Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Delay and backlog bounds for an aggregation system in wireless networks

Damien Breck *,** **Jean-Philippe Georges** *,** **Thierry Divoux** *,**

* *University of Lorraine, CRAN, UMR 7039, Campus Sciences, BP 70239, Vandœuvre lès Nancy Cedex, 54506, France*
** *CNRS, CRAN, UMR 7039, France*

damien.breck@univ-lorraine.fr

Abstract: The IEEE 802.11 standard is one of the most used wireless local area network technology. Even if it is not a deterministic protocol, it is possible to bound its performances, and thus to provide some quality of service. Its evolution 802.11n integrates an aggregation system which improves its efficiency under specific conditions. Particularly, such a system could be interesting in a wireless sensor and actuator network, where data are gathered before being forwarded to a sink. We propose to model this aggregation system since it significantly impacts the frames delay. The network calculus theory is used to compute worst-case performances. Rather than considering the aggregates in a macroscopic way and providing global performances indicators, this paper aims at defining the service offered to each incoming flow by evaluating backlogs and end-to-end delays.

Keywords: performance evaluation, aggregation system, network calculus, IEEE 802.11n

1. INTRODUCTION

Different ways exist to carry goods: individually (which implies as many travel as goods) or grouped. This is true in logistic as in communication networks for data transmission. Let us consider a truck with a half full trailer. An opposition appears between carrier and customer requirements. The first one wants to wait in order to fill the trailer. The second one prefers the truck to leave immediately in order to be delivered as soon as possible. Information is needed to make a good choice. What delay can the customer tolerate? When do the next goods arrive? What kind of goods (size, . . .)? There is a compromise to find between trailer filling and customer requirements. Two thresholds can be defined: a maximum leaving date and a minimum filling level. We name *aggregation system*, a system which stores things before sending them according to these two thresholds. Our purpose is to evaluate its performances from a specific customer point of view.

This kind of system exists in some communication networks protocols. It is used in the Synchronous Digital Hierarchy (SDH) and in the Asynchronous Transfer Mode (ATM) for example. This system could also be used in a wireless sensor network, where measurements done by several nodes are gathered by a cluster head before being together forwarded to a sink. Aggregating offers two advantages: improve the network lifetime by reducing the number of transmissions, and minimize the consumed bandwidth. IEEE-802.11n (2012) standard includes an aggregation system which is our case study. The 802.11 Working Group initial purpose was to reach a 100 Mbps data throughput. Many improvements have been made on the physical layer to achieve this goal. Nevertheless, frames aggregation remains one of the major enhancements made on the Medium Access Control (MAC) layer that enables to reach this bit rate. This system is located at the input of the MAC layer where there is the most to gain, because of collisions or variable bit rate which is a function of the distance to the access point. Therefore, when the medium is won, it is interesting to keep it as long as possible in order to transmit the maximum amount of data. Aggregation is a way to reduce the number of transmissions and thus the number of competitions for the medium access.

Some authors have studied the 802.11n aggregation performances by using various theories:

- Queuing theory (throughput, channel utilization, aggregate latency) Lin and Wong (2006); Kuppa and Dattatreya (2006).
- Analytic study (throughput) Ginzburg and Kesselman (2007).
- Simulations (channel utilization, aggregate latency) Skordoulis et al. (2008); Wang and Wei (2009).

These works suffer from several drawbacks. First, chosen performances indicators are macroscopic. They mainly deal with the delay for an aggregate to cross a given topology. It is then impossible to quantify the impact of the aggregation system on one of the aggregated flows. Aggregating isn't always appropriate, especially when considering critical applications. Indeed, some frames are voluntarily delayed, in order to wait for others with which they will be aggregated. Furthermore, many authors provide average values for performance indicators (delay, throughput), when worst case values may be required. Obtaining *a priori* bounds is necessary for critical applications. Some authors computed them like Ginzburg and Kesselman

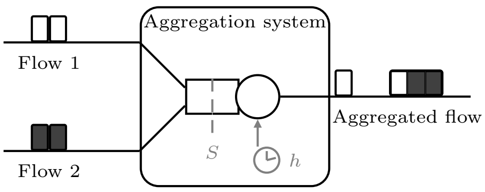

Fig. 1. Functional model of aggregation system

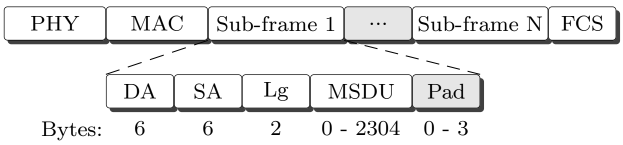

Fig. 2. A-MSDU frame format

(2007). However, their results need a detailed knowledge of incoming traffics which is often impossible to get in practice. Indeed, even in a simple case study, the number of scenarios increases quickly without any hypothesis on the input flows (packets size, inter-arrival, jitter, ...) and on the system thresholds.

