Fault prognosis for Discrete Manufacturing Processes

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Abstract: This paper deals with a fault prognosis method, based on the extraction of a health indicator (HI) from a large amount of raw sensors data, applied to Discrete Manufacturing Processes (DMP). The HI is extracted by locating the significant points of machine which are related to the degradation. The dynamics of HI is then analysed and modelled using an appropriate stochastic process. The adaptive aspect of the prediction model allows the updating of the Remaining Usesul Life (RUL) estimation. The developed approach is applied on a real case provided by ST-Microelectronics, where experimental result shows its efficiency.

Keywords: Health indicator, fault prognosis, stochastic process

1 Introduction

Fault prognosis of industrial systems is one of central issues of Condition Based Maintenance (CBM). It is important to minimize the downtime of machinery and production, and thus to increase efficiency of operations and manufacturing. Till now, the production process in almost industries (e.g. pharmaceuticals, foods, semiconductors, auto-mobiles, etc.) use a strategy of Preventive and Corrective Maintenance which is less efficient than the CBM. The development of methodologies to predict equipment failure will enable these industries to replace the PCM by the CBM, but currently few studies are conducted on this subject. This is because of their complex operations, which involve a multiple-step sequence at different manufacturing stages. The processes are highly non-linear, time varying, subject to significant disturbances and usually exhibit unit-to-unit variations.

Fault prognosis focuses on predicting the time at which a system or a component will no longer perform its intended function, the time length from the current moment to that time is defined as the RUL. It is a random variable and it depends on health/deterioration information of the asset. Generally, to predict the RUL, many works are either based on a health indicator (HI) which is already available, such as in Bakker and van Noortwijk (2004), Lawless and Crowder (2004), Tseng and Peng (2007) or based on some mathematical hypotheses of HI profile, such as in Bagdonavicius and Nikulin (2001), Le Son et al. (2012a) or propose a method to extract HI in their specific study, such as in Le Son et al. (2012b), Benkedjouh et al. (2013). In semiconductor manufacturing, a survey of datadriven prognosis in this field of Alexis Thieullen (2012) shows that, almost the health indicators are calculated from multivariate analysis such as multiway PCA, principal component based k-nearest neighbour or principal

components-based Gaussian mixture model.

This paper proposes a new fault prognosis method for DMP, as illustrated in Fig. 1. An off-line analysis is executed to support the on-line supervision. In off-line analysis, historical data is analysed to identify the significant points of machine which are related to the degradation then extracting a health indicator. Next, the dynamics of this indicator is analysed to construct a prediction model and to identify the model parameters. In on-line supervision, the significant points are observed to calculate the real time health indicator. An adaptive predictive model is launched to provide the RUL.

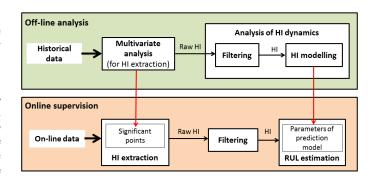
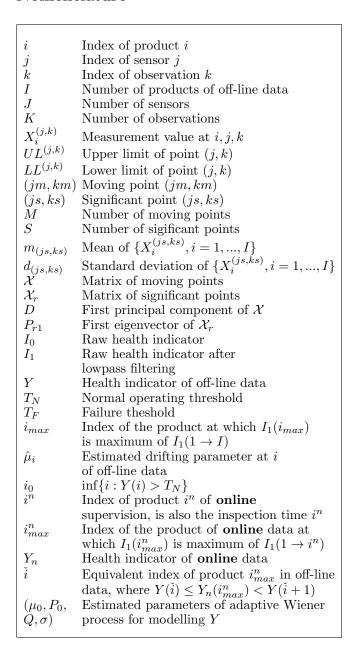


Fig. 1. Schema of fault prognosis

The remaining of this paper is organised as follows. Section 2 presents the off-line analysis where 2.1 provides the formulation of multivariate analysis to contribute a health indicator and 2.2 depicts the degradation modelling based on an adaptive Wiener process. The online supervision procedure is proposed in section 3. In section 4, a real case application using data collected in STMicroelectronics is presented to illustrate the efficiency of the proposed method. The conclusion is given in the last section 5.

