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Consensus of Singular Multi-agent Systems Based on Networked Predictive Control ⋆

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Abstract: This brief studies the causal consensus problem of singular (descriptor) multi-agent systems with networked communication delays and agents described by general singular systems. For the studied systems, only the information of outputs are available through the network. An observer-based networked predictive control scheme is employed to compensate for the communication delays actively. Furthermore, based on the output feedback, observer and the networked predictive control scheme, a novel protocol is proposed in this paper. By using the tools of graph, algebra and singular system theory, the necessary and sufficient conditions are given for the existence of the proposed protocol to solve the considered consensus problem. The given conditions depend on not only the topologies of singular multi-agent systems but also the structure properties of each agent dynamics. Moreover, a consensus algorithm is proposed to design the novel observer-based predictive protocol. A numerical example demonstrates the effectiveness of compensation for networked delays using the provided consensus algorithm.

1. INTRODUCTION

Distributed coordination control of multi-agent systems (MASs) has attracted considerable attention due to board applications including formation control, distributed sensor networks, flocking and congestion control in communication networks (Reynolds [1987], Olfati-Saber and Murray [2002], Fax and Murray [2004]).

Consensus is one of the most fundamental distributed coordination control problems in MASs. Consensus means that multiple agents reach an agreement on a common value which might be, for example, the altitude in multi-spacecraft alignment, heading direction in flocking behavior, or average in distributed computation (Lin and Jia [2010]). At present, numerous results have been obtained for consensus problems of MASs. For MASs with state and measurement disturbances, Liu et al. [2009] has provided a distributed dynamic compensator to solve the consensus problem with H_∞ performance using H_∞ techniques. Zhai et al. [2011] has dealt with the consensus problem for MASs via a decentralized dynamic compensator by reducing this problem to solving one strict matrix inequality. Consensus problems of multi-agent networks with directed communication graphs have been discussed in Guan et al. [2012]. Moreover, several conditions are proposed to guarantee that all agents achieve consensus and satisfy robust H_∞ performance. Wen et al. [2012] has investigated stochastic consensus problem for nonlinear MASs with repairable actuator failures and state-dependent noise perturbations.

It is well known that singular systems provide a more natural description of dynamical systems than state space systems (Luenberger and Arbel [1977], Hill and Maareels [1990]). Moreover, the concept of singular multi-agent systems (SMASs) has been introduced in Yang and Liu [2012]. Taking a SMAS consisting of several singular systems such as three-link manipulators through networks for example, achieving consensus means that differences of states between different manipulators tend to zero, respectively. Roughly speaking, these manipulators move in the very similar trajectories, but these trajectories do not overlap. In the last two decades, many results of state space systems have been extended to singular systems (Dai [1989], Yang et al. [2010]). But rare works have been published to deal with consensus of SMASs. For SMASs with agents described by homogenous or heterogenous singular systems, consensus conditions have been proposed using the tools of graph, algebra and singular system theory (Yang and Liu [2011]). Yang and Liu [2012] has provided the necessary and sufficient consensus conditions with respect to a set of admissible consensus protocols for SMASs with fixed topologies. However, Yang and Liu [2011] and Yang and Liu [2012] do not consider the networked communication delays. Due to finite speed of transmissions and limited bandwidth of communication channels, it is inevitable that networked communication delays occur when information exchanges among agents in networked MASs through the shared network. It is worth mentioning that networked delays often degrade the performance of MASs and destroy the stability of systems (Wu et al. [2011]). Thus, it is essential to eliminate or reduce the negative effect of networked delays. Therefore, this paper concerns the causal consensus problem of networked singular multi-agent systems (NSMASs) with fixed topologies and agents

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described by general singular systems. A observer-based networked predictive control scheme is employed to compensate for the communication delays actively and effectively. Based on the output feedback, observer and the networked predictive control scheme (NPCS), a novel protocol is proposed to solve the consensus problem of studied SMASs. Furthermore, the consensus algorithm is proposed to design this novel observer-based predictive protocol. The provided numerical example demonstrates the effectiveness of compensation for communication delays.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Preliminaries

For the given vector x, $\|x\|$ stands for the Euclidean norm. $\mathcal{R}$ and $\mathcal{C}$ represent the real plane and the complex plane, respectively. Let $\otimes$ denote the Kronecker product of matrices $A = [a_{ij}] \in \mathcal{R}^{m\times n}$ and $B \in \mathcal{R}^{p\times q}$, which is defined as

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

and satisfies the properties

$$(A \otimes B)(C \otimes D) = (AC) \otimes (BD),$$
$$A \otimes B + A \otimes C = A \otimes (B + C).$$

A matrix $H \in \mathcal{R}^{n\times n}$ is said to be Schur stable if $\sigma(H) \subseteq D(0, 1)$, where $D(0, 1)$ expresses the interior of an identity circle whose center is the origin, and $\sigma(H) = \{s|\det(sI_n - H) = 0\}$.

