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# On the Distinguishability of Positive Linear Time-Invariant Systems with Affine Parametric Uncertainties

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**Abstract:** Studies of distinguishability have focused on the case of Linear Time-Invariant systems without uncertainties. In this work, distinguishability is studied for Positive Linear Time-Invariant systems with affine parametric uncertainties in the state space model. We propose a definition of distinguishability adapted to this new context and give a characterization of this notion. The approach used is based on the estimate of the reachable output space of the systems. Under suitable assumptions, a sufficient condition for distinguishability is established.

*Keywords:* Distinguishability, Uncertain system, Parametric uncertainty, Positive system

## 1. INTRODUCTION

Consider the two uncertain Linear-Time Invariant (LTI) systems described by the following set of differential-algebraic equations :

$$S_k \begin{cases} \dot{x}_k(t) = \mathbb{A}_k(\theta_k)\, x_k(t) + B_k\, u(t), \\ y_k(t) = C_k\, x_k(t), \\ x_k(0) = x_k^o \in \mathbb{X}_k^o, \end{cases} \tag{1}$$

with $k = 1, 2$ and where $x_k(t) \in \mathbb{E}_k \subseteq \mathbb{R}^n$, $y_k(t) \in \mathbb{R}^m$ and $u(t) \in \mathbb{R}^l$ are respectively the state vector, the output vector and the input vector of the system $S_k$ ; $\mathbb{E}_k$ is the state space of $S_k$ and the set $\mathbb{X}_k^o$ is such that $\mathbb{X}_k^o \subseteq \mathbb{E}_k$ ; the matrix $B_k$ is the input matrix and $C_k$ is the output matrix ; $\theta_k = [\theta_{k,1}\ \theta_{k,2}\ \cdots\ \theta_{k,p_k}]^T$ is a vector of uncertain real parameters $\theta_{k,r}$, $r = 1, 2, \ldots, p_k$ and the state matrix $\mathbb{A}_k(\theta_k)$ has the form

$$\mathbb{A}_k(\theta_k) = A_k + \theta_{k,1}\, A_{k,1} + \theta_{k,2}\, A_{k,2} + \cdots + \theta_{k,p_k}\, A_{k,p_k},$$

where $A_k$, $A_{k,1}, \ldots, A_{k,p_k}$ are known matrices. The lower bound $\underline{\boldsymbol{\theta}}_{k,r}$ and the upper bound $\overline{\boldsymbol{\theta}}_{k,r}$ of each real parameter $\theta_{k,r} \in [\underline{\boldsymbol{\theta}}_{k,r}\,; \overline{\boldsymbol{\theta}}_{k,r}]$ are assumed to be known. The systems $S_1$ and $S_2$ can represent two distinct operating modes of an affine switched system with uncertain parameters.

This paper is concerned with the property of distinguishability between the uncertain systems $S_1$ and $S_2$. Most distinguishability problems are discussed for LTI systems without uncertainties (Avdeenko and Kargin (2000), Cocquempot et al. (2004), Grewal and Glover (1976), Lou and Si (2009), Lou and Yang (2011), Motchon et al. (2013)) and such studies have applications in Fault Detection and Isolation (FDI) (Bayoudh et al. (2008),Cocquempot et al. (2004)), and in active mode detection in switched affine systems (Domlan et al. (2007), Hakem et al. (2010)). But it is well known that to describe a system with a linear model, approximations have generally to be made leading to uncertainties in the parameters of the linear model. The motivation of this study is to take such uncertainties into account in the analysis of the distinguishability property. In the sequel,

- $\mathcal{U}$ is a sub-vector space of $L^1([0\,; \mathcal{T}], \mathbb{R}^m)$.
- $\mathbb{U} \subseteq L^1([0\,; \mathcal{T}], \mathbb{R}^m)$ denotes the space of admissible inputs of the systems $S_1$ and $S_2$.
- $\mathbb{X}^o = \mathbb{X}_1^o \times \mathbb{X}_2^o$ denotes the set of all admissible initial state pairs $(x_1^o, x_2^o)$ of systems $S_1$ and $S_2$.
- $[\underline{\boldsymbol{\theta}}_k\,; \overline{\boldsymbol{\theta}}_k] := [\underline{\boldsymbol{\theta}}_{k,1}\,; \overline{\boldsymbol{\theta}}_{k,1}] \times \cdots \times [\underline{\boldsymbol{\theta}}_{k,p_k}\,; \overline{\boldsymbol{\theta}}_{k,p_k}]$ denotes the set of all admissible values of the vector $\theta_k$.
- $\Theta := [\underline{\boldsymbol{\theta}}_1\,; \overline{\boldsymbol{\theta}}_1] \times [\underline{\boldsymbol{\theta}}_2\,; \overline{\boldsymbol{\theta}}_2] \subset \mathbb{R}^{p_1} \times \mathbb{R}^{p_2}$ is the set of all admissible parameter pairs $(\theta_1, \theta_2)$ of systems $S_1$ and $S_2$.

The output vector $y_k(t)$ of $S_k$ can be expressed as follows (Larminat (2007)):

$$y_k(t) = C_k \left( \mathrm{e}^{t\,\mathbb{A}_k(\theta_k)}\, x_k^o + \int_0^t \mathrm{e}^{(t-\tau)\,\mathbb{A}_k(\theta_k)}\, B_k\, u(\tau)\, d\tau \right).$$

Consequently, $y_k(t)$ is a function of the initial state vector $x_k^o$, the input signal $u$ and the vector $\theta_k$. This function is denoted by $y_k(t) \equiv y_k(t, x_k^o, u, \theta_k)$.

