A GNSS Interference Identification and Tracking based on Adaptive Fading Kalman Filter

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Abstract: A time domain signal tracking algorithm is proposed to estimate the frequency of a chirp type global navigation satellite system (GNSS) interference. The unknown interference frequency of GNSS is estimated by the received signal using properties of trigonometrical functions, but this value has a lot of error caused by measurement noise and the frequency change of the interference. In order to reduce the ierror, an adaptive fading Kalman filter is applied to the proposed algorithm. Furthermore, a low pass differentiator (LPD) and a pattern enhancement algorithm are used to estimate the sweep period of chirp type interference. The performance of the tracking algorithm is verified by comparing to conventional algorithms. By analyzing the results of comparisons, the proposed algorithm has better tracking performance than conventional algorithms when the jammer to signal ratio (J/S) of GNSS interference signal is more than 30dB, which seriously affects the accuracy of the GNSS.

1. INTRODUCTION

According to increasing the reliability on GNSS and purchasing the portable jammer easily online, the potential threat of GNSS interference has arisen in recent years. By investigation, South Korea has experienced intentional GPS jamming on several occasions. The types of jamming signals were single-tone continuous wave interference (CWI) on L1, and chirp type interference on L2 and L5 frequency bands (Kang *et al.*, 2012).

In order to deal with the threat, detecting and tracking interference are important for safe GPS operation in all countries. Previous work (Kang *et al.*, 2012; Borio *et al.*, 2008; Choi *et al.*, 2002) have focused on detecting the existence of CWI using an adaptive notch filter. These approaches have a limitation on detection and tracking of chirp type interference because its fast sweep rate degrades the signal tracking performance of the adaptive notch filter. Thus it is necessary to use different detection and tracking algorithm for chirp type interference.

A frequency tracking algorithm is designed and employed to estimate the instantaneous frequency of chirp type interference in this paper. The adaptive fading Kalman filter is applied and pattern recognition algorithm with LPD is used to estimate the sweep period of chirp type GNSS interference, which is used to reset the filter parameter for improving the performance of frequency tracking and identifying the type of interference.

The performance of the proposed algorithm is simulated for scenarios of GPS signal in the presence of chirp type interference and is analyzed by using software GPS and interference simulator data studied on the previous work (Yang, 2013). Through theoretical analysis and comparing simulation results with conventional algorithms, the feasibility and performance of the proposed methods are shown.

The rest of this paper is organized as follows. Section 2 refers to the intermediate frequency model of GPS and interference. In addition, the frequency estimation method is briefly reviewed. The interference tracking algorithm is proposed in section 3. The main components of the proposed algorithm which are the adaptive fading Kalman filter, the sweep period estimation algorithm with pattern recognition and enhancement method are also explained. Simulation results verifying the performance of the proposed algorithm are shown in section 4. Finally, conclusions are in section 5.

2. INSTANTANEOUS FREQUENCY ESTIMATION

Instantaneous frequency (IF) provides important information about the change of frequency in non-stationary signals. The concept of IF is popularly used in various technical fields and applications such as radar, sonar, and communications (Boashash, 2003). There are many approaches for IF estimation: time-frequency representation approaches (linear, quadratic approaches), IF estimation methods using analytic signal through the Hilbert transform, etc. During the past five years, IF estimation methods based on these approaches have been modified and upgraded in order to improve the accuracy of IF estimation. One of these methods is the S-transform which combines short-time Fourier analysis and wavelet analysis. The S-transform based IF estimator for signals with additive white Gaussian noise is proposed in the other work (Sejdic *et al.*, 2008). In this paper, IF of a chirp type civilian GNSS interference is computed by numerical summation of received signal samples through properties of trigonometrical functions because it would generate IF estimation results rapidly and could be relatively easily interpreted in the real-time system.

