

# Bode-like control loop performance index evaluated for a class of fractional-order processes

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**Abstract:** Control loop performance assessment techniques are crucial for optimizing any plant or machine. They can bring huge energy and material savings and increase product quality. Usually, the performance is compared to minimum variance controller. It is known that when optimizing process controllers having fixed structure (e.g. PIDs) different concepts must be applied. In this paper, the systematic approach for a class of fractional-order processes is presented. Inspired by the model free design techniques, only a minimum *a priori* information about the process is assumed. A novel performance index based on 'ideal' shape of sensitivity function is proposed. Two fundamental limits are considered: available loop bandwidth and robustness  $M_S$ -index. The best possible performance is computed for all processes belonging to the fractional-order model set controlled by PID-type controllers. The authors believe that the presented ideas may be utilized by both academic and industrial sphere.

*Keywords:* Control loop performance assessment, process control, fractional-order systems, loop bandwidth, Bode theorem, sensitivity function, Fourier transform

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## 1. INTRODUCTION

*Control loop performance assessment* (CLPA) is often sketched as a key asset-management technology<sup>1</sup>. Since 1970, it became an integral part of large distributed control systems – especially in refineries, oil and chemical sectors. It was observed that correct CLPA application leads into huge energy and material savings and increases overall product quality (Desborough and Miller (2002)). Therefore, CLPA faces growing interest in both research and engineering community. Several surveys of existing CLPA approaches has been done e.g. in Harris et al. (1999); Åström and Hägglund (2006); Shardt et al. (2012); Jelali (2013).

Despite the positive CLPA impact, the utilization of CLPA is still undervalued. The majority of controllers are tuned only once, they often work in manual mode or with default parameters. Even when the controllers are initially well tuned they must be continuously monitored because of process dynamics variations and the sensors and actuators time-degradation. The renowned studies estimate that about 70% of control loops are not properly tuned also due to the lack of tools based on exact problem formulation.

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\* This work was supported by the Technology Agency of the Czech Republic – project No. TA02010152 and by the European Regional Development Fund (ERDF), project NTIS – New Technologies for Information Society, European Centre of Excellence, CZ.1.05/1.1.00/02.0090.. The support is gratefully acknowledged.

<sup>1</sup> Sometimes the synonyms *loop auditing* or *loop management* are used

The automation complexity is growing also in other industrial sectors (food and paper industry, etc.) and in energetics. Thus, it is supposed that CLPA techniques will penetrate into smaller plants, machines or home devices (Jämsä-Jounela et al. (2003); Jelali (2007)). They will become an integral part of control systems and even compact controllers. Several global automation leaders (e.g. ABB, Honeywell) has already implemented loop diagnostic methods directly into their compact controllers.

The CLPA importance rises as the overall world production tends to be demand-driven. Consequently, the process working points often change and the production lines must be quickly reconfigured.

Recap that in large-scale process industries, often the independent monitoring system is used which analysis off-line data mined from a signal database. Those traditional concepts must be now revised. Especially, the tighter interaction with process controllers needs to be formed to reach the maximum performance and reliability of CLPA methods.

Usually, the actual control quality is compared to the best linear controller (minimum variance – see e.g. Lynch and Dumont (1996); Harris et al. (1999)). Unfortunately, such approach gives no insight what is the best performance achievable by the controller currently integrated in the loop which has typically with fixed structure – PI or PID. This challenge was addressed earlier e.g. in Qin (1998); Ko and Edgar (1998); Goradia et al. (2005); Grimbale (2003). However, only the low order plant models are used there and the maximum achievable performance

is computed numerically. In Thyagarajan et al. (2003), the relay is added to the control loop to assess the PI controller performance. Such an experiment can be quite time consuming and the process must be disturbed from the normal operation. The more pragmatic approach can be examined in Huang (2003) where a trade-off curve between input and output variance is taken into account. Unfortunately, the authors also restrict themselves to only second order plant model.

In this paper, the novel concise approach is presented that eliminates those drawbacks. It is based on exactly defined class of fractional-order systems covering majority of process control plants and fundamental feedback loop limits (Bode theorem) and design objectives. Based on the *a priori* information about the process and a fixed controller structure an ideal shape of sensitivity function is defined by two fundamental limits: available loop bandwidth  $\Omega_a$  and robustness  $M_s$ -index. Afterwards, the performance index is formulated as a ratio of the ideal to actual sensitivity function. As a main paper result, the best achievable performance is evaluated for all processes belonging to the fractional-order model set controlled by PID-type controllers. Up to the authors' knowledge, such systematic performance evaluation for a class of fractional-order processes has not been done before.