For a given value $\theta_k^\star \in [\underline{\boldsymbol{\theta}}_k\,; \overline{\boldsymbol{\theta}}_k]$ of the vector $\theta_k$, let $\mathbf{S}(\theta_k^\star)$ be the system defined as follows:

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$$\mathbf{S}(\theta_k^\star)\colon \begin{cases} \dot{x}_k(t) = \mathbb{A}_k(\theta_k^\star)\, x_k(t) + B_k\, u(t), \\ y_k(t) = C_k\, x_k(t), \\ x_k(0) = x_k^o. \end{cases}$$

According to the definition proposed in Lou and Si (2009), two systems $\mathbf{S}(\theta_1^\star)$ and $\mathbf{S}(\theta_2^\star)$ are distinguishable on $[0\,;\mathcal{T}]$ if and only if for all triplets $(x_1^o, x_2^o, u) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{U}$ such that $(x_1^o, x_2^o, u) \neq (0, 0, 0)$, the outputs $y_1(\cdot, x_1^o, u, \theta_1^\star)$ and $y_2(\cdot, x_2^o, u, \theta_2^\star)$ are not identical on $[0\,;\mathcal{T}]$. In particular, given $u \neq 0$, no output of $\mathbf{S}(\theta_1^\star)$ can be equal to an output of $\mathbf{S}(\theta_2^\star)$. A generalization of this notion to the case of LTI systems with affine uncertainties is the $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishability that we define as follows:

*Definition 1.* Systems $S_1$ and $S_2$ are said to be $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishable on $[0\,;\mathcal{T}]$ if for all $(x_1^o, x_2^o) \in \mathbb{X}^o$, all $u \in \mathbb{U}$ and all $(\theta_1, \theta_2) \in \Theta$ with $(x_1^o, x_2^o, u) \neq (0, 0, 0)$, the functions $y_1(\cdot, x_1^o, u, \theta_1)$ and $y_2(\cdot, x_2^o, u, \theta_2)$ are not identical on $[0\,;\mathcal{T}]$. If not, the two systems are said to be $(\mathbb{X}^o, \mathbb{U}, \Theta)$-indistinguishable on $[0\,;\mathcal{T}]$.

This paper proposes a sufficient condition for $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishability. The condition is based on an estimate of the reachable output spaces of $S_1$ and $S_2$ which is obtained with a method developed in Kieffer and Walter (2006) and Meslem (2008). When $S_1$ and $S_2$ are positive systems and under some assumptions on the spaces $\mathbb{X}_k^o$, $\mathbb{U}$ and the matrices $A_{k,r}$, it is straightforward to obtain this estimate.

In Section 2 some basic notations and definitions are given and the estimation technique is recalled. Section 3 is devoted to the proof of the sufficient condition for distinguishability. An illustration of this result with an academic example is given. Some remarks and open problems conclude the work.

## 2. PRELIMINARIES

### 2.1 Some basic notation and definitions

Let $A = (a_{ij}) \in \mathbb{R}^{\mathbf{n}\times\mathbf{m}}$ be a matrix and, $v$, $w$, $\underline{\boldsymbol{w}}$ and $\overline{\boldsymbol{w}}$ be four vectors of $\mathbb{R}^{\mathbf{p}}$.

- $A$ is said to be a **Metzler-matrix** if $\forall i \neq j$, $a_{ij} \geq 0$.
- $A$ is said to be a **non-negative matrix** ($A \succeq 0$) if $\forall i, j$, $a_{ij} \geq 0$ and $\mathbb{R}_+^{\mathbf{n}\times\mathbf{m}} := \{\mathrm{M} \in \mathbb{R}^{\mathbf{n}\times\mathbf{m}} \colon \mathrm{M} \succeq 0\}$ denotes the set of all non-negative matrices of $\mathbb{R}^{\mathbf{n}\times\mathbf{m}}$.
- $v$ is said to be a **non-negative** vector ($v \succeq 0$) if $\forall i$, $v_i \geq 0$ and $\mathbb{R}_+^{\mathbf{p}} := \{\xi \in \mathbb{R}^{\mathbf{p}} \colon \xi \succeq 0\}$ denotes the set of all non-negative vectors of $\mathbb{R}^{\mathbf{p}}$.
- $v \preceq w$ if $w - v \succeq 0$ ; $w \in [\underline{\boldsymbol{w}}\,;\overline{\boldsymbol{w}}]$ if $\underline{\boldsymbol{w}} \preceq w$ and $w \preceq \overline{\boldsymbol{w}}$.
- $v \succ 0$ if $v \succeq 0$ and $\exists i \in \{1, 2, \ldots \mathbf{p}\}$ such that $v_i > 0$.
- $v \succ w$ if $v - w \succ 0$.

### 2.2 Positive linear dynamical systems : definition and characterization

Positive dynamical systems are a class of dynamical systems which is studied in detail in Farina and Rinaldi (2000). We only recall the definition and one basic property of these systems.

*Definition 2.* A continuous linear time-invariant system

$$\begin{cases} \dot{x}(t) = A\, x(t) + B\, u(t), \\ y(t) = C\, x(t) + D\, u(t), \\ x(0) = x^o, \end{cases} \tag{2}$$

is said to be positive if for all positive initial states $x_i^o$ ($x_i^o \succeq 0$) and all input signals $u$ such that $\forall t \in \mathbb{R}_+$, $u(t) \succeq 0$, the state vector $x(t)$ and the output vector $y(t)$ are both positive ($x(t) \succeq 0$ and $y(t) \succeq 0$) for all $t \in \mathbb{R}_+$.

*Theorem 1.* (Farina and Rinaldi (2000)) The linear dynamical system (2) is positive iff $A$ is a Metzler-matrix, $B \succeq 0$, $C \succeq 0$ and $D \succeq 0$.

### 2.3 Estimate of the reachable state of uncertain dynamical systems

Several methods have been developed in the literature to estimate the reachable state space of uncertain systems. In this paper, we have chosen to focus on the one developed in Kieffer and Walter (2006) and Meslem (2008). It is based on the theory of differential inequalities and allows to bound the states of an uncertain system by those of two coupled systems without uncertainties.

