Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Robust semiclassical internal model based regulation for a class of hybrid linear systems

D. Carnevale * S. Galeani * L. Menini * M. Sassano *

* *Dipartimento di Ingegneria Civile e Ingegneria Informatica, Università di Roma "Tor Vergata", Via del Politecnico, 1 00133 Roma, Italy (Email: menini@disp.uniroma2.it, [sergio.galeani, daniele.carnevale, mario.sassano]@uniroma2.it).*

Abstract: A new robust error-feedback regulator is proposed for a class of uncertain hybrid systems with periodic jumps, using a hybrid extension of the classical internal model principle. The plant need not be minimum phase, square nor SISO. The proposed regulator contains an internal model composed by two main units, a *flow internal model* providing the correct input to achieve regulation during flows, and a *jump internal model* resetting the state of the regulator at each period. Such a structure reminds internal model design for ripple-free regulation of sampled-data systems; an important difference is that here the jump internal model must contain more copies of the relevant dynamics of the exosystem with respect to the flow internal model.

1. INTRODUCTION

Control of hybrid systems, with interacting continuous-time and discrete-time dynamics ("flow dynamics" and "jump dynamics", Goebel et al. [2012]), is a topic widely studied, see, *e.g.*, Liberzon [2003], Johansson [2004], Sun and Ge [2005], Sanfelice et al. [2007]. Output regulation has been addressed for different classes of hybrid systems, see, *e.g.*, Menini and Tornambè [2001], Galeani et al. [2008a,b], Moraescu and Brogliato [2010], Marconi and Teel [2010], Biemond et al. [2013], Cox et al. [2012], Marconi and Teel [2013], Galeani et al. [2012].

The class of systems considered here is the largest class of hybrid linear plants with periodic jumps for which robust (with respect to independent parameter variations) output regulation can be achieved by using an internal model of the exosystem alone. As discussed by Carnevale et al. [2012a, 2013b], the fact that the plant has more inputs than outputs is essentially necessary for the robust existence of admissible trajectories for the state, that correspond to the desired output. Hence, since robustness of the regulation is desired here versus the largest possible class of parameter variations, not only we do not assume that the plant is square, but we exploit the presence of more inputs than controlled outputs. The class of systems considered here is physically motivated: it strictly includes systems composed by uncertain subsystems that evolve separately during flows and interact (possibly, in an uncertain manner) at jumps: see the discussion and the physical example by Carnevale et al. [2013c,a]. Other two strengths of the paper are that it is not required that the exosystem is Poisson stable (the results apply for unbounded exogenous signals) and that exponential stability of the free responses of the closed-loop system is guaranteed (implying the weaker property that trajectories are bounded when the exosystem is Poisson stable).

The proposed compensator has many conceptual similarities with the ones proposed many years ago for ripple-free control of sampled-data systems, *e.g.*, by Yamamoto [1994], Grasselli et al. [1996, 2002], in the sense that, here and there, the internal model is provided by a continuous-time component (here called the *flow internal model*, needed to ensure output regulation during flows, and possibly obtained through the use of generalized hold functions), and a discrete-time component (here called the *jump internal model*, needed to ensure output regulation at periodically sampled time instants). A quite surprising novelty here with respect to the mentioned results is that, due to the rich class of uncertainties taken into account (affecting both the flow and the jump dynamics), the jump internal model must contain as many copies of the equivalent discrete-time dynamics of the exosystem as the sum of the dimensions of the states of the observable dynamics of the plant and of the flow internal model; on the other hand, the flow internal model must contain only as many copies of the flow dynamics of the exosystem as the number of regulated outputs. In both cases, the mentioned number of copies is the maximum that still allows to obtain exponential stability (instead of simply boundedness of trajectories) for the closed-loop system.

Notation. The acronyms GES/GIS are used for Global Exponential/Incremental Stability. $\mathbb{C}_g := \{s \in \mathbb{C} : |s| < 1\}$. The Kronecker product is denoted by $\otimes$, and the spectrum of matrix M by $\Lambda(M)$.

2. PRELIMINARIES AND PROBLEM DEFINITION

Consider the *hybrid time domain*

$$\mathcal{T} := \{(t,k) : t \in [k\tau_M, (k+1)\tau_M], k \in \mathbb{N}\}, \qquad (1)$$

with $\tau_M > 0$ given. The time variable t measures the flow of continuous time, whereas the time variable k counts

⋆ This work was supported in part by ENEA-Euratom and MIUR.

the number of times that the solution of the system has jumped. The derivative with respect to t is indicated by a dot, like in $\dot{x}$; the pushforward operation (with respect to jump times) is denoted by a plus sign, so that $x^+(t,k) := x(t,k+1)$, whenever both (t,k) and $(t,k+1)$ belong to the relevant time domain. The notation we use here is that presented in Goebel et al. [2012], but, since the time domain $\mathcal{T}$ is a priori fixed, in equations like (2), for simplicity, we do not explicitly write the flow and jump sets. The equations where time derivatives appear (as in (2a)) are supposed to hold in the intervals $[k\tau_M, (k+1)\tau_M)$, $k \in \mathbb{Z}$, whereas equations where the pushforward operator appears (as in (2b)) are supposed to hold at hybrid times $(k\tau_M, k-1)$, for $k \in \mathbb{Z}$.

