Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

On MINLP Heuristics for Solving Shale-well Scheduling Problems ⋆

Brage R. Knudsen * **Shaurya Sharma** * **Bjarne Foss** *

* *Department of Engineering Cybernetics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway.*

Abstract: This paper describes mixed integer nonlinear programming (MINLP) heuristics for solving dynamic scheduling problems in complex petroleum production systems with a network topology. We modify the Feasibility Pump heuristic for convex MINLPs [Bonami and Gonçalves, 2010] by formulating a multiobjective problem, in which we aim at balancing the two goals of quickly obtaining a feasible solution and preserving solution quality with respect to the objective value. We further present a simple linearization-based heuristic, only aimed at quickly generating feasible solutions. The MINLP heuristics are applied to a dynamic multi-pipeline shale well and compressor scheduling problem, targeted on application in decision-support tools for improving operations in large shale-gas systems. Developing efficient and robust heuristics are important for the applicability of these tools, in the sense that low computation times are often more important than global optima. A computational study shows that the proposed objective-oriented Feasibility Pump is competitive both in terms of solution quality and computation time compared to other heuristics and the branch-and-bound method.

Keywords: Heuristics, Scheduling, Mixed-integer programming, Shale-gas production.

1. INTRODUCTION

Heuristic methods are widely used in the upstream petroleum industry to improve the operational performance and to increase the economic value of the assets. The use of heuristics in this context can be divided into two groups: the first is practical empirical-based heuristics, including intuition and rule-based decisions, case studies and trial-error based analysis [Vasantharajan et al., 2006]. The second group consists of heuristic optimization techniques used to find approximate solutions to computationally demanding problems for use in model-based decision-support tools (DSTs) and knowledge-based (expert) systems. The latter category of optimization heuristics are used for solving a wide range of petroleum related optimization problems, including field development and well-placement, model-fitting for long-term reservoir planning, scheduling of well maintenance, optimization of gas-lift allocation, and scheduling and routing of well flows. Common for several of these applications is that the decisions often are made by groups of operators and engineers with time constraints in the decision process, and hence limited acceptance of waiting on termination of exact (global) optimization algorithms. Computing good feasible solutions in a short timeframe is therefore important for companies' and operators' acceptance and integration of DSTs in the workflow.

The focus in this paper is on optimization problems for scheduling and routing of well flows modeled by MINLPs. The complexity of these MINLPs depend on the size of the system considered, whether dynamics of parts or the entire system is included, the length of the planning horizon and possibly model uncertainty. For some systems it may be possible to exploit structure or sparsity, and hence solve the scheduling problem to global optimality by an exact algorithm. However, for these systems as well, it may be desirable to run a heuristic in parallel to quickly generate a feasible solution in case the global algorithm fails to solve the problem within the available timeframe.

In this paper, we extend and modify the Feasibility Pump (FP) heuristic [Fischetti et al., 2005, Bonami and Gonçalves, 2010]. This heuristic is different from *meta-heuristics* typically used in DSTs such as Genetic algorithms, Tabu Search and Simulated Annealing, which are algorithms typically developed through experimental learning, and aimed at combining a robust search of the solution space and local search strategies for preventing the algorithm from being trapped in local optima [Glover and Kochenberger, 2003]. These algorithms normally make no assumption or requirements on the underlying models. In contrast, the FP is a pure mathematical programming heuristic based on iterating between solving a continuous and a mixed-integer relaxation of the original problem, hence requiring explicit access and knowledge of the model. The FP is used both as a standalone heuristic, as well as integrated in different versions in various MI(N)LP codes to aid the solvers in quickly obtaining feasible solutions. Although the FP has been shown to be a very good and robust heuristic for quickly generating feasible solutions for difficult mixed integer programs, the heuristic discards the original objective in all except for the first iteration. Consequently, the quality of the solution obtained by the FP is often quite poor [Achterberg and Berthold, 2007]. To address this issue when applying the

⋆ This work was supported by the Center for Integrated Operations in the Petroleum Industry, Trondheim, Norway.

FP on the MINLP shale-well scheduling model, we propose an Objective Feasibility Pump (OFP) inspired by a similar technique for the FP applied on MILPs [Achterberg and Berthold, 2007]. Note that we will not further consider metaheuristics in this paper.