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Nomenclature



2 Off-line analysis

2.1 Heath indicator extraction

Hypothesis: the degradation of machine is gradual over time. From this hypothesis, the first products are considered respecting the *good quality* norm.

A discrete manufacturing process is the equipment which processes/produces the distinct items or separate unit of products such as in the industry of automobiles, semiconductors, toys, etc. Therefore, the obtained measurement of a discrete process during processing a set of products is a data of three dimensional matrix $I \times J \times K$, respectively

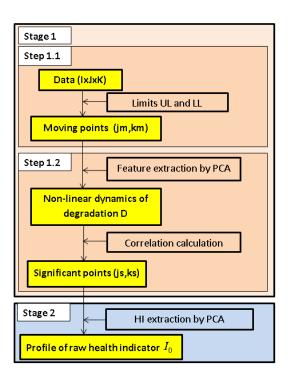


Fig. 2. Schema of health indicator extraction

I is number of products, J is the number of sensors and K is the number of observation (sampling time). Each point of this matrix data is signed $X_i^{(j,k)}$, where $i \in \{1,...,I\}, j \in \{1,...,J\}, k \in \{1,...,K\}$ are respectively the index of product, variable and observation.

A new method of heath indicator extraction is proposed in this section and is depicted in Fig. 2. To build a health indicator, a two-stages method is proposed with the main ideas:

Stage 1: identifying the significant points (js, ks) of sensor and observation which demonstrate the dynamics of degradation on machine from whole points (j, k). This stage consists two steps: step (1.1) identifying moving points and step (1.2) identifying significant points.

Step 1.1:

The two objectives of this step are:

- Eliminating the zero-variance points for the use of Principal Component Analysis (PCA) in next step
- Identifying the points which have a significant variation between the first products (considered as *good quality*) and the degraded one

A large enough number n, (n < I) of first products (considered as good quality products) is chosen to build an upper limit UL and a lower limit LL for each point (j,k):

$$UL^{(j,k)} = max((X_i^{(j,k)}, i = 1, ..., n))$$
 (1)

and

$$LL^{(j,k)} = \min((X_i^{(j,k)}, i=1,...,n) \tag{2}$$

There are two cases:

<u>Case 1</u>: If an arbitrary product N, $n < N \le I$ which is verified *bad quality* by a measure test is available, it is used to compare with the limits to select the *moving points* (j,k) which satisfy the condition:

$$X_N^{(j,k)} > UL^{(j,k)} \quad or \quad X_N^{(j,k)} < LL^{(j,k)}$$
 (3)

Case 2: If there is no product which is verified bad quality, the last product I, is considered as degraded according to the hypothesis 2.1 and is used to select the moving points (j,k) which satisfy the condition:

$$X_I^{(j,k)} > UL^{(j,k)} \quad or \quad X_I^{(j,k)} < LL^{(j,k)}$$
 (4)

After this step, every identified point is signed $X_i^{(jm,km)}$, $i \in \{1,...,I\}$, $(jm,km) \in \{(1_{jm},1_{km}),...,(M_{jm},M_{km})\}$, M is the number of identified points. They are arranged in a new matrix \mathcal{X} :

$$\mathcal{X} = \begin{pmatrix} X_1^{(1_{jm}, 1_{km})} & X_1^{(2_{jm}, 2_{km})} & \dots & X_1^{(M_{jm}, M_{km})} \\ X_2^{(1_{jm}, 1_{km})} & X_2^{(2_{jm}, 2_{km})} & \dots & X_2^{(M_{jm}, M_{km})} \\ \vdots & \vdots & \ddots & \vdots \\ X_I^{(1_{jm}, 1_{km})} & X_I^{(2_{jm}, 2_{km})} & \dots & X_I^{(M_{jm}, M_{km})} \end{pmatrix}$$
(5)

Step 1.2:

 $\overline{\mathcal{X}}$ is then mean-centered and unit-deviation scaled and is decomposed by PCA:

$$\mathcal{X} = T \times P^T \tag{6}$$

where T is score matrix and P^T is transpose matrix of P, P is loading matrix.

