\ &= \left(\begin{bmatrix} X_{i,1} \\ U_i \\ Y_i \end{bmatrix} \begin{bmatrix} X_{i,0} \\ X_{1,0} \\ \vdots \\ X_{i-1,0} \\ X_{i+1,0} \\ \vdots \\ X_{L,0} \\ R_i \end{bmatrix}^\top \right) \left(\begin{bmatrix} X_{i,0} \\ X_{1,0} \\ \vdots \\ X_{i-1,0} \\ X_{i+1,0} \\ \vdots \\ X_{L,0} \\ R_{i,0|0} \end{bmatrix} \begin{bmatrix} X_{i,0} \\ X_{1,0} \\ \vdots \\ X_{i-1,0} \\ X_{i+1,0} \\ \vdots \\ X_{L,0} \\ R_i \end{bmatrix}^\top \right)^{-1} \end{aligned} \tag{18}$$

by the least-squares method. Then, the matrices A_i and C_i are computed by

$$A_i = A_{i,\mathrm{cl}} - B_i K_{ii} \text{ and } C_i = C_{i,\mathrm{cl}} - D_i K_{ii}, \qquad (19)$$

respectively. Summarizing the above discussions, we have the following theorem for the identification via the closed-loop approach.

Theorem 2. Assume that Assumption 2 holds. Let $U_i \in \mathbb{R}^{m_i \times N}$, $R_i \in \mathbb{R}^{m_i \times N}$ and $Y_i \in \mathbb{R}^{p_i \times N}$ $(i = 1, \cdots, L)$ be given by (11), (12) and (13), respectively. Suppose that $X_{i,0} \in \mathbb{R}^{n \times N}$ $(i = 1, \cdots, L)$ is estimated by the factorization (15). Then, the system matrices $\{A_i, B_i, C_i, D_i\}$ $(i = 1, \cdots, L)$ and the network matrix K are estimated by the formula (18) and (19).

Remark 4. In the formula (18), since the size of the equation is larger than the corresponding equations (16) and (17) in the open-loop approach, the closed-loop approach requires some extent of computational amounts. However, it can estimate both single subsystem and corresponding part of the network system simultaneously without any biases.

5. CONCLUSIONS

In this paper, we have formulated the identification problem for a hierarchical network system based on dissipation equalities. As a main result, we have shown identification methods via both open-loop and closed-loop approaches using the least-squares method.

As a future work, we need to give a theoretical comparison between open-loop and closed-loop approaches and give a clear characterization comparing with conventional methods. Moreover, we should also derive an identification method which can tackle noises and disturbances added to the input and output variables.

REFERENCES

Fax, J. and Murray, R. (2004). Information Flow and Cooperative Control of Vehicle Forations. *IEEE Transactions on Automatic Control*, AC-49(9), 1465–1476.

Hara, S., Hayakawa, T., and Sugata, H. (2009a). LTI Systems with Generalized Frequency Variables: A Unified Framework for Homogeneous Multi-agent Dynamical Systems. *SICE Journal of Control, Measurement, and System Integration*, 2, 299–306.

Hara, S., Imura, J., and Tsumura, K. (2008). Multi-Resolved Dynamical System Theory for Large Scale Complex Systems. In *Proceedings of the SICE Annual Conference 2008*. Chofu, Japan.

Hara, S., Shimizu, H., and Kim, T.H. (2009b). Consensus in Hierarchical Multi-agent Dynamical Systems with Low-rank Interconnections: Analysis of stability and convergence rates. In *Proceedings of 2009 American Control Conference*. St. Louis, USA.

Julius, A., Zavlanos, M., Boyd, S., and Pappas, G. (2009). Genetic Network Identification Using Convex Programming. *IET Systems Biology*, 3(3), 155.166.

Kaneko, O. and Fujii, T. (2000). Discrete Time Average Positivity and Spectral Factorization in a Behavioral Framework. *Systems and Control Letters*, 39(1), 31–44.

Kaneko, O. and Fujii, T. (2003). When is a Storage Function a State Function in Discrete Time? *SIAM Journal on Control and Optimization*, 42(4), 1374–1394.

Katayama, T. (2005). *Subspace Methods for System Identification*. Springer.

Kojima, C., Hashimoto, R., and Nakano, S. (2012). Hierarchical Network Identification of Large-Scale Systems - Homogeneous Case. In *Proceedings of the 2012 American Control Conference*, 4034–4039. Montreal, Canada.

Massioni, P. and Verhaegen, M. (2008). Subspace Identification of Circulant Systems. *Automatica*, 44(11), 2825–2833.

Massioni, P. and Verhaegen, M. (2010). Subspace Identification of Distributed, Decomposable Systems. In *Proceedings of the combined 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 3364–3369. Shanghai, China.

Olfati-Saber, R., Fax, J., and Murray, R. (2007). Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1), 215–233.

Rapisarda, P. and Trentelman, H. (2011). Identification and Data-Driven Model Reduction of State-Space Representations of Lossless and Dissipative Systems from Noise-Free Data. *Automatica*, 47, 1721–1728.

Sandberg, H. and Murray, R. (2009). Model Reduction of Interconnected Linear Systems. *Optimal Control Applications and Methods*, 30, 225–245.

Shimizu, H. and Hara, S. (2009). Rank Properties of Interlayer Incidence Matrix and Convergence Performance in Hierarchical Consensus. *Transactions of the Society of Instrument and Control Engineers*, 45(9), 476–483. (in Japanese).

Smith, R. (2005). A Hierarchical Cyclic Pursuit Scheme for Vehicle Networks. *Automatica*, 41, 1045–1053.

Takahashi, N., Azuma, T., and Adachi, S. (2010). Estimation of Protein Networks for Cell Cycle in Yeast Based on Least Squares Method Using Periodic Signals. In *Proceedings the 19th International Symposium on Mathematical Theory of Networks and Systems*, 455–461. Budapest, Hungary.

Van Overschee, P. and Moor, B.D. (1993). N4SID - Subsace Algorithms for the Stochastic Identification Problem. *Automatica*, 29(3), 649–660.

Verhaegen, M. and Dewilde, P. (1992a). Subspace Model Identification, Part 1: The Output-Error State-Space Model Identification Class of Algorithms. *International Journal of Control*, 56(5), 1187–1210.

Verhaegen, M. and Dewilde, P. (1992b). Subspace Model Identification, Part 2: Analysis of the Elementary Output-Error State-Space Model Identification Algorithm. *International Journal of Control*, 56(5), 1211–1241.

Willems, J. (1971). Least Squares Stationary Optimal Control and the Algebraic Riccati Equation. *IEEE Transactions on Automatic Control*, AC-16(6), 621–634.

Willems, J. (1991). Paradigms and Puzzles in the Theory of Dynamical Systems. *IEEE Transactions on Automatic Control*, AC-36(11), 259–294.

Willems, J. and Trentelman, H. (1998). On Quadratic Differential Forms. *SIAM Journal on Control and Optimization*, 36(5), 1703–1749.