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Adaptive non-linear control of three-phase four-wire Shunt active power filters for unbalanced and non-linear loads.

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Abstract: The problem of controlling three-phase, four-leg shunt active power filters (SAPF) is addressed in presence of nonlinear and/or unbalanced loads. A new control model for four-leg SAPF, taking into account for the electrical grid line impedance, is developed. The control objective is threefold: (i) compensating for the current harmonics and the reactive power absorbed by the nonlinear load; (ii) annulations of neutral line current (iii) regulating the inverter DC capacitor voltage. To this end, based on the new model, a nonlinear controller is developed, using the Lyapunov technical design. It is therefore able to ensure good performances over a wide range variation of the load current. Moreover, the controller is made adaptive for compensating the uncertainty on the switching loss power. The performances of the proposed adaptive controller are formally analyzed using tools from the Lyapunov stability and averaging theory. The supremacy of the proposed controller with respect to standard control solutions is illustrated through simulation

Keywords: Three phase four wire distribution systems, 4 Leg APFs, Harmonics and reactive currents compensation, Neutral line current, p-q-r reference frames, Lyapunov stability, Average analysis.

1. INTRODUCTION

Power grids are generally distributed through a structure with three phase four-wire. In several industrial and residential applications, the non linear and unbalanced loads (such as switching power supplies, motor drivers, single or three phase rectifiers,...) brings a serious power quality problems and network performances degradation. Indeed, non linear loads involve nonlinear dynamics that entail the generation of current harmonics and the consumption of reactive power. Moreover, unbalanced loads produce a neutral current which can reach large amplitudes. If not appropriately compensated for, the current harmonics, the reactive power and the neutral current are likely to cause several harmful effects e.g. the distortion of the voltage waveform at the point of common coupling (PCC), and the overheating of neutral conductor, transformers and distribution lines. In addition the neutral current makes harder the synchronization with voltage network in application requiring such synchronization (D.Sreenivasarao et al., 2012).

The three-phases APF has sparked a great deal of interest recently to cope with harmonics current, reactive power and unbalanced load. There are various APF configurations but the most widely implemented in industrial scale products are the shunt configurations (El Habrouk et al., 2000). The principle of shunt active power filters (SAPF) is to cancel the harmonics and reactive currents (generated by the non linear loads) and the neutral current (generated by the unbalanced loads) by injecting a compensation current at the PCC (Fig. 1)). In addition, there is a third operational voltage control objective to be achieved. Accordingly, the DC voltage at the energy storage capacitor, placed next to the SAPF inverter, must be tightly regulated in order to make the SAPF conveniently operate.

A great deal of interest has been paid to this control problem over the last decade. However, most available solutions are limited to the case of three-phase three-Leg SAPF (Naimish Zaveri et al, 2012). The point is that three-Leg SAPFs are only useful in balanced load applications. Indeed they cannot meet the objective of cancelling the current in the neutral line. To overcome this shortage the Four-leg APFs are introduced.

The problem of controlling three-phase four-leg SAPF has been dealt with following three approaches. The first consists in using hysteresis operators or fuzzy logics (Chakphed et al., 2002). These methods do not make use of the exact nonlinear for-leg SAPF model in the control design. Consequently, the obtained controllers are generally not backed by formal stability analysis and their performances are generally illustrated by simulations. The second control approach consists in using linear controllers (e.g. Li Bin, Tong Minyong 2011). With linear controllers, optimal performances cannot be guaranteed on a wide range variation of the operation point, due to the nonlinear nature of the controlled system dynamics. The third category includes non-linear controllers, designed based on the system accurate nonlinear models. The control design techniques used a passivity approach, sliding mode control and Lyapunov design (Juming CHEN et al., 2006, Farid Hamoudi et al, 2011, Bouzidi Mansour et al, 2013). However, the proposed nonlinear controllers have only consisted in current loops designed to meet the harmonic and neutral current compensation requirement. Without an explicit voltage regulation loop, the DC voltage regulation objective cannot be achieved in general operation conditions. Besides, the previously proposed nonlinear controllers are designed assuming that line impedance and the switching losses in the inverter to be negligible. The point is that the line impedance introduces in practice a voltage deformation at the PCC and the switching losses act on the DC bus voltage.

This paper is devoted for the control of energy systems that involve four-legs SAPFs with nonlinear and unbalanced loads. A new control strategy is developed to simultaneously meet the previously discussed control requirement

- A satisfactory compensation of harmonics, and reactive currents absorbed by nonlinear loads.
- Cancelation of neutral current generated by unbalanced loads
- A tight regulation of the inverter DC capacitor voltage.