2.1 GNSS signal and interference models

A MATLAB SIMULINK-based GPS L1 signal and interference simulator (Yang, 2013) is used to generate intermediate frequency signal data. The received GPS L1 signal in the presence of interference (Kaplan *et al.*, 2006) can be modeled as

$$r_n = \sqrt{2PD_{n-\tau}c_{n-\tau}}\cos\left(2\pi\left(f_{IF} + f_d\right)n + \theta\right) + v_n + i_n \tag{1}$$

where *P* is an intermediate frequency signal power, D_n is navigation data, c_n is a coarse acquisition code (C/A code) assigned to gps satellite, f_{IF} is an intermediate frequency of 9.548 MHz, f_d is the doppler frequency shift, v_n is noise, i_n is interference, τ is the code phase of the received signal, and θ is the phase. $n = t/f_s$, where *t* is time and f_s is the sampling frequency of 38.192MHz.

Previous work (Mitch *et al.*, 2011) surveyed the signal properties of commercial GPS jammers based on experimental data. The test of jammers provided information on the characteristics of current civil GPS jammer signals. The majority of the jammers used chirp signals, all jammed L1 band, only six jammed L2 band and none jammed L5 band. The sweep rate of the jammers is on average about 1×10^{12} Hz/sec. According to the results of the previous work, chirp type GNSS interference used in this paper is modeled as

$$i_{chirp_n} = \sqrt{2P_i} \cos(2\pi n \left(S_{t_n}\right) + \phi_i)$$
⁽²⁾

where P_i is the interference signal power, and ϕ_i is the initial phase. S_{t_i} can be expressed as follows:

$$S_{t_{-}n} = f_0 + \frac{a_i}{2f_s}n = f_0 + \frac{\Delta f}{2}n$$
(3)

where f_0 and a_i refer to the initial frequency and sweep rate of the interference, respectively. For simplicity, $\frac{a_i}{f_s}$ is expressed by Δf which means variation of digital frequency. *n* varies within the boundary *T* which is the sweep period of chirp type interference. The frequency of the chirp type interference is modeled as

$$f_n = f_0 + \Delta f n \tag{4}$$

If *n* exceeds *T*, f_n changes from eq. 4 to

$$f_n = f_0 + \Delta f \times (n - T), \quad n \ge T$$
(5)

The f_n is shown in Fig. 1b. The sweep rate is set to 1×10^{12} Hz/sec and initial frequency is configured by 0.1832 in digital frequency.

The single-tone CWI is modeled as

$$i_{\text{single}_n} = \sqrt{2P_i} \cos(2\pi n f_i + \phi_i)$$
(6)

where f_i is the intermediate frequency of interference which is set to 9.548MHz and shown in Fig. 1a.

2.2 Estimation of interference frequency

To estimate IF of the interference, received signal whose the three samples refer to r_{n-1} , r_n , r_{n+1} and some assumptions are needed. If signal strength of i_n is greater than GNSS signal and measurement noise, the signal model of received signal can be expressed by $r_n \approx i_n + v_n$. In addition, if i_n is chirp type interference and the measurement noise is ignored, the signal model of three signal samples can be characterized as

$$r_{n+1} \approx \cos\left(2\pi (n+1)S_{t_{n+1}}\right)$$

$$r_{n} \approx \cos\left(2\pi n \left(S_{t_{n}}\right)\right)$$

$$r_{n-1} \approx \cos\left(2\pi (n-1)S_{t_{n-1}}\right)$$
(7)

where interference signal power is assumed to '0.5' and initial phase is '0' for convenient analyzing. Supposing that $\Delta f = 0$ which refer to the case of single-tone CWI (Saliu, 2000), the frequency of received signal can be calculated by

$$\frac{r_{n+1} + r_{n-1}}{2r_n} = \frac{\cos(2\pi (n+1)S_{t_n+1}) + \cos(2\pi (n-1)S_{t_n+1}))}{2\cos(2\pi n(S_{t_n}))}$$
$$= \cos(2\pi (S_{t_n}))$$
$$\therefore S_{t_n} = f_n = \frac{1}{2\pi}\cos^{-1}\left(\frac{r_{n+1} + r_{n-1}}{2r_n}\right)$$
(8)

where $S_{t_n} = f_n = f_0$, due to $\Delta f = 0$.