Definition 1. (Yang et al. [2004]) Let $E, A \in \mathcal{R}^{n\times n}$.

(i) The pair (E, A) is said to be regular if $\det(sE - A)$ is not identically zero for some $s \in \mathcal{C}$;
(ii) The pair (E, A) is said to be causal if (E, A) is regular and $\mathrm{degdet}(sE - A) = \mathrm{rank}E$ for $\forall s \in \mathcal{C}$;
(iii) Singular discrete-time system

$$Ex(k + 1) = Ax(k)$$

is said to be regular and causal, if the pair (E, A) is regular and causal.

Definition 2. (Yang et al. [2004]) Singular discrete-time system

$$Ex(k + 1) = Ax(k) + Bu(k), \tag{1a}$$
$$y(k) = Cx(k), \tag{1b}$$

is said to be Y-controllable, if there exists a state feedback $u(k) = Kx(k) + v(k)$ such that the closed-loop system

$$Ex(k + 1) = (A + BK)x(k) + Bv(k) \tag{2}$$

is causal, where $v(k)$ is a new input.

Definition 3. (Yang et al. [2004]) System (1) is said to be Y-observable, if at arbitrary time k, $x(k)$ is uniquely determined by the initial condition and $\{u(i),\ y(i),\ i = 0, 1, \cdots, k\}$.

Lemma 1. (Yang et al. [2004]) System (1) is Y-controllable if and only if

$$\mathrm{rank}\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = \mathrm{rank}(E) + n;$$

System (1) is Y-observable if and only if

$$\mathrm{rank}\begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = \mathrm{rank}(E) + n.$$

Lemma 2. (Yang et al. [2004]) For system (1), there exists an output feedback $u(k) = Fy(k) + v(k)$ such that the closed-loop system (2) via $u(k)$ is causal if and only if system (1) is Y-controllable and Y-observable.

In general, information exchanges among agents are achieved through a network for (singular) multi-agent systems. The networked communication topology of multiple agents can be modeled by directed or undirected graphs (Skelon et al. [1998]). Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted digraph with the set of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$ denoting the agents, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and the nonnegative weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$. In $\mathcal{G}$, a directed edge $e_{ij} = (i, j) \in \mathcal{E}$ means that j-th agent can receive information from i-th agent directly, where node j and node i are called child and parent node, respectively. The neighbor set of the i-th agent is denoted by $N_i = \{j \in \mathcal{V}|e_{ji} \in \mathcal{E}\}$. The adjacency elements $a_{ii} = 0$, $a_{ij} > 0 \Leftrightarrow j \in N_i$ associated with e_{ji}, otherwise $a_{ij} = 0$. The Laplacian matrix $\mathcal{L}_\mathcal{G}$ of the weighted digraph $\mathcal{G}$ is defined as

$$\mathcal{L}_\mathcal{G} = \mathcal{D} - \mathcal{A}, \tag{3}$$

where

$$\mathcal{D} = \mathrm{diag}[\mathrm{d_{in}}(1), \cdots, \mathrm{d_{in}}(N)], \tag{4}$$

$\mathrm{d_{in}}(i) = \sum_{j=1}^{N} a_{ij}$. Clearly, all row-sums of $\mathcal{L}_\mathcal{G}$ are zero, which implies that $\mathcal{L}_\mathcal{G}$ has at least one zero eigenvalue and corresponding the right eigenvector ∞_N, where $\infty_N = [1\ 1\ \cdots 1]^T \in \mathcal{R}^N$.

A directed tree is such a directed graph whose every node has exactly one parent node except the root node. A spanning tree of the digraph is a directed tree containing all nodes of the digraph. A graph is said to contain a spanning tree if a subset of the edges forms a spanning tree.

Lemma 3. (Ren and Beard [2005])The Laplacian matrix $\mathcal{L}_\mathcal{G}$ of a directed graph $\mathcal{G}$ has at least one zero eigenvalue and all non-zero eigenvalues are in the open left-half plane. Furthermore, $\mathcal{L}_\mathcal{G}$ has exactly one zero eigenvalue if and only if $\mathcal{G}$ contains a directed spanning tree.