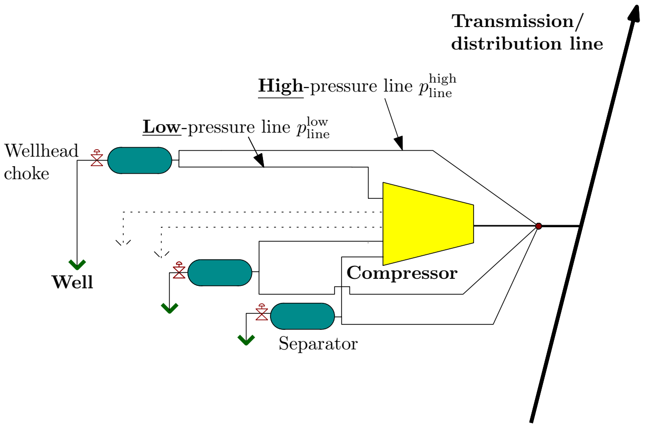


Fig. 1. Illustration of surface gathering systems.

The MINLP heuristics developed in this paper are applied to a complex dry-gas shale well and compressor scheduling problem illustrated in Fig. 1. The problem consists of multiple dynamics shale-gas wells, with pipeline routing and shut-in decisions, integrated in production planning with a several months planning horizon. We present the MINLP model for the scheduling problem in section 2, while section 2.1 describes a new compact shale well and reservoir model. A brief description of the FP, together with the novel OFP and linearization-based heuristics are given in section 3. Section 4 describes the performance tools we use to evaluate the numerical results given in section 5. Concluding remarks ends the paper in section 6.

2. PROBLEM FORMULATION

Consider the shale-well and compressor system illustrated in Fig. 1. The system consists of $|\mathcal{J}|$ distributed wells, each with its own wellhead choke. With spread well locations, it is quite common for each well to have a small separator tank for separation of any co-produced liquids. Once separation is performed, the gas can either be routed to a low-pressure line which feeds the gas to a shared compressor, or the gas can be bypassed the compressor and routed to a high-pressure line, which feeds the gas directly onto a transmission or distribution line. The compression is either performed by a mid-stream company, requiring a fraction δ_{G} of the gas sales price G, or the compression is performed by the well operator, assuming an equivalent compression cost δ_{G}. In contrast, the operator receives the full sales price if the gas is routed directly to the high-pressure line. Routing the gas flow to the low-pressure line, however, increases the well deliverability, since the pressure gradient between the reservoir and the wellhead increases. The compressor requires a minimum inflow rate $q_{\text{tot}}^{\text{low}}$ to avoid surge, and has a maximum load capacity $q_{\text{tot}}^{\text{up}}$. Wells are normally continuously drilled and added to a shared surface gathering and compression systems during a year-long field development of shale-gas assets. Consequently, when the number of wells grows, the total production may eventually exceed the compressor capacity.

Let $y_{jk}^1 = 1$ if a well is routed to the low-pressure line leading the gas to the compressor, and $y_{jk}^1 = 0$ if the well is routed to the high-pressure line. When a well is routed from the low-pressure line to the high-pressure line, the well may have to be shut in for a certain time to avoid backflow in the well, and by such increasing the well pressure to eventually obtain a positive flow. However, the gas flowrate must always be kept higher than a critical rate $q_{\text{gc}}(p)$ to avoid liquid loading [Turner et al., 1969], which is one of the major operational concerns in shale-gas production [Al Ahmadi et al., 2010]. The critical rate is a nonlinear function of pressure, and is normally evaluated at wellhead conditions [Turner et al., 1969]. As the pressure-drop over the wellhead choke normally is small in shale-gas wells, i.e. the wells operate on wellhead pressures close to the line pressure, we evaluate $q_{\text{gc}}(p)$ at the line pressures $p_{\text{line}}^{\text{low}}$ and $p_{\text{line}}^{\text{high}}$, respectively. Let $y_{jk}^2 = \{0,1\}$ be a binary variable used to model whether a well is shut in or producing. Combining the specifications of the routing of the wellflows, the compressor properties, the minimum line pressures and required minimum (critical) flowrates, we formulate the following nonconvex MINLP for shale-well scheduling over a planning horizon $\mathcal{K}$:

$$\max\ G \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}\setminus K} (1-\delta_{\text{G}} y_{jk}^1)\, q_{jk}\Delta k, \tag{1a}$$

s.t.