However, in case of chirp type interference, which means, $\Delta f \neq 0$. For that reason, eq. 8 is not valid and has additional terms due to Δf .

$$\frac{r_{n+1}+r_{n-1}}{2r_n} = \frac{\cos\left(2\pi n\left(S_{t_n}\right)+2\pi\frac{\Delta f}{2}\right)\cos\left(2\pi n\left(S_{t_n}\right)+2\pi\frac{\Delta f}{2}n\right)}{2\cos\left(2\pi n\left(S_{t_n}\right)\right)}$$
$$= \left[\cos\left(2\pi\frac{\Delta f}{2}\right)-\tan\left(2\pi n\left(S_{t_n}\right)\right)\sin\left(2\pi\frac{\Delta f}{2}\right)\right]\cos\left(2\pi S_{t_n}+2\pi\frac{\Delta f}{2}n\right)$$
(9)

If Δf is very small value, $\cos\left(2\pi\frac{\Delta f}{2}\right) = 1$, $\sin\left(2\pi\frac{\Delta f}{2}\right) = \pi\Delta f$.

By using these values, eq. 9 is rearranged by

$$\frac{r_{n+1} + r_{n-1}}{2r_n} = \left[1 - (\pi \Delta f) tan \left(2\pi n \left(S_{t_n} \right) \right) \right] \cos \left(2\pi S_{t_n} + 2\pi \frac{\Delta f}{2} n \right)$$
$$= \left[1 - (\pi \Delta f) tan \left(2\pi n \left(S_{t_n} \right) \right) \right] \cos \left(2\pi f_n \right)$$
$$\therefore f_n = \frac{1}{2\pi} \cos^{-1} \left(\frac{1}{\left[1 - (\pi \Delta f) tan \left(2\pi n \left(S_{t_n} \right) \right) \right]} \frac{r_{n+1} + r_{n-1}}{2r_n} \right)$$
(10)

In eq. 10, tangent term causes a calculation error when the phase reaches to $\frac{\pi}{2}(2n-1)$ period regardless of measurement noise. If measurement noise is comprised in eq. 10, the unexpected calculation error occurs in the whole interval as shown in Fig. 2. In order to estimate the interference frequency correctly, the calculation error can be reduced by a model based filtering method. In this paper, the adaptive fading Kalman filter is used to reduce the error and track the interference frequency.



Fig. 1. The frequency of the GNSS interference



b) The measurement of chirp type interference (J/S 30dB)

Fig. 2. The frequency measurement of the GNSS interference

3. INTERFERENCE TRACKING ALGORITHM

In this section, the proposed frequency tracking algorithm is introduced for chirp type interference detection. The main feature of chirp type interference is that its frequency changes periodically. In general, the interference frequency increases linearly within one period and is reinitialized at the beginning of every period as mentioned in the previous section. Thus, estimation of an initial frequency of interference, sweep rate and sweep period of the interference are very important factors in tracking the chirp type interference.

An adaptive fading Kalman filter (Xia *et al.*, 1994; Kim *et al.* 2009) is applied to estimate the frequency and sweep rate of interference in this paper. A pattern recognition algorithm (Oikonomou *et al.*, 2006) with LPD (Laquna *et al.*, 1990) is also used to estimate the sweep period of the interference. Moreover, in order to improve tracking performance of the filter, the state and initial error covariance of the filter are reinitialized at the beginning of the estimated sweep period.