Consider an uncertain dynamical system described by:

$$\begin{cases} \dot{z} = g(z, u, \theta), \\ z(0) = z^o, \end{cases} \tag{3}$$

where $z(t) \in \mathbb{D} \subseteq \mathbb{R}^n$ is the state vector, $u$ an input signal and $\theta \in [\underline{\boldsymbol{\theta}}\,;\overline{\boldsymbol{\theta}}] \subset \mathbb{R}^p$ a parameter vector of the system ; the vectors $\underline{\boldsymbol{\theta}}$ and $\overline{\boldsymbol{\theta}}$ are the bounds of the parameter vector $\theta$. The input signal is assumed to belong to the set

$$\mathbb{V}(\underline{\mathbf{u}}, \overline{\mathbf{u}}) = \{u \colon u(t) \in [\underline{\mathbf{u}}(t)\,;\overline{\mathbf{u}}(t)],\ \forall t \in [0\,;\mathcal{T}]\},$$

where the signals $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$ are both known and satisfy the relation $\underline{\mathbf{u}}(t) \preceq \overline{\mathbf{u}}(t)$, $\forall t \in [0\,;\mathcal{T}]$. In the case where $l = 1$, the Figure 1 below illustrates an example of an input signal $u$ that belongs to the set $\mathbb{V}(\underline{\mathbf{u}}, \overline{\mathbf{u}})$ and an example of an input signal $u$ which does not belong to $\mathbb{V}(\underline{\mathbf{u}}, \overline{\mathbf{u}})$.

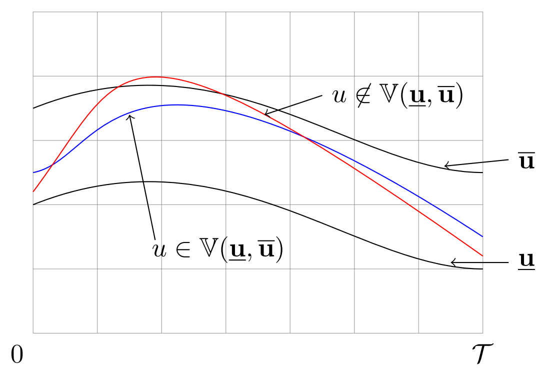


Fig. 1. An illustration of the set $\mathbb{V}(\underline{\mathbf{u}}, \overline{\mathbf{u}})$ in the case where $l = 1$

Let $\mathbb{F} \subseteq \{v \in \mathbb{R}^l \colon \exists t \in [0\,;\mathcal{T}],\ v \in [\underline{\mathbf{u}}(t)\,;\overline{\mathbf{u}}(t)]\}$. For the signals $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$ of Figure 1, $\mathbb{F}$ is a subset of the interval $[\min_{t\in[0;\mathcal{T}]} \underline{\mathbf{u}}(t)\,;\ \max_{t\in[0;\mathcal{T}]} \overline{\mathbf{u}}(t)]$. Assume that the vector field $g$ is continuously differentiable on $\mathbb{D} \times \mathbb{F} \times [\underline{\boldsymbol{\theta}}\,;\overline{\boldsymbol{\theta}}]$ and that the sign of functions $\frac{\partial g_i}{\partial z_j}$ $i \neq j$, $\frac{\partial g_i}{\partial u_j}$ and $\frac{\partial g_i}{\partial \theta_j}$ does not change on the domain $\mathbb{D} \times \mathbb{F} \times [\underline{\boldsymbol{\theta}}\,;\overline{\boldsymbol{\theta}}]$. Then as shown in Kieffer and Walter (2006) and Meslem (2008), one can determine two vector fields $\underline{\boldsymbol{g}} := \underline{\boldsymbol{g}}(\underline{\boldsymbol{z}}, \overline{\boldsymbol{z}}, \underline{\mathbf{u}}, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}})$ and $\overline{\boldsymbol{g}} := \overline{\boldsymbol{g}}(\underline{\boldsymbol{z}}, \overline{\boldsymbol{z}}, \underline{\mathbf{u}}, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}})$ for which the following two coupled systems

$$\dot{\underline{\boldsymbol{z}}} = \underline{\boldsymbol{g}}(\underline{\boldsymbol{z}}, \overline{\boldsymbol{z}}, \underline{\mathbf{u}}, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}) \text{ and } \dot{\overline{\boldsymbol{z}}} = \overline{\boldsymbol{g}}(\underline{\boldsymbol{z}}, \overline{\boldsymbol{z}}, \underline{\mathbf{u}}, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}})$$

are such that if $\underline{\boldsymbol{z}}(0) \preceq \overline{\boldsymbol{z}}(0)$ then, for all $t \in [0\,;\mathcal{T}]$ and all triplets $(z^o, u, \theta) \in [\underline{\boldsymbol{z}}(0)\,;\overline{\boldsymbol{z}}(0)] \times \mathbb{V}(\underline{\mathbf{u}}, \overline{\mathbf{u}}) \times [\underline{\boldsymbol{\theta}}\,;\overline{\boldsymbol{\theta}}]$,

$$\underline{\boldsymbol{z}}(t) \preceq z(t) \preceq \overline{\boldsymbol{z}}(t).$$

The components $\underline{\boldsymbol{g}}_i$, and $\overline{\boldsymbol{g}}_i$, $i = 1, 2, \ldots, n$ of the vectors fields $\underline{\boldsymbol{g}}$ and $\overline{\boldsymbol{g}}$ are constructed as follows:

- $\underline{\boldsymbol{g}}_i(\underline{\boldsymbol{z}}, \overline{\boldsymbol{z}}, \underline{\mathbf{u}}, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}) = g_i\left(\underline{\boldsymbol{\xi}}^i, \underline{\boldsymbol{v}}^i, \underline{\boldsymbol{\vartheta}}^i\right)$, where the function $\underline{\boldsymbol{\xi}}^i$ (resp. $\underline{\boldsymbol{v}}^i$) defined on $\mathbb{R}$ with values in $\mathbb{R}^n$ (resp. $\mathbb{R}^l$) and the vector $\underline{\boldsymbol{\vartheta}}^i \in \mathbb{R}^p$ are computed as follows:

$$\underline{\boldsymbol{\xi}}^i_j = \begin{cases} \underline{\boldsymbol{z}}_i & \text{if } i = j, \\ \underline{\boldsymbol{z}}_j & \text{if } j \neq i \text{ and } \frac{\partial g_i}{\partial z_j} \geq 0, \\ \overline{\boldsymbol{z}}_j & \text{if } j \neq i \text{ and } \frac{\partial g_i}{\partial z_j} < 0, \end{cases}$$

$$\underline{\boldsymbol{v}}^i_j = \begin{cases} \underline{\mathbf{u}}_j & \text{if } \frac{\partial g_i}{\partial u_j} \geq 0, \\ \overline{\mathbf{u}}_j & \text{if } \frac{\partial g_i}{\partial u_j} < 0, \end{cases}$$

$$\underline{\boldsymbol{\vartheta}}^i_j = \begin{cases} \underline{\boldsymbol{\theta}}_j & \text{if } \frac{\partial g_i}{\partial \theta_j} \geq 0, \\ \overline{\boldsymbol{\theta}}_j & \text{if } \frac{\partial g_i}{\partial \theta_j} < 0. \end{cases}$$