2.2 Problem Formulation

Consider a NSMAS consisting of N agents indexed by $1, 2, \cdots, N$, respectively. The dynamics of the i-th agent are described by a singular discrete-time system:

$$Ex_i(k + 1) = Ax_i(k) + Bu_i(k), \tag{5a}$$
$$y_i(k) = Cx_i(k), \tag{5b}$$

where $x_i(k)$ is the state, $u_i(k)$ is the control input, $y_i(k)$ is the measured output, $E, A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times q}$, $C \in \mathcal{R}^{m\times n}$ and $\mathrm{rank}E = r \leq n$. The communication topology is described by a directed digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a set of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and a nonnegative weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$.

In order to guarantee the feasibility of the proposed approach designing protocols, some assumptions can be reasonably made:

Assumption 1. (i) The each agent system is Y-controllable and Y-observable;

(ii) The state of each agent system can not be measured, but the output is available through the shared network;

(iii) The networked communication delay d is a known and constant positive integer.

Definition 4. For NSMAS (5), protocol $u_i(k),\ i \in \mathcal{V}$ is said to solve the causal consensus problem (or NSMAS (5) achieves causal consensus via protocol $u_i(k)$) if the closed-loop system via $u_i(k)$ is causal, and the following condition holds:

$$\lim_{k\to\infty} \|x_j(k) - x_i(k)\| = 0,\ \forall i, j \in \mathcal{V}. \tag{6}$$

The aim of this paper is to solve the following consensus problem.

Problem 1. For NSMAS (5) with the directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, design protocol $u_i(k)$ to solve the causal consensus problem.

Adopt an output feedback

$$u_i(k) = Fy_i(k) + v_i(k),\ i \in \mathcal{V}, \tag{7}$$

where $F \in \mathcal{R}^{n\times q}$ is designed to guarantee that the closed-loop system

$$Ex_i(k+1) = (A + BFC)x_i(k) + Bv_i(k) \tag{8}$$

is causal (Yang et al. [2004]), and $v_i(k)$ will be designed as follows. There exist two nonsingular matrices P and Q such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},\ P(A+BFC)Q = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},\ CQ = [\,C_1\ C_2\,],\ Q^{-1}x_i(k) = \begin{bmatrix} z_{i1}(k) \\ z_{i2}(k) \end{bmatrix},$$

where A_{22} such that $\det(A_{22}) \neq 0$ which implies system (8) is causal (Yang et al. [2004]). Then the restricted equivalent form of the system (8) is obtained:

$$z_{i1}(k+1) = A_{11}z_{i1}(k) + A_{12}z_{i2}(k) + B_1v_i(k), \tag{9a}$$

$$0 = A_{21}z_{i1}(k) + A_{22}z_{i2}(k) + B_2v_i(k), \tag{9b}$$

$$\begin{aligned} &y_{i1}(k) = C_1z_{i1}(k),\ y_{i2}(k) = C_2z_{i2}(k), \\ &y_i(k) = y_{i1}(k) + y_{i2}(k). \end{aligned} \tag{9c}$$

Hence (6) holds if and only if

$$\lim_{k\to\infty} \|z_{j1}(k) - z_{i1}(k)\| = 0 \tag{10}$$

and

$$\lim_{k\to\infty} \|z_{j2}(k) - z_{i2}(k)\| = 0 \tag{11}$$

hold simultaneously.

It can be obtained from (9b) that

$$z_{i2}(k) = -A_{22}^{-1}[A_{21}z_{i1}(k) + B_2v_i(k)]. \tag{12}$$

Substituting (12) into (9a) derives

$$z_{i1}(k+1) = \bar{A}z_{i1}(k) + \bar{B}v_i(k),\ y_{i1}(k) = C_1z_{i1}(k), \tag{13}$$

where $\bar{A} = A_{11} - A_{12}A_{22}^{-1}A_{21},\ \bar{B} = B_1 - A_{12}A_{22}^{-1}B_2$.

Due to the state of each agent system can not be measured, an observer is adopted:

$$\hat{z}_{i1}(k+1) = \bar{A}\hat{z}_{i1}(k) + \bar{B}v_i(k) + L[y_{i1}(k) - C_1\hat{z}_{i1}(k)], \tag{14}$$

where $\hat{z}_{i1}(k)$ and $v_i(k)$ are the state and control input of the observer, respectively. Using observer design approaches (Yang et al. [2004]), design L to guarantee that $\lim_{k\to\infty} \|\hat{z}_{i1}(k) - z_{i1}(k)\| = 0,\ i \in \mathcal{V}$.