Point out that all ideas presented in this paper are consistent with the authors' previous achievements in the area of controller autotuning Schlegel and Čech (2005); Čech and Schlegel (2011). Based on those ideas, a successful industrial PID autotuner has been recently developed and implemented in various commercial control systems and compact controllers. There exists a strong demand for continuous monitoring of the loops controlled by these PIDs. It is the main driver for the work documented below.

The paper is organized as follows: Section 2 clarifies the *Available bandwidth* paradigm in simulation and real-time environment. Section 3 formulates the problem of performance indices definition and evaluation for fractional-order model set. Section 4 summarizes the results of computing optimal indices for all processes from fractional-order model set. The practical method for evaluating actual performance index is demonstrated on real example in Section 5. Conclusions and ideas for further development are recalled in Section 6.

## 2. BODE THEOREM AND AVAILABLE LOOP BANDWIDTH

Key fact about physical systems is that they do not exhibit good frequency response fidelity beyond a certain bandwidth  $\Omega_a$ . It is influenced by various factors when working in different environments<sup>2</sup> (see Tab. 1). Let us call that  $\Omega_a$  *Available bandwidth*, to distinguish it from other bandwidths such as crossover or 3-dB magnitude loss. In today's popular robust control jargon,  $\Omega_a$  is the frequency range over which unstructured multiplicative perturbations are substantially less than unity; see Stein (2003). Further remind, that the *Bode Theorem* is a cornerstone of linear feedback control theory. It expresses

<sup>2</sup> MIL – Model-in-the-Loop; SIL – Software-in-the-Loop; PIL – Processor-in-the-Loop; HIL – Hardware-in-the-Loop

Table 1. Typical factors influencing available bandwidth  $\Omega_a$  in different environments

Environment	factors influencing available bandwidth $\Omega_a$
1. MIL	number representation precision
2. SIL	computer achievable sampling time
3. PIL	communication speed
4. HIL	D/A A/D converter resolution, signal noise ratio, saturations
5. real plant / machine	additional non-linearities, sensor/actuator speed and precision, unmodeled dynamics

the limits of feedback loop to reject disturbances and track signals. When designing the control loop, one should always consider the modified relation respecting available bandwidth in the form

$$\int_0^{\Omega_a} \ln(|S(j\omega)|) d\omega \doteq 0, \quad (1)$$

where  $S(j\omega)$  is the loop sensitivity function.

*Remark 1.* The Table 1 should be viewed as a cumulative set of factors that can restrict the available bandwidth when going through individual steps of control development cycle. Hence, e.g. despite working on real plant, the communication speed could be the main restrictive factor.

*Assumption 1.* In the following, it is assumed that the value of  $\Omega_a$  is known and can be taken as a key design parameter. In practice, it can be estimated e.g. using discrete Fourier transform.

## 3. PROBLEM FORMULATION

In this Section, it is explained how the model set is constructed and how the 'optimal' controller is chosen for each process belonging to the model set.

### 3.1 *A priori* admissible systems

It was shown in Charef et al. (1992) that to cover the huge number of real processes, one has *a priori* to consider the transfer function in the form

$$P(s) = \frac{K}{\prod_{i=1}^p (\tau_i s + 1)^{n_i}}, \quad (2)$$

where  $p$  is arbitrary integer number and  $K, \tau_i, n_i$   $i = 1, 2, \dots, p$  are positive real numbers. The transfer function (2) describes very well the majority of essentially monotone processes (see Åström and Hägglund (2004) for definition).

*Remark 2.* If  $p \rightarrow \infty$  then the set of all transfer functions (2) contains also processes with dead time and approximates well several processes with transcendent transfer functions (like heat transfer, chemical processes, etc.).