To this end, a two-loop cascade nonlinear controller is developed using the Lyapunov techniques. The inner loop involves a current regulator designed to cope with harmonics compensation and neutral current cancelation. The outer-loop involves a voltage regulator that aims at regulating the DC line voltage. Furthermore, the controller is provided with a parameter adaptation capability to cope with the uncertainty that prevails on the switching losses in the inverter. It is formally shown using tools from Lyapunov stability and system averaging theory that, all control objectives are actually achieved. This theoretical result confirmed by numerical simulations, which illustrate further controller features

The paper is organized as follows: the four-legs SAPF modeling, is described in section 2; the harmonic, reactive and neutral currents references extraction is dealt with in section 3; the control problem formulation, and the adaptive nonlinear controller design is achieved in section 4, the theoretical analysis results are confirmed by simulation in section 5. A conclusion and a references list end the paper.

2. MODELING OF THREE-PHASE FOUR-WIRE SAPF

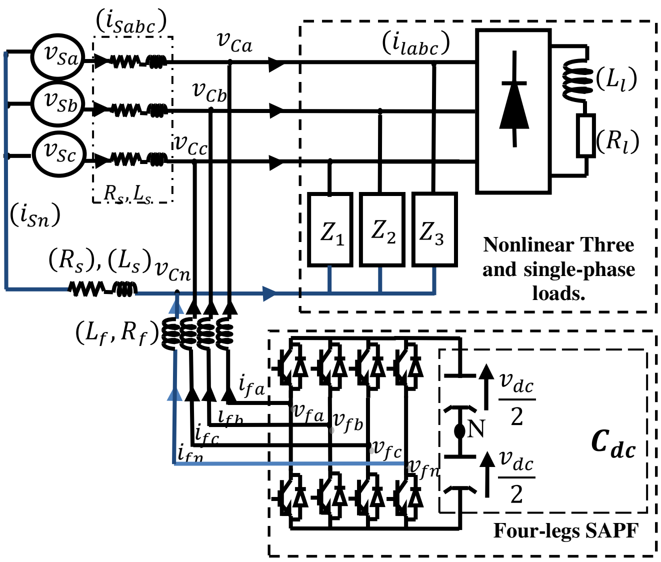

Fig 1: Four-wire SAPF associated to nonlinear and unbalanced loads

The structure of the 4 leg SAPF is presented in fig.1, which consists of a 4 leg full-bridge inverter uses 1-leg specially to compensate zero sequence current, and an energy storage capacitor C_{dc} placed at DC side providing a power source of the inverter. From the AC side, the 4 leg SAPF is connected to the network through a filtering inductor (L_f, R_f) for reducing the circulation of the harmonics currents due to the inverter switching. The DC-AC inverter operates in accordance to the well known of Pulse Width Modulation principle (PWM).

The notations of Table I are used in the description of the system model.

Table I. System Model Notations	
$(v_{saN}, v_{sbN}, v_{scN}, v_{snN})$	PCC network voltage
$(i_{sa}, i_{sb}, i_{sc}, i_{sn})$	PCC network currents
$(v_{faN}, v_{fbN}, v_{fcN}, v_{fnN})$	AC inverter voltages.
$(v_{f\alpha}, v_{f\beta}, v_{f0})$	AC inverter voltages in *α-β-0* coordinate
$(i_{fa}, i_{fb}, i_{fc}, i_{fn})$	AC inverter currents.
(v_α, v_β, v_0)	PCC voltage in *α-β-0* coordinate
(i_{la}, i_{lb}, i_{lc})	Load current.
$(i_{l\alpha}, i_{l\beta}, i_{l0})$	Load current in *α-β-0* coordinate
$(\overline{i_{fa}}, \overline{i_{fb}}, \overline{i_{fc}}, \overline{i_{fn}})$	Fundamental AC inverter current component.
$(\widetilde{i_{fa}}, \widetilde{i_{fb}}, \widetilde{i_{fc}}, \widetilde{i_{fn}})$	Harmonics AC inverter current component.
$(\mu_a, \mu_b, \mu_c, \mu_n)$	PWM inverter control voltage
v_{dc}	DC bus voltage
L_f	Decoupling filter induction.
R_f	Decoupling filter resistor
C_{dc}	DC bus capacitor.
R_{dc}	C_{dc} Leak resistance.
R_s	Grid line resistor at the PCC
L_s	Grid line inductor at the PCC