3.1 Adaptive fading Kalman filter

The concept of an adaptive fading Kalman filter is to apply a factor to the Kalman gain to adjust tracking performance of the filter. In most adaptive fading Kalman filters, the factor is calculated by the relation between the calculated innovation covariance and an estimated innovation covariance because the innovation covariance of the filter can present the effect of unaccounted errors (Kim *et al.*, 2009). The innovation covariance of the filter is

$$C_n = \mathbf{E}\left[\left(z_n - \hat{z}_n^{-}\right)\left(z_n - \hat{z}_n^{-}\right)^{T}\right] = \mathbf{H}\mathbf{P}_n^{-}\mathbf{H}^{T} + \mathbf{R}_n$$
(11)

where C_n refers to the calculated innovation covariance and \hat{z}_n^- is an estimation of measurement, $\mathbf{H}\hat{\mathbf{x}}_n^-$. **H** is a

measurement matrix, \mathbf{P}_n^- is a prior predicted error covariance and \mathbf{R}_n is a measurement noise covariance of the filter.

The estimated innovation covariance can be expressed by

$$\hat{C}_{n} = \frac{1}{M-1} \sum_{i=n-M+1}^{n} \left(z_{i} - \hat{z}_{i}^{-} \right) \left(z_{i} - \hat{z}_{i}^{-} \right)^{T}$$
(12)

where M is a window size. By using the eq. 11 and 12, the factor which adjusts the Kalman gain is obtained as follows:

$$\alpha_n = \max\left\{1, \frac{trace(\hat{C}_n)}{trace(C_n)}\right\}$$
(13)

The factor is used to compensate the measurement error by decreasing the Kalman gain. The Kalman gain is reduced by

$$\mathbf{K}_{n} = \frac{1}{\alpha_{n}} \mathbf{P}_{n}^{-} \mathbf{H}^{T} \left[\mathbf{H} \mathbf{P}_{n}^{-} \mathbf{H}^{T} + \mathbf{R}_{n} \right]^{-1}$$
(14)

In case of normal operation, α_n will be set to one because $trace(\hat{C}_n)$ is smaller than $trace(C_n)$ and Kalman gain is updated using the general formular. If the measurement error occurs severely, the α_n exceeds 1. In that case, Kalman gain decreases by the $1/\alpha_n$ in order to depend less on measurement information. The rest of the filter equations are the same as the conventional discrete Kalman filter (Kim *et al.*, 2009).

$$\hat{\mathbf{x}}_n^- = \mathbf{F} \hat{\mathbf{x}}_n^+ \tag{15}$$

$$\mathbf{P}_n^- = \mathbf{F} \mathbf{P}_{n-1}^+ \mathbf{F}^T + \mathbf{Q}_n \tag{16}$$

$$\mathbf{P}_{n}^{+} = \left(\mathbf{I} - \mathbf{K}_{n}\mathbf{H}\right)\mathbf{P}_{n}^{-} \tag{17}$$

$$\hat{\mathbf{x}}_{n}^{+} = \hat{\mathbf{x}}_{n}^{-} + \mathbf{K}_{n} \left(z_{n} - \mathbf{H} \hat{\mathbf{x}}_{n}^{-} \right)$$
(18)

The state and measurement model of the Kalman filter are

$$\begin{bmatrix} f_{n+1} \\ \Delta f_{n+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_n \\ \Delta f_n \end{bmatrix} + \begin{bmatrix} w_n \\ 0 \end{bmatrix}$$
(19)

$$z_n = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} f_n \\ \Delta f_n \end{bmatrix} + v_n$$
(20)

where the states are frequency and variation of frequency which are expressed as $[f_n, \Delta f_n]^T$. w_n is the process noise which is white Gaussian noise ($w_n \approx N(0, \sigma_w^2)$). v_n is the measurement noise whose distribution is $N(0, \sigma_v^2)$. The proposed system is observable as the rank of observability matrix is full.