### 3.2 *Characteristic numbers – experimental data*

Three-parameter time domain process description is well accepted in the control community. The authors' previous works vindicate the usage of first three moments  $m_i$  of the impulse response  $h(t)$  instead of numbers obtained from the step response using its tangent line in the inflexion point. The application of time moments in control

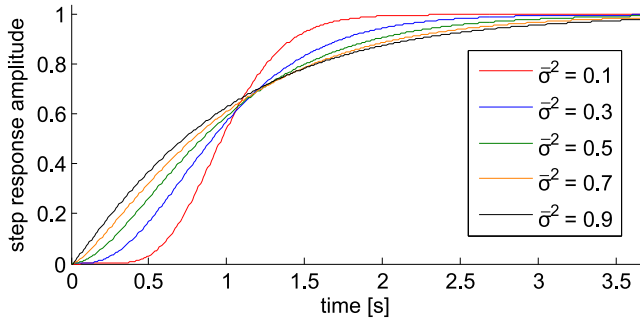


Fig. 1. Step response shaping by parameter  $\bar{\sigma}^2$

appeared firstly in Maamri and Trigeassou (1993). They are defined as

$$m_i = \int_0^{\infty} t^i h(t) dt, \quad i = 0, 1, 2 \quad (3)$$

and may be converted to another more suitable group of numbers  $\{\kappa, \mu, \sigma^2\}$  (Schlegel et al. (2003)) defined as

$$\kappa = \int_0^{\infty} h(t) dt = m_0, \quad \mu = \frac{\int_0^{\infty} t h(t) dt}{\int_0^{\infty} h(t) dt} = \frac{m_1}{m_0},$$

$$\sigma^2 = \frac{\int_0^{\infty} (t - \mu)^2 h(t) dt}{\int_0^{\infty} h(t) dt} = \frac{m_2}{m_0} - \frac{m_1^2}{m_0^2}. \quad (4)$$

It can be proved (Čech (2008)) that for transfer function (2), it holds

$$\kappa = K, \quad \mu = \sum_{i=1}^p \tau_i n_i, \quad \sigma^2 = \sum_{i=1}^p \tau_i^2 n_i. \quad (5)$$

From a control point of view,  $\kappa$  is equal to process static gain and  $\mu$  represents the residual time constant. Without loss of generality, the process can be normalized in gain and time, thus  $\bar{\kappa} = 1$  and  $\bar{\mu} = 1$ . The remaining parameter  $\bar{\sigma}^2$  then has a meaning similar to normalized dead time. It shapes the step response from first order to pure dead time process as shown in Fig. 1. <sup>3</sup>

*Remark 3.* The impulse response moments (3) or equivalently the numbers (4) can be obtained from the process step response or rectangular pulse response (Schlegel et al. (2003)). They may be also estimated from process input/output data.

*Assumption 2.* In the following let us assume that we have measured precisely the numbers (4) and other information about the process is not available.

### 3.3 Model set, extremal, vertex and ultimate processes

To make the paper more self-contained let us briefly remind basic definitions and lemmas.

<sup>3</sup> Optimized Charef's method (Charef et al. (1992)) is used for time-domain realization of (2).

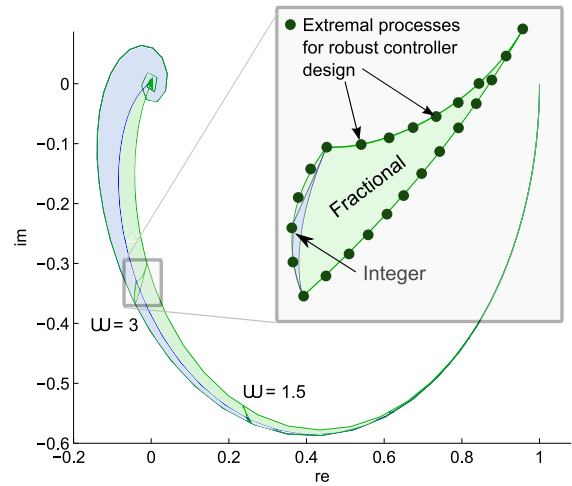


Fig. 2. Comparison of integer (blue) and fractional order (green) model set uncertainty in frequency domain ( $\sigma^2 = 0.7$ )

*Definition 1.* (Model set). The transfer function  $P(s)$  is *admissible* if and only if

(i)  $P(s)$  is in the form (2),  $n_i \geq m, \forall i, \sum_{i=1}^p n_i \leq n$ , where  $n \in \mathbb{R}^+$  is the total order of the process and  $m \in \mathbb{R}^+$  is the minimum allowed order of each fractional pole.

(ii)  $P(s)$  is consistent with experimental data, thus fulfills (5). The set of all admissible transfer functions will be called *model set* and denoted as  $\mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$ .

The following lemma answers the question, when the model set is not empty.