Applying the usual electric laws to the 4-Leg shunt APF with considering, the line impedance one easily gets:

$$\begin{bmatrix} v_{faN}(t) \\ v_{fbN}(t) \\ v_{fcN}(t) \\ v_{fnN}(t) \end{bmatrix} - \begin{bmatrix} v_{Ca}(t) \\ v_{Cb}(t) \\ v_{Cc}(t) \\ v_{Cn}(t) \end{bmatrix} = L_f \frac{d}{dt} \begin{bmatrix} i_{fa}(t) \\ i_{fb}(t) \\ i_{fc}(t) \\ i_{fn}(t) \end{bmatrix} + R_f \begin{bmatrix} i_{fa}(t) \\ i_{fb}(t) \\ i_{fc}(t) \\ i_{fn}(t) \end{bmatrix} \quad (1)$$

with:

$$\begin{bmatrix} v_{Ca}(t) \\ v_{Cb}(t) \\ v_{Cc}(t) \\ v_{Cn}(t) \end{bmatrix} = \begin{bmatrix} v_{saN}(t) \\ v_{sbN}(t) \\ v_{scN}(t) \\ v_{snN}(t) \end{bmatrix} - R_s \begin{bmatrix} i_{sa}(t) \\ i_{sb}(t) \\ i_{sc}(t) \\ i_{sn}(t) \end{bmatrix} - L_s \frac{d}{dt} \begin{bmatrix} i_{sa}(t) \\ i_{sb}(t) \\ i_{sc}(t) \\ i_{sn}(t) \end{bmatrix} \quad (2)$$

On the other hand, we recall that the output voltages and output currents of the DC-AC inverter are given by:

$$\begin{pmatrix} V_{faN} \\ V_{fbN} \\ V_{fcN} \\ V_{fnN} \end{pmatrix} = \frac{v_{dc}}{4} \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} \mu_a \\ \mu_b \\ \mu_c \\ \mu_n \end{pmatrix} \quad (3)$$

$$C_{dc}\frac{dv_{dc}}{dt} = \frac{-1}{2}\left(\mu_a i_{fa} + \mu_b i_{fb} + \mu_c i_{fc} + \mu_n i_{fn}\right) - \frac{v_{dc}}{R_{dc}} \quad (4)$$

Where the inverter switching functions μ_i *(i =a, b, c or n)* are defined by:

$$u_i = \begin{cases} 1 & if \quad S_{i1} \quad is \quad ON \ ; \quad S_{i2} \quad is \quad OFF \\ -1 & if \quad S_{i1} \quad is \quad OFF \ ; \quad S_{i2} \quad is \quad ON \end{cases}$$

Substituting (2) into (1), the filter equations in a-b-c coordinates becomes:

$$\frac{d}{dt}\begin{bmatrix} i_{fa} \\ i_{fb} \\ i_{fc} \\ i_{fn} \end{bmatrix} = \frac{-R_f}{L_f}\begin{bmatrix} i_{fa} \\ i_{fb} \\ i_{fc} \\ i_{fn} \end{bmatrix} + \frac{v_{dc}}{L_f}\begin{bmatrix} \mu_a \\ \mu_b \\ \mu_c \\ \mu_n \end{bmatrix} - \frac{1}{L_f}W_a(t) \quad (5)$$

$$\frac{dv_{dc}}{dt} = -\frac{1}{2C_{dc}}\left(\mu_a i_{fa} + \mu_b i_{fb} + \mu_c i_{fc} + \mu_n i_{fn}\right) - \frac{v_{dc}}{C_{dc}R_{dc}} \quad (6)$$

where $W_a(t) = \begin{bmatrix} v_{saN}(t) \\ v_{sbN}(t) \\ v_{scN}(t) \\ v_{snN}(t) \end{bmatrix} - R_s\begin{bmatrix} i_{sa}(t) \\ i_{sb}(t) \\ i_{sc}(t) \\ i_{sn}(t) \end{bmatrix} - L_s\frac{d}{dt}\begin{bmatrix} i_{sa}(t) \\ i_{sb}(t) \\ i_{sc}(t) \\ i_{sn}(t) \end{bmatrix}$

Note that in the rest of the paper, the amount $W_a(t)$ is considered measurable because it depends directly on the acquisition of the line current (and its derivative) which is not a problem in practice. The equations (5-6) are useful for building up an accurate simulator of the four-leg SAPF. However, it cannot be based upon in the control design as it involves a binary control input, namely $(\mu_a, \mu_b, \mu_c, \mu_n)$. This kind of difficulty is generally coped with by resorting to average models. Signal averaging is performed over cutting intervals (e.g. Krein et al., 1990).

The obtained average model is the following:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{-R_f}{L_f}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \frac{v_{dc}}{L_f}\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_n \end{bmatrix} - \frac{1}{L_f}\left[\begin{bmatrix} v_{saN}(t) \\ v_{sbN}(t) \\ v_{scN}(t) \\ v_{snN}(t) \end{bmatrix} - W_a(t)\right] \quad (7)$$