Consider the following hybrid linear plant $\mathcal{P}$:

$$\dot{x} = Ax + Bu + Pw\,, \tag{2a}$$
$$e = Cx + Qw\,, \tag{2b}$$
$$x^+ = Ex + Rw\,, \tag{2c}$$

with state $x(t,k) \in \mathbb{R}^n$, control input $u(t,k) \in \mathbb{R}^m$ and output $e(t,k) \in \mathbb{R}^p$, $p \leq m$, where the exogenous signal $w(t,k) \in \mathbb{R}^q$ is generated by the exosystem $\mathcal{E}$

$$\dot{w} = Sw\,, \tag{3a}$$
$$w^+ = Jw\,. \tag{3b}$$

The following assumption avoids trivialities.

Assumption 1. No eigenvalue of $\tilde{J} = Je^{S\tau_M}$ is in $\mathbb{C}_g$. ■

2.1 The considered class of hybrid systems

In this paper, robust regulation is achieved for a class of plants subject to arbitrary parameter uncertainties under a structural condition. Such a condition holds in particular for systems composed by two or more subsystems evolving separately during flows and coupled only at jumps; see the discussion and the physical example by Carnevale et al. [2013c,a]. The results in Carnevale et al. [2013b] show that this structural condition captures the largest class of systems for which an internal model based regulator (ensuring robust regulation for arbitrary perturbations of the nonzero elements in the prescribed structure) can be designed containing only a model of the exosystem (that is, the *zero dynamics internal model principle* is trivially satisfied in this class of plants).

Assumption 2. Plant $\mathcal{P}$ is uncertain and belongs to a family of plants $\mathcal{F}$ such that for each $\mathcal{P} \in \mathcal{F}$, the matrices in the description (2) have the form

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ 0 & A_{22} & A_{23} \\ 0 & 0 & A_{33} \end{bmatrix}, B = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \\ 0 & B_{32} \end{bmatrix}, C = [0\ \ 0\ \ C_3], \tag{4}$$

where $A_{11} \in \mathbb{R}^{n_1 \times n_1}$, $A_{22} \in \mathbb{R}^{n_2 \times n_2}$, $A_{33} \in \mathbb{R}^{n_3 \times n_3}$, $B_{11} \in \mathbb{R}^{n_1 \times m_1}$, $B_{32} \in \mathbb{R}^{n_3 \times m_2}$ with $m_2 \geq p$. The nominal plant and its state space description matrices are denoted by a superscript as follows: $\mathcal{P}^\circ \in \mathcal{F}$, A°, B°, ... ■

Remark 1. The state and input partitions $x = [x_1'\ \ x_2'\ \ x_3']'$ and $u = [u_1'\ u_2']'$ (with u_1 having no effect on the regulated output e during flows, and x_1 corresponding to the reachable subspace of the subsystem having state (x_1, x_2) and input u_1, as will be specified later) used in (4) induce similar partitions on the other matrices in (2):

$$E = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}, R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}, P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix},$$

which however have no specific structure (*i.e.*, zero blocks) as in (4). Considering (2) with the constraint imposed by the fixed zero entries in (4), there remain exactly

$$\begin{aligned} z = [n(n+m+q) - (n_1+m_1)(n_2+n_3) - n_2n_3] + \\ + p(n_3+q) + n(n+m+q) \end{aligned}$$

free parameters, so that each plant in $\mathcal{F}$ can be uniquely identified with a vector of parameters $f \in \mathbb{R}^z$, with f° corresponding to $\mathcal{P}^\circ$. It follows that each open neighborhood of f° in $\mathbb{R}^z$ defines an open neighborhood of $\mathcal{P}^\circ$ in $\mathcal{F}$. ■

In previous works, *e.g.* Carnevale et al. [2012a,b, 2013b], the form (4) was obtained by coordinate and feedback transformations implying also additional properties, *e.g.*, $m_2 = p$, pairwise disjoint spectra for A_{11}, A_{22}, A_{33} and S, and the subsystem (A_{33}, B_{32}, C_3) being square and without finite invariant zero. Here, on the contrary, as in Carnevale et al. [2013a,c], the structure in (4) is based on physical reasons (instead of preliminary transformations) and then such additional properties are not guaranteed.

2.2 Problem definition and main solvability assumption

The robust output regulation problem is now formalized, assuming that the plant $\mathcal{P}$ in (2) belongs to the family $\mathcal{F}$ and has a nominal description $\mathcal{P}^\circ$; next, more assumptions on $\mathcal{P}^\circ$ will be given under which the problem is solvable.

Problem 1. Given the nominal plant $\mathcal{P}^\circ \in \mathcal{F}$ as in (2) and (4), and the exosystem $\mathcal{E}$ in (3), find, if possible, a *robust output regulator* using only measurements of e which achieves

- (GES) global exponential stability of the closed loop;
- (OR) $\lim\limits_{t+k \to +\infty} e(t,k) = 0$ for all initial conditions;

for any $\mathcal{P} \in \mathcal{F}_0 \subset \mathcal{F}$, where $\mathcal{F}_0$ contains an open neighborhood of $\mathcal{P}^\circ$. ■

Remark 2. In Problem 1, one could require simply that all signals remain bounded instead of insisting on the stronger GES requirement. At the price of a more careful design of the internal model of the exosystem (which has to guarantee that some structural properties of the plant are preserved when it is augmented with the internal model), achieving GES provides the added benefits that the exosystem need not be restricted to be Poisson stable, and, more importantly, there is no risk of "drifts" associated to internal closed loop dynamics which, although bounded and unobservable from the regulation error, might grow with time due to noise or modeling errors (*e.g.* unaccounted changes of reference signal, which appear as unmodeled jumps). ■