*Lemma 1.* Let  $n \geq 2m$ , then the model set  $\mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$  is not empty if and only if

$$\frac{1}{n} \leq \frac{\sigma^2}{\mu^2} \leq \frac{1}{m}. \quad (6)$$

The proof is omitted for brevity. If the inequality (6) is satisfied then the model set contains for given characteristic numbers  $\kappa, \mu, \sigma^2$  infinite number of processes. Fortunately, these processes create after mapping into frequency domain a connected area called value set for each frequency  $\omega > 0$ .

*Definition 2.* (Value set). The set  $\mathcal{V}_{\omega}^{n,m}(\kappa, \mu, \sigma^2) = \{P(j\omega) : P(s) \in \mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)\}$  will be called the *value set* of  $\mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$  at the frequency  $\omega > 0$ .

The value set boundary is generated by so called extremal transfer functions.

*Definition 3.* (Extremal transfer function). The admissible transfer function  $P(s) \in \mathcal{S}^{n,m}(\kappa, \mu, \sigma^2)$  will be called *extremal*, if there exists  $\omega > 0$  such, that  $P(j\omega) \in \partial \mathcal{V}_{\omega}^{n,m}(\kappa, \mu, \sigma^2)$ , where  $\partial \mathcal{V}_{\omega}^{n,m}(\kappa, \mu, \sigma^2)$  denotes the value set boundary in complex plane. Let us denote the set of all extremal transfer functions as  $\mathcal{S}_E^{n,m}(\kappa, \mu, \sigma^2)$ .

*Remark 4.* For the a priori assumption (2) and condition (5) the set  $\mathcal{S}_E^{n,m}(\kappa, \mu, \sigma^2)$  is independent on frequency  $\omega$ .

The value set boundary is composed of finite number of smooth curves. In authors previous works (Čech (2008); Schlegel et al. (2003)), the analytical relations for com-

puting value set boundaries (extremal processes) were derived for both integer-order (IO) and fractional-order (FO) model set. In Fig. 2, one can examine that omitting fractional-order processes reduces the uncertainty markedly<sup>4</sup>.

*Remark 5.* It is acceptable to define the minimum pole order as  $m = 1$ . Processes of order less than one do not have an equivalent in the real world. Besides, they extend more and more the model set uncertainty. On the contrary, the maximum process order need not to be restricted because the model set uncertainty (value sets size) converges very quickly for  $n \rightarrow \infty$  and the generated extremal processes are quite easier to simulate in the time domain. Therefore, the normalized model set dependent only on  $\bar{\sigma}^2$  and denoted as  $\mathcal{S}^{\infty,1}(\bar{\sigma}^2)$  and the set of processes creating its value set boundary  $\mathcal{S}_E^{\infty,1}(\bar{\sigma}^2)$  will be further considered. It is also worth to mention, that this approach is much less conservative compared to popular  $H_\infty$  techniques where the uncertainty has a circle shape for each frequency.

*Corollary 1.* It flows out from Lemma 1 that  $\mathcal{S}^{\infty,1}(\bar{\sigma}^2)$  is nonempty if and only if  $\sigma^2 \in \langle 0, 1 \rangle$ .

### 3.4 Control loop design specifications

Respecting majority of robust design methods, the upper limit of sensitivity function  $S(j\omega)$  amplitude is specified as

$$\sup_{\omega} |S(j\omega)| \leq M_S. \quad (7)$$

In addition, we want to find the fastest loop minimizing the criterion of control error  $e(t)$

$$J = \int_0^{\infty} e(t) dt \quad (8)$$

when the step in reference signal is applied to the loop.

### 3.5 Controller form

Lets consider PI/PID controller in the form

$$\begin{aligned} C_{PI}(s) &= K \left( 1 + \frac{1}{T_i s} \right), \quad K_i = K/T_i, \\ C_{PID}(s) &= K \left( 1 + \frac{1}{T_i s} + \frac{T_d s}{(T_d/N)s + 1} \right), \end{aligned} \quad (9)$$

where  $K$ ,  $T_i$ ,  $T_d$ ,  $N$  are static gain, integral time constant, derivative time constant and derivative filter, respectively.

*Remark 6.* The controllers (9) are the most employed controllers in industrial practice, hence the systematic evaluation of their maximum achievable performance can bring a new insight into numerous control loops.