Since the machine degradation is supposed to be gradual, at least one among the first principal components depicts the machine features which is progressively decreasing or increasing. The principal features of machine over time are: gradual drifts of degradation, abrupt drifts, noises, disturbances; among them, only gradual drifts of degradation and noises always occur on all the products, so, they are depicted by the first principal component (PC) (first column of T) called D.

The significant points are then identified. Noting that each column of \mathcal{X} , signed $\mathcal{X}^{(jm,km)}$, depicts the evolution of the point (jm,km) chronologically. Among them, some points degrade with ascending tendency meanwhile some other points degrade with descending tendency, the remaining points are those whose evolution do not describe the degradation. Because D carries the degradation dynamics thus the absolute value of correlation between $\mathcal{X}^{(jm,km)}$ of the points which do not correspond to the degradation and D are smaller than those between $\mathcal{X}^{(jm,km)}$ of other points and D. The absolute value of correlation between each $\mathcal{X}^{(jm,km)}$ and D, signed $c^{(jm,km)}$, is calculated as in equation (7):

$$c^{(jm,km)} =$$

$$\frac{1}{I} \left| \frac{(\mathcal{X}^{(jm,km)^T} - m_{(jm,km)} I_{(1,I)}) \times (D - m_D I_{(I,1)})}{d_{(jm,km)} d_D} \right| \quad (7)$$

where $m_{(jm,km)}$ is mean of column $\mathcal{X}^{(jm,km)}$, $d_{(jm,km)}$ is standard deviation of column $\mathcal{X}^{(jm,km)}$, m_D is mean of D, d_D is standard deviation of D, $I_{(1,I)}$ is the identity row of size $1 \times I$, $I_{(I,1)}$ is the identity column of size $I \times 1$ and I is the number of products. Because \mathcal{X} is meancentered and unit-deviation scaled thus $m_{(jm,km)} = 0$, $d_{(jm,km)} = 1 \quad \forall (jm,km)$ and $m_D = 0$ as D is the first PC of \mathcal{X} , $c^{(jm,km)}$ is rewritten as:

$$c^{(jm,km)} = a|\mathcal{X}^{(jm,km)}^T \times D| \tag{8}$$

where $a = 1/(I * d_D)$ is a constant. Signing C is the set of all $c^{(jm,km)}$. To separate the points (jm,km) which do not correspond to degradation from others, a percentile p^{th} of C is calculated and considered as a lower limit to identify the significant points which satisfy:

$$(jm, km): c^{(jm,km)} > percentile_{p^{th}}\{C\}$$
 (9)

The value of p^{th} percentile depends on the application case.

The significant points are signed (js,ks) and their measure value is $X_i^{(js,ks)}$, $i \in \{1,...,I\},(js,ks) \in \{(1_{js},1_{ks}),(2_{js},2_{ks}),...,(S_{js},S_{ks})\}$, the number of significant points is S. They are then arranged in a new matrix \mathcal{X}_r which is called *reduced matrix* and is given as follows:

$$\mathcal{X}_{r} = \begin{pmatrix}
X_{1}^{(1_{js}, 1_{ks})} & X_{1}^{(2_{js}, 2_{ks})} & \dots & X_{1}^{(S_{js}, S_{ks})} \\
X_{2}^{(1_{js}, 1_{ks})} & X_{2}^{(2_{js}, 2_{ks})} & \dots & X_{2}^{(S_{js}, S_{ks})} \\
\vdots & \vdots & \ddots & \vdots \\
X_{I}^{(1_{js}, 1_{ks})} & X_{I}^{(2_{js}, 2_{ks})} & \dots & X_{I}^{(S_{js}, S_{ks})}
\end{pmatrix} (10)$$

Signing $m_{(js,ks)}$ and $d_{(js,ks)}$ are respectively mean and standard deviation of column (js,ks) of matrix \mathcal{X}_r . They are used later (equation 21) to scale the online data for online supervision.

Stage 2: extracting the principal feature from these significant points.