The needed structural properties of the hybrid system (2), (4) can be tested by using the following matrices, defined for any plant $\mathcal{P} \in \mathcal{F}$:

$$\tilde{A} := \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} e^{\left[\begin{smallmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{smallmatrix}\right]\tau_M},\ \tilde{B} := \begin{bmatrix} E_{11} \\ E_{21} \end{bmatrix}, \tag{5a}$$

$$\tilde{C} := [E_{31}\ \ E_{32}]\, e^{\left[\begin{smallmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{smallmatrix}\right]\tau_M}, \tilde{D} := E_{31}, \tag{5b}$$

and considering the following notations:

- the *flow system matrix* $P_F(s)$

$$P_F(s) := \begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix}, \tag{6}$$

- the *partial hybrid system matrix* $P_H(s)$

$$P_H(s) := \begin{bmatrix} \tilde{A} - sI & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix}, \tag{7}$$

- the *flow reachability matrix*

$$R_F := \begin{bmatrix} B & AB & A^2B \cdots A^{n-1}B \end{bmatrix}, \tag{8}$$

- the *flow observability matrix*

$$O_F := \begin{bmatrix} C' & (CA)' & (CA^2)' \cdots (CA^{n-1})' \end{bmatrix}', \tag{9}$$

- the *hybrid PBH stabilizability matrix* $R_H(s)$

$$R_H(s) := \begin{bmatrix} Ee^{A\tau_M} - sI & ER_F \end{bmatrix}, \tag{10}$$

- the *hybrid PBH observability matrix* $O_H(s)$

$$O_H(s) := \begin{bmatrix} (Ee^{A\tau_M})' - sI & O_F' \end{bmatrix}'. \tag{11}$$

As stated before, for each of the defined matrices, a $^\circ$ superscript denotes nominal values. Define also:

$$R_{F,1} := \begin{bmatrix} B_{11} & A_{11}B_{11} & A_{11}^2 B_{11} \cdots A_{11}^{n_1-1}B_{11} \end{bmatrix},$$

$$O_{F,3} := \begin{bmatrix} C_3' & (C_3A_{33})' & (C_3A_{33}^2)' \cdots (C_3A_{33}^{n-1})' \end{bmatrix}'.$$

Assumption 3. The nominal plant $\mathcal{P}^\circ \in \mathcal{F}$ satisfies:

$$\operatorname{rank}(R_{F,1}^\circ) = n_1, \tag{12a}$$

$$\operatorname{rank}(O_{F,3}^\circ) = n_3, \tag{12b}$$

$$\operatorname{rank}(R_H^\circ(s)) = n, \qquad \forall s \notin \mathbb{C}_g, \tag{12c}$$

$$\operatorname{rank}(O_H^\circ(s)) = n, \qquad \forall s \notin \mathbb{C}_g, \tag{12d}$$

$$\operatorname{rank}(P_F^\circ(s)) = n + p, \qquad \forall s \in \Lambda(S), \tag{12e}$$

$$\operatorname{rank}(P_H^\circ(s)) = n, \qquad \forall s \in \Lambda(\tilde{J}), \tag{12f}$$

where $\tilde{J} = Je^{S\tau_M}$. ■

Note that Assumption 3 on the nominal plant $\mathcal{P}^\circ$ implies the existence of a neighborhood $\mathcal{F}_1 \subset \mathcal{F}$ where (12) hold.

3. REGULATOR ARCHITECTURE AND DESIGN

The overall regulator is composed of three main dynamic blocks, as in Figs. 1 and 2:

- a *jump internal model* $\mathcal{I}_\mathcal{J}$, with a trivial flow dynamics, which ensures that the states of the subsystem (A_{33}, B_{32}, C_3) and $\mathcal{I}_\mathcal{F}$ have correct values after jumps;
- a *flow internal model* $\mathcal{I}_\mathcal{F}$, with a trivial jump dynamics, which, suitably reinitialized at each jump time, generates the input required for output regulation during flow intervals;
- a dynamic *stabilizer* $\mathcal{K}$, which guarantees GES for the closed loop.

The overall internal model $\mathcal{I}_\mathcal{M}$ is obtained by interconnecting $\mathcal{I}_\mathcal{J}$ and $\mathcal{I}_\mathcal{F}$ as in Fig. 2. If the cascade-like interconnection of $\mathcal{I}_\mathcal{M}$ and $\mathcal{P}$ is shown to be hybrid-detectable and hybrid-stabilizable, then the design of $\mathcal{K}$ is easily performed (*e.g.* by using an approach similar to the one used in Carnevale et al. [2012b]); hence, the key point is to show how to design $\mathcal{I}_\mathcal{J}$ and $\mathcal{I}_\mathcal{F}$ in such a way to achieve output regulation and to preserve the structural properties of hybrid-detectability and hybrid-stabilizability already enjoyed by $\mathcal{P}^\circ$ (and then by all "sufficiently close" $\mathcal{P} \in \mathcal{F}$) according to (12c), (12d). Note that such a property could be destroyed, even in the SISO case, if the internal model were designed by simply copying the exosystem dynamics (3) several times.

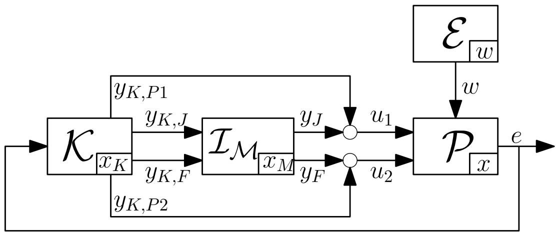


Fig. 1. The internal model based regulator.