### 3.6 Controller design procedure

Firstly, let us describe the two-step design procedure for PI controller for particular value of  $\bar{\sigma}^2$ :

- (1) Using robustness regions method described earlier in Čech and Schlegel (2013), the set of all pairs of controller parameters  $K$ ,  $T_i$  ensuring condition (7)

<sup>4</sup> The dynamics of real distributed parameter processes is closed to fractional. Hence considering only IO processes in robust design may lead to non-satisfactory closed loop behavior.

for any  $P(s) \in \mathcal{S}_E^{\infty,1}(\bar{\sigma}^2)$  was determined. The set can be drawn as a compact region  $\mathcal{R}$  in  $K - K_i$  plane.

- (2) The fastest controller  $C \in \mathcal{R}$  is selected as a point with maximum  $K_i$  coordinate. It can be proven that such controller minimizes the criterion (8).

The procedure remains the same even for PID controller. However, one has to choose *a priori* the derivative filter constant  $N$  and the ratio  $f = T_i/T_d$ , usually close to 1/4. In such a way one can obtain an optimal controller for each  $\bar{\sigma}^2$ . In the following the control loop performance will be evaluated for both PI and PID controllers for each  $\bar{\sigma}^2$ .

## 4. OPTIMAL PERFORMANCE INDICES

### 4.1 Ideal shape of sensitivity function and performance index

*Claim 1.* The reference sensitivity function should respect the Bode theorem (1) and should be specified by minimum set of parameters.

Therefore, only the robustness index (7) and the loop bandwidth  $\Omega_a$  are used for reference sensitivity function parametrization. When adding the assumption that the controller  $C(s)$  contains an integrator and that the process is – in concordance with (2) – essentially monotone, the reference sensitivity function shape arises as shown in Fig. 3 (simplified shape proposed by the authors).

Applying Bode's theorem to the reference shape leads to

$$\begin{aligned} \int_0^{\Omega_a} \ln(|S(j\omega)|) d\omega &= \\ &= \int_0^{\Omega_1} \ln\left(\frac{M_S \omega}{\Omega_1}\right) d\omega + \int_{\Omega_1}^{\Omega_a} \ln\left(M_S - \frac{(M_S - 1)(\omega - \Omega_1)}{\Omega_a - \Omega_1}\right) d\omega \quad (10) \\ &= \Omega_1 (\ln(M_S) - 1) + \frac{(\Omega_a - \Omega_1) (\ln(M_S)M_S - M_S + 1)}{M_S - 1} \doteq 0 \end{aligned}$$

Consequently  $\Omega_1 \doteq \frac{\Omega_a (\ln(M_S)M_S - M_S + 1)}{\ln(M_S)}$  and  $\Omega_0 = \Omega_1/M_S$ . Then the performance index enumerates the ratio of the ideal to actual sensitivity function at some frequency from interval  $\omega_d \in (0, \Omega_0)$  (see Fig. 3. It can be defined as

$$I_p = \frac{M_S \omega_d}{\Omega_1 |S(j\omega_d)|} \doteq \frac{M_S \omega_d \ln(M_S)}{\Omega_a (\ln(M_S)M_S - M_S + 1)} \cdot \frac{1}{|S(j\omega_d)|}. \quad (11)$$

*Remark 7.* The substantial advantage of index (11) is that it provides information whether the controller is too sluggish ( $I_p \ll 1$ ) or too aggressive ( $I_p \gg 1$ ), i.e. can handle to robustness/performance trade-off.

### 4.2 Index estimation method – sketch

Any plant disturbance can be equivalently considered as plant output disturbance  $d_o$ . Assume, that one has some knowledge about its spectral density. Consequently, one can select a frequency  $\omega_d$  with sufficiently high energy in the interval  $(0, \Omega_0)$ . The spectrum amplitude of this frequency is determined for both closed ( $A_y$ ) and open ( $A_d$ ) loop. It can be done f.e. using running discrete Fourier transform. From equation

$$\begin{aligned} y_d(t) &= |S(j\omega_d)|d(t) = |S(j\omega_d)|A_d \sin(\omega_d t + \varphi) = \\ &= A_y \sin(\omega_d t + \varphi) \end{aligned} \quad (12)$$

