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Active Fault Isolation in MIMO Systems

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Abstract: Active fault isolation of parametric faults in closed-loop MIMO systems are considered in this paper. The fault isolation consists of two steps. The first step is group-wise fault isolation. Here, a group of faults is isolated from other possible faults in the system. The group-wise fault isolation is based directly on the input/output signals applied for the fault detection. It is guaranteed that the fault group includes the fault that had occurred in the system. The second step is individual fault isolation in the fault group. Both types of isolation are obtained by applying dedicated auxiliary inputs and the associated residual outputs.

Keywords: Active fault isolation, MIMO systems, YJBK parameterization.

1. INTRODUCTION

Fault diagnosis can be divided into two groups, passive based methods and active based methods. In the first group, the fault diagnosis is based on passive observations of the systems. There exist various methods for both deterministic or stochastic based diagnosis, see e.g. Basseville and Nikiforov [1993], Blanke et al. [2006], Campbell and Nikoukhah [2004], Chen and Patton [1998], Gertler [1998], Gustafsson [2000] for mention some of the books in the area of passive fault diagnosis. Methods based on an active approach, the diagnosis is still based on observations of the system, but auxiliary inputs are injected to get a faster diagnosis of faults in the system or get a diagnosis at a specified time, i.e. when an auxiliary input is injected at the system. The area of active fault diagnosis (AFD) has not been investigated so much as passive based methods. Some relevant references in the area of active fault diagnosis are e.g. Ashari et al. [2011, 2012], Campbell and Nikoukhah [2004], Kerestecioglu [1993], Niemann [2006], Simandl and Puncochar [2009], Zhang [1989].

It is here important to point out that there are some differences between the two types of diagnosis methods. The active methods can be applied when the faults in the system occurs as changes of parameters or dynamic in the system, i.e. parametric faults or multiplicative faults. It will then be possible to see a change of the signature from the auxiliary input in the outputs from the systems. In the case of additive faults, active methods cannot be applied. Additive faults will not change the signature from the auxiliary input in the output from the system. On the hand, passive based methods can handle both types of faults. However, using a passive method, parametric or multiplicative faults will first be detected when a signal is injected to the system, so it is possible to see an effect from the fault. This input signal can be a disturbance or reference input etc. Active fault diagnosis is therefore relevant when we need to have a guarantee for detection and isolation of a change in the system due to parametric faults within a certain time after it has occurred.

The main focus in this paper is isolation of parametric faults in MIMO systems by using active methods. One of the main challenges in connection with fault diagnosis in MIMO systems is the selections of input and output directions. The fault detection problem for MIMO system has been considered in Niemann and Poulsen [2014]. The main result from this paper is an analysis of the design of optimal auxiliary inputs and associated residual signals such that it is possible to detect all parametric faults in the system.

The isolation problem is a two steps problem, divided into a group-wise fault isolation followed by individual fault isolation. The group-wise fault isolation step is to isolate a group of faults including the fault that had occurred in the system. For the individual fault isolation problem, both gain and phase information will be used. Different faults will give different signature in the complex plane that can be used for fault isolation.

The rest of this paper is organized as follows. In Section 2, the system set-up is given together with some preliminary results for active fault diagnosis. Active fault detection for MIMO systems is shortly described in Section 3 following by Section 4 where active fault isolation for MIMO systems is considered. An example is given in Section 5. The paper is closed with a conclusion in Section 6.

2. SET-UP

The needed system set-up and the applied set-up for active fault diagnosis (AFD) are shortly introduced in the following.