- $\overline{\boldsymbol{g}}_i(\underline{\boldsymbol{z}}, \overline{\boldsymbol{z}}, \underline{\mathbf{u}}, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}) = g_i\left(\overline{\boldsymbol{\xi}}^i, \overline{\boldsymbol{v}}^i, \overline{\boldsymbol{\vartheta}}^i\right)$ where the function $\overline{\boldsymbol{\xi}}^i$ (resp. $\overline{\boldsymbol{v}}^i$) defined on $\mathbb{R}$ with values in $\mathbb{R}^n$ (resp. $\mathbb{R}^l$) and the vector $\overline{\boldsymbol{\vartheta}}^i \in \mathbb{R}^p$ are computed as follows:

$$\overline{\boldsymbol{\xi}}^i_j = \begin{cases} \overline{\boldsymbol{z}}_i & \text{if } i = j, \\ \overline{\boldsymbol{z}}_j & \text{if } j \neq i \text{ and } \frac{\partial g_i}{\partial z_j} \geq 0, \\ \underline{\boldsymbol{z}}_j & \text{if } j \neq i \text{ and } \frac{\partial g_i}{\partial z_j} < 0, \end{cases}$$

$$\overline{\boldsymbol{v}}^i_j = \begin{cases} \overline{\mathbf{u}}_j & \text{if } \frac{\partial g_i}{\partial u_j} \geq 0, \\ \underline{\mathbf{u}}_j & \text{if } \frac{\partial g_i}{\partial u_j} < 0, \end{cases}$$

$$\overline{\boldsymbol{\vartheta}}^i_j = \begin{cases} \overline{\boldsymbol{\theta}}_j & \text{if } \frac{\partial g_i}{\partial \theta_j} \geq 0, \\ \underline{\boldsymbol{\theta}}_j & \text{if } \frac{\partial g_i}{\partial \theta_j} < 0. \end{cases}$$

## 3. MAIN RESULTS

### *3.1 An estimate of the reachable output space of the uncertain system $S_k$ in* (1)

The goal of this section is to give an estimate of the reachable output space of systems $S_1$ and $S_2$. As recalled in the previous section, under some assumptions, the reachable state space of a given uncertain system can be estimated by the state vectors of two specific systems. Some assumptions are made first in this part. Under these assumptions, we show that for each system $S_k$, there exist two LTI systems $\underline{\boldsymbol{\Sigma}}_k$ and $\overline{\boldsymbol{\Sigma}}_k$ with outputs $\underline{\boldsymbol{y}}_k$ and $\overline{\boldsymbol{y}}_k$ that are respectively a lower and an upper bound of $y_k$.

Let $f^k$, $k = 1, 2$ be the two vectors fields associated with the systems $S_k$ defined as follows:

$$f^k(x_k, u, \theta_k) := \left(A_k + \sum_{r=1}^{p_k} \theta_{k,r}\, A_{k,r}\right) x_k + B_k\, u.$$

The dependence in $t$ of functions $x$ and $u$ is omitted in the expression of $f^k$ for simplicity. Jacobian matrices $\frac{\partial f^k}{\partial x_k} = \left(\frac{\partial f^k_i}{\partial x_{k,j}}\right)$ and $\frac{\partial f^k}{\partial u} = \left(\frac{\partial f^k_i}{\partial u_j}\right)$ are given by:

$$\frac{\partial f^k}{\partial x_k} = A_k + \sum_{r=1}^{p_k} \theta_{k,r}\, A_{k,r} \text{ and } \frac{\partial f^k}{\partial u} = B_k.$$

Clearly, signs of $\frac{\partial f^k_i}{\partial x_{k,j}}$ and $\frac{\partial f^k_i}{\partial u_j}$ do not change on any subset of $\mathbb{R}^n \times \mathbb{R}^l \times \mathbb{R}^{p_k}$. Therefore, to estimate the reachable state space of systems $S_k$, $k = 1, 2$ using the method in Kieffer and Walter (2006) and Meslem (2008), only the signs of the functions $\frac{\partial f^k_i}{\partial \theta_{k,r}}$, $i = 1, 2, \ldots, n$, $r = 1, 2, \ldots, p_k$ need further consideration. As seen in Proposition 1 below, under Assumptions 1, 2 and 3 on the system $S_k$, these signs do not change on the domain $\mathbb{E}_k \times \mathbb{R}^l \times \mathbb{R}^{p_k}$.

*Assumption 1.* The admissible input space is of the form $\mathbb{U} \subseteq \mathbb{V}(\underline{\mathbf{u}}, \overline{\mathbf{u}})$ for two signals $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$ satisfying the relation

$0 \preceq \underline{\mathbf{u}}(t) \prec \overline{\mathbf{u}}(t)$, $\forall t \in [0\,;\mathcal{T}]$. Moreover, $\mathbb{X}_k^o \subseteq [\underline{\mathbf{x}}_k^o\,;\overline{\mathbf{x}}_k^o]$ where $\underline{\mathbf{x}}_k^o$ and $\overline{\mathbf{x}}_k^o$ are two non-negative vectors of $\mathbb{R}^n$.

*Assumption 2.* The systems $\mathbf{S}(\theta_k)$ are positive for all parameter vectors $\theta_k \in [\underline{\boldsymbol{\theta}}_k\,;\overline{\boldsymbol{\theta}}_k]$.