3.1 The flow internal model $\mathcal{I}_\mathcal{F}$

The subcompensator $\mathcal{I}_\mathcal{F}$ can be designed focusing exclusively on the flow-only subsystem (A_{33}, B_{32}, C_3) and the flow-only exosystem. Since this system is possibly nonsquare, a squaring down static time invariant gain is introduced (as part of $\mathcal{I}_\mathcal{F}$), to guarantee that the cascade between $\mathcal{I}_\mathcal{F}$ and (A_{33}, B_{32}, C_3) preserves the flow observability property of the latter system.

Algorithm 1. **Design of $\mathcal{I}_\mathcal{F}$ (see Fig. 2)**

Step 1 Let $\mu_S(s)$ be the minimal polynomial of S, having degree n_S.
Step 2 Define A_{F0} as a lower companion matrix having characteristic polynomial $\mu_S(s)$.
Step 3 Define $C_{F0} = [1\ 0 \cdots\ 0] \in \mathbb{R}^{1 \times n_S}$.
Step 4 Define the matrices

$$A_F = I_p \otimes A_{F0}, \qquad C_F = I_p \otimes C_{F0}, \tag{13}$$

with A_F having size $n_F = p \cdot n_S$.
Step 5 Define $\mathcal{I}_{\mathcal{F}0}$ according to the equations

$$\dot{x}_F = A_F x_F, \tag{14a}$$

$$x_F^+ = u_F, \tag{14b}$$

$$y_{F0} = C_F x_F. \tag{14c}$$

Step 6 Define the matrix $M_2 \in \mathbb{R}^{m_2 \times p}$, such that

$$\operatorname{rank}\left(\begin{bmatrix} A_{33} - sI & B_{32}M_2 \\ C_3 & 0 \end{bmatrix}\right) = n_3 + p,\ \forall s \in \Lambda(S)$$

Step 7 Define $\mathcal{I}_\mathcal{F}$ as the cascade interconnection of $\mathcal{I}_{\mathcal{F}0}$ and M_2, *i.e.* let $y_F = M_2 y_{F0} = M_2 C_F x_F$ and let $u_F = y_{JF} + y_{K,F}$. ■

Note that the role of M_2 consists in *squaring down* the flow dynamics described by (A_{33}, B_{32}, C_3), so that the "squared" system $(A_{33}, B_{32}M_2, C_3)$ has exactly p inputs and p outputs and has no invariant zero in the set $\Lambda(S)$, thus guaranteeing that the cascade of the "core" flow internal model $\mathcal{I}_{\mathcal{F}0}$ (containing exactly p copies of the essential flow dynamics of the exosystem, as characterized by its minimal polynomial $\mu_S(s)$) and system $(A_{33}, B_{32}M_2, C_3)$ is observable. Such a "squaring down" is possible, under Assumption 3, by exploiting, *e.g.*, algebraic duals of the results in Kouvaritakis and MacFarlane [1976].

In turn, the flow observability property of the mentioned cascade of $\mathcal{I}_{\mathcal{F}0}$ and $(A_{33}, B_{32}M_2, C_3)$ (equivalently, of $\mathcal{I}_\mathcal{F}$ and (A_{33}, B_{32}, C_3)) and classic output regulation theory imply the existence of a unique state

$$\begin{bmatrix} x_3(k\tau_M, k) \\ x_F(k\tau_M, k) \end{bmatrix} = \begin{bmatrix} \Pi_3 \\ \Pi_F \end{bmatrix} w(k\tau_M, k), \tag{15}$$

(where Π_3, Π_F solve a suitable Francis equation) such that $e(t,k) \equiv 0$ for all $t \in [k\tau_M, (k+1)\tau_M]$.

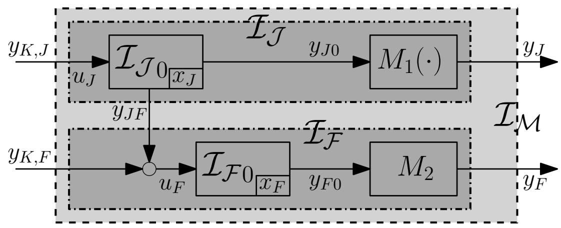


Fig. 2. Detailed view of the internal structure of the internal model $\mathcal{I}_{\mathcal{M}}$.

Clearly, the exact values of Π_3 and Π_F depend on the (unknown) plant parameters. Hence, in order to have condition (15) satisfied for all k, an ancillary robust output regulation problem must be solved, having a purely discrete time nature and for a "regulated output"

$$e_a(k) := \begin{bmatrix} x_3(k\tau_M, k) \\ x_F(k\tau_M, k) \end{bmatrix} - \begin{bmatrix} \Pi_3 \\ \Pi_F \end{bmatrix} w(k\tau_M, k), \qquad (16)$$

of dimension $n_3 + n_F$ (the number of scalar equations in (15)). These considerations give a qualitative motivation for the design of $\mathcal{I}_{\mathcal{J}}$ in the following subsection.