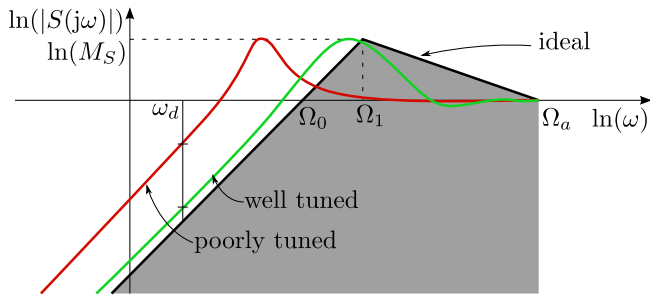


Fig. 3. Ideal (reference) and real shapes of sensitivity functions

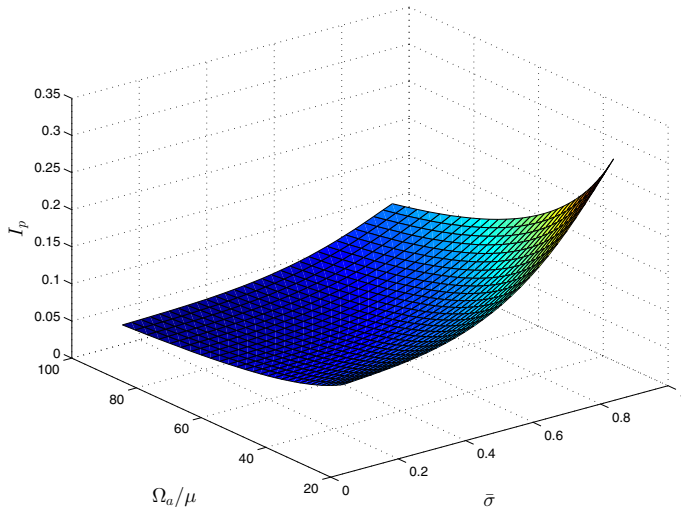


Fig. 4. Maximum achievable PI controller performance for normalized model set,  $M_s = 2$  and different bandwidth  $\Omega_a$ .

it is evident that the ratio of those amplitudes defines the actual value of sensitivity function  $|S(j\omega_d)|$  as follows

$$|S(j\omega_d)| = \frac{A_y}{A_d}. \quad (13)$$

When the energy of disturbances is not sufficient, the testing harmonic signal may be injected into the loop. The details are omitted for brevity.

#### 4.3 Control loop performance evaluation

When the process model is known, the index (11) can be directly evaluated. The evaluation was done for the whole normalized model set respecting Corollary 1, thus  $\bar{\sigma}^2 \in \langle 0, 1 \rangle^5$ . For each  $\bar{\sigma}^2$ , the set of representative extremal processes  $\mathcal{S}_E^{\infty,1}(\bar{\sigma}^2)$  was generated. Then the controller was computed according to steps provided in subsection 3.6. Note, that the evaluation is done at frequency  $\omega_d < \Omega_0$ , where the frequency domain uncertainty (value sets area) is very small. Consequently, one gets almost the same value of  $I_p$  for all extremal processes and taking an average value seems to be correct. The values of performance index were computed for various  $\Omega_a$  and are depicted in Figures 4 and 5.

<sup>5</sup> Hence the performance index is computed based on the knowledge of process normalized dead-time which is consistent with minimum variance control where the process dead-time must be known.

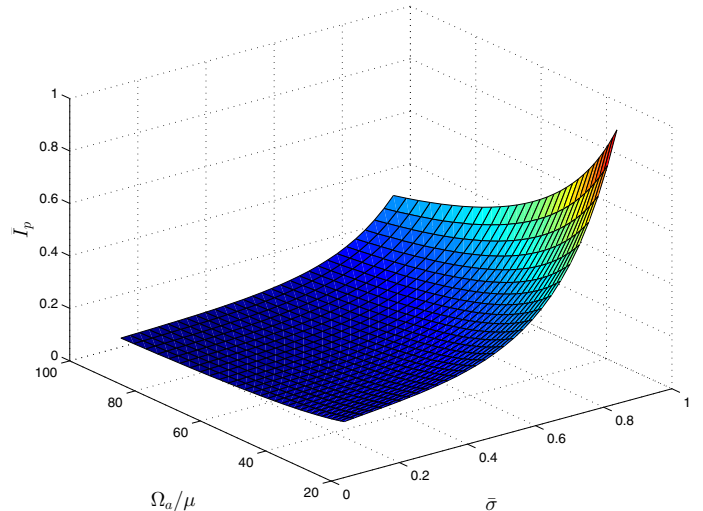


Fig. 5. Maximum achievable PID controller performance for normalized model set,  $M_s = 2$  and different bandwidth  $\Omega_a$ .