Since information exchanges with the networked communication delay d, at time k, i-th agent only receives information from j-th agent at time $k-d$. In order to compensate the networked communication delay actively and effectively, NPCS proposed in Liu et al. [2007] is employed to the studied consensus problem. Based on the output data from j-th agent up to time $k-d$, construct the state predictions of j-th agent from time $k-d$ to time t as follows:

$$\begin{aligned} \hat{z}_{j1}(k-d+1|k-d) =& \bar{A}\hat{z}_{j1}(k-d) + \bar{B}v_j(k-d) \\ &+ L[y_{j1}(k-d) - C_1\hat{z}_{j1}(k-d)], \\ \hat{z}_{j1}(k-d+2|k-d) =& \bar{A}\hat{z}_{j1}(k-d+1|k-d) \\ &+ \bar{B}v_j(k-d+1), \\ &\vdots \\ \hat{z}_{j1}(k|k-d) =& \bar{A}\hat{z}_{j1}(k-1|k-d) + \bar{B}v_j(k-1), \end{aligned}$$

By the way of iteration,

$$\begin{aligned} \hat{z}_{j1}(k|k-d) =& \bar{A}^{d-1}(\bar{A} - LC_1)\hat{z}_{j1}(k-d) \\ &+ \sum_{s=1}^{d} \bar{A}^{d-s}\bar{B}v_j(k-d+s-1) \\ &+ \bar{A}^{d-1}Ly_{j1}(k-d),\ j \in \mathcal{V}. \end{aligned} \tag{15}$$

For NMAS (13) with the communication delay d, the protocol based on NPCS is adopted as

$$v_i(k) = v_i(k|k-d) = K\sum_{j\in N_i} a_{ij}[\hat{z}_{j1}(k|k-d) - \hat{z}_{i1}(k)], \tag{16}$$

where K is a weighted constant matrix to be designed.

Definition 5. For NMAS (13), protocol (16) based on observer (14) is said to solve the consensus problem (or NMAS (13) achieves consensus via protocol (16) based on observer (14)) if the following conditions hold:

$$\lim_{k\to\infty} \|z_{j1}(k) - z_{i1}(k)\| = 0,$$

$$\lim_{k\to\infty} \|e_i(k)\| = 0,\ \forall i, j \in \mathcal{V},$$

where $e_i(k) = \hat{z}_{i1}(k) - z_{i1}(k)$.

Based on the previous preparation, solving Problem 1 has been converted to solving the following Problem 2.

Problem 2. Design a matrix F and protocol (16) to guarantee that $\det(A_{22}) \neq 0$, (10) and (11) hold.

3. CONSENSUS OF MASS BASED ON NPCS

3.1 Analysis of Consensus Conditions Based on NPCS

Theorem 1. For NMAS (13) with the directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, if $\bar{A}$ is not Schur stable, then protocol (16) solves the consensus problem if and only if $\mathcal{G}$ contains a directed spanning tree, and $\bar{A} - \lambda_i\bar{B}K$ and $\bar{A} - LC_1$ are Schur stable, where $\lambda_i,\ i \in \mathcal{V}\backslash\{1\}$ are the non-zero eigenvalues of the Laplacian matrix $\mathcal{L}_{\mathcal{G}}$.

Proof. Denote

$$\delta_i(k) = z_{i1}(k) - z_{11}(k),\ i \in \mathcal{V},$$
$$\delta(k) = [\delta_2^T(k)\ \delta_3^T(k)\ \cdots \delta_N^T(k)]^T,$$
$$e(k) = [e_2^T(k)\ e_3^T(k)\ \cdots e_N^T(k)]^T.$$

It can be concluded from Definition 5 that protocol (16) solves the consensus problem if and only if $\lim_{k\to\infty} \delta(k) = 0$ and $\lim_{k\to\infty} e(k) = 0$ hold simultaneously. By the iteration of system (13), the state $z_{j1}(k)$ can also be written as

$$z_{j1}(k) = \bar{A}^d z_{j1}(k-d) + \sum_{s=1}^{d} \bar{A}^{d-s}\bar{B}v_j(k-d+s-1). \quad (17)$$

From (13) and (14), it can be obtained:

$$e_i(k+1) = (\bar{A} - LC_1)^{d-1} e_i(k-d+1). \quad (18)$$

Then it follows that

$$\hat{z}_{i1}(k) = z_{i1}(k) + (\bar{A} - LC_1)^{d-1} e_i(k-d+1). \quad (19)$$

Combining with (15), (17) and (18) gives

$$\begin{aligned}\hat{z}_{j1}(k|k-d) =& \bar{A}^{d-1}(\bar{A} - LC_1)\hat{z}_{j1}(k-d) + z_{j1}(k)\\ &- \bar{A}^d z_{j1}(k-d) + \bar{A}^{d-1}LC_1 z_{j1}(k-d)\\ =& z_{j1}(k) + \bar{A}^{d-1}e_j(k-d+1).\end{aligned} \quad (20)$$