3.2 The jump internal model $\mathcal{I}_{\mathcal{J}}$

The jump internal model $\mathcal{I}_{\mathcal{J}}$ is designed focusing exclusively on the one period equivalent dynamics of the cascade of $\mathcal{I}_{\mathcal{F}}$ and $\mathcal{P}$, and the one period equivalent dynamics of the exosystem, given by $w((k+1)\tau_M, k+1) = \tilde{J}w(k\tau_M, k)$. The internal model in $\mathcal{I}_{\mathcal{J}}$ is designed to provide at each period the correct initialization to $x_3(k)$ and $x_F(k)$ to ensure $e \equiv 0$ during the flow interval $(k\tau_M, (k+1)\tau_M)$. Again, in order to deal with possible nonsquareness and to ensure that the internal model $\mathcal{I}_{\mathcal{J}}$ is hybrid observable from e, a multiplexing/squaring static time-varying (possibly, piecewise constant) gain is introduced (as part of $\mathcal{I}_{\mathcal{J}}$).

Algorithm 2. **Design of $\mathcal{I}_{\mathcal{J}}$ (see Fig. 2)**

Step 1 Let $\mu_{\tilde{J}}(s)$ be the minimal polynomial of $\tilde{J}$, having degree $n_{\tilde{J}}$.
Step 2 Define E_{J0} as a lower companion matrix having characteristic polynomial $\mu_{\tilde{J}}(s)$.
Step 3 Define $C_{J0} = [1 \ 0 \ \cdots \ 0] \in \mathbb{R}^{1\times n_{\tilde{J}}}$.
Step 4 Define the matrices

$$E_J = I_{n_3+n_F} \otimes E_{J0}, \qquad (17a)$$
$$C_J = I_{n_3+n_F} \otimes C_{J0}, \qquad (17b)$$

with E_J having size $n_J = (n_3 + n_F) \cdot n_{\tilde{J}}$.
Step 5 Define $\mathcal{I}_{\mathcal{J}0}$ according to the equations

$$\dot{x}_J = 0, \qquad (18a)$$
$$x_J^+ = E_J x_J + u_J, \qquad (18b)$$
$$\begin{bmatrix} y_{J0} \\ y_{JF} \end{bmatrix} = C_J x_J = \begin{bmatrix} C_{J1} \\ C_{J2} \end{bmatrix} x_J. \qquad (18c)$$

with $C_{J1} \in \mathbb{R}^{n_3\times n_J}$
Step 6 Choose $\bar{M}_1 \in \mathbb{R}^{n_1\times n_3}$ such that

$$\operatorname{rank}\left(\begin{bmatrix} \tilde{A} - sI & \tilde{B}\bar{M}_1 \\ \tilde{C} & \tilde{D}\bar{M}_1 \end{bmatrix}\right) = n, \ \forall s \in \Lambda(\tilde{J}), \qquad (19)$$

and define $M_1(\tau) \in \mathbb{R}^{m_1\times n_3}$, $\tau \in [0, \tau_M]$, such that[1] $\bar{M}_1 = \int_0^{\tau_M} e^{A_{11}(\tau_M-\tau)} B_{11} M_1(\tau) d\tau$.
Step 7 Define $\mathcal{I}_{\mathcal{J}}$ as the cascade of $\mathcal{I}_{\mathcal{J}0}$ and $M_1(\cdot)$, *i.e.* let $y_J(t,k) = M_1(t - k\tau_M) y_{J0}(t,k)$. ■

It is worth noting that the time-varying gain $M_1(\cdot)$ serves a double purpose. Let $\bar{B}_{11} := \int_0^{\tau_M} e^{A_{11}(\tau_M-\tau)} B_{11} d\tau$. If $\operatorname{Im}(\bar{M}_1) \subset \operatorname{Im}(\bar{B}_{11})$, then a constant (instead of time-varying) M_1 can be chosen with the only role of performing a squaring down fully similar to the one performed by M_2 (cfr the discussion in Sec. 3.2); such a constant M_1 can be chosen as $M_1 = \bar{B}_{11}^{\sharp}\bar{M}_1$, where $\bar{B}_{11}^{\sharp}$ is the Moore-Penrose pseudoinverse of $\bar{B}_{11}$. Since (A_{11}, B_{11}) is a reachable pair, then $m_1 \geq n_1$ is a sufficient condition for $\operatorname{Im}(\bar{M}_1) \subset \operatorname{Im}(\bar{B}_{11})$ to happen for almost all τ_M.[2] On the other hand, if $\operatorname{Im}(\bar{M}_1) \not\subset \operatorname{Im}(\bar{B}_{11})$, then the role of time variation in $M_1(\cdot)$ is crucial in order to achieve $\bar{M}_1 = \int_0^{\tau_M} e^{A_{11}(\tau_M-\tau)} B_{11} M_1(\tau) d\tau$. In this case $M_1(\cdot)$ has the effect of "squaring up" (opposite to squaring down), that is the effect of virtually "enlarging" the number of inputs to the x_1 dynamics; this is particularly evident when $\operatorname{rank}(\bar{M}_1) = n_3 > m_1$, in which case a constant solution is not possible since it should be $\bar{M}_1 = \bar{B}_{11} M_1$ but $\operatorname{rank}(\bar{B}_{11}) \leq m_1$. In any case, the choice of $M_1(\cdot)$ allows to design $\mathcal{I}_{\mathcal{J}0}$ as an internal model for a square system having $n_3 + n_F$ inputs and outputs (cfr (16)).

3.3 Main result: efficacy of the proposed solution

Under our assumptions, it can be proven that the interconnection of $\mathcal{I}_{\mathcal{M}}$ (designed as in Fig. 2 by Algorithms 1 and 2) and $\mathcal{P}$ with input $y_K = [y_{K,P1} \ y_{K,P2} \ y_{K,J} \ y_{K,F}]$ and output e is a detectable and stabilizable hybrid system. Similarly to what done by Carnevale et al. [2012b] (see also Carnevale et al. [2014]), such properties ensure that it is possible to design an hybrid linear output feedback stabilizer (the block $\mathcal{K}$ in Fig. 1) providing GES and GIS, and based on a suitable separation principle. The following result can be proven.