To overcome this multitude of cases, we propose to use the network calculus theory Cruz (1991); Chang (2000); Le Boudec and Thiran (2001). The network calculus enables to compute delay and backlog bounds by using a worst case analysis. Our study can help to decide if using an aggregation system is compliant with the applications constraints.

Section 2 defines precisely the aggregation system in 802.11n. In section 3 we remind the fundamentals of the network calculus, we propose a model of the service provided to a particular flow, and we discuss its relevance. Section 4 gives delay and backlog bounds. We also compare these bounds with the values obtained by using the method promoted by Ginzburg and Kesselman (2007).

2. AGGREGATION SYSTEM

The aggregation system collects frames issued from different flows and creates a new "super flow". Cumulated data are sent in the network according to two criteria. On one hand if cumulated data reaches a size threshold. On the other hand if the time spent in the system by the first collected frame reaches a temporal threshold. The Fig. 1 represents the aggregation system with the size threshold s and h the temporal one. It aggregates several Mac Service Data Units (MSDU) to form an Aggregated-MSDU (A-MSDU) encapsulated in a 802.11n frame (Fig. 2). One of the purpose of the aggregation was to provide better compatibility between IEEE 802.3 (Ethernet) and IEEE 802.11n (Wi-Fi n) standards. There are Ethernet pseudo-headers in the aggregation headers which have been preserved in order to reduce switching latencies.

It is important to note that our study is restricted to an aggregation system which only considers the maximal size threshold S.

3. MODELLING OF AGGREGATION SYSTEMS

3.1 Introduction to network calculus

The network calculus fundamentals can be found in Cruz (1991), Chang (2000) and Le Boudec and Thiran (2001). The network modelling is achieved by non-negative and non-decreasing functions which characterize an amount of data at a time t.

The first use of such functions is for the input $R(t)$ and the output $R^*(t)$, which cumulatively count the number of bits that respectively enter and go out of a system. As $R(t)$ is unknown, the network calculus introduces the concept of arrival curve as follows. Given a flow with an input function $R(t)$, a wide-sense increasing function α is an arrival curve for $R(t)$ if and only if:

$$\forall t, s \geq 0,\ t \geq s : R(t) - R(t-s) \leq \alpha(s)$$

In the same way, the service offered by a system to a flow is modelled by a (non-negative and non-decreasing) minimum service curve β if $\beta(0) = 0$ and:

$$\forall t \geq 0,\ R^*(t) \geq \inf_{s \leq t} \{R(s) + \beta(t-s)\}$$

In addition, β is called a strict service curve for the system if during any backlogged period $]s,t]$, at least $\beta(t-s)$ data is served. These arrival and service curves allow to compute the following bounds.

Theorem 1. (Chang (2000); Le Boudec and Thiran (2001)). Assume a flow, constrained by arrival curve α, traverses a system that offers a service curve of β ($\alpha \leq \beta$) and that stores input data in a FIFO-ordered queue. The backlog b and the delay d are defined as:

$$b(t) \leq \sup\{s \geq 0 \mid \alpha(s) - \beta(s)\} \quad (1)$$

$$d(t) \leq \sup\{s \geq 0 \mid \inf\{\tau \geq 0 \mid \alpha(s) \leq \beta(s+\tau)\}\} \quad (2)$$

Lemma 2. (Residual service: Le Boudec and Thiran (2001); Schmitt et al. (2008)). Let us consider a server node offering a strict service curve β to two input flows, with respective arrival curves α_1 and α_2. A service curve for the flow 1 is:

$$\beta_1 = (\beta - \alpha_2)^+ = \max(0, \beta - \alpha_2) \quad (3)$$

In this section, our purpose is to compute a service curve for a 802.11n aggregation system. This curve represents the service provided to a particular input flow. Our study highlights limitations of the residual service (lemma 2).

3.2 Aggregate departure

Let us consider the aggregation system of Fig. 1. Assume first that the incoming traffic is merged in a "super flow" formed by the two flows such as $R(t) = R_1(t) + R_2(t)$.

Assumption 3. The aggregation system sends an aggregate with a throughput C when its backlog is greater than or equal to a maximal size threshold S. The aggregate's size varies between $S - l_{max}$ and S, with l_{max} the maximum packet size at the system input.

As shown in Fig. 3 the backlog varies with frames arrival and decreases only when reaching the critical size S, i.e $R(t) - R^*(t) \geq S$. Consequently, this system is non work conserving and its service varies with data arrival (unlike the systems with a constant bit rate service $\beta(t) = Ct$).

