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Adaptive Finite-time Observer for a Nonlinear and Flexible Space Launch Vehicle

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Abstract: In this paper we propose a new finite-time state observer for a nonlinear space launch vehicle with flexible dynamics and uncertain parameters. Indeed, flexible states are required to ensure nonlinear control objectives of both reference path tracking and bending mode damping. Our main contribution is to show that it is enough to observe a sublinear uncertain system to ensure a finite-time convergence of the whole state. For that purpose, a Luenberger observer is mixed with a parameter and initial state estimator, based on algebraic estimation tools. Closed loop simulations show the effectiveness of the observer in combination with a backstepping control design (extended to flexible systems).

Keywords: Hybrid observer, Nonlinear control, Parameter estimation, Flexible mode

NOMENCLATURE

ψ	Attitude angle, rad
β	Gimbal deflection angle, rad
η	Mode shape temporal coordinate
q	Pitch rate, rad/s
h	Flexible displacement, m
r	Flexible rotation, rad
r_{ci}	Inertial unit flexible rotation, rad
r_{gy}	Rategyro flexible rotation, rad
T	Thrust, kg.m/s^2
L	Lift, kg.m/s^2
G_L	Launcher center of mass
C_T	Gimbal joint
F_L	Aerodynamic center
I_L	Launcher body inertia, kg.m^2
L_T	Algebraic distance from G_L to C_T, m
l_{aero}	Algebraic distance from F_L to G_L, m
$\bar{q}$	Dynamic pressure, Pa
S	Reference area of the vehicle, m^2
ω	Natural frequency of the first bending mode, rad/s
ξ	Natural damping of the first bending mode
$\mathcal{R}_n$	Frame (G_L, x_n, y_n) linked to the reference trajectory
$\mathcal{R}_L$	Frame (G_L, x_L, y_L) linked to the launcher

1. INTRODUCTION

The problem of finite-time observation for linear and nonlinear systems has been widely investigated over the past decades. Two classes of observers emerge in the series of methods that achieve finite-time convergence. The first one, based on the use of delays has deserved a lot of attention [Menold (2003)],[Engel (2002)]. Recently in [Karafyllis (2011)], a novel hybrid dead-beat observer which uses delays has been proposed. The history of the output is used in order to estimate the state of the system. Sliding mode observers that contain large study in the literature make the second class. (see [Shtessel (2010b)], [Ahmed-Ali (1999)] for instance). More recently, homogeneous finite-time observers have been developed for a specific class of nonlinear systems [Perruquetti (2008)]. Most of these approaches make the assumption that the system structure and parameters are known.

In this paper we propose to design a finite-time observer for a space launch vehicle which belongs to the class of uncertain nonlinear systems. Due to mass constraints, space vehicles tend to have lightweight and flexible structures with low natural frequencies, distorting sensors measurement and adding stability problems during flight. Researchers have recently investigated this subject in the field of nonlinear control. Several solutions have been proposed. Some of them, on the one hand, addressed the problem of unknown parameters and uncertainties using direct-adaptive [Fiorentini (2009)], or time varying controller [Hervas (2012)]. Nevertheless, only rigid states, (that means measured states), are used in these proposed methods. On the other hand, using sliding mode state observer, the authors of [Shtessel (2010a)] reconstruct the flexible states in order to remove the undesirable dynamics from the measurements. This approach requires unfortunately a strong knowledge of the mathematical model of the system, in particular flexible modes parameters. However, to the best of our knowledge, the design of a finite-time observer has not been achieved on uncertain nonlinear aerospace models.

As far as we are concerned, we recently designed a Lyapunov-based nonlinear controller, which uses the flexible states, to ensure control objectives of both reference path tracking and bending mode damping for a class of nonlinear and flexible system [Duraffourg (2013a)], [Duraffourg (2013b)], [Burlion (2013)]. Assuming that the whole state is available, this control law has been applied to the rotational dynamics of a space launch vehicle. [Duraffourg (2013c)]. Such assumptions do not hold in practical applications since flexible states are generally not measured. Consequently we need to reconstruct the flexible states. Besides, noting that flexible parameters are subject to uncertainties or variation during flight, this paper proposes to extend existing theory by proposing an indirect adaptive hybrid observer that no longer requires system parameter knowledge. The proposed approach consists in estimating flexible parameters (natural frequency and damping), and state initial conditions using algebraic tools [Fliess (2003)]. The first ones improve the accuracy and the robustness of the observer through indirect adaptive feature. The second ones are used to regularly update the estimated state and so guarantee a finite-time convergence.