Theorem 1. Under Assumptions 1, 2 and 3, Problem 1 is solved by any regulator having the structure in Fig. 1, with the subcompensators $\mathcal{I}_{\mathcal{F}}$ and $\mathcal{I}_{\mathcal{J}}$ designed as in Sections 3.1 and 3.2, respectively, and with the subcompensator $\mathcal{K}$ guaranteeing GES for the closed-loop system. ■

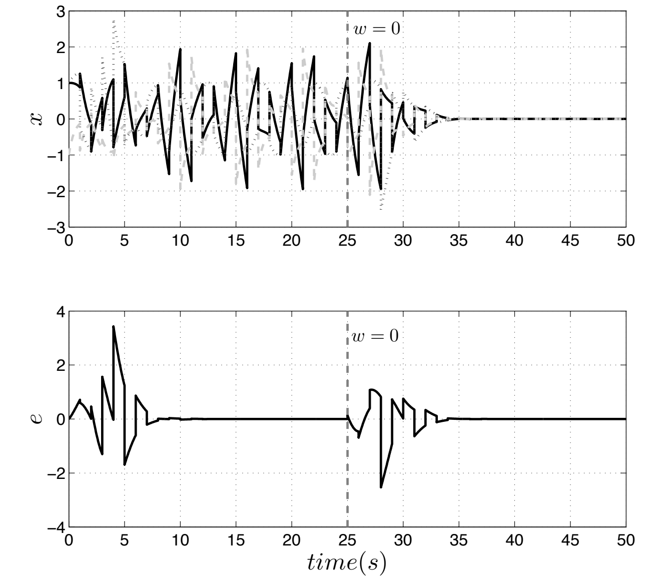
As for classical linear output regulation theory, the convergence to zero of the regulated output is ensured by the presence of the internal model as far as the closed loop remains asymptotically stable (which is equivalent to GES/GIS for the considered class of closed loop systems, see *e.g.* Carnevale et al. [2012b]); in fact, the proof of Theorem 1 yields the following corollary.

Corollary 1. Under Assumptions 1, 2 and 3, any regulator designed as above ensures requirement (OR) in Problem 1 for all $\mathcal{P} \in \mathcal{F}$ for which it achieves requirement (GES). ■

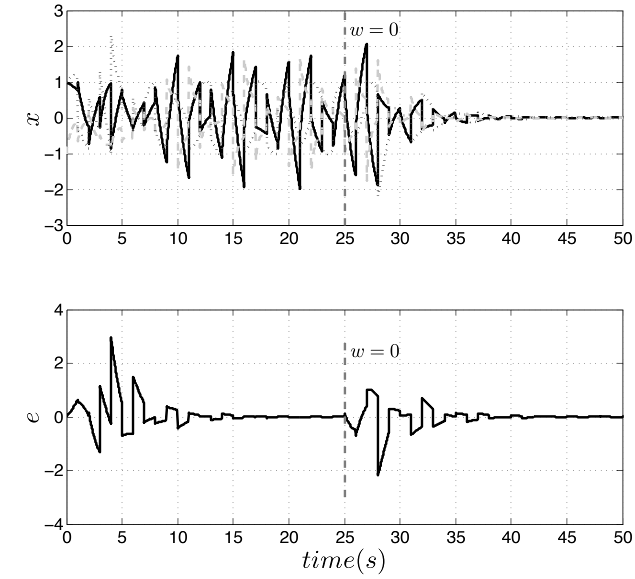
In other words, given a regulator as in Theorem 1, and denoted by $\mathcal{F}_s \subset \mathcal{F}$ the set of plants such that $\mathcal{K}$ provides GES for the closed-loop system, the same regulator solves Problem 1 on the whole set $\mathcal{F}_s$. Note that $\mathcal{F}_s$ includes arbitrary large variations of matrices P, Q, R in (2).

[1] By (12a), the reachability Gramian G_1 of the pair (A_{11}, B_{11}) is invertible, so $M_1(\tau) := B_{11}' e^{A_{11}'(\tau_M-\tau)} G_1^{-1} \bar{M}_1$ is a possible choice; however, more easily implementable alternative choices exist, *e.g.* with $M_1(\tau)$ piecewise constant (cfr [Carnevale et al., 2013c, Sec. V]).

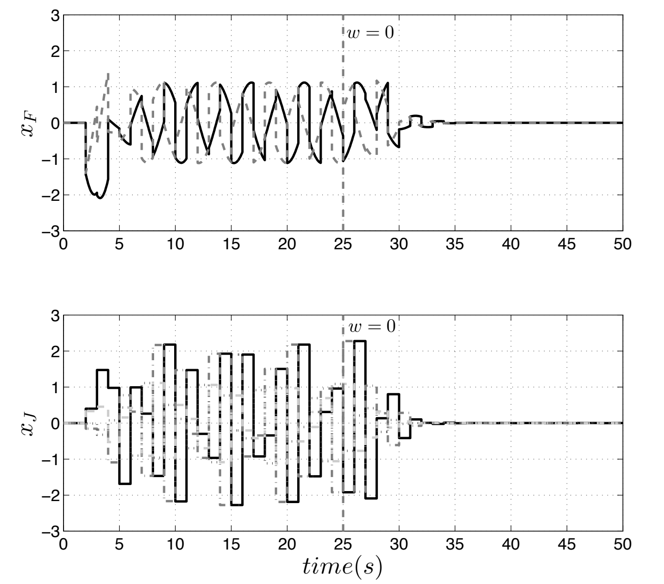
[2] Pathological cases for values of τ_M in a set of measure zero can happen, *e.g.* $A_{11} = \left[\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right]$, $B_{11} = I$, $\tau_M = 2\pi$, yielding $\bar{B}_{11} = 0$.