Substituting (19) and (20) into (16) yields

$$\begin{aligned}&v_i(k)\\ =&K\sum_{j=1}^{N}\{a_{ij}[\delta_j(k) + \bar{A}^{d-1}e_j(k-d+1)]\} - \mathrm{d_{in}}(i)K[\delta_i(k)\\ &+ (\bar{A} - LC_1)^{d-1}e_i(k-d+1)].\end{aligned} \quad (21)$$

The closed-loop system of system (13) via protocol (21) can be described as

$$\begin{aligned}&z_{i1}(k+1)\\ =&\bar{A}z_{i1}(k) + \bar{B}K\sum_{j=1}^{N}\{a_{ij}[\delta_j + \bar{A}^{d-1}e_j(k-d+1)]\}\\ &- \mathrm{d_{in}}(i)\bar{B}K[\delta_i(k) + (\bar{A} - LC_1)^{d-1}e_i(k-d+1)].\end{aligned}$$

Thus,

$$\begin{aligned}&\delta_i(k+1)\\ =&(\bar{A} - \mathrm{d_{in}}(i)\bar{B}K)\delta_i(k) + [(\gamma_i - \gamma_1)\otimes(\bar{B}K)]\delta(k)\\ &+ [(\mathcal{A}_i - \mathcal{A}_1)\otimes(\bar{B}K\bar{A}^{d-1})]e(k-d+1)\\ &+ \{(\mathcal{D}_1 - \mathcal{D}_i)\otimes[\bar{B}K(\bar{A} - LC_1)^{d-1}]\}e(k-d+1),\end{aligned}$$

where $\mathcal{A}_i,\ \mathcal{D}_i,\ \gamma_i,\ i \in \mathcal{V}$ such that $[\mathcal{A}_1\ \mathcal{A}_2\ \cdots\ \mathcal{A}_N] = \mathcal{A}^T,\ [\mathcal{D}_1\ \mathcal{D}_2\ \cdots\ \mathcal{D}_N] = \mathcal{D}^T,\ \gamma_i = [a_{i2}\ a_{i3} \cdots\ a_{iN}]$, and $\mathcal{D}$ is defined by (4).

Hence

$$\delta(k+1) = \Gamma\delta(k) + \Xi e(k-d+1),$$

where

$$\begin{aligned}\Gamma &= I_{N-1}\otimes\bar{A} - (\mathcal{L}_{22} - \infty_{N-1}\mathcal{L}_{12})\otimes(\bar{B}K),\\ \Xi &= (\infty_{N-1}\mathcal{D}_1 - \tilde{\mathcal{D}})\otimes[\bar{B}K(\bar{A} - LC_1)^{d-1}],\\ &\quad + (\tilde{\mathcal{A}} - \infty_{N-1}\mathcal{A}_1)\otimes(\bar{B}K\bar{A}^{d-1}),\\ \tilde{\mathcal{D}} &= [\mathcal{D}_2^T\ \mathcal{D}_3^T\ \cdots\ \mathcal{D}_N^T]^T,\ \tilde{\mathcal{A}} = [\mathcal{A}_2^T\ \mathcal{A}_3^T\ \cdots\ \mathcal{A}_N^T]^T,\\ &\begin{bmatrix}\mathcal{L}_{11} & \mathcal{L}_{12}\\ \mathcal{L}_{21} & \mathcal{L}_{22}\end{bmatrix} = \mathcal{L}_{\mathcal{G}},\ \infty_{N-1} = [1\ 1\ \cdots\ 1]^T \in \mathcal{R}^{N-1}.\end{aligned}$$

Then it follows that

$$\begin{bmatrix}\delta(k+1)\\ e(k-d+2)\end{bmatrix} = \begin{bmatrix}\Gamma & \Xi\\ 0 & I_N\otimes(\bar{A} - LC_1)\end{bmatrix}\begin{bmatrix}\delta(k)\\ e(k-d+1)\end{bmatrix}. \quad (22)$$

Furthermore, protocol (16) solves the consensus problem if and only if system (22) is Schur stable, which implies that Γ and $\bar{A} - LC_1$ are Schur stable.