 \mathcal{X}_r is then mean-centered and unit-deviation scaled and is decomposed by PCA:

$$\mathcal{X}_r = T_r \times P_r^T \tag{11}$$

The first PC of \mathcal{X}_r , signed I_0 is considered as the health indicator of machine because it represents the first principal dynamics of all significant points, which are determined describing the degradation dynamics in $Stage\ 1$.

$$I_0 = \mathcal{X}_r \times P_{r1} \tag{12}$$

where P_{r1} is the first column of P_r

2.2 Analysis of health indicator dynamics

2.2.1 Filtering: Applying the health indicator extraction presented at previous section, a common form of the indicator is provided in Fig. 3.a, called I_0 (applied on a real data provided by STMicroelectronics). It is highly

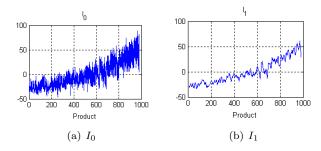


Fig. 3. Raw health indicator with lowpass filtering

noisy with a large variance all the time which will lead to a highly noisy degradation rate if I_0 is modelled. Thus a low-pass filter (e.g. an average filter with a window size of 10) is used to eliminate high frequency noises, the result is called I_1 and presented in the Fig. 3.b.

A real health indicator is always monotonous over time because the degradation is not reversible. However, under the influence of perturbations of machine, of environment and significant disturbances of quality of input products, I_1 is not monotonous. If I1 increases progressively, the higher values reflect the degradation better than their lower neighbour values and inversely if I1 decreases progressively. Therefore, an algorithm is proposed to eliminate disturbances and to monotonize the indicator: I_1 is analysed to structure a top curve I_t which is then considered as health indicator if I1 increases or a bottom-curve I_b if I1 decreases. This algorithm is presented for an increasing indicator as follows (for a decreasing indicator it is the same but replacing "maximum" by "minimum" and replacing the signs by their opposite sign):

Step 1: Searching the maximum peaks of I_1 $\overline{\{I_1(i), i=1 \to I\}}$ is divided into several subsets: $\{I_{1,u}(i), i=1+wu \to w+wu\}$, u, w are integers w>1 (e.g:w=10), u=0,1...,[I/w]

• If $\exists u : max(I_{1,u}(i)) > max(I_{1,u-1}(i), I_{1,u+1}(i))$ $\Longrightarrow max(I_{1,u}(i))$ is a maximum peak $\Longrightarrow I_t = I_t \cup max(I_{1,u}(i))$

Step 2: Monotonizing I_t

- Eliminating minimum peaks of I_t : $I_t(i) \leq min(I_t(i-1), I_t(i+1))$ (this step is executed several times till there is no minimum peak on I_t)
- Eliminating $I_t(end)$ if $I_t(end) \leq I_t(end-1)$

After this step, the last value of I_t is the maximum. Signing i_{max} is the index of product of this last value. $I_t(i_{max}) = I_1(i_{max})$ and $I_1(i_{max})$ is also the maximum value of I_1

Step 3: Interpolating and extrapolating I_t by linear method for all product $i, i \in \{1, ..., i_{max}\}$

One result of this algorithm on a real data (given by STMicroelectronics) is given in Fig. 7 in section 4.

2.2.2 Health indicator modelling: The time unit here is the duration of processing a product on machine. A normal operating threshold T_N is predefined. The filtered health indicator from the previous section is signed Y. Supposing

that Y(i) is increasing, signing i_0 as $i_0 = \inf\{i : Y(i) > T_N\}$. (if Y(i) is decreasing, $i_0 = \inf\{i : Y(i) < T_N\}$). $Y(i), i_0 < i < i_{max}$ is then modelled using an adequate stochastic process.

The Wiener process is considered inadequate in modelling degradation which is monotone, according to Gorjian et al. (2010). In spite of that, it is possibly used in this case because fault prognosis pays more attention to the indicator trend modelling than to the diffusion aspect of the process. Moreover, the health indicator Y(i) has the degradation rate changes with an unknown function. Therefore, the profile of Y(i) can be modelled by an adaptive Wiener process described in Wang et al. (2011). It is presented as below:

Y(j) is the Condition Monitoring (CM) reading at time j, it is modelled as:

$$Y(j) = Y(i) + \mu_i(j-i) + \sigma \epsilon_{i,j}, \quad j > i$$
 (13)

where Y(i) is the i^{th} available CM point, μ_i is the updated drifting parameter at i after observing Y(i), σ is a constant, $\sigma\epsilon_{i,j}$ is the error term which is normally distributed and $\epsilon_{i,j} \sim N(0,j-i)$. μ_i is updated by Kalman filtering. At time i, the system equation is described:

$$\mu_i = \mu_{i-1} + \nu \tag{14}$$

$$Y(i) - Y(i-1) = \mu_{i-1} + \sigma \epsilon_{i-1,i}$$
 (15)

where the error terms are distributed as $\nu \sim N(0,Q)$ and $\epsilon_{i-1,i} \sim N(0,1)$. The updated estimate of μ_i at time i is given as:

$$\hat{\mu}_i = \hat{\mu}_{i-1} + P_{i|i-1}F_i^{-1}(Y(i) - Y(i-1) - \hat{\mu}_{i-1})$$
 (16)

where

$$P_{i|i-1} = P_{i-1} + Q (17)$$

and

$$F_i = P_{i|i-1} + \sigma^2 \tag{18}$$

and

$$P_i = P_{i|i-1} - P_{i|i-1}F_i^{-1}P_{i|i-1}$$
(19)

The parameters (μ_0, P_0, Q, σ) are estimated using the expectation-maximization algorithm given in Wang et al. (2011). All the obtained drifting parameter values are signed $\hat{\mu}_i, i_0 \leq i \leq i_{max}$.

3 On-line supervision

The off-line analysis and on-line supervision can be resumed in the Fig. 4

For on-line supervision: signing i^n is the index of product. For a new product i^n processed on machine, the obtained data is used to calculate the health indicator and to estimate the RUL. We repeat again that the *time unit* here is the duration of processing a product on machine, thus, it is also the index of product.

3.1 Extraction of HI and filtering

From the equation (12), the value of raw health indicator at product i^n is calculated as:

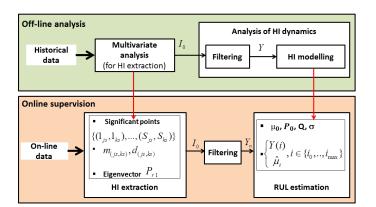


Fig. 4. Off-line and online prognosis

$$I_0(i^n) = \mathcal{X}_r(i^n) \times P_{r1} \tag{20}$$

where $\mathcal{X}_r(i^n) = \left(\bar{X}_{i^n}^{(1_{js},1_{ks})} \bar{X}_{i^n}^{(2_{js},2_{ks})} \dots \bar{X}_{i^n}^{(S_{js},S_{ks})}\right)$, each value $\bar{X}_{i^n}^{(js,ks)}$ is computed from the raw measurement value $X_{i^n}^{(js,ks)}$ of online data as follows:

$$\bar{X}_{in}^{(js,ks)} = \frac{X_{in}^{(js,ks)} - m_{(js,ks)}}{d_{(js,ks)}}$$
(21)

 $m_{(js,ks)}, d_{(js,ks)}$ are respectively mean and standard deviation of the significant points (js,ks) of off-line data, P_{r1} is the eigenvector given in subsection 2.1.

The curve I_0 is then similarly filtered and the obtained health indicator called Y_n (see 2.2.1).

3.2 RUL estimation

A failure threshold T_F is predefined. Supposing that the health indicator is increasing (if it decreases, the method is the same but with opposite signs). When Y_n exceeds the normal operating threshold T_N , the prognosis model is launched. The main idea of RUL estimation is establishing the expected evolution of Y_n based on establishing the predicted evolution of drifting parameter μ_{i^n} on taking into account the maximum value $Y_n(i^n_{max})$.