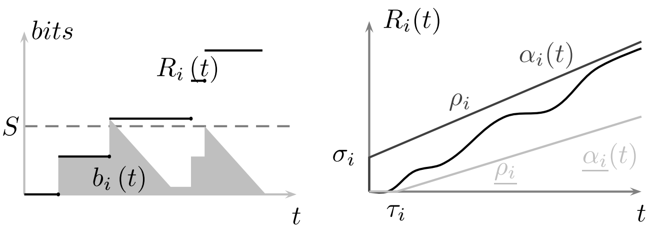

Fig. 3. Backlog and incoming traffic envelopment

As sending an aggregate depends on the amount of input traffic, we propose to low-bound the input flows with a minimal arrival curve like in Real-Time Calculus (Thiele et al. (2000)). In this study, we assume to consider only affine arrival curves (Fig. 3). Thus $\forall 0 \leq s \leq t$:

$$\underline{\alpha_i}(t-s) \leq R_i(t) - R_i(s) \leq \alpha_i(t-s)$$
$$\underline{\rho_i}(t-s-\tau_i)^+ \leq R_i(t) - R_i(s) \leq \sigma_i + \rho_i(t-s) \qquad (4)$$

The arrival curves parameters α_i and ρ_i may be deduced from the output of some traffic regulators such as a leaky bucket. In the same way, $\underline{\alpha_i}$ and $\underline{\rho_i}$ are deduced from the traffic generation characteristics. The model of the service offered to the "super flow" is given below.

Proposition 4. Given an aggregation system with two input flows. Each of them is constrained by two arrival curves $\underline{\alpha_k}(t)$ and $\alpha_k\,(t)$ with $k = 1, 2$. The service curve $\beta\,(t)$ becomes:

$$\beta\,(t) = R\,(t-\Delta)^+ \qquad (5)$$

with a rate $R = \underline{\rho} = \underline{\rho_i} + \underline{\rho_j}$ with $i = \arg\min_{i=1,2} \tau_i$ and $j = \arg\max_{j=1,2} \tau_j$, and a latency $\Delta = \tau_i + S/\underline{\rho_i}$ if $S/\underline{\rho_i} \leq \tau_j - \tau_i$ or else $\Delta = \tau_i + \Big(S + \underline{\rho_j}\,(\tau_j - \tau_i)\Big) \,/\, \Big(\underline{\rho_i} + \underline{\rho_j}\Big)$.

Two variables must be computed in this model, the service rate R and the latency Δ. Our reasons for choosing an rate-latency service curve and the computation of R can be found in appendix A. The computation of the maximal time between the departure of two aggregates, Δ can be found in appendix B.

The minimal service rate R is expressed as the sum of the minimal arrival rates. This is one of the particularity of the aggregation systems, because the offered service depends on the incoming traffic. Thus, the other flows which are usually considered as rivals may be now recognized as friends, since they enable the aggregate to be sent earlier. It is also interesting to note that the service rate and the temporal performances are bad for a flow if it is not "helped" by an other incoming flow. This is an example of the opposition between customer and provider interests.

3.3 Service dedicated to a flow

After having formulated the service offered to $R_1(t)+R_2(t)$ (proposition 4), we will now identify a service curve noted $\beta_1\,(t)$ for $R_1(t)$ only. As our service curve from proposition 4 is strict by definition of Δ, applying the residual service lemma (equation (3)) leads to:

$$\beta_1(t) = (\beta(t) - \alpha_2(t))^+ = \big((\underline{\rho_1} + \underline{\rho_2})(t-\Delta) - (\sigma_2 + \rho_2 t)\big)$$
$$\forall t > \frac{\sigma_2 + \Delta(\underline{\rho_1} + \underline{\rho_2})}{\underline{\rho_1} + \underline{\rho_2} - \rho_2} \qquad (6)$$

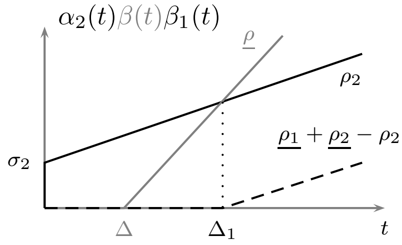

Fig. 4. Service curve dedicated to flow 1

It means that the maximal delay before the flow 1 begins to be served is:

$$\Delta_1 = \frac{\sigma_2 + \Delta(\underline{\rho_1} + \underline{\rho_2})}{\underline{\rho_1} + \underline{\rho_2} - \rho_2}$$

We can also identify the service rate for the flow 1:

$$\beta_1(t) = \big((\underline{\rho_1} + \underline{\rho_2})(t-\Delta) - (\sigma_2 + \rho_2 t)\big)$$
$$= \big(\underline{\rho_1} + \underline{\rho_2} - \rho_2\big)\,(t-\Delta) - \sigma_2$$

When $t > \Delta_1$, the service offered to the flow 1 is $\underline{\rho_1} + \underline{\rho_2} - \rho_2$. The Fig. 4 shows the graphical resolution of the equation (6). Below is the proposition arising from these developments.