$$\sum_{j\in\mathcal{J}} q_{jk} y_{jk}^1 \le q_{\text{tot}}^{\text{up}}, \qquad \forall k \in \mathcal{K} \tag{1b}$$

$$\sum_{j\in\mathcal{J}} q_{jk} y_{jk}^1 \ge q_{\text{tot}}^{\text{low}}, \qquad \forall k \in \mathcal{K} \tag{1c}$$

$$F_j(p_{jk+1}, q_{jk+1}, p_{\text{t},jk}, p_{\text{wf},jk}) = 0, \qquad \forall j \in \mathcal{J}, k \in \mathcal{K}\setminus K \tag{1d}$$

$$p_{j0} = p_j^{\text{init}}, \qquad \forall j \in \mathcal{J} \tag{1e}$$

$$p_{\text{t},jk} \ge p_{\text{line}}^{\text{low}} y_{jk}^1 + (1-y_{jk}^1) p_{\text{line}}^{\text{high}}, \qquad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{1f}$$

$$q_{jk} \ge y_{jk}^1 y_{jk}^2 q_{\text{gc}}^{\text{low}} + (1-y_{jk}^1) y_{jk}^2 q_{\text{gc}}^{\text{high}}, \qquad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{1g}$$

$$g_{\text{sw}}(y_{jk}^1, y_{jk}^2) = 0, \qquad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{1h}$$

$$y_{jk}^1, y_{jk}^2 \in \{0,1\},$$

$$q_{jk}, p_{\text{t},jk}, p_{\text{wf},jk} \in \mathbb{R}, \quad p_{jk} \in \mathbb{R}^I.$$

In (1), p is reservoir pressure, p_{t} is tubinghead pressure, p_{wf} is flowing bottomhole pressure and q is the gas rate. We include $g_{\text{sw}}(y_{jk}^1, y_{jk}^2)$ for representing a general set of constraints on the binaries for requiring minimum stay-times for the pipeline routing and minimum shut-in and production times for the wells. Note that the degrees-of-freedom in (1) are y_{jk}^1, y_{jk}^2 and $p_{\text{t},jk}$. $F_j(\cdot)$ is a I-dimensional vector-valued function representing a discretized dynamic reservoir and well model for each well $j \in \mathcal{J}$. The form and the size of this model greatly impacts the tractability of the above nonconvex MINLP.

2.1 Shale Well Modeling

Hydraulically fractured shale and tight gas reservoirs are mainly modeled using either a dual-porosity system (see e.g. Al Ahmadi et al. [2010]), or as fully discretized single-porosity dual-permeability models [Cipolla et al., 2010].

The former, idealized modeling scheme is often used to derive static production forecasting tools by assuming steady-state operations, while the latter scheme normally leads to complex, numerically demanding models. Knudsen and Foss [2013] presents a simple shale well and reservoir proxy model together with a tuning scheme to achieve good transient fit of the model when performing well shut-ins. This model is however designed to capture the dominating dynamics during short cyclic shut-ins to prevent well liquid-loading, and may therefore loose accuracy for longer prediction horizons [Knudsen et al., 2014]. Consequently, since (1) includes longer planning horizons, we derive a slightly different model for the current application.

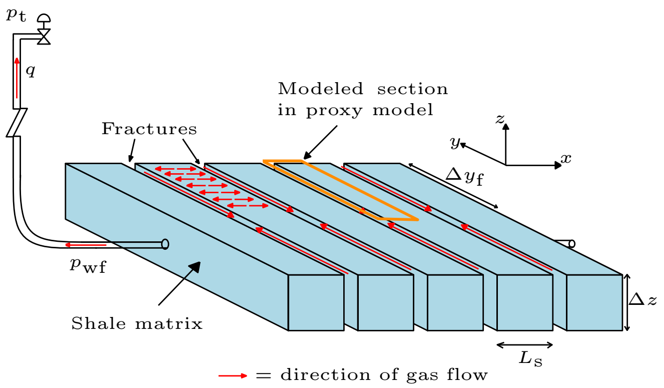


Fig. 2. Illustration of reservoir and proxy model.

Consider Fig. 2 illustrating the geometry of a so-called multi-fracture slab matrix model [Al Ahmadi et al., 2010], consisting of shale matrix blocks and vertical intersecting fractures, where the fractures are orthogonal to the horizontal wellbore and assumed to penetrate the entire organic-rich formation. By further assuming that the fractures are symmetric around the wellbore and equally spaced, we reduce the size of the model by only considering a quarter section of the slab-fracture system as illustrated by the orange rectangle in Fig. 2. The dominating direction of the flow in the shale matrix is orthogonal to the fractures, i.e. in the x-direction, while the pressure drop in the fractures are negligible due to very high fracture conductivity. We assume that the gas is dry (i.e. single-phase gas), and we use a single layer and a spatially dependent permeability $k(x)$. We further use an integral transformation from pressure p to *pseudopressure* $m(p)$ [Al-Hussainy et al., 1966],