Assumption 2 means that the matrix $\mathbb{A}(\theta_k)$ is a Metzler-matrix for all parameter vector $\theta_k \in [\underline{\boldsymbol{\theta}}_k\,;\overline{\boldsymbol{\theta}}_k]$ and that the matrices $B_k$ and $C_k$ are non-negative. Under Assumptions 1 and 2 on the system $S_k$, without loss of generality, we take $\mathbb{E}_k \subseteq \mathbb{R}^n_+$. Indeed, according to the definition of a positive system recalled in Definition 2 and under Assumptions 1 and 2 on the system $S_k$, the vector $x_k(t) := x_k(t, x_k^o, u, \theta_k)$ is non-negative for all triplets $(x_k^o, u, \theta_k) \in \mathbb{X}_k^o \times \mathbb{U} \times [\underline{\boldsymbol{\theta}}_k\,;\overline{\boldsymbol{\theta}}_k]$.

*Assumption 3.* Each row $A^i_{k,r}$, $i \in \{1, 2, \ldots, n\}$ of the matrix $A_{k,r} = \left[\left(A^1_{k,r}\right)^T \left(A^2_{k,r}\right)^T \cdots \left(A^n_{k,r}\right)^T\right]^T$ is such that $A^i_{k,r} \succeq 0$ or $A^i_{k,r} \preceq 0$.

*Proposition 1.* Suppose the systems $S_k$ described by (1) satisfy the Assumptions 1, 2 and 3. For all $i \in \{1, 2, \ldots, n\}$ and all $r \in \{1, 2, \ldots, p_k\}$, the sign of the function $\dfrac{\partial f_i^k}{\partial \theta_{k,r}}$ does not change on the domain $\mathbb{E}_k \times \mathbb{R}^l \times \mathbb{R}^{p_k}$.

**Proof.** As $\mathbb{E}_k \subseteq \mathbb{R}^n_+$, if $A^i_{k,r} \succeq 0$ then,

$$\frac{\partial f_i^k}{\partial \theta_{k,r}}(\xi, u, \theta) = \left\langle \left(A^i_{k,r}\right)^T, \xi \right\rangle \geq 0, \quad \forall \xi \in \mathbb{E}_k,$$

where $\langle \cdot, \cdot \rangle$ denotes the usual scalar product on $\mathbb{R}^n$. A similar reasoning applied to the case $A^i_{k,r} \preceq 0$ implies that $\forall \xi \in \mathbb{E}_k$, $\dfrac{\partial f_i^k}{\partial \theta_{k,r}}(\xi, u, \theta_k) \leq 0$.

*Lemma 1.* Consider the uncertain LTI systems $S_k$ described by (1) as follow:

$$S_k \begin{cases} \dot{x}_k(t) = \mathbb{A}_k(\theta_k)\, x_k(t) + B_k\, u(t), \\ y_k(t) = C_k\, x_k(t), \\ x_k(0) = x_k^o \in \mathbb{X}_k^o. \end{cases}$$

with

$$\mathbb{A}_k(\theta_k) = A_k + \theta_{k,1}\, A_{k,1} + \theta_{k,2}\, A_{k,2} + \cdots + \theta_{k,p_k}\, A_{k,p_k},$$

and suppose the systems $S_k$ satisfy the Assumptions 1, 2 and 3. Then, there exist two LTI systems

$$\underline{\mathbf{\Sigma}}_k \begin{cases} \dot{\underline{\boldsymbol{x}}}_k(t) = \underline{A}_k(\underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)\, \underline{\boldsymbol{x}}_k(t) + B_k\, \underline{\mathbf{u}}(t), \\ \underline{\boldsymbol{y}}_k(t) = C_k\, \underline{\boldsymbol{x}}_k(t), \\ \underline{\boldsymbol{x}}_k(0) = \underline{\mathbf{x}}_k^o, \end{cases}$$

and

$$\overline{\mathbf{\Sigma}}_k \begin{cases} \dot{\overline{\boldsymbol{x}}}_k(t) = \overline{A}_k(\underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)\, \overline{\boldsymbol{x}}_k(t) + B_k\, \overline{\mathbf{u}}(t), \\ \overline{\boldsymbol{y}}_k(t) = C_k\, \overline{\boldsymbol{x}}_k(t), \\ \overline{\boldsymbol{x}}_k(0) = \overline{\mathbf{x}}_k^o, \end{cases}$$

such that the following relation

$$\underline{\boldsymbol{y}}_k(t) \preceq y_k(t, x_k^o, u, \theta_k) \preceq \overline{\boldsymbol{y}}_k(t)$$

holds for all $x_k^o \in \mathbb{X}_k^o$, all $u \in \mathbb{U}$, all $\theta_k \in [\underline{\boldsymbol{\theta}}_k\,;\overline{\boldsymbol{\theta}}_k]$ and all $t \in [0\,;\mathcal{T}]$.

**Proof.** One can determine with the rules recalled in Section 3 two systems

$$\begin{cases} \dot{\underline{\boldsymbol{x}}}_k(t) = \underline{A}_k(\underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)\, \underline{\boldsymbol{x}}_k(t) + B_k\, \underline{\mathbf{u}}(t), \\ \underline{\boldsymbol{x}}_k(0) = \underline{\mathbf{x}}_k^o, \end{cases}$$

and

$$\begin{cases} \dot{\overline{\boldsymbol{x}}}_k(t) = \overline{A}_k(\underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)\, \overline{\boldsymbol{x}}_k(t) + B_k\, \overline{\mathbf{u}}(t), \\ \overline{\boldsymbol{x}}_k(0) = \overline{\mathbf{x}}_k^o, \end{cases}$$

such that the relation

$$\underline{\boldsymbol{x}}_k(t) \preceq x_k(t) \preceq \overline{\boldsymbol{x}}_k(t)$$

holds for all triplets $(x_k^o, u, \theta_k) \in \mathbb{X}_k^o \times \mathbb{U} \times [\underline{\boldsymbol{\theta}}_k\,;\overline{\boldsymbol{\theta}}_k]$ and all $t \in [0\,;\mathcal{T}]$. Therefore, as $C_k$ is a non-negative matrix then,

$$C_k\, \underline{\boldsymbol{x}}_k(t) \preceq C_k\, x_k(t) \preceq C_k\, \overline{\boldsymbol{x}}_k(t).$$