However, this model is only satisfied when the sample index, n is within the sweep period because the frequency of chirp type interference is f_0 at the beginning of every sweep period. In order to estimate the frequency of the interference accurately, it is needed to reset the filter state, f_n to f_0 when n = T. In this paper, f_0 is calculated by estimated states of the filter when n = T - 1 which almost converges to true value of f_n and Δf_n .

$$\hat{f}_{0} = \hat{f}_{T-1} - (T-1)\Delta\hat{f}_{T-1}$$

$$= \left(f_{T-1} + \tilde{f}_{T-1}\right) - (T-1)\left(\Delta f + \Delta\tilde{f}_{T-1}\right)$$

$$= f_{T-1} - (T-1)\Delta f + \tilde{f}_{T-1} - (T-1)\Delta\tilde{f}_{T-1}$$

$$= f_{0} + \tilde{f}_{T-1} - (T-1)\Delta\tilde{f}_{T-1} = f_{0} + \tilde{f}_{0}$$
(21)

where \hat{f}_0 is the estimation of f_0 which is an unknown value depending on the property of the received interference. \hat{f}_n and $\Delta \hat{f}_n$ are estimated states. The estimation errors refer to \tilde{f}_n and $\Delta \tilde{f}_n$.

The initial error covariance, $\mathbf{P}_{0_{-T}}(1,1)$ of the filter is also reinitialized at the beginning of the sweep period. $\mathbf{P}_{0_{-T}}(1,1)$ is expressed by

$$\mathbf{P}_{0_{T}}(1,1) = E\left[\left(f_{0} - \hat{f}_{0}\right)^{2}\right]$$
$$= E\left[\left(\tilde{f}_{T-1} - (T-1)\Delta\tilde{f}_{T-1}\right)^{2}\right] = E\left[\tilde{f}_{T-1}^{2}\right] + (T-1)^{2}E\left[\Delta\tilde{f}_{T-1}^{2}\right]$$
$$\approx \mathbf{P}_{T-1}^{+}(1,1) + (T-1)^{2}\mathbf{P}_{T-1}^{+}(2,2)$$
(22)

where \tilde{f}_n and $\Delta \tilde{f}_n$ are uncorrelated and \mathbf{P}_{T-1}^+ is a posterior predicted error covariance.

3.2 Sweep period estimation by pattern recognition

The sweep period of the interference is used to reset the filter state and initial error covariance to track the chirp type interference. To estimate the sweep period the measurement passing through LPD (Laquna *et al.*, 1990) is normalized and expressed by

$$z_{G_n} = \gamma z_{G_n-1} - (1-\gamma) \left(\frac{z_n - z_{n-6}}{\beta \times \left(\frac{f_{max} - f_{min}}{f_s} \right)} \right)$$
(23)

where γ is the forgetting factor and β is the normalization factor. These values are design parameters for enhancement of the peak pattern. f_{\min} and f_{\max} are minimum and maximum frequency of observable frequency bandwidth. The transfer function of LPD is $G(z) = 1 - z^{-6}$. Fig. 3 shows the value of z_{G_n} when the chirp type interference is received. The z_{G_n} has a peak at the beginning of the sweep period where the frequency of interference changes radically. If the time index at the peak is measured, the sweep period can be estimated.

However, the peak of z_{G_n} is hard to recognize by a certain threshold because z_{G_n} has a lot of noise due to the measurement error of z_n . In this paper, a pattern enhancement algorithm (Oikonomou *et al.*, 2006) is used to detect the peak effectively for reducing the effect of noise. The algorithm is based on an autoregressive (AR) model and the AR coefficients are estimated with the use of Kalman filter. The z_{G_n} is described by an AR model and the model is given as

$$z_{G_n} = \sum_{i=1}^{p} \gamma_{i_n} z_{G_n(n-i)} + v_{AR_n}$$
(24)

where *p* is the order of the model which is set to 10 for considering computation load, γ_{t_i} is the AR coefficient and v_{AR_n} is the white Gaussian noise with zero mean and variance σ_{AR}^2 . To estimate the AR coefficient, the state and measurement models of Kalman filter are

$$\Gamma_n = \mathbf{A}\Gamma_{n-1} + \boldsymbol{\omega}_n \tag{25}$$

$$z_{G_n} = \mathbf{C}_n^T \mathbf{\Gamma}_n + v_{AR_n} \tag{26}$$

where $\Gamma_n = [\gamma_{1_n}, ..., \gamma_{p_n}]^T$, $\mathbf{A} = diag(0.2, ..., 0.2)$ which is a diagonal matrix and its dimension is $p \times p$ for smoothing and