$$m(p) := 2\int_{p_b}^{p} \frac{p'}{\mu(p')Z(p')}dp', \tag{2}$$

to incorporate pressure variations of the gas viscosity $\mu(p)$ and the gas compressibility factor $Z(p)$, where p_b is a low base pressure. Using the pseudopressure transformation (2) reduces the nonlinearity of the governing partial differential equation (PDE). Including an initial pressure and Neumann boundary conditions, we formulate the shale-gas reservoir proxy model as the following one-dimensional initial-boundary value problem (IBVP):

$$\phi\mu c\frac{\partial m}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial m}{\partial x}\right), \tag{3a}$$

$$\left.\frac{\partial m}{\partial x}\right|_0 = q\frac{2Tp_{sc}}{T_{sc}\Delta z\Delta y_f k_f}, \tag{3b}$$

$$\left.\frac{\partial m}{\partial x}\right|_{\frac{L_s}{2}} = 0, \tag{3c}$$

$$m(x,0) = m^{init}. \tag{3d}$$

In (3), ϕ is the porosity, c is the total (pressure dependent) compressibility, q is the gas rate and T is temperature. Spatial references are shown in Fig. 2. The subscript sc refers to evaluation at standard surface conditions. An I-dimensional spatial discretization of (3) is constructed by using central difference approximations, while we apply backward Euler approximation for time discretization of (3a). This leads to the discretized reservoir proxy model

$$Am_{k+1} = m_k + Bq_{k+1}, \qquad \forall k \in \mathcal{K}\setminus K \tag{4a}$$

$$m_0 = m^{init}. \tag{4b}$$

The pressure drop from the bottomhole to the surface of the well is modeled by using the static tubing-model [Katz and Lee, 1990],

$$\frac{1}{C_t^2}q_k^2 + p_{t,k}^2 = e^{-S}p_{wf,k}^2, \tag{5}$$

where p_t is the tubinghead pressure, p_{wf} is the bottomhole pressure and C_t and S are tubing specific constants. The first term in (5) models the tubing friction, which for some wells may cause a significant pressuredrop, while the term e^S yields the hydrostatic head of the gas column. The gas rate the well can deliver is for a given tubinghead pressure p_t found by the intersection of (5) and the well inflow from the reservoir, given by

$$q_k = y_{jk}^2\beta\left(m_{k1} - m_{wf,k}\right), \tag{6a}$$

$$m_{wf,k} := \tilde{a}_1\bar{p}_k, +\tilde{a}_2, \tag{6b}$$

where m_{k1} is the pseudopressure in the gridblock adjacent to the fracture, m_{wf} is the bottomhole pseudopressure, $\bar{p}$ is the *square* of the bottomhole pressure (i.e. $\bar{p} := p_{wf}^2$), and $\beta, \tilde{a}_1$ and $\tilde{a}_2$ are constants. The conversion (6b) of the bottomhole pressure squared to pseudopressure is obtained by considering μZ as constant at low pressures [Al-Hussainy et al., 1966], and constructing a linear fit of the map $p^2 \mapsto m$ using the definition (2). As the well model (5)–(6) only requires computing the square of the bottomhole pressure, we can substitute $\bar{p}$ for p_{wf}^2 in (5), hence reducing the nonlinearity of the tubing model. Observe that p_t serves as the boundary condition of the aggregated single-well and reservoir proxy model.

The proxy model (4)–(6) is tuned and validated against a high-fidelity numerical multi-fracture reference model (MFR) with full-scale geometry as illustrated in Fig. 2, using the reservoir modeling scheme described in Cipolla et al. [2010]. We apply a similar parameter estimation technique as described in Knudsen and Foss [2013] and Knudsen et al. [2014]. The parameter estimation is based on a filtering of the prediction errors through Butterworth bandpass filters, fitting the proxy model such that it captures the dominating dynamics during switching of pipelines and during shut-ins. Fig. 3 shows estimation and validation of the gas rate q_k and the bottomhole pressure $p_{wf,k}$, with sets of alternating switching from the high-pressure to the low-pressure line, and a shut-in followed by switching from the low-pressure to the high-pressure. We use $I = 4$ grid blocks for the proxy model. Both the estimation and the cross-validation shows that the proxy model gives a good match of the rate transients and the pressure build-up in the reference model, however by sacrificing some accuracy in the peak rates due to the design of the prefilters and the simplicity of the proxy model.