Proposition 5. The aggregation system defined in the proposition 4 offers to the flow $R_1\,(t)$ a minimal service curve:

$$\beta_1(t) = (\underline{\rho_1} + \underline{\rho_2} - \rho_2)\,(t - \Delta_1)^+ \qquad (7)$$

Since the forwarding rate might be inferior to the arrival rate (i.e $\underline{\rho_1}+\underline{\rho_2}-\rho_2 \leq \rho_1$) a stability condition is necessary: $\underline{\rho_i} = \rho_i$. So, the service rate for the flow 1 becomes ρ_1. It means that the worst service rate is obtained when an flow is alone. Besides, the delay Δ_1 doesn't ensure that data from flow 1 was arrived. Furthermore, Δ_1 is increased by a flow 2 burst which doesn't appear in the expression of $\beta(t)$. To avoid the pessimism introduced by these observations we propose to replace Δ_1 in equation (7).

We search $\Delta_1' \geq \Delta$, the maximal inter-leaving time between two aggregates that contain data from flow 1. Following the FIFO delivery, if the flow 1 arrives before the flow 2 ($\tau_1 < \tau_2$), then $\Delta_1' = \Delta$. Otherwise, it means that we can have $\Delta \leq \tau_1$ and it is necessary to wait for the next aggregate (sent after τ_1). This new inter-leaving time Δ_1' consists in two parts: the delay time τ_1 and the departure date of the first aggregate including at least a packet of the flow 1. This last part depends on the threshold S, on the maximum number of aggregates only based on the flow 2 $\lfloor \frac{\tau_1-\tau_2}{S/\rho_2} \rfloor$, and on the quantity of data of the flow 2 continuously arriving $\rho_2(\tau_1 - \tau_2)$. It gives:

$$\Delta_1' = \tau_1 + \frac{S - \rho_2(\tau_1 - \tau_2) + \lfloor \frac{\tau_1-\tau_2}{S/\rho_2} \rfloor S}{\rho_2 + \rho_1}$$

We have all the elements for a new proposition.

Proposition 6. The aggregation system defined in the proposition 4 offers to the flow $R_1\,(t)$ a minimal service curve:

$$\beta_1'(t) = \rho_1\,(t - \Delta_1')^+ \;\text{ with }\; \Delta_1' = \max\,(\Delta, \delta_1')$$

3.4 Discussion

Let us consider the case study of the Fig. 5. Two traffics

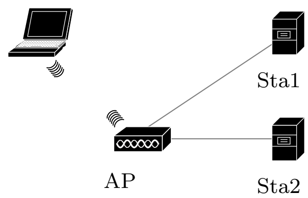

Fig. 5. The topology of our case study

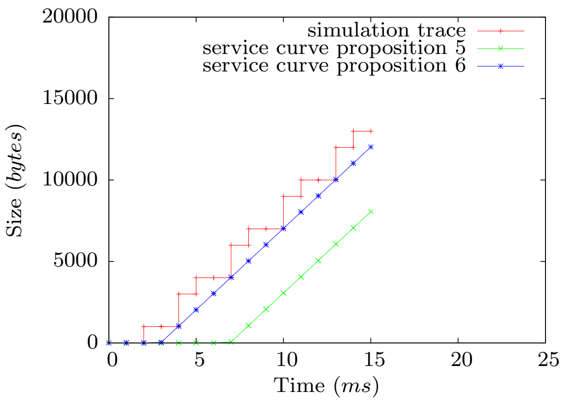

Fig. 6. $\beta_1'(t)$ and $\beta_1(t)$ compared to a simulation trace

are characterized with a 1 ms period, 1000 bytes input packets, and a size threshold of 3839 bytes (defined in 802.11n). We will compare the efficiency of propositions 5 and 6 with values provided by a very simple simulator we have developed which reproduces the aggregation system behaviour. This tool generates a trace of the aggregates (departure time and size).

Arrival curves are calculated according to packet size and minimal inter-arrival time as mentioned in Chakraborty et al. (2000). The minimal arrival rate is $\underline{\rho_i} = \rho_i = 8\ Mbps$ and the maximal burst size is $\sigma_i = 1000$ bytes.

The Fig. 6 pictures the service dedicated to a flow following the propositions 5 and 6 compared to the results given by the trace. It shows that for this scenario, the proposition 6 is the closest to the simulation. Residual service leads here to a modeling pessimism. System performances for an user-flow are better when other flows give traffic. The residual service only considers other flows as rivals. That leads to a pessimistic approximation of the service for one flow. This is why we propose an upgrade of the residual service which reduces the pessimism for such systems.

4. DELAY AND BACKLOG BOUNDS

4.1 QoS bounds computation

Let us now compute the backlog and delay bounds by using equations (1) and (2). The Fig. 7 shows that they correspond respectively to the vertical and the horizontal maximal distance between the arrival and service curves. The vertical distance is maximal at $t = T$ as shown in Fig. 7. Hence $T = \Delta$, and the backlog bound is then,

$$b = \alpha(\Delta) - \beta(\Delta) = \sigma_1 + \sigma_2 + \Delta(\rho_1 + \rho_2) \qquad (8)$$

To return to the introduction analogy, the backlog bound computation sizes the trailer filling. In 802.11n, this value sizes the transmission buffers.