This paper is organised as follows. Section 2 describes the space launch vehicle and gives the problem statement. Section 3 develops a parameter estimator and a state observer which are then mixed to design a hybrid adaptive finite-time observer. In section 4, estimated state is used in a nonlinear control law and a closed-loop simulation is presented. Finally Section 5 contains our conclusions and future research directions.

2. PROBLEM STATEMENT

2.1 Launcher mathematical model

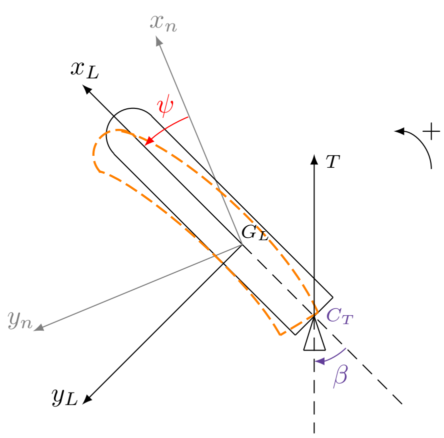

Fig. 1. Schematic model for a flexible launcher

The rotational dynamics of a nonlinear flexible launch vehicle is extracted from [Duraffourg (2013c)], where Lagrange's formalism was developed to get the full-mathematical model. The equations of motion are given by:

$$\begin{cases} \dot{\psi} = q \\ \dot{q} = -\dfrac{l_{\text{aero}}}{I_L} L(\psi) + \dfrac{T}{I_L}(L_T r - h)\eta + \dfrac{T L_T}{I_L}\beta \\ \ddot{\eta} = -(\omega^2 - hrT)\eta - 2\xi\omega\dot{\eta} + hT\beta \end{cases} \tag{1}$$

where ψ and η are real variables and the lift L is a nonlinear function of the attitude, given by:

$$L(\psi) = \bar{q}S\big(C_L^1\psi - C_L^2\psi^2\big)$$

The launch vehicle is equipped with an inertial unit and a rategyro that give attitude and pitch rate informations. As the first bending mode distorts the sensors measurement, the available outputs are:

$$\begin{cases} y_1 = \psi + r_{\text{ci}}\eta \\ y_2 = q + r_{\text{gy}}\dot{\eta} \end{cases} \tag{2}$$

Measurement corruption terms make the control law design more difficult.

2.2 Nonlinear control law

A nonlinear control law, denoted Flexible Backstepping, that achieves the control objectives of both reference path tracking and bending mode damping has been developed in [Duraffourg (2013c)]. This controller limits the interaction of the rigid-dynamics on the transient of the flexible dynamics, and so, improves the damping of the bending mode. However, it is based on the deep knowledge of the flexible states and parameters.

Using the notation $y_3 = y_2 + (r_{\text{ci}} - r_{\text{gy}})\dot{\eta}$, system (1) can be reformulated as follow:

$$\begin{cases} \dot{y}_1 = y_3 \\ \dot{y}_3 = \bar{g}(y_1, N)y_1 + KN + \dfrac{hT}{C_\beta}\beta + O(\eta^2) \\ \ddot{\eta} = -\bar{\omega}^2\eta - 2\xi\omega\dot{\eta} + hT\beta \end{cases} \tag{3}$$

where:

$$\bar{\omega}^2 = \omega^2 - hrT \quad C_\beta = \frac{I_L h}{L_T + I_L r_{\text{ci}} h}$$

$$\bar{g}(y_1, N) = -\frac{l_{\text{aero}}\bar{q}S}{I_L}\Big(C_L^1 - C_L^2 y_1 + 2C_L^2 r_{\text{ci}}\eta\Big)$$

$$K = \left(\frac{l_{\text{aero}}\bar{q}SC_L^1 r_{\text{ci}}}{I_L} + \tfrac{T}{I_L}(L_T r - h) - r_{\text{ci}}\bar{\omega}^2 \quad -2\xi\omega r_{\text{ci}}\right)$$