Where x_1, x_2 , x_3, x_4, V_{dc} , u_a, u_b, u_c and u_n. denote the average values, over cutting periods, of the signals i_{fa},i_{fb} , i_{fc}, i_{fn}, v_{dc}, μ_a, μ_b, μ_c and μ_n. The mean value(u_a, u_b, u_c,u_n) of (μ_a,μ_b, μ_c, μ_n) turns out to be the control input for system (7). To carry out the DC bus voltage control, the system modeling must be completed with a third equation describing the energy stored in the capacitor ($E_{dc} = \frac{1}{2}C_{dc}v_{dc}^2$). To this end, consider the total power (P_{DC}) at the DC bus:

$$P_{DC} = \frac{d}{dt}\left(\frac{1}{2}C_{dc}v_{dc}^2\right) \quad (8)$$

assuming that the filter switches are ideal, the introduction of (4) into (8) leads to:

$$P_{DC} = \frac{v_{dc}}{2}\left(\mu_a i_{fa} + \mu_b i_{fb} + \mu_c i_{fc} + \mu_n i_{fn}\right) + \frac{v_{dc}^2}{R_{DC}} \quad (9)$$

In practice, the switching losses in the power converter cannot be ignored. So the power balance, presented in (9) must be modified according to the following equation:

$$P_{DC} = \frac{v_{dc}}{2}\left(\mu_a i_{fa} + \mu_b i_{fb} + \mu_c i_{fc} + \mu_n i_{fn}\right) + \frac{v_{dc}^2}{R_{DC}} + P_{sc} \quad (10)$$

where P_{sc}, denotes the switching inverter losses (considered unknown). Now let x_5 denotes the averaged capacitor energy, using (10) one gets by operating the usual averaging (over cutting periods):

$$\dot{x}_5 = \frac{V_{dc}}{2}\left(u_a i_{fa} + u_b i_{fb} + u_c i_{fc} + u_n i_{fn}\right) + \frac{V_{dc}^2}{R_{DC}} + P_{sc} \quad (11)$$

For convenience, the model equations (7) and (11) are rewritten altogether

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \frac{-R_f}{L_f}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \frac{1}{L_f}\sqrt{\frac{2x_5}{C_{dc}}}\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_n \end{bmatrix} - \frac{1}{L_f}\left[\begin{bmatrix} v_{saN}(t) \\ v_{sbN}(t) \\ v_{scN}(t) \\ v_{snN}(t) \end{bmatrix} - W_a(t)\right] \quad (12)$$

$$\dot{x}_5 = \sqrt{\frac{x_5}{2C_{dc}}}\left(u_a x_1 + u_b x_2 + u_c x_3 + u_n x_4\right) + \frac{2}{C_{dc}R_{dc}}x_5 + P_{sc} \quad (13)$$

3. CURRENTS REFERENCES EXTRACTION:

Asymmetric load current can be decomposed into three components: positive, negative and zero sequences.

In order to identify and extract this components we making use of the so-called instantaneous power technique, which enjoys a good compromise between accuracy and computational complexity. The use of p-q-r reference frames leads to extract the one instantaneous active power P and two reactive power q_q *and* q_r (see Akagi et al, 1999).

In the *α-β-0* coordinate the instantaneous real load power (P), imaginary power (q) and zero sequence power (P_0) are given by (Mehmet Ucar, Engin Ozdemir (2008)) :

$$\begin{bmatrix} P_0 \\ P \\ q \end{bmatrix} = \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & -v_\beta & v_\alpha \end{bmatrix}\begin{bmatrix} i_{l0} \\ i_{l\alpha} \\ i_{l\beta} \end{bmatrix} \quad (14)$$

To obtain the currents references, the load current are transformed from *α-β-0* coordinate to p-q-r coordinates according to the following equation:

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \frac{1}{v_{0\alpha\beta}}\begin{bmatrix} v_0 & v_\alpha & v_\beta \\ 0 & -\frac{v_{0\alpha\beta}v_\beta}{v_{\alpha\beta}} & \frac{v_{0\alpha\beta}v_\alpha}{v_{\alpha\beta}} \\ v_{\alpha\beta} & -\frac{v_0 v_\alpha}{v_{\alpha\beta}} & -\frac{v_0 v_\beta}{v_{\alpha\beta}} \end{bmatrix}\begin{bmatrix} i_{l0} \\ i_{l\alpha} \\ i_{l\beta} \end{bmatrix} \quad (15)$$

Where: $v_{0\alpha\beta} = \sqrt{v_0^2 + v_\alpha^2 + v_\beta^2}$, $\quad v_{\alpha\beta} = \sqrt{v_\alpha^2 + v_\beta^2}$

To construct the current references compensation in a-b-c coordinate one can easily get from (15) (see Mehmet Ucar, Engin Ozdemir (2008)).

$$\begin{bmatrix} i_{ca}^* \\ i_{cb}^* \\ i_{cc}^* \end{bmatrix} = \frac{1}{v_{0\alpha\beta}}\begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \widetilde{i_p} \\ i_q \\ i_r + \frac{i_p v_0}{v_{\alpha\beta}} \end{bmatrix} \quad (16)$$

The neutral current reference is determinate as the following:

$$i_{cn}^* = i_{ca}^* + i_{cb}^* + i_{cc}^* \tag{17}$$

4. CONROL DESIGN:

1.4. Control objective reformulation

We seek the achievement of the three following control objectives:

- Controlling the filter current (i_{fa}, i_{fb} and i_{fc}) so that the load current harmonics and the load reactive currents are well compensated for.
- Canceling the current in the neutral conductor
- Regulating the DC bus voltage (V_{dc}) to maintain the capacitor charge at a suitable level so that the filter operates properly.