2.1 System set-up

The dynamic system is given by:

$$\Sigma_P \ : \ \begin{cases} z = G_{zw}w + G_{zd}d + G_{zu}u \\ e = G_{ew}w + G_{ed}d + G_{eu}u \\ y = G_{yw}w + G_{yd}d + G_{yu}u \end{cases} \quad (1)$$

where $d \in \mathcal{R}^{r_d}$ is an external disturbance input vector, $u \in \mathcal{R}^m$ the control input signal vector, $e \in \mathcal{R}^{r_e}$ is the external output signal vector to be controlled and $y \in \mathcal{R}^p$ is the measurement vector. Further, $w \in \mathcal{R}^k$ and $z \in \mathcal{R}^k$ are external input and output vectors. The connection between z and w is given by:

$$w = \theta z$$

where θ is a diagonal matrix represents the parametric faults in the system. $\theta_i,\ i = 1, \cdots, k$, in the diagonal of θ represent the k single parametric faults in the system. $\theta = 0$ represent the fault free case. For further description of the fault modeling, see e.g. Niemann [2012].

Closing the loop from w to z in Σ_P by θ, the system can be realized as an upper linear fractional transformation (LFT) in θ given by

$$\Sigma_{P,\theta} = \mathcal{F}_u(\Sigma_P, \theta)$$

where $\Sigma_{P,\theta}$ is given by

$$\Sigma_{P,\theta} \ : \ \begin{cases} e = G_{ed}(\theta)d + G_{eu}(\theta)u \\ y = G_{yd}(\theta)d + G_{yu}(\theta)u \end{cases} \quad (2)$$

It is assumed in the rest of this paper that only a single fault can occur at the time.

The system is controlled by a stabilizing feedback controller given by:

$$\Sigma_C \ : \ \{ \ u = Ky \quad (3)$$

2.2 Active fault detection

We will in this paper use an AFD approach based on a closed-loop setup for the detection and isolation described in e.g. Niemann [2006, 2012], Poulsen and Niemann [2008]. The AFD set-up includes a nominal feedback controller.

The set-up is based directly on the Youla-Jabr-Bongiorno-Kucera (YJBK) parameterization and the dual YJBK parameterization.

Before the setup is given, the coprime factorization of the nominal system G_{yu} from (1) and the stabilizing controller K from (3) are given. These are:

$$\begin{aligned} G_{yu} &= NM^{-1} = \tilde{M}^{-1}\tilde{N},\ N, M, \tilde{N}, \tilde{M} \in \mathcal{RH}_\infty \\ K &= UV^{-1} = \tilde{V}^{-1}\tilde{U}, \qquad U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty \end{aligned} \quad (4)$$

where the eight matrices in (4) must satisfy the double Bezout equation given in e.g. Tay et al. [1997].

It is now possible to give a parameterization of all controllers that stabilize the system in terms of a stable matrix transfer function Q, i.e. all stabilizing controllers are given by, Tay et al. [1997]:

$$K(Q) = (\tilde{V} + Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M}),\ Q \in \mathcal{RH}_\infty \quad (5)$$

The above controller parameterization can be realized as a lower LFT in the parameter Q:

$$K(Q) = \mathcal{F}_l\left(\begin{pmatrix} UV^{-1} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1}N \end{pmatrix}, Q\right) = \mathcal{F}_l(J_K, Q) \quad (6)$$

Equivalently, it is possible to derive a parameterization in terms of a stable matrix transfer function S of all systems that are stabilized by one controller, i.e. the dual YJBK parameterization. The parameterization is given by Tay et al. [1997]:

$$G_{yu}(S) = (\tilde{M} + S\tilde{U})^{-1}(\tilde{N} + S\tilde{V}),\ S \in \mathcal{RH}_\infty \quad (7)$$

An LFT representation of (7) is given by:

$$\begin{aligned} G_{yu}(S) &= \mathcal{F}_l\left(\begin{pmatrix} NM^{-1} & \tilde{M}^{-1} \\ M^{-1} & -M^{-1}U \end{pmatrix}, S\right) \\ &= \mathcal{F}_l(J_G, S) \end{aligned} \quad (8)$$