$$N = (\eta \ \dot{\eta})^T$$

Then, the following change of coordinates is applied to the flexible-dynamics:

$$z = N - C_\beta \begin{pmatrix} y_1 \\ y_3 \end{pmatrix} \tag{4}$$

System (3) becomes:

$$\begin{cases} \dot{y}_1 = y_3 \\ \dot{y}_3 = \bar{g}(y_1, N)y_1 + KN + \dfrac{hT}{C_\beta}\beta + O(\eta^2) \\ \dot{z} = A_z z + \bar{F}(y_1, N)y_1 + Gy_3 + O(\eta^2) \end{cases} \tag{5}$$

with:

$$A_z = \begin{pmatrix} 0 & 1 \\ -\bar{\omega}^2 & -2\xi\omega \end{pmatrix} - \begin{pmatrix} 0 \\ C_\beta \end{pmatrix} K$$

$$G = A_z \begin{pmatrix} 0 \\ C_\beta \end{pmatrix} - \begin{pmatrix} C_\beta \\ 0 \end{pmatrix}$$

$$\bar{F}(y_1, N) = A_z \begin{pmatrix} C_\beta \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ C_\beta \end{pmatrix} \bar{g}(y_1, N)$$

A_z being Hurwitz, system (5) belongs to the nonlinear class of system on which a flexible backstepping controller can be developed (see [Duraffourg (2013a)], [Duraffourg (2013b)]). The design of this flexible backstepping control law requires the whole state. Hence, outputs (y_1 and y_2) and the flexible states (η and $\dot{\eta}$) are required.

Remark 1. The exact expression of the y_3-dynamics is:

$$\dot{y}_3 = \bar{g}(y_1, \eta)y_1 + KN + \frac{hT}{C_\beta}\beta + \frac{l_{\text{aero}}\bar{q}SC_L^2 r_{\text{ci}}^2}{I_L}\eta^2$$

Inertial flexible rotation r_{ci} takes very low values (in the order of 10^{-4}). Thus the last term can be seen as an error term and is represented by a $O(\eta^2)$.

3. PARAMETERS AND STATE ESTIMATION

Since the bending mode is not measured and the flexible parameters are generally distorted, this section proposes a way to estimate the flexible states and parameters and so to make the use of this control law possible via output feedback.

3.1 State observer design with known parameter (ideal case)

The flexible states η and $\dot{\eta}$ must be estimated. They are described by linear differential equations that result from system (1). Since the outputs (2) involve both the rigid states and the flexible ones, the idea is to consider the augmented state $X = [\psi\ \eta\ \dot{\eta}]^T$ instead of just the required flexible states. **It is important to note that this system is linear, contrary to the original nonlinear one (1)**.

Working with this augmented state gives the possibility to design a linear observer, by focusing on:

$$\begin{cases} \dot{X} = AX + B_y y_2 + B_\beta \beta \\ y_1 = CX \end{cases} \tag{6}$$

where y_2 acts as an input and:

$$A = \begin{pmatrix} 0 & 0 & -r_{\text{gy}} \\ 0 & 0 & 1 \\ 0 & -\bar{\omega}^2 & -2\xi\omega \end{pmatrix} \quad B_y = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad B_\beta = \begin{pmatrix} 0 \\ 0 \\ hT \end{pmatrix}$$
$$C = (1\ r_{\text{ci}}\ 0)$$

Observability conditions holds for the pair $(A,\ C)$ and the estimated state $\hat{X} = [\hat{\psi}\ \hat{\eta}\ \hat{\dot{\eta}}]^T$ is given by the classical Luenberger observer:

$$\dot{\hat{X}} = A\hat{X} + B_y y_2 + B_\beta \beta + L(y_1 - C\hat{X}) \tag{7}$$

where $L \in \mathbb{R}^{3\times 1}$ is chosen such that $A - LC$ is stable.

3.2 Parameters estimation

Bending mode natural damping and pulsation are generally subject to uncertainties and variations during the flight. Besides, since our flight control law intends to attenuate the oscillations of the bending mode, that is, to add damping, it is important to know these parameters accurately.

Moreover the accuracy and convergence time of the state observer can be improved by a better knowledge of the state initial conditions.