Note that, there are five variables that need to be controlled (i.e. $i_{fa}, i_{fb}, i_{fc}, i_{fn}$ and V_{dc}), while one has only four control inputs (i.e. u_a, u_b, u_c, and u_n). This is coped with by considering a cascade control strategy involving two loops (Fig. 2). The outer control loop aims at regulating the DC bus voltage. The control signals generated by the outer loop regulator, denoted $(\overline{i_{fa}}, \overline{i_{fb}}, \overline{i_{fc}}, \overline{i_{fn}})$, serve as the desired fundamental components of the output current filter. These components are augmented with the (load current) harmonic and reactive components, next denoted $(i_{fa}^*, i_{fb}^*, i_{fc}^*, i_{fn}^*)$, to constitute the final AC current references x_1^*, x_2^*, x_3^* and x_4^*.

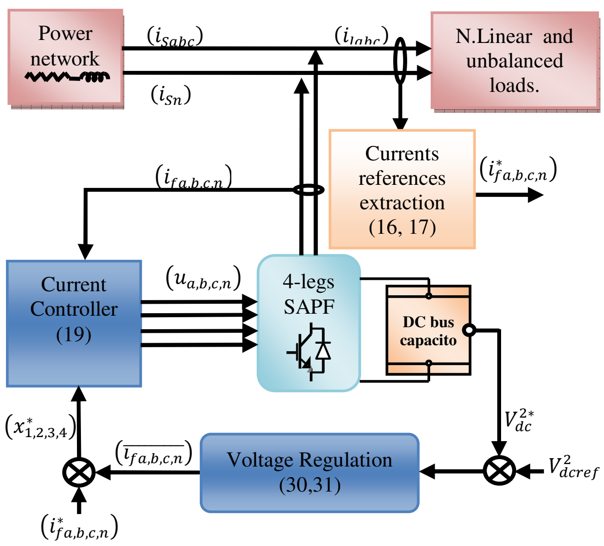

Fig.2: synoptic scheme of the proposal control strategy

2.4. Inner control loop design

The inner loop is designed to make the current tracking errors,

$$\begin{aligned} z_1 &= x_1 - x_1^* \\ z_2 &= x_2 - x_2^* \\ z_3 &= x_3 - x_3^* \\ z_4 &= x_4 - x_4^* \end{aligned}$$

as small as possible. To this end, the dynamics of these errors needs to be determined. It follows, using the model equations (12-13), that the errors undergo the following equations:

$$\begin{bmatrix} \dot z_1 \\ \dot z_2 \\ \dot z_3 \\ \dot z_4 \end{bmatrix} = \frac{-R_f}{L_f}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \frac{1}{L_f}\sqrt{\frac{2x_5}{C_{dc}}}\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_n \end{bmatrix} - \frac{1}{L_f}\left[\begin{bmatrix} v_{saN}(t) \\ v_{sbN}(t) \\ v_{scN}(t) \\ v_{snN}(t) \end{bmatrix} - W_a(t)\right] - \begin{bmatrix} \dot x_1^* \\ \dot x_2^* \\ \dot x_3^* \\ \dot x_4^* \end{bmatrix} \tag{18}$$

To ensure the asymptotic stability of the equilibrium$(z_1, z_2, z_3, z_4) = (0,0,0,0)$, equation (18) suggests that the control inputs (u_a, u_b, u_c, u_n), should be chosen so that:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_n \end{bmatrix} = \sqrt{\frac{C_{dc}}{2x_5}}\left(\frac{R_f}{L_f}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \frac{1}{L_f}\left[\begin{bmatrix} v_{saN}(t) \\ v_{sbN}(t) \\ v_{scN}(t) \\ v_{snN}(t) \end{bmatrix} - W_a(t)\right] + \begin{bmatrix} \dot x_1^* \\ \dot x_2^* \\ \dot x_3^* \\ \dot x_4^* \end{bmatrix} - \begin{bmatrix} c_1 z_1 \\ c_2 z_2 \\ c_3 z_3 \\ c_4 z_4 \end{bmatrix}\right) \tag{19}$$

Combining equations (19) and (18) one gets the following equations describing the inner closed loop:

$$\begin{bmatrix} \dot z_1 \\ \dot z_2 \\ \dot z_3 \\ \dot z_4 \end{bmatrix} = -\begin{bmatrix} c_1 z_1 \\ c_2 z_2 \\ c_3 z_3 \\ c_4 z_4 \end{bmatrix}$$

It readily follows that:

$$\begin{aligned} &z_1(t) = z_1(0)e^{-c_1 t}, z_2(t) = z_2(0)e^{-c_2 t}, \\ &z_3(t) = z_3(0)e^{-c_3 t}, z_4(t) = z_4(0)e^{-c_4 t} \end{aligned} \tag{20}$$

This shows that the errors are globally exponentially vanishing, that is the objective of cancelling the load current harmonics, load current reactive component and the neutral line current is well established.