Note that the output vector $\underline{\boldsymbol{y}}_k(t)$ (resp. $\overline{\boldsymbol{y}}_k(t)$) of system $\underline{\mathbf{\Sigma}}_k$ (resp. $\overline{\mathbf{\Sigma}}_k$) is a function of variables $\underline{\mathbf{x}}_k^o$ (resp. $\overline{\mathbf{x}}_k^o$), $\underline{\mathbf{u}}$ (resp. $\overline{\mathbf{u}}$), $\underline{\boldsymbol{\theta}}_k$ and $\overline{\boldsymbol{\theta}}_k$ i.e $\underline{\boldsymbol{y}}_k(t) := \underline{\boldsymbol{y}}_k(t, \underline{\mathbf{x}}_k^o, \underline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)$ (resp. $\overline{\boldsymbol{y}}_k(t) := \overline{\boldsymbol{y}}_k(t, \overline{\mathbf{x}}_k^o, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)$). The output vectors $\underline{\boldsymbol{y}}_k(t)$ and $\overline{\boldsymbol{y}}_k(t)$ are respectively a lower and an upper bound of $y_k(t)$.

### 3.2 Sufficient condition for $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishability

Theorem 2 below gives a sufficient condition for $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishability.

*Theorem 2.* Consider the uncertain LTI systems $S_k$ describbed by (1). Suppose $S_k$ satisfy the Assumptions 1, 2 and 3, and consider the output signals $\underline{\boldsymbol{y}}_k(\cdot, \underline{\mathbf{x}}_k^o, \underline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)$ and $\overline{\boldsymbol{y}}_k(\cdot, \overline{\mathbf{x}}_k^o, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_k, \overline{\boldsymbol{\theta}}_k)$ of systems $\underline{\mathbf{\Sigma}}_k$ and $\overline{\mathbf{\Sigma}}_k$ obtained in Lemma 1. If there exists an interval $I \subseteq [0\,;\mathcal{T}]$ such that for all $t \in I$,

$$\overline{\boldsymbol{y}}_2(t, , \overline{\mathbf{x}}_2^o, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_2, \overline{\boldsymbol{\theta}}_2) \prec \underline{\boldsymbol{y}}_1(t, , \underline{\mathbf{x}}_1^o, \underline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_1, \overline{\boldsymbol{\theta}}_1), \tag{4}$$

or

$$\overline{\boldsymbol{y}}_1(t, \overline{\mathbf{x}}_1^o, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_1, \overline{\boldsymbol{\theta}}_1) \prec \underline{\boldsymbol{y}}_2(t, \underline{\mathbf{x}}_2^o, \underline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_2, \overline{\boldsymbol{\theta}}_2), \tag{5}$$

then, systems $S_1$ and $S_2$ are $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishable on $[0\,;\mathcal{T}]$.

**Proof.** For all $(\theta_1, \theta_2) \in \Theta$ and all $(x_1^o, x_2^o) \in \mathbb{X}$,

$$\begin{cases} y_2(t, x_2^o, u, \theta_2) \preceq \overline{\boldsymbol{y}}_2(t, \overline{\mathbf{x}}_2^o, \overline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_2, \overline{\boldsymbol{\theta}}_2), \\ \underline{\boldsymbol{y}}_1(t, \underline{\mathbf{x}}_1^o, \underline{\mathbf{u}}, \underline{\boldsymbol{\theta}}_1, \overline{\boldsymbol{\theta}}_1) \preceq y_1(t, x_1^o, u, \theta_1), \end{cases} \quad \forall t \in [0\,;\mathcal{T}]$$

Therefore, if $\forall t \in I$ relation (4) holds then

$$y_2(t, x_2^o, u, \theta_2) \prec y_1(t, x_1^o, u, \theta_1), \quad \forall t \in I.$$

Hence, $y_1(\cdot, x_1^o, u, \theta_1) \neq y_2(\cdot, x_2^o, u, \theta_2)$. If relation (5) holds on $I$, then the conclusion is the same as previously. It suffices to remark in this case that:

$$\begin{cases} y_1(t, x_1^o, u, \theta_1) \preceq \overline{\boldsymbol{y}}_1(t, \overline{\mathbf{x}}_1^o, u, \underline{\boldsymbol{\theta}}_1, \overline{\boldsymbol{\theta}}_1), \\ \underline{\boldsymbol{y}}_2(t, \underline{\mathbf{x}}_2^o, u, \underline{\boldsymbol{\theta}}_2, \overline{\boldsymbol{\theta}}_2) \preceq y_2(t, x_2^o, u, \theta_2), \end{cases} \quad \forall t \in [0\,;\mathcal{T}].$$

The sufficient condition for $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishability of Theorem 2 concerns with positive LTI systems, which constitute an important class of LTI systems (Farina and Rinaldi (2000)) and which includes in particular many physical systems (networks of reservoirs, heat exchangers and distillation columns,...). If this condition is not satisfied, one cannot draw any conclusion on the distinguishability/indistinguishability between $S_1$ and $S_2$.

### 3.3 An academic example

Figure 2 below illustrates a system $S_1$ of two tanks.