Fig. 3. The results of period estimation in case of chirp type interference

enhancement of the peak pattern. $\mathbf{C}_n = \left[z_{G_{-}(n-1)}, \dots, z_{G_{-}(n-p)} \right]^T$, $\boldsymbol{\omega}_n$ is white Gaussian noise with zero mean and covariance \mathbf{Q}_{AR} . As a results, the enhancement pattern of z_{G_n} can be expressed by

$$z_{AR_n} = \mathbf{C}_n^T \hat{\mathbf{\Gamma}}_n \tag{27}$$

where $\hat{\Gamma}_n$ is the estimated AR coefficient by the Kalman filter and z_{AR_n} is shown in Fig. 3. The peak of z_{AR_n} is detected by the maximum value above the selected threshold which is set to $\beta_{en} \times \left(\frac{f_{max} - f_{min}}{f_s}\right)$ in this paper. The β_{en} is an enhancement rate. Consequently, the estimated time index at the peak is sent to the filter and used to reset the filter.

4. SIMULATION

The performance of the proposed tracking algorithm is verified by a simulation which is compared to other methods namely a conventional discrete Kalman filter (KF), adaptive fading filter without period estimation (AKF) (Kim *et al.*, 2009) whose models are the same as that of proposed algorithm. In case of AKF, a Kalman gain is increased by α_n , which refer to $\mathbf{K}_n = \alpha_n \mathbf{P}_n^- \mathbf{H}^T [\mathbf{H} \mathbf{P}_n^- \mathbf{H}^T + \mathbf{R}_n]^{-1}$. The increase

which refer to $\mathbf{K}_n = \alpha_n \mathbf{P}_n \mathbf{H} [\mathbf{H}\mathbf{P}_n \mathbf{H} + \mathbf{K}_n]$. The increase of the Kalman gain means to depend more on measurement information and consequently increase the sensitivity of the response to frequency change. Thus, the filter can track the chirp type interference except the proposed sweep period estimation method.

In this simulation, the chirp type interference is only received and the signal power of received GNSS signal is -130dBm, the overall noise floor is at -114dB/MHz and the J/S is set to 30dB.



Fig. 4. The signal tracking results of Simulation

Tracking results of the simulation are shown in Fig. 4. KF tracks the interference frequency well in the first period, but the tracking error increases after the first period because the convergence speed of the filter is slower than the change rate of frequency near the beginning of the period. In case of AKF, the convergence rate of the filter is improved by increasing the Kalman gain and prior predicted error covariance so the tracking error is less than KF. However, increasing Kalman gain also leads to estimation error caused by depending too much on the measurement which has a lot of noise as shown in Fig. 2. Comparing the results of the frequency estimation, the proposed algorithm has the better tracking performance than KF and AKF as the proposed algorithm reset the filter at the beginning of the sweep period by using the period estimation method. The frequency estimation RMSE of the proposed algorithm is 0.0071, that of KF is 0.0220 and AKF is 0.0115.

5. CONCLUSIONS

An interference tracking algorithm is proposed by utilizing an adaptive fading Kalman filter and sweep period estimation method using LPD to estimate the frequency of chirp type interference whose sweep rate is 1×10^{12} Hz/sec and J/S is above 30dB. Simulation results show that the proposed algorithm can more efficiently track the interference compared to conventional methods.

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