In this way we choose to estimate the following parameters:

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} -2\xi\omega \\ -\bar{\omega}^2 \\ y_3(0) \\ \dot{y}_3(0) - \dot{y}_2(0) + 2\xi\omega y_3(0) \end{pmatrix} \tag{8}$$

The first two parameters (θ_1 and θ_2) give the natural damping and pulsation of the bending mode. The flexible state initial conditions depend on the four parameters θ.

$$\eta(0) = \frac{1}{\theta_2}\left(\frac{\theta_4 + \theta_1 y_2(0)}{r_{\text{ci}} - r_{\text{gy}}} - hT\beta(0)\right)$$
$$\dot{\eta}(0) = \frac{\theta_3 - y_2(0)}{r_{\text{ci}} - r_{\text{gy}}}$$

r_{ci} being small, we use the first initial output to approximate the state initial condition $\psi(0)$:

$$\psi(0) = y_1(0) - r_{\text{ci}}\eta(0) \simeq y_1(0) \tag{9}$$

The flexible state initial conditions must be estimated through the outputs and their time derivatives. Noting that $\dot{\eta} = \dfrac{\dot{y}_1 - y_2}{r_{\text{ci}} - r_{\text{gy}}}$, it comes the following equation that links the outputs and their time derivatives to the input derivative, and consists in the basic equation of the parameter estimation:

$$y_1^{(3)} = \theta_2(\dot{y}_1 - y_2) + \theta_1(\ddot{y}_1 - \dot{y}_2) + hT(r_{\text{ci}} - r_{\text{gy}})\dot{\beta} + \ddot{y}_2 \tag{10}$$

An algebraic methodology for parameter identification is described in [Fliess (2003)] . This approach was used here to estimate θ. The main steps are presented here:

(1) Take the Laplace transformation of (10) to reveal the four parameters to be estimated. It comes:

$$\begin{aligned} \theta_1\big[-s^2Y_1 + s\big(Y_2 + y_1(0)s\big) - y_2(0)\big] + \theta_2\big[-sY_1 + Y_2 + y_1(0)\big] \\ - s\theta_3 - \theta_4 = hT(r_{\text{ci}} - r_{\text{gy}})\big[sB - \beta(0)\big] \\ - s^3Y_1 + s^2\big[Y_2 + y_1(0)\big] - sy_2(0) \end{aligned} \tag{11}$$

where s represents the Laplace variable and Y_i (resp. B) is the Laplace transformation of signal y_i ($i \in \{1, 2\}$) (resp. β).

(2) Take derivatives with respect to s, (three times) to get as equations as unknown parameters.
(3) Multiply by s^{-3} both sides to avoid time derivations.
(4) Come back to time domain using inverse transformations.
(5) Define time dependent matrices $P \in \mathbb{R}^{4\times 4}$ and $Q \in \mathbb{R}^{4\times 1}$ such that:

$$P(t)\theta = Q(t) \tag{12}$$

Assumption 2. Persistence of excitation condition: $\exists\, \tau > 0$ such that:

$$\forall\ u \geq \tau, \quad \int_{u-\tau}^{\tau} P(s)^T P(s) ds > 0 \tag{13}$$

Under Assumption 2, θ is finally given by:

$$\theta = \frac{\int_{u-\tau}^{\tau} P(s)^T Q(s) ds}{\int_{u-\tau}^{\tau} P(s)^T P(s) ds} \tag{14}$$

Since τ is unknown, we compute estimated parameters $\hat{\theta}$ and then $\hat{A}$ and $\hat{X}_0$ at discrete time instants τ_k where $\{\tau_k\}_{k=0}^{+\infty}$ is a partition of $\mathbb{R}^+$, using:

$$\hat{\theta}(\tau_k) = \frac{\int_0^{\tau_k} P(s)^T Q(s) ds}{\int_0^{\tau_k} P(s)^T P(s) ds} \tag{15}$$

$$\hat{A}(\tau_k) = \hat{A}_k = \begin{pmatrix} 0 & 0 & -r_{\mathrm{gy}} \\ 0 & 0 & 1 \\ 0 & \hat{\theta}_2(\tau_k) & \hat{\theta}_1(\tau_k) \end{pmatrix} \tag{16}$$

$$\hat{X}_0(\tau_k) = \begin{pmatrix} y_1(0) \\ \frac{1}{\hat{\theta}_2(\tau_k)}\left(\frac{\hat{\theta}_4(\tau_k) + \hat{\theta}_1(\tau_k) y_2(0)}{r_{\mathrm{ci}} - r_{\mathrm{gy}}} - hT\beta(0)\right) \\ \frac{\hat{\theta}_3(\tau_k) - y_2(0)}{r_{\mathrm{ci}} - r_{\mathrm{gy}}} \end{pmatrix} \tag{17}$$