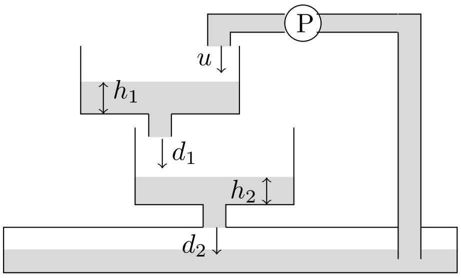


Fig. 2. System $S_1$ of two tanks

The two tanks have both the same section $\sigma = 1$. Assume that the flows $d_i$, $i = 1, 2$ of the fluid are such that $d_i = a_i\, h_i$ where $a_i = a_i^o + \alpha_i$, $\alpha_i \in [\underline{\boldsymbol{\alpha}}_i\,; \overline{\boldsymbol{\alpha}}_i]$ and $a_i^o - \underline{\alpha}_i \geq 0$. Choosing

$$x_1(t) = \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} \text{ and } y_1(t) = h_2(t).$$

Therefore, a state space representation of $S_1$ is

$$S_1 : \begin{cases} \dot{x}_1(t) = (A_1 + \alpha_1\, A_{1,1} + \alpha_2\, A_{1,2})\; x_1(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t), \\ y_1(t) = [0\ 1]\; x_1(t), \\ x_1(0) = x_1^o \in [\underline{\mathbf{x}}_1^o\,; \overline{\mathbf{x}}_1^o], \end{cases}$$

where

$$A_1 = \begin{bmatrix} -a_1^o & 0 \\ a_1^o & -a_2^o \end{bmatrix}, \quad A_{1,1} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \text{ and } A_{1,2} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}.$$

For all $(\alpha_1, \alpha_2) \in [\underline{\boldsymbol{\alpha}}_1\,; \overline{\boldsymbol{\alpha}}_1] \times [\underline{\boldsymbol{\alpha}}_2\,; \overline{\boldsymbol{\alpha}}_2]$, the matrix $\mathbb{A}_1(\alpha) = A_1 + \alpha_1\, A_{1,1} + \alpha_2\, A_{1,2}$ with $\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ is a Metzler-matrix and matrices $A_{1,i}$, $i = 1, 2$ satisfied the conditions of Assumption 3. A simple application of the computation rules of $\underline{\Sigma}_1$ and $\overline{\Sigma}_1$ gives:

$$\underline{\boldsymbol{\Sigma}}_1 : \begin{cases} \dot{\underline{\boldsymbol{x}}}_1(t) = \begin{bmatrix} -a_1^o - \overline{\boldsymbol{\alpha}}_1 & 0 \\ a_1^o + \underline{\boldsymbol{\alpha}}_1 & -a_2^o - \overline{\boldsymbol{\alpha}}_2 \end{bmatrix} \underline{\boldsymbol{x}}_1(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \underline{\mathbf{u}}(t), \\ \underline{\boldsymbol{y}}_1(t) = [0\ 1]\; \underline{\boldsymbol{x}}_1(t), \\ \underline{\boldsymbol{x}}_1(0) = \underline{\mathbf{x}}_1^o, \end{cases}$$

and

$$\overline{\boldsymbol{\Sigma}}_1 : \begin{cases} \dot{\overline{\boldsymbol{x}}}_1(t) = \begin{bmatrix} -a_1^o - \underline{\boldsymbol{\alpha}}_1 & 0 \\ a_1^o + \overline{\boldsymbol{\alpha}}_1 & -a_2^o - \underline{\boldsymbol{\alpha}}_2 \end{bmatrix} \overline{\boldsymbol{x}}_1(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \overline{\mathbf{u}}(t), \\ \overline{\boldsymbol{y}}_1(t) = [0\ 1]\; \overline{\boldsymbol{x}}(t), \\ \overline{\boldsymbol{x}}(0) = \overline{\mathbf{x}}_1^o. \end{cases}$$

In addition to system $S_1$, consider the two tanks system $S_2$ of Figure 3 below. Assume that

- the two tanks have both the same section $\tilde{\sigma} = 1$.
- the flows $\tilde{d}_i$, $i = 1, 2$ and $\tilde{d}_{12}$ are such that $\tilde{d}_i = b_i\, \tilde{h}_i$, $i = i, 2$ and $\tilde{d}_{12} = b_{12}\, \left(\tilde{h}_2 - \tilde{h}_1\right)$.
- $b_2 = b_2^o + \beta$ where $\beta \in [\underline{\boldsymbol{\beta}}\,; \overline{\boldsymbol{\beta}}]$.

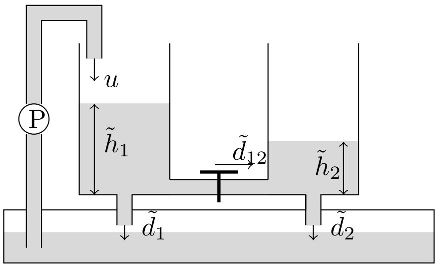


Fig. 3. System $S_2$ of two tanks

Choosing

$$x_2(t) = \begin{bmatrix} \tilde{h}_2(t) \\ \tilde{h}_1(t) \end{bmatrix} \text{ and } y_2(t) = \tilde{h}_1(t).$$

Therefore, a state space representation of $S_2$ is

$$S_2 : \begin{cases} \dot{x}_2(t) = (A_2 + \beta\, A_{2,1})\, x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \\ y_2(t) = [0\ 1]\; x_2(t), \\ x_2(0) = x_2^o \in [\underline{\mathbf{x}}_2^o\,; \overline{\mathbf{x}}_2^o], \end{cases}$$

where

$$A_2 = \begin{bmatrix} -b_2^o - b_{12} & b_{12} \\ b_{12} & -b_{12} - b_1 \end{bmatrix} \text{ and } A_{2,1} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}.$$

The systems $\underline{\boldsymbol{\Sigma}}_2$ and $\overline{\boldsymbol{\Sigma}}_2$ which bound $S_2$ are defined as follows:

$$\underline{\boldsymbol{\Sigma}}_2 : \begin{cases} \dot{\underline{\boldsymbol{x}}}_2(t) = \underline{A}_2(\overline{\boldsymbol{\beta}})\, \underline{\boldsymbol{x}}_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \underline{\mathbf{u}}(t), \\ \underline{\boldsymbol{y}}_2(t) = [0\ 1]\; \underline{\boldsymbol{x}}_2(t), \\ \underline{\boldsymbol{x}}_2(0) = \underline{\mathbf{x}}_2^o \end{cases}$$

and

$$\overline{\boldsymbol{\Sigma}}_2 : \begin{cases} \dot{\overline{\boldsymbol{x}}}_2(t) = \overline{A}_2(\underline{\boldsymbol{\beta}})\, \overline{\boldsymbol{x}}_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \overline{\mathbf{u}}(t), \\ \overline{\boldsymbol{y}}_2(t) = [0\ 1]\; \overline{\boldsymbol{x}}_2(t), \\ \overline{\boldsymbol{x}}_2(0) = \overline{\mathbf{x}}_2^o, \end{cases}$$

where

$$\underline{A}_2(\overline{\boldsymbol{\beta}}) = \begin{bmatrix} -b_2^o - b_{12} - \overline{\boldsymbol{\beta}} & b_{12} \\ b_{12} & -b_{12} - b_1 \end{bmatrix}$$

and

$$\overline{A}_2(\underline{\boldsymbol{\beta}}) = \begin{bmatrix} -b_2^o - b_{12} - \underline{\boldsymbol{\beta}} & b_{12} \\ b_{12} & -b_{12} - b_1 \end{bmatrix}$$