3. MINLP HEURISTICS

Heuristics for MINLPs are algorithms that are designed to assist in the solution process within an MINLP solver. As such, heuristics for MINLPs are often divided into two categories: *construction heuristics* and *improvement heuristics*. Both of these classes are primal heuristics, while the distinguishing feature between them is that construction heuristics can be used as standalone methods to find feasible solutions, while improvement heuristics require a feasible solution to search for improved solutions within a local area of the solution space. One of the benefits of construction heuristics is that they often can be used to compute feasible solutions within a fraction of the time required by full-space MINLP algorithms [Bonami and Gonçalves, 2010].

Initial testing with algorithms within the MINLP solver `BONMIN` [Bonami et. al, 2008] revealed that the solver required a prohibitively large amount of time to compute a feasible solution to (1). This motivated the search for and development of efficient MINLP heuristics as an approach to find high quality feasible solutions in a relatively short computation time. To this end, we have adapted and applied four MINLP heuristics for solving (1), the basic FP and its objective-oriented variant, OFP [Sharma, 2013], and two linearization-based heuristics.

3.1 The Feasibility Pump

For describing the main steps of the diving heuristic FP, consider the following general MINLP

$$\begin{aligned} \min_{x,y} \quad & f(x,y) \\ \text{s.t.} \quad & g(x,y) \le 0, \\ & x \in X \subset \mathbb{R}^n, \quad y \in \{0,1\}^q \end{aligned} \tag{7}$$

where $f : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}$ and $g : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^p$ are smooth functions and the set X is a bounded polyhedron. The FP starts by computing a constraint feasible point $(\bar{x}^0, \bar{y}^0)$ by solving the continuous NLP relaxation of (7), obtained by relaxing the integrality restriction $y \in \{0,1\}^q$ to $y \in [0,1]^q$. The relaxed solution $(\bar{x}^0, \bar{y}^0)$ will generally be integer infeasible. Subsequently each binary variable y is rounded to the nearest integer point $\tilde{y} := [\bar{y}^0]$, and the current ℓ_1 projection problem

$$\begin{aligned} \min_{x,y} \quad & ||y - \tilde{y}||_1 \\ \text{s.t} \quad & g(x,y) \le 0, \\ & x \in X \subset \mathbb{R}^n, \\ & y \in [0,1]^q, \end{aligned} \tag{8}$$

is solved. If the solution to (8) is feasible with an objective value equal to zero, then FP terminates and returns a feasible solution to (7). Otherwise it rounds the new solution, and solves the NLP (8) in an iterative manner by rounding the solutions and solving the projection problem until it converges to a point where the objective value is zero, or a termination criterion such as an iteration limit is met.

3.2 An Objective Feasibility Pump

The objective FP seeks to overcome the aforementioned poor solution quality often obtained by the basic FP by including the original objective $f(x, y)$ in the search for a feasible solution [Sharma, 2013] . The OFP uses the exact same steps as the FP described above, but implements a weighted sum of the original objective of (7), $f(x, y)$, and the ℓ_1 objective used in (8), formulated as the NLP

$$\begin{aligned} \min_{x,y} \quad & (1-\alpha_i)\eta_1 \, ||y - \tilde{y}||_1 + \alpha_i \eta_2 f(x,y) \\ \text{s.t.} \quad & g(x,y) \le 0, \\ & x \in X, \quad y \in [0,1]^q \end{aligned} \tag{9}$$

where $\alpha_i \in [0, 1]$ is a weighting factor geometrically reduced at each iteration, and η_1 and η_2 are normalization factors that are computed by expressing (9) as a multiobjective optimization problem. Note that finding optimal normalization factors η_1 and η_2 are significantly more difficult for nonlinear functions $f(x, y)$ than for linear objectives as considered in Achterberg and Berthold [2007].

3.3 Linearization Heuristic

In addition to the FP and the OFP, we propose a simple linearization-based heuristic for solving (1), in which we apply Glover-type reformulations [Glover, 1975] of the bilinear binary-continuous products in (1b)–(1c) and the pure binary products in (1f)–(1g), together with a linearization of the tubing model (5), eventually converting (1) to an approximated, increased-size MILP. Once a feasible solution to this MILP approximation is obtained, we fix the binary variables to their values at this solution, and solve (1) as a nonconvex NLP. A pseudocode description of this heuristic is given in Algorithm 1.