Remark 3. It is important to note that $\hat{A}$ and $\hat{X}_0$ are not completely described by estimated parameters. Parameters r_{gy}, r_{ci}, h and T are supposed to be known. This may be assumed for flexible displacements and rotations since they take low values, and cannot vary significantly during the flight.

Figure 2 shows in blue (resp. in red) the evolution of the flexible estimated parameters ($\hat{\theta}_1$, $\hat{\theta}_2$) and state initial conditions $\hat{X}_0 = \left(\hat{\psi}(0), \hat{\eta}(0), \hat{\dot{\eta}}(0)\right)$ when the sensors give ideal (resp. noisy) measurements. The red curve was obtained applying a zero-mean periodic noise on signals y_1 and y_2.

Remark 4. With noisy measurements, estimated parameter $\hat{\dot{\eta}}(0)$ is biased and the estimation error $\tilde{\dot{\eta}} = \dot{\eta}(0) - \hat{\dot{\eta}}(0)$ is quite important (about 1.5). This problem is detailed in section 3.3.

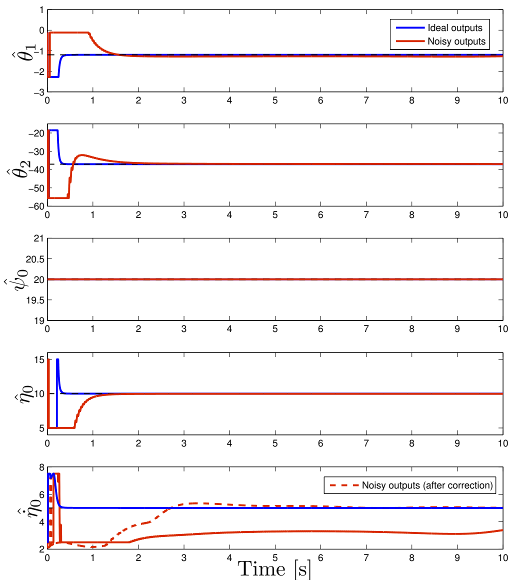

Fig. 2. Estimated parameters

3.3 Robustness improvement of the observer with respect to noisy measurements

An estimation bias appears in simulation on estimated parameter $\hat{\dot{\eta}}(0)$ when measurements are noisy (solid red curve on the last plot of figure 1). It is given by:

$$\tilde{\dot{\eta}}(0) = \dot{\eta}(0) - \hat{\dot{\eta}}(0) = \frac{\theta_3 - \hat{\theta}_3}{r_{\mathrm{ci}} - r_{\mathrm{gy}}} \tag{18}$$

This bias term is due to the low values of $r_{\mathrm{ci}} - r_{\mathrm{gy}}$ which accentuates the (low) difference between θ_3 and $\hat{\theta}_3$. This section proposes a way to identify this estimation bias $\tilde{\dot{\eta}}(0)$ and thus to correct $\hat{X}_0(\tau_k)$.

From system (6), it comes:

$$y_1(t) = CX(t) = Ce^{At}X_0(t) + C\int_0^t e^{A(t-u)}\Big[B_y y_2(u) + B_\beta \beta(u)\Big]du \tag{19}$$

Similarly with the estimated state:

$$\hat{X}(t) = e^{\hat{A}_k t}\hat{X}_0(t) + \int_0^t e^{\hat{A}_k(t-u)}\Big[B_y y_2(u) + B_\beta \beta(u)\Big]du \tag{20}$$

Under persistence of excitation condition, algebraic parameter estimation still converges in finite-time. Thus, there exists $k^* > 0$ such that for all $k \geq k^*$

$$\hat{A}_k = A \text{ and } \hat{X}_0 = \left(\psi(0)\ \eta(0)\ \dot{\eta}(0) - \tilde{\dot{\eta}}(0)\right)^T \tag{21}$$

This last equation underlines the fact that only estimated initial condition $\dot{\eta}$ is biased.