3.4.Outer control loop design

The outer loop aims at making the voltage tracking error,

$$z_5 = x_5 - x_5^* \tag{21}$$

as small as possible, where x_5^* is the reference value of the DC bus voltage. These together with its first time-derivative are assumed to be known and bounded.

Furthermore, by respecting the notations presented in table I, the AC filter currents and voltages verify:

$$\begin{bmatrix} i_{fa} \\ i_{fb} \\ i_{fc} \\ i_{fn} \end{bmatrix} = \begin{bmatrix} \overline{i_{fa}} \\ \overline{i_{fb}} \\ \overline{i_{fc}} \\ \overline{i_{fn}} \end{bmatrix} + \begin{bmatrix} \widetilde{i_{fa}} \\ \widetilde{i_{fb}} \\ \widetilde{i_{fc}} \\ \widetilde{i_{fn}} \end{bmatrix} \tag{22}$$

By introducing (22) and (3) into (13), the DC side equation becomes:

$$\begin{aligned} \dot x_5 = &\left(v_{fa}\overline{i_{fa}} + v_{fb}\overline{i_{fb}} + v_{fc}\overline{i_{fc}} + v_{fn}\overline{i_{fn}}\right) + \\ &+\left(v_{fa}\widetilde{i_{fa}} + v_{fb}\widetilde{i_{fb}} + v_{fc}\widetilde{i_{fc}} + v_{fn}\widetilde{i_{fn}}\right) + \frac{2}{C_{dc}R_{dc}}x_5 + P_{sc} \end{aligned} \tag{23}$$

On the other hand, in practice state variable x_5 present a very slow dynamic (associated to the DC bus) then the AC current components $(\widetilde{i_{fa}}, \widetilde{i_{fb}}, \widetilde{i_{fc}})$ and $(\widetilde{i_{fn}})$. Indeed, the latter are varying at harmonics load current frequency. Consequently, the control design is based on the average model, obtained from (23) letting there:

$$\langle v_{fa}\widetilde{i_{fa}} + v_{fb}\widetilde{i_{fb}} + v_{fc}\widetilde{i_{fc}} + v_{fn}\widetilde{i_{fn}} \rangle = 0$$

It turns out that the average model is given by:

$$\dot x_5 = \left(v_{fa}\overline{i_{fa}} + v_{fb}\overline{i_{fb}} + v_{fc}\overline{i_{fc}} + v_{fn}\overline{i_{fn}}\right) + \frac{2}{C_{dc}R_{dc}}x_5 + P_{sc} \tag{24}$$

Time derivative of the error voltage gives, using (21) and (24):

$$\dot{z}_5 = v_{fa}\overline{i_{fa}} + v_{fb}\overline{i_{fb}} + v_{fc}\overline{i_{fc}} + v_{fn}\overline{i_{fn}} - \frac{2x_5}{C_{dc}R_{dc}} - P_{sc} - \dot{x}_5^* \quad (25)$$

The switching-loss power (P_{sc}) is seen as an unknown parameter in (25). Indeed, the latter is mainly depending on the load which presently is assumed to undergo a piecewise constant variation. On the other hand, the quantity $v_{fa}\overline{i_{fa}} + v_{fb}\overline{i_{fb}} + v_{fc}\overline{i_{fc}} + v_{fn}\overline{i_{fn}}$ stands in (25) as a virtual control. Interestingly, this quantity is nothing other than the electric network power, denoted P_{net}, transmitted to control the voltage DC bus.

$$P_{net} = v_{fa}\overline{i_{fa}} + v_{fb}\overline{i_{fb}} + v_{fc}\overline{i_{fc}} + v_{fn}\overline{i_{fn}} \quad (26)$$

In order to obtain a stabilizing control law of the error system (25), let us introduce the following Lyapunov function candidate:

$$V_a = \frac{z_5^2}{2} + \frac{1}{2\gamma}\tilde{P}_{sc}^2$$

Where $\hat{P}_{sc}$denotes the online estimate of P_{sc} and $\tilde{P}_{sc} = P_{sc} - \hat{P}_{sc}$ is the corresponding estimation error; γ is a positive design parameter. Deriving V_a along (25) yields:

$$\dot{V}_a = z_5\left(P_{net} - \frac{2x_5}{C_{dc}R_{dc}} - \hat{P}_{sc} - \dot{x}_5^*\right) + \tilde{P}_{sc}\left(\frac{1}{\gamma}\dot{\hat{P}}_{sc} - z_5\right) \quad (27)$$