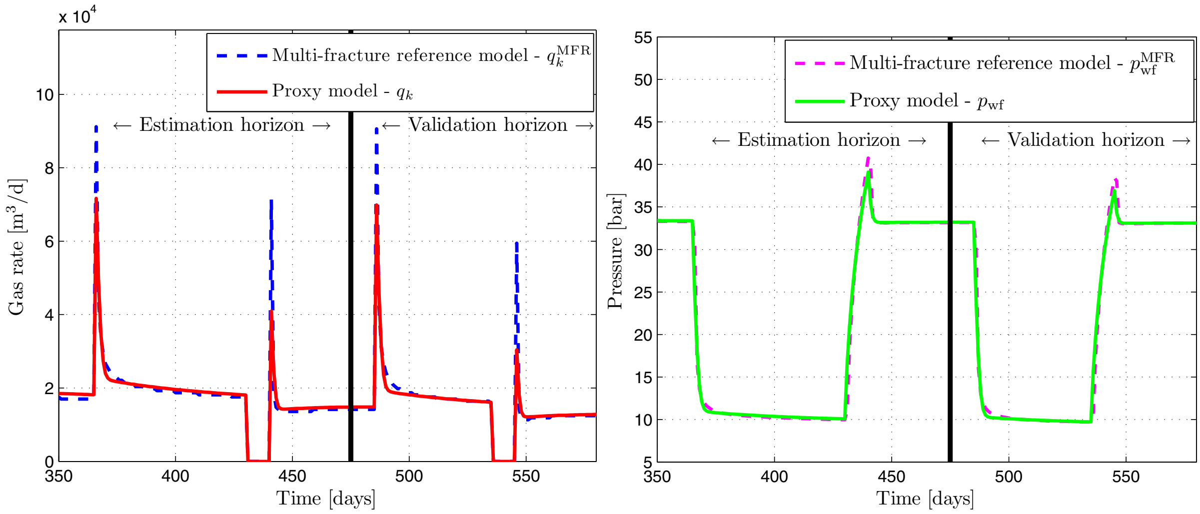
Algorithm 1 Linearization heuristic

```
begin
1:  solve a MILP approximation of (1) to feasibility
2:  fix the binary variables to the solution y*
3:  solve the sub-NLP of (1) with fixed binary
    variables
end
```

We consider two techniques for linearization of the nonlinear constraint (5), rendering two MILP formulations: a first-order Taylor series approximation (FOTA), and a piecewise linear approximation (PWL).

4. PERFORMANCE PROFILES

The state-of-the-art tool for evaluating and comparing the performance of optimization solvers on a set of test-problems is the so-called *performance profiles* [Dolan and Moré, 2002]. The performance profile for a solver is the cumulative distribution function for a *performance metric*, such as the CPU time, the number of nodes, the number of function evaluations or the objective value. Given a set



(a) Gas Rates.

(b) Bottomhole pressures.

Fig. 3. Parameter estimation and cross validation of proxy model with $I = 4$ grid blocks.

of problems $\mathcal{P}$ where $|\mathcal{P}| = n_p$ and a set of solvers $\mathcal{S}$ where $|\mathcal{S}| = n_s$, the performance profile is generated by comparing the results of applying all solvers $s \in \mathcal{S}$ on all problems $p \in \mathcal{P}$.

For each problem p and solver s, the performance $t_{p,s}$ is defined as

$t_{p,s}$:= Performance metric on problem p by solver s.

The performance on problem p by solver s is compared with the best performance by any solver s on this problem by defining a *performance ratio*

$$r_{p,s} = \frac{t_{p,s}}{\min \{t_{p,s} : s \in \mathcal{S}\}}. \tag{10}$$

A parameter $r_M \geq r_{p,s}$ is specified for all p, s, by defining $r_M = r_{p,s}$ if and only if solver s does *not* solve problem p. The cumulative distribution function for the performance ratio is defined as

$$\psi_s(\kappa) = \frac{1}{n_p} \text{size}\{p \in \mathcal{P} : r_{p,s} \leq \kappa\}. \tag{11}$$

Hence, $\psi_s(\kappa)$ is the probability that a solver s yields a performance ratio $r_{p,s}$ that is at most worse by a factor of κ of the best ratio.

5. COMPUTATIONAL STUDY

The performance of the 4 heuristics are tested and compared with a standard branch-and-bound method (BB) on a set of 15 test cases of the nonconvex MINLP (1). The test sets consist of $|\mathcal{J}| = 6$ wells, and a two-month planning horizon $\mathcal{K}$ with a fixed 2-day time step. Each test problem differs only in the value of the initial pseudopressure m_j^{init}, which is randomly generated from the time span in the production profiles shown in Fig 3. The discretized shale-well proxy model (4) naturally replaces the constraints $F_j(\cdot)$ in (1). We further use a Big-M reformulation [Nemhauser and Wolsey, 1988] to omit the products of y_{jk}^2 and m in (6a).