Then,

$$\forall t > t_{k^*} \quad y_1(t) - C\hat{X}(t) = Ce^{At}e_3\,\tilde{\dot{\eta}}(0) \tag{22}$$

with $e_3 = (0\ 0\ 1)^T$. That can be written as:

$$\int_0^t (Ce^{Au}e_3)^2 du\ \tilde{\dot{\eta}}(0) = \int_0^t Ce^{Au}e_3\Big[y_1(u) - C\hat{X}(u)\Big]du \tag{23}$$

Supposing that $\int_0^t (Ce^{Au}e_3)^2 du \neq 0$, estimation error is given by:

$$\tilde{\dot{\eta}}(0) = \frac{\int_0^t Ce^{Au}e_3\Big[y_1(u) - C\hat{X}(u)\Big]du}{\int_0^t (Ce^{Au}e_3)^2 du} \tag{24}$$

This term is added on the last simulation. The result is represented by the dashed red line on figure 1.

3.4 Adaptive Finite-time observer

Mixing the results of the last subsections, it is now possible to design an observer when the parameters are unknown.

Estimated parameters and initial conditions are used to improve the accuracy of the observer. In particular, $\hat{\theta}_1$ and $\hat{\theta}_2$ are used in the design such that the observer no longer depends on the natural damping and pulsation of the bending mode, that are subject to variations.

Proposition 5. Under Assumption 2, the following hybrid observer converges in finite-time.

$$\begin{cases} \forall t \in [\tau_k, \tau_{k+1}[\\ \quad \dot{\hat{X}} \;= \hat{A}_k\hat{X} + B_y y_2 + B_\beta\beta + L(y_1 - C\hat{X}) \\ \text{when } t = \tau_k \\ \hat{X}(\tau_k) \;= e^{\hat{A}_k\tau_k}\hat{X}_0(\tau_k) \\ \qquad\quad + \int_0^{\tau_k} e^{\hat{A}_k(\tau_k - u)}\Big[B_y y_2(u) + B_\beta\beta(u)\Big]du \end{cases} \tag{25}$$

At each τ_k, the state is updated and the dynamics ($\hat{A}_k$) as well.

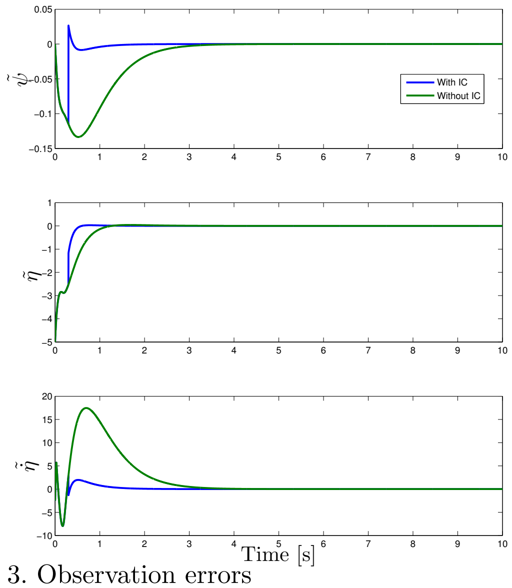

Fig. 3. Observation errors

Proof. Using the notations $\tilde{X} = X - \hat{X}$ and $\tilde{A} = A - \hat{A}$, it comes:

$$\dot{\tilde{X}} = AX - \hat{A}\hat{X} - LC\tilde{X} = (A - LC)\tilde{X} - \tilde{A}\hat{X} \tag{26}$$

Because of the persistence of excitation condition, algebraic parameter estimation converges in finite-time. Thus, there exists k^* such that $\tau_{k^*} > \tau > 0$ and

$$\forall\ k > k^*, \quad \hat{\theta}(\tau_k) = \theta \tag{27}$$

Consequently $\forall\ k > k^*$, $\hat{A}_k = A$ and the estimation error satisfies:

$$\forall\ t \geq \tau_k^*, \quad \dot{\tilde{X}} = (A - LC)\tilde{X} \tag{28}$$

$A - LC$ being Hurwitz, the estimation error $\tilde{X}$ converges asymptotically to zero.