Further, S is given as an upper LFT by, Tay et al. [1997]:

$$S = \mathcal{F}_u(J_K, G_{yu}(S)) \quad (9)$$

The matrix transfer function S is a function of the system variations. Here variations in the system in terms of the parametric faults described by θ, i.e. $S = S(\theta)$ will be considered. Assuming that $\theta = 0$ is the nominal value of θ, then there exist the following simple relation, Niemann [2003]:

$$S(\theta) = 0,\ \text{for } \theta = 0 \quad (10)$$

This connection between the YJBK and the dual YJBK parameterization in (6) and (9) is a central element in the active fault diagnosis approach described in Niemann [2006, 2012], Poulsen and Niemann [2008]. By testing if $S(\theta)$ is zero or non-zero, parametric faults can be detected. From (9) we have that S is given directly as the matrix transfer function between the lower inputs and outputs of J_K. This is shown in Fig. 1.

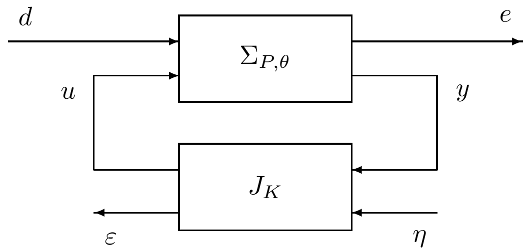

Fig. 1. The setup for AFD in closed loop system. The auxiliary input vector is η and the external output vector is ε, i.e. the residual vector.

It has been shown in Niemann [2006, 2012] that ε is a residual vector satisfying the decoupling conditions. ε can be used as a residual vector in connection with AFD. Therefor S is the matrix transfer function between the auxiliary input η and the residual output ε. As a consequence of the important of $S(\theta)$ for both fault detection, isolation and also estimation, it is called the *fault signature matrix*, Niemann [2006, 2012]. This name will be used in the following.

From the set-up that is shown in Fig. 1, the closed-loop system is given by:

$$\Sigma_{P,K} \ : \ \begin{cases} e = P_{ed}(\theta)d + P_{e\eta}(\theta)\eta \\ \varepsilon = P_{\varepsilon d}(\theta)d + \ S(\theta)\eta \end{cases} \quad (11)$$

where the matrix transfer functions can be found in Niemann [2012], Niemann and Poulsen [2014].

3. ACTIVE FAULT DETECTION

An explicit expression of the dependence of θ in $S(\theta)$ is given as, Niemann [2003], Niemann and Poulsen [2014]:

$$S(\theta) = \tilde{M}G_{yw}\theta(I - (G_{zw} + G_{zu}U\tilde{M}G_{yw})\theta)^{-1}G_{zu}M \quad (12)$$

Based on this explicit equation for $S(\theta)$ given in (12) together with the focus of detecting small parametric faults in closed-loop systems, it is possible to make a Taylor expansion of the fault signature matrix around the nominal value $\theta_0 = 0$ for obtaining a linear function of the parametric faults θ_i, i.e. $S(\theta)$ is given by:

$$S(\theta) \approx \sum_{i=1}^{k} \left(\frac{\partial}{\partial \theta_i} S(\theta)|_{\theta=0} \right) \theta_i \quad (13)$$

Let the system matrices G_{yw} and G_{zu} be partitioned into k columns and k rows, respectively, given by:

$$G_{yw} = [(G_{yw})_{:,1} \cdots (G_{yw})_{:,k}]$$
$$G_{zu} = \begin{bmatrix} (G_{zu})_{1,:} \\ \vdots \\ (G_{zu})_{k,:} \end{bmatrix} \quad (14)$$

The Taylor expansion in (13) is then given by, Niemann and Poulsen [2014]:

$$S(\theta) \approx \tilde{M} \sum_{i=1}^{k} (G_{yw})_{:,i}(G_{zu})_{i,:}M\theta_i = \sum_{i=1}^{k} \bar{S}_i \theta_i \quad (15)$$

where $\bar{S}_i = \tilde{M}(G_{yw})_{:,i}(G_{zu})_{i,:}M$.