Equation (27) suggests the following control law:

$$P_{net} = -c_5 z_5 + \frac{2x_5}{C_{dc}R_{dc}} + \hat{P}_{sc} + \dot{x}_5^* \quad (28)$$

and the following parameter adaptation law:

$$\dot{\hat{P}}_{sc} = \gamma z_5 \quad (29)$$

Now, as P_{net} is a virtual control input, we make use of equation (26) to obtain:

$$\begin{bmatrix} \overline{i_{fa}} \\ \overline{i_{fb}} \\ \overline{i_{fc}} \\ \overline{i_{fn}} \end{bmatrix} = \frac{P_{net}}{{v_{f\alpha}}^2 + {v_{f\beta}}^2 + {v_{f0}}^2} \begin{bmatrix} v_{faN} & 0 & 0 & 0 \\ 0 & v_{fbN} & 0 & 0 \\ 0 & 0 & v_{fcN} & 0 \\ 0 & 0 & 0 & v_{fnN} \end{bmatrix} \quad (30)$$

Substituting (28) into (30) one finds the following expression of the fundamental current references:

$$\begin{bmatrix} \overline{i_{fa}} \\ \overline{i_{fb}} \\ \overline{i_{fc}} \\ \overline{i_{fn}} \end{bmatrix} = \frac{1}{{v_{f\alpha}}^2 + {v_{f\beta}}^2 + {v_{f0}}^2} \begin{bmatrix} v_{faN}\left(-c_5 z_5 + \frac{2x_5}{C_{dc}R_{dc}} + \hat{P}_{sc} + \dot{x}_5^*\right) \\ v_{fbN}\left(-c_5 z_5 + \frac{2x_5}{C_{dc}R_{dc}} + \hat{P}_{sc} + \dot{x}_5^*\right) \\ v_{fcN}\left(-c_5 z_5 + \frac{2x_5}{C_{dc}R_{dc}} + \hat{P}_{sc} + \dot{x}_5^*\right) \\ v_{fnN}\left(-c_5 z_5 + \frac{2x_5}{C_{dc}R_{dc}} + \hat{P}_{sc} + \dot{x}_5^*\right) \end{bmatrix} \quad (31)$$

Substituting the control law equation (28) and the parameter adaptation law (29) one gets the following time derivative of the Lyapunov function candidate:

$$\dot{V}_a = -c_5 z_5^2$$

This shows that the voltage tracking error is globally exponentially vanishing,.

Remark 1. Although the (inner and outer) control laws (19) and (31) involve a division by x_5 and $({v_{f\alpha}}^2 + {v_{f\beta}}^2 + {v_{f0}}^2)$, respectively, this entails no singularity in practice. Indeed, the active filter cannot work if these quantities are zero. In other words, the latter are nonzero as long as the active filter is carrying non-identically null currents.

5-SIMULATION AND DISCUSSION OF RESULTS:

We now evaluate the new four-leg SPAF regulator including the current control loop (19) and the adaptive voltage control loop ((28), (29), (31)). The non linear load is constituted by an AC-DC three-phase converter. The load asymmetry was simulated by introducing (at time 0.1s) different mono-phase resistor at each phase. The typical load and filter characteristics are summarized in Table 2. With this simulation protocol, the obtained load current shape is presented in Fig.3, which shows the strong harmonic distortion and asymmetry of this current. In the other hand Fig.4 shows that before the introduction of the load asymmetry ($t \in [0\ 0.1s]$) the neutral current is equal to zero but it's not the case for $t \geq 0.1s$.

TABLE 2. SHUNT APF CHARACTERISTICS

Power active filter	L_f	0,22 H.
	C_f	1000 μF.
	R_f	0,2 Ω.
Rectifier-Load	L	*10* mH.
	R	100 Ω
Single loads	*R1*	*100* Ω
	R2	*150* Ω
	R3	*800* Ω

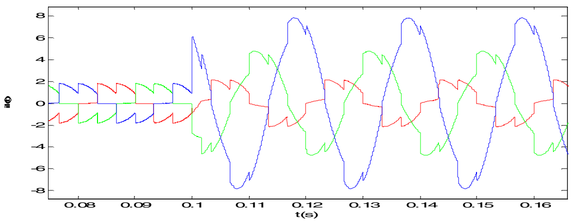

Fig.3: The load current.

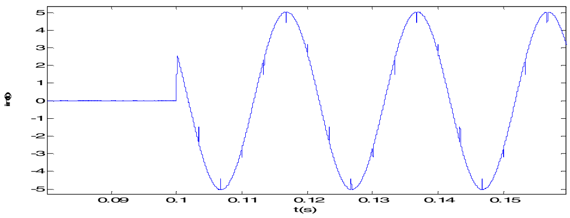

Fig.4: The neutral line current

5.1 Inner loop performances

The control design parameters are given the following numerical values of Table 3, which proved to be convenient.