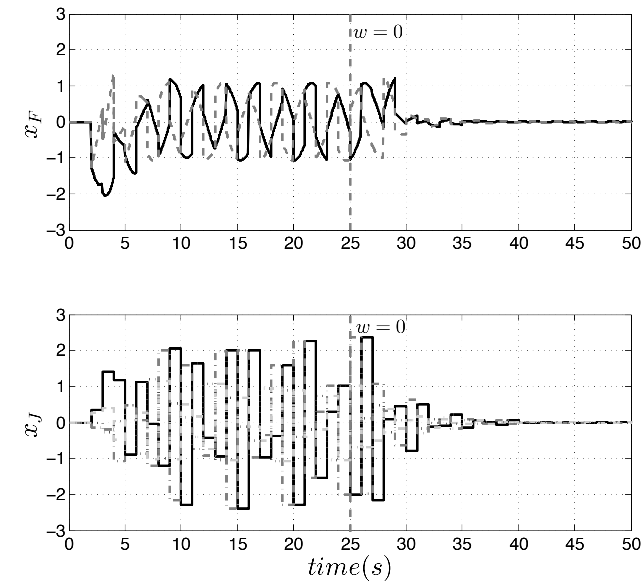
(a) Nominal plant $\mathcal{P}^{\circ}$. Time histories of the state x (top) and of the regulation error e (bottom).



(b) Perturbed plant $\mathcal{P}$. Time histories of the state x (top) and of the regulation error e (bottom).



(c) Nominal plant $\mathcal{P}^{\circ}$. Time histories of the states x_F of the flow internal model $\mathcal{I}_{\mathcal{F}}$ defined in Section 3.1 (top) and x_J of the jump internal model $\mathcal{I}_{\mathcal{J}}$ defined in Section 3.2 (bottom).



(d) Perturbed plant $\mathcal{P}$. Time histories of the states x_F of the flow internal model $\mathcal{I}_{\mathcal{F}}$ defined in Section 3.1 (top) and x_J of the jump internal model $\mathcal{I}_{\mathcal{J}}$ defined in Section 3.2 (bottom).

Fig. 3. Response of the nominal plant $\mathcal{P}^{\circ}$ defined in of [Carnevale et al., 2013c, Example 1] (left column, Figures 3a and 3c) and a perturbed plant $\mathcal{P}$ (right column, Figures 3b and 3d) interconnected with the compensator described in 3. Global exponential stability of the closed-loop system ensures that all states converge to zero after the state of the exosystem is reset to zero at time $t = 25$ (cfr Remark 2).

4. SIMULATIONS

The first example introduced in Carnevale et al. [2013c] is revisited here by means of the construction in the previous section, which yields a robust regulator. The flow and jump internal models are defined according to the procedures in Sections 3.1 and 3.2, respectively, and interconnected to the plant according to Figs. 1 and 2, with a constant $M_1(\tau) = 1/\int_0^{\tau_M} e^{-(\tau_M-\tau)}d\tau$ and $M_2 = 1$. The stabilizer $\mathcal{K}$ is designed to obtain (GES) for the closed-loop system.

In the following simulations, Problem 1 is solved for the nominal plant $\mathcal{P}^{\circ}$ defined in Example 1 of Carnevale et al. [2013c], and for a generic element $\mathcal{P}$ of the family $\mathcal{F}$ of perturbed plants. The plant $\mathcal{P}$ was obtained by perturbing the entries of A, B and E of $\mathcal{P}^{\circ}$ between 15% and 20% of their nominal values. The top graph of Fig. 3a shows the time history of the state of the nominal plant $\mathcal{P}^{\circ}$ defined in Example 1 of Carnevale et al. [2013c] interconnected with the compensator proposed in this paper, whereas the time history of the corresponding regulation error is depicted in the bottom graph. The time history of the state of the flow and jump internal models, introduced in Sections 3.1 and 3.2, respectively, is displayed in Fig. 3c, top and bottom graph, respectively. Note that the exosystem injecting exogenous signals to the plant is *switched off* at $t = 25s$. As expected, the state of the internal model asymptotically converges to zero.

As for the perturbed scenario, the top and bottom graphs of Fig. 3b show the time history of the state of the perturbed plant $\mathcal{P}$ interconnected with the compensator designed for the nominal plant $\mathcal{P}^{\circ}$, and the time history of the corresponding regulation error e, respectively. Finally, similarly to Fig. 3c, the time history of the state of the flow and jump internal models is shown in Fig. 3d.

5. CONCLUSIONS

The robust output regulation problem has been solved for the largest class of linear hybrid systems with periodic jumps for which the internal model design can be performed based on the exosystem dynamics only. The proposed design also achieves global exponential stability of the closed-loop system and applies to MIMO, possibly nonsquare and nonminimum phase systems. The proposed design is based on two internal models of the exosystem, one taking care of regulation during flows and the other solving a related regulation problem at jumps.