As auxiliary inputs, periodic inputs are applied. It turns out in Niemann and Poulsen [2014] that it will not in general be possible to detect all faults just using a single auxiliary input vector. Instead a number of auxiliary inputs need to be applied. The single inputs are given by:

$$\eta = hv\sin(\omega t) = v\bar{\eta} \quad (16)$$

where $v \in \mathcal{R}^m$, the input direction, $h \in \mathcal{R}$, the gain and ω, the frequency need to be selected. $\bar{\eta}$ is a scalar input. The design of the single parameters in (16) can be developed by using a singular value decomposition (SVD), of $\bar{S}_i$. Let the maximal gain through $\bar{S}_i$ be given by $\sigma_{max}(\bar{S}_i)$ and it is obtained at the frequency ω_i. Further, the associated input and output directions for $\sigma_{max}(\bar{S}_i(\omega_i))$ are given by v_i and u_i, respectively. This gives the following auxiliary input η_i:

$$\eta_i = h_i v_i \sin(\omega_i t) = v_i \bar{\eta}_i \quad (17)$$

where h_i is a scalar constant to be designed. The design of h_i can be done with respect to the effect from the auxiliary input is allowed in the external output e in the nominal case. For simplifying the following fault isolation, it will be assumed that all $h_i = 1$. This is without loss of generality. Further, it is assumed that the given frequency ω_i is inside a relevant frequency range. Further, the optimal residual signal $\bar{\varepsilon}_i$ with respect to the given auxiliary input in (17) is given by

$$\bar{\varepsilon}_i = u_i = \sigma_{max}(\bar{S}_i(\omega_i))\theta_i \sin(\omega_i t + \phi_{ii}) \quad (18)$$

where $u_i \in \mathcal{R}^p$ is the output direction, ϕ_{ii} is the phase lag through $\bar{S}_i(\omega_i)$ with respect to θ_i.

The organization of the auxiliary inputs and which are needed is discussed in Niemann and Poulsen [2014].

4. ACTIVE FAULT ISOLATION

The isolation step consists of two steps, a group-wise fault isolation followed by isolation in the fault group.

4.1 Group-wise Fault Isolation

For simplifying the following group-wise fault isolation, it will in the following be assume that we have k set of auxiliary inputs and associated residual signals given by $(\bar{\eta}_i, \bar{\varepsilon}_i)$. This is without loss of generality. Some of the signal sets $(\bar{\eta}_i, \bar{\varepsilon}_i)$ might be identical or almost identical.

Given an auxiliary input $\bar{\eta}_i$ and the associated residual signal $\bar{\varepsilon}_i$. Further let the fault θ_j occur in the system. Using the linear approximation of S given in (15), the residual signal $\bar{\varepsilon}_i$ is then given by:

$$\begin{aligned} \bar{\varepsilon}_i &= u_i \bar{S}_j v_i \theta_j \bar{\eta}_i \\ &= [\, u_{i,1} \cdots u_{i,p} \,] \begin{bmatrix} \bar{S}_{j,11} & \cdots & \bar{S}_{j,1m} \\ \vdots & & \vdots \\ \bar{S}_{j,p1} & \cdots & \bar{S}_{j,pm} \end{bmatrix} \begin{bmatrix} v_{1,i} \\ \vdots \\ v_{m,i} \end{bmatrix} \theta_j \bar{\eta}_i \\ &= \sum_{r=1}^{p} \sum_{t=1}^{m} u_{i,r} \bar{S}_{j,rt} v_{t,i} \theta_j \bar{\eta}_i \\ &= \Psi_{ij} e^{j\phi_{ij}} \theta_j \bar{\eta}_i \end{aligned} \quad (19)$$

where Ψ_{ij} is the fault signature gain and ϕ_{ij} is the phase shift of the transfer function from $\bar{\eta}_i$ to $\bar{\varepsilon}_i$ with respect to fault θ_j. Including the auxiliary input $\bar{\eta}_i$ from (17) in the above equation gives:

$$\bar{\varepsilon}_i = \Psi_{ij} e^{j\phi_{ij}} \theta_j \sin(\omega_i t) \quad (20)$$