The numerical results of the Figure 5 are obtained in the case where

- $a_1^o = 0.1 = a_2^o$, $\underline{\boldsymbol{\alpha}}_1 = \underline{\boldsymbol{\alpha}}_2 = -0.005$, $\overline{\boldsymbol{\alpha}}_1 = \overline{\boldsymbol{\alpha}}_2 = 0.005$.
- $b_1 = b_2^o = 0.2$, $b_{21} = 0.1$, $\underline{\boldsymbol{\beta}} = -0.005$, $\overline{\boldsymbol{\beta}} = 0.005$.
- $\underline{\mathbf{x}}_1^o = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \underline{\mathbf{x}}_2^o$, $\overline{\mathbf{x}}_1^o = \begin{pmatrix} 0.02 \\ 0.02 \end{pmatrix} = \overline{\mathbf{x}}_2^o$, $\mathcal{T} = 500$.
- the space $\mathbb{U} = \mathbb{V}(\underline{\mathbf{u}}, \overline{\mathbf{u}})$ of admissible inputs of the systems $S_1$ and $S_2$ is defined by the piecewise constant signals $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$ of the Figure 4 below.

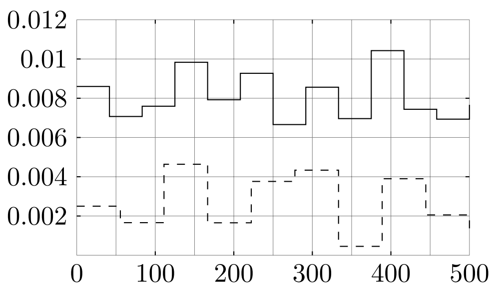


Fig. 4. Space $\mathbb{U}$ of admissible inputs of systems $S_1$ and $S_2$

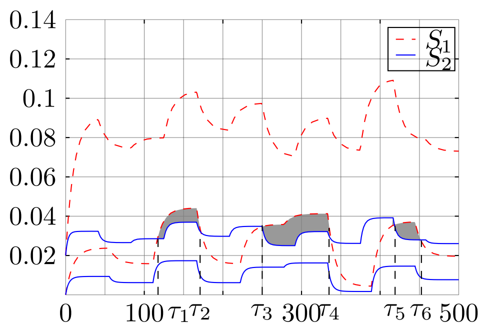


Fig. 5. An estimate of the reachable output spaces of $S_1$ and $S_2$

The dotted (resp. solid) curves of Figure 5 represent the lower bound, $\underline{\boldsymbol{y}}_1$ (resp. $\underline{\boldsymbol{y}}_2$) and the upper bound, $\overline{\boldsymbol{y}}_1$ (resp. $\overline{\boldsymbol{y}}_2$) of the output $y_1$ (resp. $y_2$). For $I = [\tau_1\,;\tau_2]$ or $I = [\tau_3\,;\tau_4]$ or $I = [\tau_4\,;\tau_5]$ with $\tau_1 = 121.4$, $\tau_2 = 172.2$, $\tau_3 = 249.9$, $\tau_4 = 336.2$, $\tau_5 = 418.5$ and $\tau_6 = 447.2$, the condition (4) of Theorem 2 is satisfied by outputs signals $\overline{\boldsymbol{y}}_1$ and $\underline{\boldsymbol{y}}_2$ of systems $\overline{\boldsymbol{\Sigma}}_1$ and $\underline{\boldsymbol{\Sigma}}_2$ respectively. Systems $S_1$ and $S_2$ are $(\mathbb{X}^o, \mathbb{U}, \Theta)$-distinguishable on $[0\,;500]$ with:

- $\mathbb{X}^o = \left[\begin{pmatrix}0\\0\end{pmatrix}\,;\begin{pmatrix}0.02\\0.02\end{pmatrix}\right] \times \left[\begin{pmatrix}0\\0\end{pmatrix}\,;\begin{pmatrix}0.02\\0.02\end{pmatrix}\right]$.
- $\mathbb{U}$ is defined by the Figure 4 and $\Theta = [-0.005\,;0.005]^3$.

## 4. CONCLUSIONS AND OPEN PROBLEMS

Distinguishability is a property that allows one to determine the active mode of a switched system by only observing its input-output data. In the literature, most of the works dealing with the subject only deal with LTI systems without uncertainties. This paper proposes to deal with linear systems with affine uncertainties in the state space model. In this new context, an adapted definition of distinguishability is given. Two systems can be distinguished when their reachable output spaces are disjoint. When the considered systems are positive, extension of the result in Kieffer and Walter (2006) and Meslem (2008) allows to bound all their admissible outputs by the outputs of two specific LTI systems. An estimate of the reachable output space of these systems in response to an input belonging to an uncertain range can then be determined. A sufficient condition for distinguishability is derived from this estimation.

As the sufficient condition that we have obtained strongly depends on the estimation of the reachable output spaces, a further work will aim to find a method to obtain a better estimate of the reachable output space of the considered systems. On the other hand, we have shown in Motchon et al. (2013) that, when uncertainties are ignored in the systems' models, the conditions of distinguishability can be deduced from some of intrinsic properties of the models. In the same way, for the class of linear systems with affine uncertainties, it would be interesting to determine the intrinsic properties which can ensure the distinguishability of two systems.

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