At each observation i^n of on-line supervision, $Y_n(1 \rightarrow i_{max}^n)$ is available. The RUL estimation at i^n is executed as:

- Searching \check{i} which satisfies $Y(\check{i}) \leq Y_n(i_{max}^n) < Y(\check{i}+1), Y$ is the HI of section 2
- Calculating $\mu_{i_{max}^n}$ of on-line indicator Y_n by the Kalman filtering using parameters (μ_0, P_0, Q, σ) and the equations (16-19)
- Calculating the residual $\Delta = \mu_{i_{max}^n} \hat{\mu}_{\tilde{i}}$
- Establishing the on-line predicted drifting parameter is $\Psi_{i^n} = \{\hat{\mu}_{\check{i}} + \Delta, \hat{\mu}_{\check{i}+1} + \Delta, ..., \hat{\mu}_{i_{max}} + \Delta\}$
- From the equation (15), the expected value of Y_n is estimated recursively: for $0 \le j < i_{max} \check{i}$:

$$E(Y_{n}(i_{max}^{n} + j + 1))$$

$$= E(Y_{n}(i_{max}^{n} + j) + (\hat{\mu}_{i+j} + \Delta) + \sigma\epsilon_{0,1})$$

$$= E(Y_{n}(i_{max}^{n} + j)) + (\hat{\mu}_{i+j} + \Delta)$$
(22)
and for $j \ge i_{max} - \check{i}$:
$$E(Y_{n}(i_{max}^{n} + j + 1))$$

$$= E(Y_{n}(i_{max}^{n} + j) + (\hat{\mu}_{i_{max}} + \Delta) + \sigma\epsilon_{0,1})$$

$$= E(Y_{n}(i_{max}^{n} + j)) + (\hat{\mu}_{i_{max}} + \Delta)$$
(23)

• Searching \hat{j} where $\hat{j} = \inf\{j : E(Y_n(i_{max}^n + j)) > T_F\},$ $\hat{j} - (i^n - i_{max}^n)$ is the estimated RUL.

4 Application

This section provides the result of application of the proposed method on a real industrial data from STMicroelectronics Rousset. Measured variables are sampled at 1 second intervals during a process, for 351 observations of totally 19 sensors for one month of production, which represents about 1000 wafers from the first wafer to the last one before a new reparation. The data is preprocessed by a synchronization step to obtain the common length trajectories.

4.1 Off-line analysis

4.1.1 Health indicator extraction The first two hundred wafers are used to contribute the upper limit UL and lower limit LL of each point (j,k) with equations (1-2). The total number of (j,k) points is 351*19=6669 points, after this step 1.1 it remains 1527 points (jm,km). The Fig. 5 shows the first 4 principal components and their correspondent eigenvalue of PCA in step 1.2. The first PC (blue color) which is progressive depicts the non-linear degradation of machine.

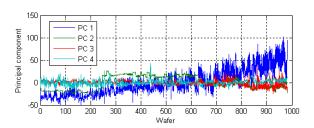


Fig. 5. The first four principal components

Fig. 6 shows 763 significant points (js, ks) which are identified with equations (8 - 9), p^{th} is chosen as 50^{th} percentile. These points relate to the sensor 1, 2, 9, 10, 18.

Finally, all these significant points are used to calculate the raw health indicator of machine with equations (11-12).

4.1.2 Analysis of health indicator dynamics Applying the filtering proposed in 2.2.1, the health indicator Y is given in Fig. 7. The normal operating threshold is predefined $T_N = -10$ and the failure threshold is predefined $T_F = 55$.

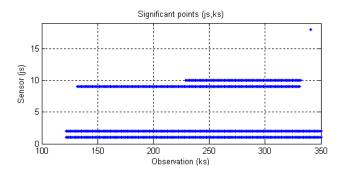


Fig. 6. Significant points

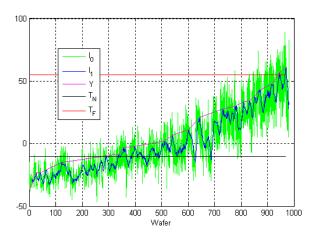


Fig. 7. Health indicator

The parameter result of health indicator modelling is $\mu_0 = 0.055$, $P_0 = 3.06 \times 10^{-6}$, $Q = 2.7 \times 10^{-5}$ and $\sigma = 0.071$. The evolution of $\hat{\mu}_i$ is given in Fig. 8, this is really an unknown function over time.