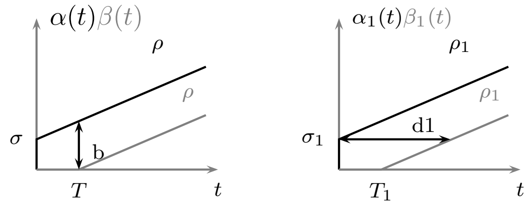

Fig. 7. Backlog and delay bounds computation

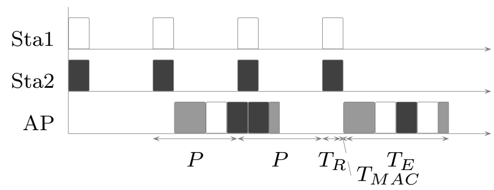

Fig. 8. A possible execution scenario

We want to compute the delay bound crossing the aggregation system. It is computed for each sub-frame of the aggregate using the service curve dedicated to a flow. As shown in Fig. 7, the delay bound is:

$$d_1(t) \leq \sup\{s \geq 0 \mid \inf\{\tau \geq 0 \mid \forall t \geq 0,\ \beta_1(s+\tau) \geq \sigma_1\}\}$$
$$d_1 = \frac{\sigma_1}{\rho_1} + \Delta_1$$

4.2 Discussion

We consider the same scenario as in section 3.4 to illustrate the bounds pessimism. That is why we need a reference value which is computed by summing the elementary latencies of the system like done by Ginzburg and Kesselman (2007). As shown in Fig. 8, we sum the time to receive all packets required to create an aggregate $(P + T_R)$, the time to send the aggregate (T_E) and the MAC layer latency (T_{MAC}) set to 2 μs (IEEE-802.11n (2012)).

There are 14 bytes of MSDU headers and 18 bytes of MAC headers. Our reference value is then,

$$D = 2P + T_R + T_E + T_{MAC} = 2.52ms$$

Let us now compute d_1. Here $s/\underline{\rho_i} > \tau_j - \tau_i$, then,

$$\Delta = \tau_2 + (s + \rho_1(\tau_1 - \tau_2)) / (\rho_1 + \rho_2).$$

We obtained $d_1 = 3.91\ ms$, with $\tau_1 = \tau_2 = 1\ ms$, $\rho_1 = \rho_2 = 1000$ bytes/ms and $\sigma_2 = 1000$ bytes. The delay d_1 well bounds the reference value with an overestimation of about 35 % with D. This might indicate that our hypothesis are too pessimistic or that our arrival model is not precise enough. However, the residual service gives $d_1 = 7.82\ ms$ which is more than 3 times superior to D. In fact, most of our pessimism is due to the maximal arrival latency τ_1 used in our model while in the scenario, the flow 1 arrived immediately $(\tau_1 = 0)$.

Moreover, this result applies only on this particular scenario. Let us now consider the input frames defined in the table bellow (compatible with the minimal arrival curves previously defined).

time (ms)	0	1	1.5	2.1	2.5	2.85
flow	1	2	2	1	1	2
size (bytes)	1000	500	1300	250	650	139

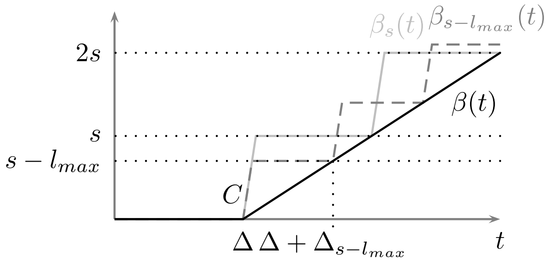

Fig. A.1. Service curves for different aggregate size

The reference value becomes $D = 3.16$ ms which reduces the overestimation to 20 %. It is mostly due to the traffic modeling quality and to the method for the delay computation. Indeed, the delay bound is the horizontal distance between the service curve based on the minimal and maximal arrival curves. There is 0.75 ms between the computed bound and the trace which is not so much compared to the 2 ms gap between the maximal and minimal arrival curves (the uncertainty on the frame arrival time). Furthermore, the error on the burst for the considered flow can impact the pessimism. The lower the gap between maximal and minimal arrival curves is, the greater the precision of the delay bound is.

The backlog bound computation gives 7839 bytes. That is more than twice the simulated backlog of 3000 bytes. For flow 1, the backlog bound is 3919 bytes compared to the 2000 bytes given by the simulation. It is not very precise, but when using the residual service curve we obtain 7839 bytes. All these values come to support the conclusions made on the delay bound.