REFERENCES

J. Biemond, N. van de Wouw, W. Heemels, and H. Nijmeijer. Tracking control for hybrid systems with state-triggered jumps. *IEEE Trans. Aut. Cont.*, 58(4):876–890, 2013. ISSN 0018-9286. doi: 10.1109/TAC.2012.2223351.

D. Carnevale, S. Galeani, and L. Menini. Output regulation for a class of linear hybrid systems. Part 1: trajectory generation. In *Conf. on Decision and Control*, pages 6151–6156, 2012a. doi: 10.1109/CDC.2012.6426497.

D. Carnevale, S. Galeani, and L. Menini. Output regulation for a class of linear hybrid systems. Part 2: stabilization. In *Conf. on Decision and Control*, pages 6157–6162, 2012b. doi: 10.1109/CDC.2012.6426495.

D. Carnevale, S. Galeani, and L. Menini. A case study for hybrid regulation: Output tracking for a spinning and bouncing disk. In *Mediterranean Control Conference*, pages 858–867, 2013a. doi: 10.1109/MED.2013.6608823.

D. Carnevale, S. Galeani, and M. Sassano. Necessary and sufficient conditions for output regulation in a class of hybrid linear systems. In *Conf. on Decision and Control*, pages 2659–2664, dec. 2013b.

D. Carnevale, S. Galeani, and M. Sassano. On semiclassical solutions of hybrid regulator equations. In *Mediterranean Control Conference*, pages 868–876, june 2013c. doi: 10.1109/MED.2013.6608824.

D. Carnevale, S. Galeani, and M. Sassano. A linear quadratic approach to linear time invariant stabilization for a class of hybrid systems. In *Mediterranean Control Conference*, pages –, 2014.

N. Cox, L. Marconi, and A. R. Teel. Hybrid internal models for robust spline tracking. In *Conf. on Decision and Control*, pages 4877 –4882, 2012. doi: 10.1109/CDC.2012.6426043.

S. Galeani, L. Menini, and A. Potini. Trajectory tracking in linear hybrid systems: An internal model principle approach. In *American Control Conf.*, pages 4627–4632, 2008a. doi: 10.1109/ACC.2008.4587225.

S. Galeani, L. Menini, A. Potini, and A. Tornambe. Trajectory tracking for a particle in elliptical billiards. *International Journal of Control*, 81(2):189–213, 2008b. doi: 10.1080/00207170701487789.

S. Galeani, L. Menini, and A. Potini. Robust trajectory tracking for a class of hybrid systems: An internal model principle approach. *IEEE Trans. Aut. Cont.*, 57(2):344–359, 2012. ISSN 0018-9286. doi: 10.1109/TAC.2011.2162884.

R. Goebel, R. Sanfelice, and A. Teel. *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*. Princeton University Press, 2012. ISBN 9781400842636.

O. Grasselli, S. Longhi, A. Tornambè, and P. Valigi. Robust ripple-free regulation and tracking for parameter-dependent sampled-data systems. *IEEE Trans. Aut. Cont.*, 41(7):1031–1037, 1996. ISSN 0018-9286. doi: 10.1109/9.508911.

O. M. Grasselli, L. Menini, and P. Valigi. Ripple-free robust output regulation and tracking for multirate sampled-data control systems. *International Journal of Control*, 75(2):80–96, 2002. doi: 10.1080/00207170110075342.

K. H. Johansson. *Encyclopedia of Life Support Systems (EOLSS)*, chapter Hybrid Control Systems. H. Unbehauen, Ed., 2004.

B. Kouvaritakis and A. G. J. MacFarlane. Geometric approach to analysis and synthesis ofsystem zeros. part 2. non-square systems. *International Journal of Control*, 23(2):167–181, 1976. doi: 10.1080/00207177608922150.

D. Liberzon. *Switching in Systems and Control*. Systems & control. SPRINGER VERLAG NY, 2003. ISBN 9780817642976.

L. Marconi and A. Teel. A note about hybrid linear regulation. In *Conf. on Decision and Control*, pages 1540–1545, 2010. doi: 10.1109/CDC.2010.5717219.

L. Marconi and A. Teel. Internal model principle for linear systems with periodic state jumps. *IEEE Trans. Aut. Cont.*, 58(11):2788–2802, 2013. ISSN 0018-9286. doi: 10.1109/TAC.2013.2272137.

L. Menini and A. Tornambè. Asymptotic tracking of periodic trajectories for a simple mechanical system subject to nonsmooth impacts. *IEEE Trans. Aut. Cont.*, 46(7):1122–1126, 2001. ISSN 0018-9286. doi: 10.1109/9.935068.

I. Moraresku and B. Brogliato. Trajectory tracking control of multiconstraint complementarity lagrangian systems. *IEEE Trans. Aut. Cont.*, 55(6):1300–1313, 2010. ISSN 0018-9286. doi: 10.1109/TAC.2010.2042231.

R. Sanfelice, R. Goebel, and A. Teel. Invariance principles for hybrid systems with connections to detectability and asymptotic stability. *IEEE Trans. Aut. Cont.*, 52(12):2282–2297, 2007. ISSN 0018-9286. doi: 10.1109/TAC.2007.910684.

Z. Sun and S. Ge. *Switched Linear Systems: Control and Design*. Springer, 2005. ISBN 9781852338930.

Y. Yamamoto. A function space approach to sampled data control systems and tracking problems. *IEEE Trans. Aut. Cont.*, 39(4):703–713, 1994. ISSN 0018-9286. doi: 10.1109/9.286247.