Let $H = \begin{pmatrix}1 & 0\\ \infty_{N-1} & I_{N-1}\end{pmatrix}$. Using the definition of Laplacian matrix $L_{\mathcal{G}}$ obtains

$$H^{-1}L_{\mathcal{G}}H = \begin{pmatrix}0 & \mathcal{L}_{12}\\ 0 & \mathcal{L}_{22} - \infty_{N-1}\mathcal{L}_{12}\end{pmatrix}. \quad (23)$$

Let $\lambda_1 = 0,\ \lambda_2,\ \cdots,\ \lambda_N$ be eigenvalues of $L_{\mathcal{G}}$. It can be obtained from (23) that $\lambda_2,\ \cdots,\ \lambda_N$ are eigenvalues of $\mathcal{L}_{22} - \infty_{N-1}\mathcal{L}_{12}$. Thus, there exists a nonsingular matrix T such that $\mathcal{L}_{22} - \infty_{N-1}\mathcal{L}_{12}$ is similar to a Jordan canonical, that is

$$T^{-1}(\mathcal{L}_{22} - \infty_{N-1}\mathcal{L}_{12})T = J = \mathrm{diag}(J_1, \cdots, J_s),$$

where $J_k,\ k = 1, 2, \cdots, s$, are upper triangular Jordan blocks. Hence

$$\begin{aligned}&I_{N-1}\otimes\bar{A} - J\otimes(\bar{B}K)\\ =&(T\otimes I_n)^{-1}[\bar{A} - (\mathcal{L}_{22} - \infty_{N-1}\mathcal{L}_{12})\otimes(\bar{B}K)](T\otimes I_n),\end{aligned}$$

which implies that eigenvalues of $I_{N-1}\otimes\bar{A} - (\mathcal{L}_{22} - \mathbf{1}_{N-1}\mathcal{L}_{12})\otimes(\bar{B}K)$ are given by all eigenvalues of $\bar{A} - \lambda_i\bar{B}K,\ i \in \mathcal{V}\backslash\{1\}$. Thus, protocol (16) solves the consensus problem if and only if $\bar{A} - \lambda_i\bar{B}K,\ i \in \mathcal{V}\setminus\{1\}$ and $\bar{A} - LC_1$ are Schur stable. The proof of sufficiency is completed.

Necessity. Based on the previous derivation, it suffices to show that $\mathrm{Re}\lambda_i > 0,\ i \in \mathcal{V}\setminus\{1\}$ and $\mathcal{G}$ contains a directed spanning tree. It can be obtained from Lemma 3 that $\lambda_i = 0$ or $\mathrm{Re}\lambda_i > 0,\ i \in \mathcal{V}\setminus\{1\}$. Assume that there exists some $j \in \mathcal{V}\setminus\{1\}$ such that $\lambda_j = 0$. Thus, $\bar{A} - \lambda_j\bar{B}K = \bar{A}$. Since $\bar{A} - \lambda_j\bar{B}K$ is Schur stable, so $\bar{A}$ is Schur stable, which is a contradiction with the precondition. Hence $\mathrm{Re}\lambda_i > 0,\ i \in \mathcal{V}\setminus\{1\}$. Therefore, it can be concluded from Lemma 3 that $\mathcal{G}$ contains a directed spanning tree. The proof is completed.

Remark 1. It can be obtained from Theorem 1 that MAS (13) achieving consensus via protocol (16) based on NPCS has no relationship with networked communication delays, and only depends on the topology of MAS (13) and the structure properties of each agent dynamics. Hence protocol (16) based on NPCS can compensate communication delays effectively.

Remark 2. When protocol (16) solves consensus problem of NMAS (13), one has

$$\lim_{k\to\infty}\delta_i(k) = 0,\ \lim_{k\to\infty}e_i(k) = 0,\ i \in \mathcal{V}.$$

which, together with (21) derives

$$\lim_{k\to\infty} v_i(k) = 0, i \in \mathcal{V}. \quad (24)$$

From (12), one obtains

$$z_{i2}(k) - z_{12}(k) = -A_{22}^{-1}A_{21}\delta_i(k) - A_{22}^{-1}B_2[v_i(k) - v_1(k)].$$

Combining $\lim_{k\to\infty}\delta_i(k) = 0$ and (24) yields

$$\lim_{k\to\infty}\|z_{j2}(k) - z_{i2}(k)\| = 0,\ \forall i,\ j \in \mathcal{V}.$$

Hence Problem 2 is solved if protocol (16) solves the consensus problem of system (13).

Corollary 1. For NMAS (13) with the directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, when protocol (16) solves consensus problem, if $\bar{A}$ is not Schur stable, then $\mathcal{G}$ contains a directed spanning tree. Furthermore, if $\bar{A}$ is nonsingular, then $(\bar{A}, \bar{B}, C_1)$ is stabilizable and detectable.