Based on $\bar{\varepsilon}_i$ given above, we will use the following illustration signal:

$$\delta_i(t) = \sqrt{\left(\int s_i(t)dt\right)^2 + \left(\int c_i(t)dt\right)^2} \quad (21)$$

where $s_i(t)$ and $c_i(t)$ are given by:

$$s_i(t) = \bar{\varepsilon}_i \sin(\omega_i t), \; c_i(t) = \bar{\varepsilon}_i \cos(\omega_i t) \quad (22)$$

The illustration signal given by (21) will be a linear function of the fault signature gain Ψ_{ij}.

All fault signature gains can be calculated with respect to input/output sets $(\bar{\eta}_i, \bar{\varepsilon}_i)$ and the parametric faults in the system. Based on these gains, Table 1 can be given.

Table 1. The fault signature gains Ψ_{ij} with respect to input/output sets $(\bar{\eta}_i, \bar{\varepsilon}_i)$ and the parametric fault θ_j.

Input/output	θ_1	$\cdots$	θ_i	$\cdots$	θ_k
$(\bar{\eta}_1, \bar{\varepsilon}_1)$	Ψ_{11}	$\cdots$	Ψ_{1i}	$\cdots$	Ψ_{1k}
$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$
$(\bar{\eta}_i, \bar{\varepsilon}_i)$	Ψ_{i1}	$\cdots$	Ψ_{ii}	$\cdots$	Ψ_{ik}
$\vdots$	$\vdots$		$\vdots$	$\ddots$	$\vdots$
$(\bar{\eta}_k, \bar{\varepsilon}_k)$	Ψ_{k1}	$\cdots$	Ψ_{ki}	$\cdots$	Ψ_{kk}

Based on Table 1, we can setup the fault signature gain matrix given by:

$$\Psi = \begin{bmatrix} \Psi_{11} & \cdots & \Psi_{1k} \\ \vdots & \ddots & \vdots \\ \Psi_{k1} & \cdots & \Psi_{kk} \end{bmatrix} \quad (23)$$

A large gain Ψ_{ij} in Table 1 or in (23) indicate that the associated auxiliary input $\bar{\eta}_i$ and the associated residual signal $\bar{\varepsilon}_i$ are sensitive to the parametric fault θ_j. A small gain indicates that residual signal is insensitive to the associated fault. Due to the design of the auxiliary input and the associated residual signal, Ψ will have a diagonal structure with the largest elements in the diagonal.

In the case when Φ has full rank, i.e.

$$rank(\Psi) = k \quad (24)$$

then it is possible to isolate all faults directly from the applied k residual signals. This will not in general be the case. Instead, group-wise fault isolation can be done based on Table 1.

Assume that fault θ_j had occurred in the system. The effect from this fault will be seen in one or more residuals signals. It will at least be able to see it in residual signal $\bar{\varepsilon}_j$ but also in others residual signals depending on the fault signature gains in column j. From Table 1, it can be decided which residual signals that are interesting in connection with fault θ_j.

Let's define the fault group in which the fault had occurred. The fault group needs to be defined out from the residual signals or the illustrations signals after a fault has been detected. A number of non-zero residual signals (or illustration signals) will be the result due to the gains given in Table 1. Select the illustration signal(s) with the largest increasing rate. The increasing rates are proportional with the fault signature gains Ψ_{ij} and the parametric fault θ_j. A non-zero residual $\bar{\varepsilon}_i$ indicate that fault θ_i might had occurred. If it is not θ_i, then there will be at least another residual signal with at least the same gain.