The OFP is implemented inside the `BONMIN` framework, while the FP exists as an algorithm in `BONMIN`. The two linearization-based heuristics are implemented in `Matlab` scripts that generate `AMPL` input files. The FP heuristic and BB are used with their default settings in `BONMIN` v. 1.6, and the MILP approximations FOTA and PWL are solved using `CBC` v. 2.7.7 in conjunction with the NLP solver `IPOPT` v. 3.10.2. All test problems are implemented in `AMPL` and computed on a personal computer running a 64-bit Ubuntu v. 12.04.3 with Intel i7-2600 3.40 GHz CPU and 16GB of RAM.

We construct performance profiles for the MINLP heuristics using two performance metrics: the CPU time to find the first integer feasible solution, and a relative quality of this solution in terms of an optimality *gap*. For the sake of this latter metric, we define the optimality gap for the heuristics as follows:

$$\text{Gap} = 100 \times \frac{|\text{Best possible BB sol.} - \text{First feasible sol.}|}{\text{Best possible BB sol.}}, \tag{12}$$

where the Best possible BB solution for each of the test problems is computed by running the BB method in `BONMIN` with default settings for three hours. We use the base-2 logarithm of κ in (11) in the performance profiles.

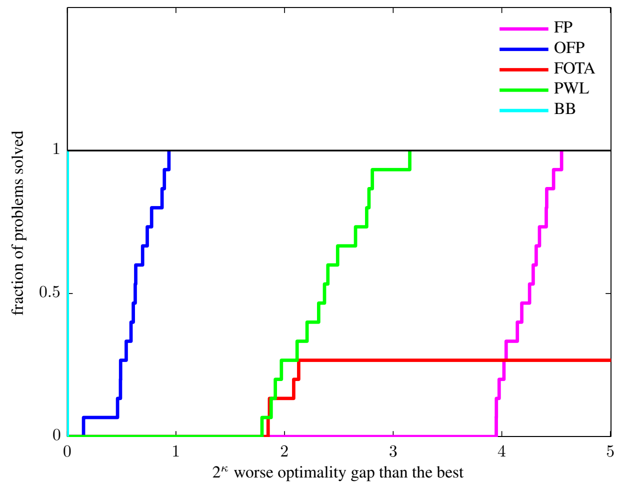


Fig. 4. Performance profile with optimality gap (12) at first feasible solution as metric.

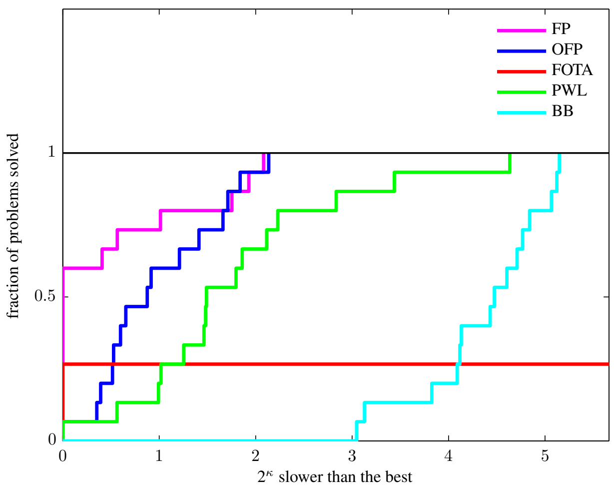


Fig. 5. Performance profile with CPU time to first feasible solution as metric.

Fig. 4 and 5 compares the performance profiles with the defined optimality gap (12) and the CPU time to first solution as metrics, respectively. Fig. 4 clearly shows that BB is the best algorithm with respect to the optimality gap (12) of the first solution found in its tree search, while Fig. 5 reveals that it is by far also the slowest algorithm. It is observed that the OFP is the second best algorithm with respect to the optimality gap, and finds a solution that has at worst twice as large gap as the solution found by BB. Fig. 5 further shows that the OFP is the second fastest of those heuristics that find a solution to all of the 15 test problems, only outperformed by the FP which in Fig. 4 and in Table 1 is seen to clearly perform worst in terms of solution quality. The two variants of the linearization heuristics in Algorithm 1, FOTA and PWL, yield solutions of similar quality as seen in Fig. 4, but the PWL formulation is seen in Fig. 5 to be significantly more robust than the FOTA formulation; the latter only finds a solution to 4 out of 15 test problems. However, in these 4 test problems where the FOTA finds a solution, it is the fastest of all the algorithms.