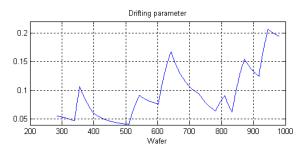


Fig. 8. Evolution of drifting parameter $\hat{\mu}_i$

4.2 Online supervision

To validate the prognosis model, the online data is generated by a simulator which takes into account the dynamics of historical data. One profile of online indicator is given in Fig. 9 compared to the off-line indicator. At each inspection time i^n , the available online data is known only for $t = 1, ..., i^n$. When $Y_n(i^n_{max}) > T_N$, (see section 3.1), the degradation alarm launches the prognosis model.

At each inspection time i^n , the real failure time is 921 thus the real RUL is $(921 - i^n)$. Hence, the estimate RUL and the real RUL can be compared as given in Fig. 10. The result shows that the RUL estimation of almost inspection

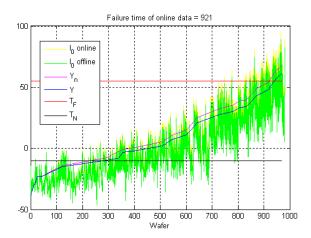


Fig. 9. Online data

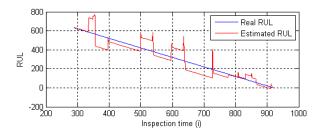


Fig. 10. Estimation error

times gives a small error excepting some periods. This is because during these periods, the online health indicator Y_n evolves much differently from the reference one Y (which can be caused practically by several phenomena of the degradation process of machine) so that provokes an important error. This error is then corrected due to the adaptive aspect of the prediction model. The root mean squared error of RUL estimation is 74 time units (equivalent to the duration of processing 74 wafers or nearly 3 lots in STMicroelectronics manufacturing) is a small error, thus, the prognosis model is validated.

5 Conclusion

This paper proposes a new method of health indicator extraction for discrete manufacturing processes from raw data, based on identifying the significant points which relate to the degradation dynamics of machine. The HI of offline data is then modelled with an adaptive Wiener process and its parameters are used to predict the evolution of HI for the on-line supervision. An application of the proposed method on a real industrial data is presented and it shows a small error of RUL estimation.

References

Alexis Thieullen, Mustapha Ouladsine, J.P. (2012). A survey of health indicators and data-driven prognosis in

- semiconductor manufacturing process. In (ed.), , volume 8, 19–24. 8th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes.
- Bagdonavicius, V. and Nikulin, M. (2001). Estimation in degradation models with explanatory variables. *Lifetime Data Analysis*, 7(1), 85–103.
- Bakker, J. and van Noortwijk, J. (2004). Inspection validation model for life-cycle analysis. In *Bridge maintenance*, safety, management and cost, proceedings of the second international conference on bridge maintenance, safety and management (IABMAS), Kyoto, Japan, 18–22. Citeseer.
- Benkedjouh, T., Medjaher, K., Zerhouni, N., and Rechak, S. (2013). Remaining useful life estimation based on nonlinear feature reduction and support vector regression. *Engineering Applications of Artificial Intelligence*.
- sion. Engineering Applications of Artificial Intelligence. Gorjian, N., Ma, L., Mittinty, M., Yarlagadda, P., and Sun, Y. (2010). A review on degradation models in reliability analysis. In D. Kiritsis, C. Emmanouilidis, A. Koronios, and J. Mathew (eds.), Engineering Asset Lifecycle Management, 369–384. Springer London.
- Lawless, J. and Crowder, M. (2004). Covariates and random effects in a gamma process model with application to degradation and failure. *Lifetime Data Analysis*, 10(3), 213–227.
- Le Son, K., Barros, A., and Fouladirad, M. (2012a). Deterioration model filtering by gibbs algorithm and rul estimation. In *Fault Detection, Supervision and Safety of Technical Processes*, volume 8, 13–18.
- Le Son, K., Fouladirad, M., Barros, A., Levrat, E., and Iung, B. (2012b). Remaining useful life estimation based on stochastic deterioration models: A comparative study. *Reliability Engineering and System Safety*.
- Tseng, S.T. and Peng, C.Y. (2007). Stochastic diffusion modeling of degradation data. *Journal of Data Science*, 5(3), 315–333.
- Wang, W., Carr, M., Xu, W., and Kobbacy, K. (2011). A model for residual life prediction based on brownian motion with an adaptive drift. *Microelectronics Reliability*, 51(2), 285 – 293.