Assume that the illustration signal $\delta_i(t)$ has the largest increasing rate given by Γ_i, i.e.

$$\delta_i(t) \approx \Gamma_i t \quad (25)$$

From this approximation of the largest illustration signal, we define a conic sector that includes the group of relevant illustrations signals. This defines the group of relevant faults that need to be considered in connection with isolation of the fault occurred in the system. Let the conic sector be bounded by the two lines $\delta_{sec,upper}$ and $\delta_{sec,lower}$ given by:

$$\begin{aligned} \delta_{sec,upper} &= \Gamma_i t \\ \delta_{sec,lower} &= \alpha_{sec}\Gamma_i t \end{aligned}, \ t \geq 0 \quad (26)$$

where $\alpha_{sec} \leq 1$ define the gab in the sector. The selection of α_{sec} need to be done with respect to the disturbance in the system as well as with respect to the non-linearity of the fault signature matrix $S(\theta)$. If $S(\theta)$ is quite non-linear, the linear approximation given in (15) is only valid for small faults.

The conic sector defined in (26) is equivalent with defining the fault group based on the fault signature gains. The fault signature gains that give illustration signals inside the conic section will satisfy

$$\Psi_{ij} \geq \alpha_{iso}\Psi_{jj} \quad (27)$$

Without loss of generality, it is assumed that the residual (illustration) signals are arranged such that the isolated fault group consist of the faults $\theta_1, \cdots, \theta_l$, $l \leq k$.

4.2 Gain based Fault Isolation

As indicated above in connection with group-wise fault isolation, it will in some cases be possible for fault isolation based directly on the fault signature gains. Let the fault signature gain matrix for the group of isolated faults given by:

$$\Psi_{group} = \begin{bmatrix} \Psi_{11} & \cdots & \Psi_{1l} \\ \vdots & \ddots & \vdots \\ \Psi_{l1} & \cdots & \Psi_{ll} \end{bmatrix} \quad (28)$$

If Ψ_{group} has full rank, i.e.

$$rank(\Psi_{group}) = l \quad (29)$$

then it is possible to isolate all faults in the group directly from the applied l residual signals. If this is not the case, phase information need to be applied for the isolates as described in the following.

4.3 Phase based Fault Isolation

From (20) it has been shown that the transfer function from an auxiliary input to a residual signal can be written as the fault signature gain and a phase shift with respect to a certain fault. Using $\bar{\varepsilon}_i$ given by (20) in the two signals $s_i(t)$ and $c_i(t)$ from (22) gives then:

$$\begin{aligned} s_i &= \Psi_{ij} e^{j\phi_{ij}} \theta_j \sin(\omega_i t)\sin(\omega_i t) \\ c_i &= \Psi_{ij} e^{j\phi_{ij}} \theta_j \sin(\omega_i t)\cos(\omega_i t) \end{aligned} \quad (30)$$

Rewriting (30) using some trigonometric relations the two equations take the following form:

$$\begin{aligned} s_i &= \frac{1}{2}\Psi_{ij}\theta_j\Big(\cos(\phi_{ij}) - \cos(2\omega_i t + \phi_{ij})\Big) \\ c_i &= \frac{1}{2}\Psi_{ij}\theta_j\Big(\sin(\phi_{ij}) + \sin(2\omega_i t + \phi_{ij})\Big) \end{aligned} \quad (31)$$

From (31) it can be seen that both s_i and c_i have a constant component and a periodic component. The periodic component is zero in average over time. The constant components are given by:

$$\begin{bmatrix} s_i \\ c_i \end{bmatrix}_{constant} = \frac{1}{2}\Psi_{ij}\theta_j \begin{bmatrix} \cos(\phi_{ij}) \\ \sin(\phi_{ij}) \end{bmatrix} \quad (32)$$

The direction of the constant part of (s_i, c_i) in the complex plane (s_i is the real part and c_i is the imaginary part) depend directly on the phase shift ϕ_{ij} from the fault signature matrix as a result of the fault θ_j. From (32) we have directly that a unit vector in the complex plane with respect to fault θ_j is given by:

$$q_{ij} = \begin{bmatrix} \cos(\phi_{ij}) \\ \sin(\phi_{ij}) \end{bmatrix} \quad (33)$$

Table 2 gives the fault signature phases based on (33).