TABLE 3. CONTROLLER PARAMETERS

Current regulator	C_1	*5.10³*
	C_2	*5.10³*
	C_3	*5.10³*
	C_4	*5.10³*
Voltage regulator	C_5	*100*
	γ	*10*

The resulting controller performances are illustrated by Figs. 5 to 7. Fig. 5 shows that the AC four-legs filter currents track well their references confirming the theoretical analysis results. Fig.6 shows that the neutral line current has been well suppressed. The resulting network line current is plotted in Fig. 7. This last shows that the line currents practically form a symmetrical three-phase sinusoidal system.

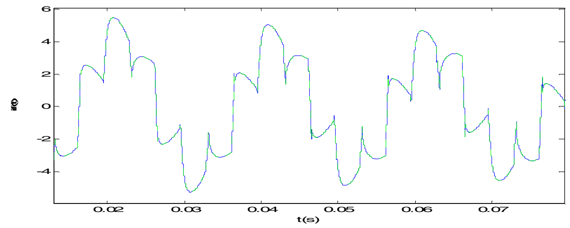

Fig.5: Current tracking

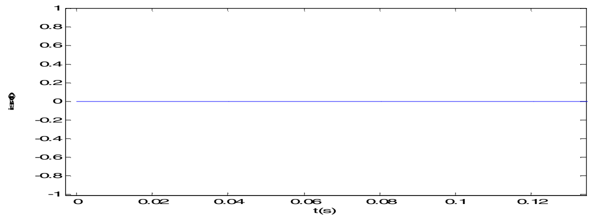

Fig.6: Neutral line current after filtering

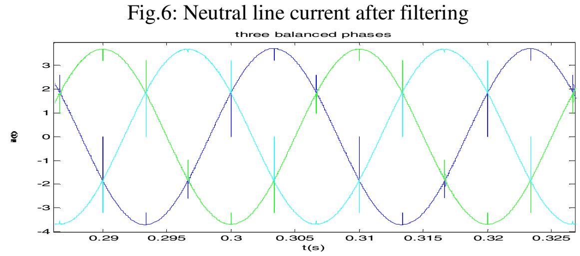

Fig.7: Line current after filtering in time domain

5.2 Outer Loop performances

The controller of the outer loop is dedicated to maintain the DC bus voltage to a reference value in order to cope with the demand of electrical energy filter. To highlight the robustness of the proposed control law, a disturbance is injected at the entrance of SAPF representing a fugitive discharge of DC bus

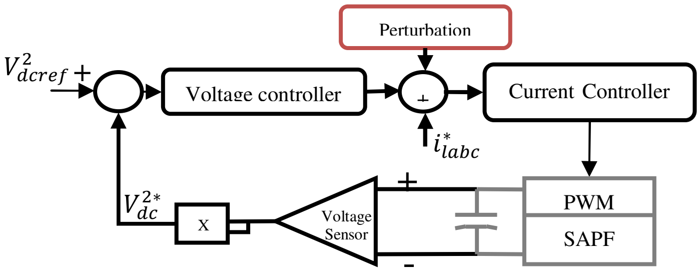

Fig.8: A simulation scheme of the DC bus regulator under a perturbation source

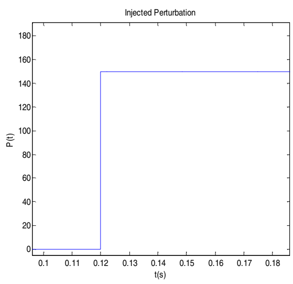

Fig.9: Injected perturbation

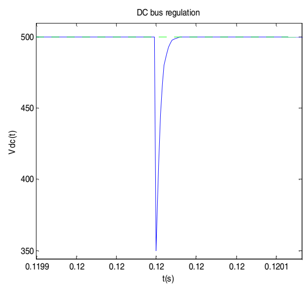

Fig .10: DC voltage and its reference.

capacitor. The perturbation introduced during this validation test appears at time 0.12s. It corresponds to a transient voltage discharge (150v variation Fig: 9) across the capacitor of the DC bus. Fig.10 shows the good behavior of the DC voltage outer loop.

6. Conclusion:

A nonlinear control strategy is proposed for four leg three phase shunt active power filters. The aim is to achieve current harmonics, reactive power and neutral current compensation, in presence of nonlinear and unbalanced loads, as well as tight voltage regulation at the inverter output capacitor. To this end, a nonlinear controller is developed and analyzed making use of advanced tools from the control theory e.g. Lyapunov stability, system averaging theory. It is formally established that the control objectives are actually achieved in average with a quite satisfactory accuracy. The formal results are confirmed by simulations.

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