Proof. It can be concluded from Theorem 1 that $\mathcal{G}$ contains a directed spanning tree, and $\bar{A} - LC_1$ is Schur stable which implies that $(\bar{A}, C_1)$ is detectable. Similar to the proof processes of [Yang and Liu, 2012, Theorem 1], it can be obtained that $\left(\begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{A} \end{bmatrix}, \begin{bmatrix} \bar{B} & 0 \\ 0 & \bar{B} \end{bmatrix} \right)$ is stabilizable. Since $\bar{A}$ is nonsingular, one obtains

$$\text{rank} \begin{bmatrix} sI_n - \bar{A} & 0 & \bar{B} & 0 \\ 0 & sI_n - \bar{A} & 0 & \bar{B} \end{bmatrix} = 2n,$$

or equivalently,

$$\text{rank} \left[sI_n - \bar{A} \ \bar{B} \right] = n, \ \forall s \in \mathcal{C} \setminus D(0, 1).$$

Hence $(\bar{A}, \bar{B})$ is stabilizable. The proof is completed.

3.2 Design of Observer-based Predictive Protocols

Based on the previous preparation, the following algorithm is provided to design observer-based predictive protocol (7) associated with observer (14) and protocol (16), which implies that Problem 1 will be solved under Assumption 1.

Algorithm 1. Input: the matrices E, $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times q}$, $C \in \mathcal{R}^{m\times n}$, $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$ and $d \in \mathcal{R}$;
Output: the gain matrices F, L and K.

(a) Find nonsingular matrices P and Q such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix};$$

(b) Compute the matrices $\hat{A}_{11} \in \mathcal{R}^{r\times r}$, $\hat{A}_{12} \in \mathcal{R}^{r\times(n-r)}$, $\hat{A}_{21} \in \mathcal{R}^{r\times(n-r)}$, $\hat{A}_{22} \in \mathcal{R}^{(n-r)\times(n-r)}$, $B_1 \in \mathcal{R}^{r\times q}$, $B_2 \in \mathcal{R}^{(n-r)\times q}$, $C_1 \in \mathcal{R}^{m\times r}$ $C_2 \in \mathcal{R}^{m\times(n-r)}$ by

$$\begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} = PAQ, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = PB,$$

$$\left[C_1 \ C_2 \right] = CQ;$$

(c) Choose a matrix F such that $\det(\hat{A}_{22} + B_2FC_2) \neq 0$;

(d) Compute the matrices A_{11}, A_{12}, A_{21}, A_{22}, $\bar{A}$ and $\bar{B}$ by

$$A_{11} = \hat{A}_{11} + B_1FC_1, \ A_{12} = \hat{A}_{12} + B_1FC_2,$$

$$A_{21} = \hat{A}_{21} + B_2FC_1, \ A_{22} = \hat{A}_{22} + B_2FC_2,$$

$$\bar{A} = A_{11} - A_{12}A_{22}^{-1}A_{21}, \ \bar{B} = B_1 - A_{12}A_{22}^{-1}B_2;$$

(e) Compute Laplacian matrix $\mathcal{L}_{\mathcal{G}}$ by (3) and the nonzero eigenvalues λ_i of $\mathcal{L}_{\mathcal{G}}$, $i = \in \mathcal{V} \setminus \{1\}$;

(f) If $(\bar{A}, C_1)$ is detectable, choose a matrix L such that $\bar{A} - LC_1$ is Schur stable. Otherwise, come back Step (c) to choose F again;

(g) Choose a matrix K such that $\bar{A} - \lambda_i\bar{B}K$, $i \in \mathcal{V} \setminus \{1\}$ are Schur stable. Then output the matrices F, L and K.

Remark 3. According to Lemma 2, Assumption 1 guarantees the existence of F in step (c) of Algorithm 1.