5. CONCLUSION AND OUTLOOK

We have proposed a first model of the service dedicated to one flow in a 802.11n aggregation system. We have identified two service curves for this system: a service curve for all input flows enabling to compute a backlog bound, and a service curve for a particular flow enabling to compute a delay bound. The residual service defined in the network calculus was not compliant with our expectations. We have then refined it to significantly reduce the pessimism introduced when modeling the aggregation system. We have compared our delay bound with two reference values issued from our scenario. Using simulations tools is interesting to improve our service curves.That is why we have developed a very simple one in the expectation of the implementation of the 802.11n aggregation mechanism now available in the Riverbed Opnet Modeler version 17.5 simulator. Future works will take into account the temporal threshold. We will evaluate the bounds pessimism evolution when the system has n input flows. This generalization could introduce a difficulty caused by the chosen scheduling for the $n-1$ rival flows.

Appendix A. MINIMAL SERVICE RATE R

Consider that aggregates are transmitted at a constant throughput C and at each Δ time units (pictured in Fig. A.1). Without fragmentation (like in 802.11n), aggregate's size can vary in $]s - l_{max}, s]$. Indeed, the worst case occurs if last packet has a maximal length l_{max} and if this packet leads backlog to exceed S of one bit. The

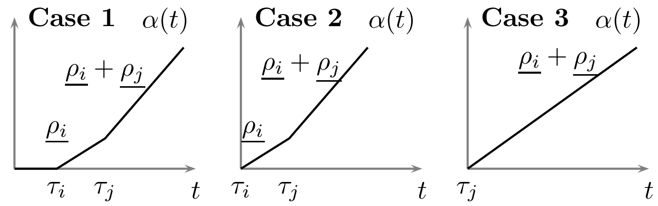

Fig. B.1. Considered cases

packet is then excluded from the aggregate. The two curves in Fig. A.1 represent alternatively worst case and best case. That is why we propose to use the rate-latency curve $\beta(t) = R(t-\Delta)$, with the expression of Δ explained in appendix B and R formulated below:

$$R = \frac{S - l_{max}}{\frac{S-l_{max}}{\underline{\rho}}} = \frac{S}{\frac{S}{\underline{\rho}}} = \underline{\rho} = \underline{\rho_1} + \underline{\rho_2}$$

This proposition assumes that the following stability condition is respected: $C \gg \sum_i \underline{\rho_i}$.

Appendix B. MAXIMAL DEPARTURE TIME BETWEEN TWO AGGREGATES, Δ

Remember that the backlog is defined as $b(t) = R(t) - R^*(t)$. Assume that during the interval $[t, t+\Delta]$ there is no offered service, the value Δ corresponds to the maximum value of δ such that we have (with $k \in \mathbb{N}$):

$$\Delta \leq \sup_{t\geq 0}\left\{\inf_{\delta\geq 0}\left\{\sum_{k=i}^{j}\left(R_k\left(t+\delta\right) - R_k^*\left(t+\delta\right)\right) \geq S\right\}\right\}$$

$\forall 0 \leq \delta \leq \Delta$, not enough traffic has been received to send an aggregate, we have thus $R_k^*(t+\delta) = R_k(t)$ and from equation (4):

$$\Delta \leq \sup_{t\geq 0}\left\{\inf_{\delta\geq 0}\left\{\sum_{k=i}^{j}\left(\underline{\alpha_k}\left(t+\delta\right) - \underline{\alpha_k}\left(t\right)\right) \geq S\right\}\right\} \quad \text{(B.1)}$$

As shown by Fig. B.1, different formula could be considered for $\underline{\alpha_k}(t)$ for a given t. We have hence:

Case 1: $\forall 0 \leq t < \tau_i$, there may be no traffic

$$\Delta \leq \sup_{0\leq t<\tau_i}\left\{\inf_{\delta\geq 0}\left\{\sum_{k=i}^{j}\left(\underline{\rho_k}\left(t+\delta-\tau_k\right)^+\right) \geq S\right\}\right\}$$

Case 1.1: $\forall S/\underline{\rho_i} \leq \tau_j - \tau_i$, flow i has enough traffic to reach the threshold

Here, $\underline{\alpha_i}\left(\tau_i + S/\underline{\rho_i}\right) = S$ whereas $\underline{\alpha_j}\left(\tau_i + S/\underline{\rho_i}\right) = 0$. Then, the threshold is reached for $\underline{\rho_j}\left(t+\delta-\tau_j\right)^+ = 0$.

$$\Delta \leq \sup_{0\leq t<\tau_i}\left\{\inf_{\delta\geq 0}\left\{\underline{\rho_i}\left(t+\delta-\tau_i\right) \geq S\right\}\right\} \leq \frac{S}{\underline{\rho_i}} + \tau_i$$

Case 1.2: $\forall S/\underline{\rho_i} > \tau_j - \tau_i$, flow i hasn't enough traffic to reach the threshold