*Remark 8.* The results confirmed the well-known fact that control performance is markedly lower when the process has large normalized dead-time ( $\bar{\sigma}^2$  close to zero). For those processes, also the employment of derivative term does not bring much improvement.

*Remark 9.* In addition, it can be observed that increasing the bandwidth  $\Omega_a$  decreases the performance index due to the own process dynamics. The potential controller which covers the full bandwidth is simply too aggressive and does not match the robustness requirement (7).

There are two potential *scenarios* how to utilize the results obtained for known  $\Omega_a$  and  $M_s$

- (1) The model (i.e.  $\bar{\sigma}^2$ ) is not known – still the interval of acceptable index value can be computed.
- (2) The value of  $\bar{\sigma}^2$  is known from the initial proper controller tuning. Let us denote the corresponding nominal performance index value as  $I_p$ . Then the actual loop performance  $I_A$  can be computed and compared to the reference one. The relative error  $E_R$  may be evaluated as  $E_R = |I_A/I_p - 1|$ . The big value (e.g.  $E_R > 50\%$ ) warns the loop manager away from some changes in the process dynamics or malfunctioning equipment.

## 5. EXAMPLE

Consider the real plant described by fractional-order transfer function

$$P(s) = \frac{1}{(0.4109s + 1)(0.2194s + 1)^{3.7423}} \quad (14)$$

having characteristic numbers  $\kappa = 1$ ,  $\mu = 1.29$ ,  $\sigma = 0.4$ . The controller is initially properly tuned for the maximum performance. After normalization  $\bar{\sigma} = \sigma/\mu = 0.31$  one gets the maximum performance evaluated for  $\Omega_a = 4$  [ $rad \cdot s^{-1}$ ] and  $M_s = 2$  as  $I_p = 0.76$ . The value of performance index is stored in the controller. Firstly assume that after some time the process dynamics has changed and can be described by

$$P_1(s) = \frac{0.5}{(0.5141s + 1)(0.244s + 1)^{2.2778}}. \quad (15)$$

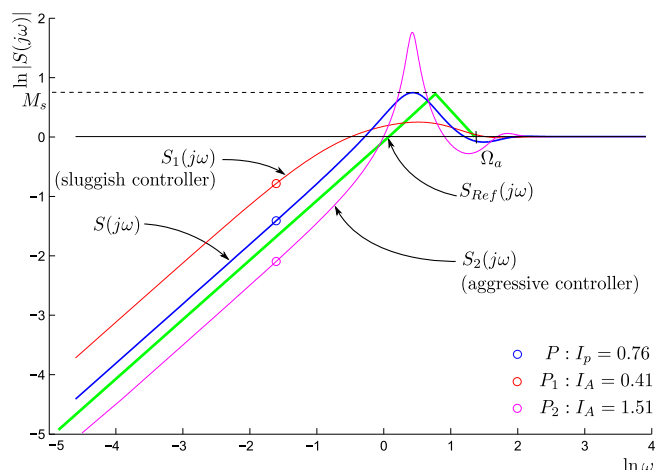


Fig. 6. Original well tuned controller and loop performance degradation caused by the changes in the process dynamics

The new evaluation gives the performance index as  $I_A = 0.41$  meaning too sluggish controller. Secondly assume that the process dynamics has changed to the transfer function

$$P_2(s) = \frac{2}{(0.4781s + 1)(0.1629s + 1)^{6.4563}}. \quad (16)$$

The new evaluation gives the performance index as  $I_A = 1.51$  signifying too aggressive controller. Both values are outside the acceptable band. Consequently, the controller informs the plant management that it should be re-tuned.

## 6. CONCLUSION

In this paper, novel approach to control loop performance assessment was proposed. The performance index is based on fundamental limits resulting into available bandwidth and robustness  $M_s$ -index. Those limits define an ideal shape of sensitivity function. The best possible performance is computed for processes belonging to the exactly defined fractional-order model set controlled by PID-type controllers. Finally, a procedure for realtime estimation of performance indices was demonstrated on practical examples. In the future, the authors plan to make similar evaluation for astatic processes and to integrate presented ideas with previously achieved world wide known results in PID controller tuning. Moreover, the effort will be put to transfer the technology into industrial practice.

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