Besides, the estimated state $\hat{X}(t)$ is updated on $\hat{X}(\tau_k)$ at each τ_k verifying $\tau_k > \tau_{k^*}$.

$$\forall t \geq \tau_k^*, \quad \hat{X}_0(t) = X_0(t) \quad \text{ie} \quad \hat{X}(\tau_k) = X(\tau_k) \tag{29}$$

Estimation error is then given by:

$$\forall t \geq \tau_k^*, \quad \tilde{X}(t) = \tilde{X}(\tau_k)e^{(A-LC)(t-\tau_k)} = 0 \tag{30}$$

Finally the estimation error vanishes in finite-time. □

Figure 3 compares this finite-time observer that updates the state from the knowledge of the initial condition (IC) (in blue) with the same Luenberger observer without any update of the estimated state (in green). In the two cases, the observer parameter L is the same. Moreover, initial conditions of the classical Luenberger observer have been chosen very close to the estimated parameters, so that the comparison is fair. This figure shows how the observer convergence time is improved. The dramatic slope on the blue curve corresponds to the moment where the estimated state is updated.

4. CLOSED-LOOP SIMULATION

Finite-time observer was designed in open-loop. In this section, the loop is closed and estimated flexible states and parameters are directly used in the controller.

As explained in Section 2.2, a flexible backstepping control law can be designed for system (5). It is extracted from [Duraffourg (2013c)], [Duraffourg (2013b)] and involved the following Lyapunov function:

$$V = \frac{c_1^{\mathrm{f}}}{2}y_1^2 + \frac{c_2^{\mathrm{f}}}{2}Z^T P_z Z + \frac{c_3^{\mathrm{f}}}{2}\tilde{y}_3^2 \tag{31}$$

where c_i^{f} are positive constants ($i \in \{1,2,3\}$), $P_z \in \mathbb{R}^{2\times 2}$ is the positive and symmetric matrix that verifies:

$$A_z^T P_z + P_z A_z = -2Q_z \tag{32}$$

with Q_z a positive and symmetric matrix of $\mathbb{R}^{2\times 2}$, and

$$A_z = \begin{pmatrix} 0 & 1 \\ \theta_2 & \theta_1 \end{pmatrix} - \begin{pmatrix} 0 \\ C_\beta \end{pmatrix} K$$

$$K = \left(\frac{l_{\text{aero}}\bar{q}SC_L^1 r_{\text{ci}}}{I_L} + \frac{T}{I_L}(L_T r - h) + r_{\text{ci}}\theta_2 \quad r_{\text{ci}}\theta_1 \right)$$

$$Z = z - Gy_1, \quad G = A_z \begin{pmatrix} 0 \\ C_\beta \end{pmatrix} - \begin{pmatrix} C_\beta \\ 0 \end{pmatrix}$$

$$\bar{F}(y_1, N) = A_z \begin{pmatrix} C_\beta \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ C_\beta \end{pmatrix} \bar{g}(y_1, N) \quad \tilde{y}_3 = y_3 - y_{3_{\text{cmd}}}$$

$$y_{3_{\text{cmd}}} = -\frac{1}{c_1^{\mathrm{f}}}\left(\lambda_{y_1}y_1 + c_2^{\mathrm{f}} F(y_1, N)^T P_z Z\right)$$

$$F(y_1, N) = A_z G + \bar{F}(y_1, N)$$

Choosing the control law as follow:

$$\beta^{\mathrm{f}}(y_1, y_3, N) = \frac{C_\beta}{hT}\Big[\dot{y}_{3_{\text{cmd}}} - \bar{g}(y_1, N)y_1 - KN$$
$$-\frac{1}{c_3^{\mathrm{f}}}(c_1^{\mathrm{f}}y_1 + \lambda_{y_3}\tilde{y}_3)\Big] \quad \lambda_{y_1} > 0 \quad \lambda_{y_3} > 0 \tag{33}$$

Lyapunov function time derivative is given by:

$$\dot{V} = -\lambda_{y_1}y_1^2 - c_2^f Z^T Q_z Z - \lambda_{y_3}\tilde{y}_3^2 \tag{34}$$

This nominal control law is then blended with the adaptive observer, using estimated flexible states $\hat{N} = (\hat{\eta}\ \hat{\dot{\eta}})$. The tested control law is thus $\beta^{\mathrm{f}}(y_1, y_3, \hat{N})$, reminding that flexible states converge in finite time.