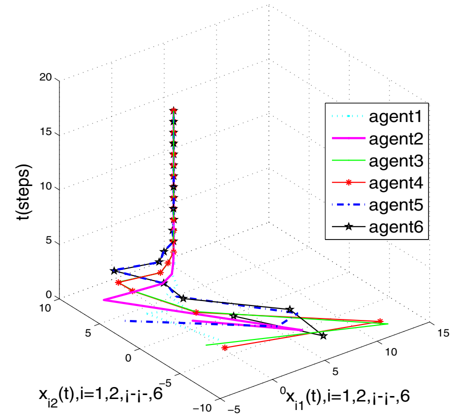

Fig. 1. Trajectories of state x_{i1}, x_{i2}, $i = 1, 2 \cdots, 6$

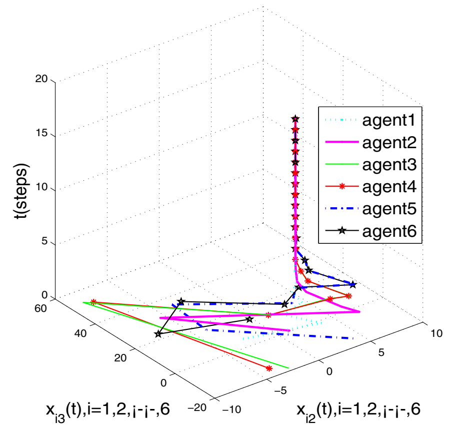

Fig. 2. Trajectories of state x_{i2}, x_{i3}, $i = 1, 2 \cdots, 6$

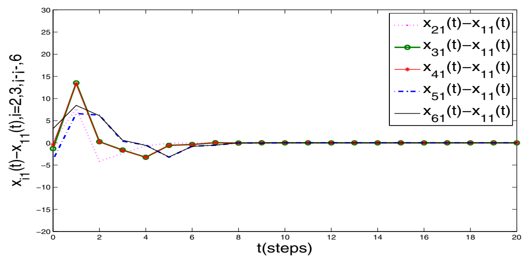

Fig. 3. Trajectories of $x_{i1} - x_{11}$, $i = 2, 3 \cdots, 6$

4. NUMERICAL EXAMPLE

Consider SMAS (5) consisting of $N = 6$, where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} 1 & 1 & -1 \\ -3.5 & -1.5 & 1 \\ -2 & 0 & 0 \end{bmatrix},$$

$$B = \left[1 \ 0 \ 1 \right]^T, \ C = \left[3 \ 2 \ 1 \right].$$

The networked communication delay $d = 3$ and elements of the adjacency matrix $\mathcal{A}$: $a_{21} = a_{32} = a_{42} = a_{54} = a_{64} = 1$, otherwise $a_{ij} = 0$.

According to the steps in Algorithm 1, the following can be obtained: $F = 1$, $L = \left[1.3 \ -1.5 \right]^T$, $K = \left[3.5 \ 2.7 \right]$.

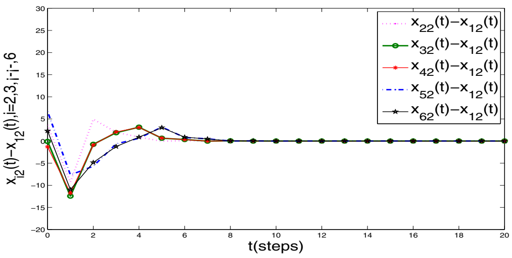

Fig. 4. Trajectories of $x_{i2} - x_{12},\ i = 2, 3 \cdots, 6$

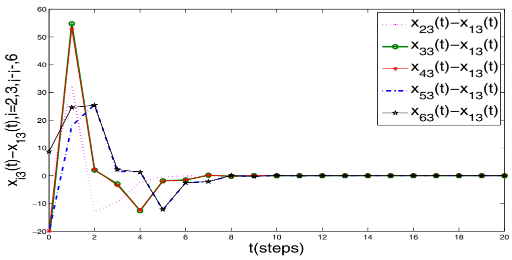

Fig. 5. Trajectories of $x_{i3} - x_{13},\ i = 2, 3 \cdots, 6$

The simulation results are presented in Figs. 1-5, respectively. Figs. 1 and 2 show state trajectories of NSMAS (5) which indicates NSMAS (5) achieves causal consensus via observer-based predictive protocol (16). Figs. 3-5 present state differences of NSMAS (5) all tend to zero which implies Problem 1 is solved by protocol (16) using Definition 4.

5. CONCLUSION

For NSMASs with directed topologies and constant communication delays, the causal consensus problem via observer-based predictive protocols has been solved. Due to only the information of outputs is available through the shared network with communication delays, an observer-based NPCS has been employed to compensate communication delays actively. Based on the output feedback, observer and NPCS, a novel protocol has been proposed in this paper. Moreover, the consensus algorithm has been provided to design this novel observer-based predictive protocol. The provided simulation results have successfully demonstrated the effectiveness of compensation for networked delays via the proposed novel protocol. However, it is worth noticing that the study of consensus for SMASs with directed topologies and constant communication delays is a basic problem, which only serves as a stepping stone to investigate time-varying networked communication delays or more complicated topologies and agent dynamics. The future research will study singular multi-agent systems with time-varying networked delays, stochastic or switching topologies, and agents described by switching systems or hybrid systems, and so on.

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