It means that $\underline{\alpha_i}(\tau_j) < S$ and thus δ exists only if $\underline{\rho_j}\left(t+\delta-\tau_j\right)^+ = \underline{\rho_j}\left(t+\delta-\tau_j\right)$.

$$\Delta \leq \sup_{0\leq t<\tau_i}\left\{\inf_{\delta\geq 0}\left\{\sum_{k=i}^{j}\left(\underline{\rho_k}\left(t+\delta-\tau_k\right)\right) \geq S\right\}\right\}$$
$$\leq \frac{S}{\underline{\rho_i}+\underline{\rho_j}} + \frac{\underline{\rho_i}\tau_i + \underline{\rho_j}\tau_j}{\underline{\rho_i}+\underline{\rho_j}} = \tau_i + \frac{S + \underline{\rho_j}\left(\tau_j - \tau_i\right)}{\underline{\rho_i}+\underline{\rho_j}}$$

Case 2: $\forall \tau_i \leq t < \tau_j$*, there may be traffic from flow* i *only*

$$\Delta \leq \sup_{\tau_i \leq t < \tau_j} \left\{ \inf_{\delta \geq 0} \left\{ \underline{\rho_i}\delta + \underline{\rho_j}\left(t + \delta - \tau_j\right)^+ \geq S \right\} \right\}$$

Case 2.1: $\forall S/\underline{\rho_i} \leq \tau_j - \tau_i$*, flow* i *has enough traffic to reach the threshold*

The inequality becomes:

$$\Delta \leq \sup_{\tau_i \leq t < \tau_j - \frac{S}{\underline{\rho_i}}} \left\{ \inf_{\delta \geq 0} \left\{ \underline{\rho_i}\delta + \underline{\rho_j}\left(t + \delta - \tau_j\right)^+ \geq S \right\} \right\}$$
$$\vee \sup_{\tau_j - \frac{S}{\underline{\rho_i}} \leq t < \tau_j} \left\{ \inf_{\delta \geq 0} \left\{ \underline{\rho_i}\delta + \underline{\rho_j}\left(t + \delta - \tau_j\right)^+ \geq S \right\} \right\} \tag{B.2}$$

For the first supremum in equation B.2, we have $\tau_j - t > S/\underline{\rho_i}$. Applying $\underline{\alpha_i}$ to this inequality gives $\underline{\alpha_i}\left(\tau_j\right) - \underline{\alpha_i}\left(t\right) > S$. As $\underline{\alpha_j}\left(\tau_j\right) = 0$, δ exists $\forall t < \tau_j - S/\underline{\rho_j}$ and $\underline{\rho_j}\left(t + \delta - \tau_j\right)^+ = 0$.

Contrariwise if $\tau_j - t \leq S/\underline{\rho_i}$, then $\underline{\alpha_i}\left(\tau_j\right) - \underline{\alpha_i}\left(t\right) \leq S$ and δ exists if $\underline{\rho_j}\left(t + \delta - \tau_j\right)^+ = \underline{\rho_j}\left(t + \delta - \tau_j\right)$.

$$\Delta \leq \sup_{\tau_i \leq t < \tau_j - \frac{S}{\underline{\rho_i}}} \left\{ \inf_{\delta \geq 0} \left\{ \underline{\rho_i}\delta \geq S \right\} \right\}$$
$$\vee \sup_{\tau_j - \frac{S}{\underline{\rho_i}} \leq t < \tau_j} \left\{ \inf_{\delta \geq 0} \left\{ \underline{\rho_i}\delta + \underline{\rho_j}\left(t + \delta - \tau_j\right) \geq S \right\} \right\}$$
$$\leq \frac{S}{\underline{\rho_i}} \vee \sup_{\tau_j - \frac{S}{\underline{\rho_i}} \leq t < \tau_j} \left\{ \frac{S - \underline{\rho_j}\left(t - \tau_j\right)}{\underline{\rho_i} + \underline{\rho_j}} \right\} \leq \frac{S}{\underline{\rho_i}}$$

Case 2.2: $\forall S/\underline{\rho_i} > \tau_j - \tau_i$*, flow* i *hasn't enough traffic to reach the threshold*

Here δ exists if $\underline{\rho_j}\left(t + \delta - \tau_j\right)^+ = \underline{\rho_j}\left(t + \delta - \tau_j\right)$. Hence:

$$\Delta \leq \sup_{\tau_i \leq t < \tau_j} \left\{ \inf_{\delta \geq 0} \left\{ \underline{\rho_i}\delta + \underline{\rho_j}\left(t + \delta - \tau_j\right) \geq S \right\} \right\} \leq \frac{S}{\underline{\rho_i}}$$