Comparing in Table 1 the geometric mean (GM) of the objective values in the test problems obtained with the different algorithms, shows that the relative difference between the BB and the OFP algorithm is only 1.2%, while the PWL difference is 9.6%. Hence, compared with the BB method, we argue that the OFP is able to produce good feasible solutions within a short computation time when when applied to the nonconvex MINLP (1). Finally, we comment that a structured reformulation of the shale-well scheduling problem formulated as a disjunctive program was initially tested, but observed to perform worse than the compact MINLP formulation (1) due to substantially increased problem size while retaining the nonlinear tubing model (5).

Table 1. Average objective values and CPU times for the algorithms on the test sets.

Algorithm	FP	OFP	FOTA	PWL	BB
GM obj [10^6 **\$**]	0.7023	1.1207	NA	1.0256	1.1347
GM time [s]	17.5	24.2	NA	42.9	252.4

6. CONCLUDING REMARKS

The computational study in this paper shows that the objective-oriented Feasibility Pump is able to quickly find good feasible solutions of complex MINLP scheduling problems such as the multi-pipeline shale-well and compressor problem. As such, we argue that the OFP may be well suited for integration in DSTs for improving the productivity in shale-gas system, which typically consists of tens to hundreds of wells, and where yet production optimization DSTs are almost absent.

REFERENCES

T. Achterberg and T. Berthold. Improving the feasibility pump. *Discrete Optimization*, 4(1):77–86, 2007.

H. Al Ahmadi, A. Almarzooq, and R. Wattenbarger. Application of linear flow analysis to shale gas wells - Field cases. In *SPE Unconventional Gas Conference*, Pittsburgh, Pennsylvania, USA, 2010.

R. Al-Hussainy, H.J. Ramey Jr., and P.B. Crawford. The flow of real gases through porous media. *J. of Petroleum Technology*, 18(5):624–636, 1966.

P. Bonami and J. P. M. Gonçalves. Heuristics for convex mixed integer nonlinear programs. *Computational Optimization and Applications*, 51(2):729–747, 2010.

C. L. Cipolla, E.P. Lolon, J.C. Erdle, and B. Rubin. Reservoir modeling in shale-gas reservoirs. *SPE Reservoir Evaluation & Engineering*, 13(4):638–653, 2010.

E. D Dolan and J. J Moré. Benchmarking optimization software with performance profiles. *Mathematical programming*, 91(2):201–213, 2002.

P. Bonami et. al. An algorithmic framework for convex mixed integer nonlinear programs. *Discrete Optimization*, 5(2):186–204, 2008.

M. Fischetti, F. Glover, and A. Lodi. The feasibility pump. *Mathematical Programming*, 104(1):91–104, 2005.

F. Glover. Improved linear integer programming formulations of nonlinear integer problems. *Management Science*, 22(4):455–460, 1975.

F. Glover and G.A. Kochenberger, editors. *Handbook of Metaheuristics*. Kluwer Academic Publishers, 2003.

D. L. Katz and R. L. Lee. *Natural Gas Engineering*. McGraw-Hill Publishing Company, New York, 1990.

B. R. Knudsen and B. Foss. Shut-in based production optimization of shale-gas systems. *Computers & Chemical Engineering*, 58:54–67, 2013.

B. R. Knudsen, I. E. Grossmann, B. Foss, and A. R. Conn. Lagrangian Relaxation Based Decomposition for Well Scheduling in Shale-gas Systems. *Computers & Chemical Engineering*, 63:234–249, 2014.

G. L. Nemhauser and L. A. Wolsey. *Integer and Combinatorial Optimization*. John Wiley & Sons, Inc., 1988.

S. Sharma. Mixed integer nonlinear programming heuristics applied to a shale gas production optimization problem. Master's thesis, NTNU, 2013. URL `daim.idi.ntnu.no/masteroppgave?id=8956`.

R.G. Turner, M.G. Hubbard, and A.E. Dukler. Analysis and prediction of minimum flow rate for the continuous removal of liquids from gas wells. *J. of Petroleum Technology*, 21(11):1475–1482, 1969.

S. Vasantharajan, R. Al-Hussainy, and R. F. Heinemann. Applying optimization technology in reservoir management. *J. of Petroleum Technology*, 58(5):82–88, 2006.