This output-feedback flexible backstepping control law is compared with a classical backstepping one, applied on the sole rigid dynamics:

$$\beta^{\mathrm{c}} = \frac{I_l}{TL_T}\Big[\dot{q}_{\text{cmd}} + \frac{L_{\text{aero}}}{I_L}L(\psi) - \frac{1}{c_2^{\mathrm{c}}}(c_1^{\mathrm{c}}\psi - \lambda_q\tilde{q})\Big] \tag{35}$$

where

$$\tilde{q} = q - q_{\text{cmd}} \quad q_{\text{cmd}} = -\lambda_\psi \psi \tag{36}$$

and c_1^{c}, c_2^{c}, λ_ψ, λ_q are positive constants. The classical backstepping law also consists of an output-feedback controller, assuming that state ψ (resp. q) approximately corresponds to output y_1 (resp. y_2). This approximation makes sense thanks to the low values of the sensors flexible rotations r_{ci} and r_{gy} in equation (2). Simulation is realised in this way.

It is worth noting that both flexible (33) and classical (35) backstepping laws converge asymptotically when the flexible mode is collocated.

Figure 4 presents a simulation result of the state temporal evolution with the two different backstepping laws. Gains have been tuned so that the time delay response is the same. Initial conditions are also the same. Regardless of the control law, attitude ψ has nearly the same behaviour. As expected, the difference appears on the flexible states: the bending mode oscillations are better damped by the flexible backstepping controller.

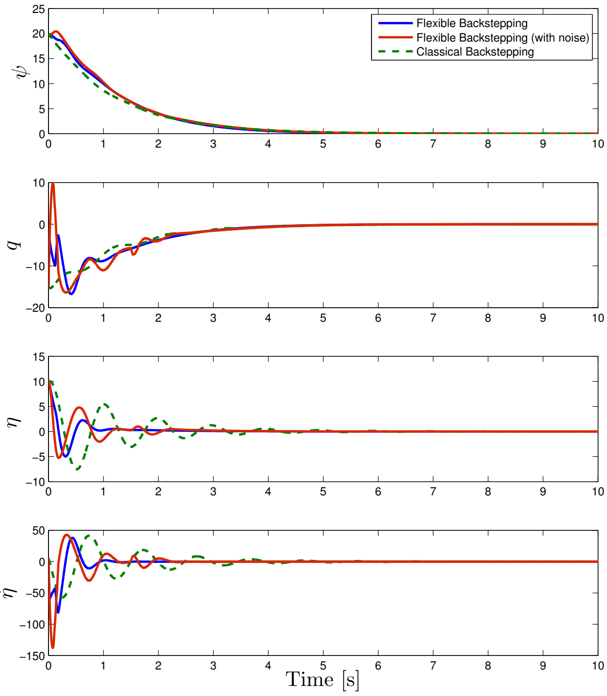

Fig. 4. Evolution of attitude, pitch rate and flexible states

However it is worth noting that peaks appear at the beginning of the simulation (on the blue curve and particularly on q and $\dot{\eta}$). This phenomenon is caused by state initial condition estimation and estimated state update. Indeed, at time equals to 0.2 s state initial conditions are obtained and estimated state is thus updated at that time. Once this process has been done, states get a smooth behaviour. The complete proof of the stability of the closed-loop will be addressed in the future.

Finally, although we have not given a formal proof, simulations have been performed with noisy measurements. More precisely, a zero-mean periodic noise has been applied on the outputs. In this case, as illustrated in red on figure 4 for the closed loop system, our finite-time convergent estimation algorithm well performs.

5. CONCLUSION

An adaptive finite-time observer has been designed in this paper. Based on a linear state observer and a parameter estimator, it identifies the unmeasured flexible states, without requiring parameter knowledge. Flexible states are then used to realise an output-feedback controller via flexible backstepping control law. Methodology and simulation results have been developed on the rotational dynamics of a space launch vehicle with only one flexible mode. Although this first flexible mode is the most relevant to consider in the design, more flexible modes will be considered in the future. In this study, flexible parameters (natural pulsation and damping) are estimated and then used in the control law. Uncertainties also affect other parameters such as aerodynamic coefficients. Theses uncertainties will be considered in our future work. Finally we will focus on the stability proof of the closed-loop system.

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