Case 3: there is traffic from both flows ($t \geq \tau_j$)

$$\Delta \leq \sup_{t \geq \tau_j} \left\{ \inf_{\delta \geq 0} \left\{ \sum_{k=i}^{j} \left(\underline{\alpha_k}\left(t + \delta\right) - \underline{\alpha_k}\left(t\right) \right) \geq S \right\} \right\}$$
$$\leq \inf_{\delta \geq 0} \left\{ \delta \left(\underline{\rho_i} + \underline{\rho_j} \right) \geq S \right\} \leq \frac{S}{\underline{\rho_i} + \underline{\rho_j}}$$

Results can be summarized as follow:

If flow i has enough traffic to reach the threshold, ($S/\underline{\rho_i} \leq \tau_j - \tau_i$):

$$\begin{array}{ll} \Delta \leq \tau_i + S/\underline{\rho_i} & \text{if } 0 \leq t < \tau_i \\ \Delta \leq S/\underline{\rho_i} & \text{if } \tau_i \leq t < \tau_j - S/\underline{\rho_i} \\ \Delta \leq \left(S + \underline{\rho_j} S/\underline{\rho_i} \right) / \left(\underline{\rho_i} + \underline{\rho_j} \right) & \text{if } \tau_j - S/\underline{\rho_i} \leq t < \tau_j \\ \Delta \leq S/\left(\underline{\rho_i} + \underline{\rho_j} \right) & \text{else } (t \geq \tau_j) \end{array}$$

else if both flows are needed to reach the threshold ($S/\underline{\rho_i} > \tau_j - \tau_i$):

$$\begin{array}{ll} \Delta \leq \tau_i + \left(S + \underline{\rho_j}\left(\tau_j - \tau_i\right) \right) / \left(\underline{\rho_i} + \underline{\rho_j} \right) & \text{if } 0 \leq t < \tau_i \\ \Delta \leq \left(S + \underline{\rho_j}\left(\tau_j - \tau_i\right) \right) / \left(\underline{\rho_i} + \underline{\rho_j} \right) & \text{if } \tau_i \leq t < \tau_j \\ \Delta \leq S/\left(\underline{\rho_i} + \underline{\rho_j} \right) & \text{else } (t \geq \tau_j) \end{array}$$

The maximal value of Δ is obtained when $0 \leq t < \tau_i$:

$$\begin{array}{lll} \Delta & = \tau_i + S/\underline{\rho_i} & \text{if } S/\underline{\rho_i} \leq \tau_j - \tau_i \\ & = \tau_i + \dfrac{\left(S + \underline{\rho_j}\left(\tau_j - \tau_i\right) \right)}{\left(\underline{\rho_i} + \underline{\rho_j} \right)} & \text{else } (S/\underline{\rho_i} > \tau_j - \tau_i) \end{array}$$

REFERENCES

Chakraborty, S., Kunzli, S., and Thiele, L. (2000). A general framework for analysing system properties in platform-based embedded system designs. 190–195. IEEE Comput. Soc.

Chang, C.S. (2000). *Performance guarantees in communication networks.* Telecommunication networks and computer systems. Springer, London ; New York.

Cruz, R. (1991). A calculus for network delay. 1 network elements in isolation. *IEEE Transactions on Information Theory*, 37(1), 114–131.

Ginzburg, B. and Kesselman, A. (2007). Performance analysis of a-MPDU and a-MSDU aggregation in IEEE 802.11n. 1–5. IEEE.

IEEE-802.11n (2012). Standard for information technology. Telecommunications and information exchange between systems local and metropolitan area networks. specific requirements part 11: Wireless LAN medium access control (MAC) and physical layer (PHY) specifications.

Kuppa, S. and Dattatreya, G. (2006). Modeling and analysis of frame aggregation in unsaturated WLANs with finite buffer stations. 967–972. IEEE.

Le Boudec, J.Y. and Thiran, P. (2001). *Network calculus: a theory of deterministic queuing systems for the Internet.* Number 2050 in Lecture notes in computer science. Springer, Berlin ; New York.

Lin, Y. and Wong, V.W.S. (2006). WSN01-1: frame aggregation and optimal frame size adaptation for IEEE 802.11n WLANs. 1–6. IEEE.

Schmitt, J.B., Zdarsky, F.A., and Fidler, M. (2008). Delay bounds under arbitrary multiplexing: When network calculus leaves you in the lurch... 1669–1677. IEEE.

Skordoulis, D., Ni, Q., Chen, H.H., Stephens, A., Liu, C., and Jamalipour, A. (2008). IEEE 802.11n MAC frame aggregation mechanisms for next-generation high-throughput WLANs. *IEEE Wireless Communications*, 15(1), 40–47.

Thiele, L., Chakraborty, S., and Naedele, M. (2000). Real-time calculus for scheduling hard real-time systems. volume 4, 101–104. Presses Polytech. Univ. Romandes.

Wang, C.Y. and Wei, H.Y. (2009). IEEE 802.11n MAC enhancement and performance evaluation. *Mobile Networks and Applications*, 14(6), 760–771.