It is possible to set up conditions for isolability of the faults in the fault group, $\theta_1, \cdots, \theta_l$. For doing this, let's define the

Table 2. The fault signature phases ϕ_{ij} with respect to input/output sets $(\bar{\eta}_i, \bar{\varepsilon}_i)$ and the parametric fault θ_j.

Input/output	θ_1	$\cdots$	θ_i	$\cdots$	θ_l
$(\bar{\eta}_1, \bar{\varepsilon}_1)$	ϕ_{11}	$\cdots$	ϕ_{1i}	$\cdots$	ϕ_{1l}
$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$
$(\bar{\eta}_i, \bar{\varepsilon}_i)$	ϕ_{i1}	$\cdots$	ϕ_{ii}	$\cdots$	ϕ_{il}
$\vdots$	$\vdots$		$\vdots$	$\ddots$	$\vdots$
$(\bar{\eta}_l, \bar{\varepsilon}_l)$	ϕ_{l1}	$\cdots$	ϕ_{ki}	$\cdots$	ϕ_{ll}

phase shift matrix with respect to the given fault group. Let the phase shift matrix Φ be given by:

$$\Phi = \begin{bmatrix} \begin{pmatrix} \cos(\phi_{11}) \\ \sin(\phi_{11}) \end{pmatrix} & \cdots & \begin{pmatrix} \cos(\phi_{1l}) \\ \sin(\phi_{1l}) \end{pmatrix} \\ \vdots & \ddots & \vdots \\ \begin{pmatrix} \cos(\phi_{l1}) \\ \sin(\phi_{l1}) \end{pmatrix} & \cdots & \begin{pmatrix} \cos(\phi_{ll}) \\ \sin(\phi_{ll}) \end{pmatrix} \end{bmatrix} \quad (34)$$

Out from the phase shift matrix Φ in (34) we have that the l faults in the given fault group can be isolated using the given l set of auxiliary input signals and residual signals, $(\bar{\eta}_i, \bar{\varepsilon}_i),\ i = 1, \cdots, l$ if Φ has full rank, i.e.

$$rank(\Phi_i\ \Phi_j) = l \quad (35)$$

This condition is too strong and will not be satisfied in general. The condition can only be satisfied if it is based on l different sets of auxiliary input signals and residual signals. Instead we need to look at the columns in Φ. Let Φ be partitioned into l columns as:

$$\Phi = [\,\Phi_1, \cdots, \Phi_l\,] \quad (36)$$

It is clear that two faults can be isolated from each other if:

$$rank(\Phi_i\ \Phi_j) = 2,\ i \neq j \quad (37)$$

(37) is equivalent with that the two set of residual vectors has different phase shift. This condition is both necessary and sufficient for isolating θ_i from θ_j based on the given sets of inputs and output signals. If the condition in (37) is satisfied for all faults, then it is possible to isolate all l faults in the given fault group.

In few cases it will not be possible to isolate the parametric faults based on the fault signature gain or the phase shift as described above, the auxiliary inputs need to be changes. This aspect will not be considered in this paper.

5. EXAMPLE

The example is a spring-mass system as shown in Fig. 2. The system include 5 masses that are connected with 6 springs and 6 dampers. The first and last mass is assumed to be connected with a spring and a damper to a fixed ground.

A detailed description of the model is given in Niemann and Poulsen [2014]. Based on the analysis given in Niemann and Poulsen [2014], only faults in spring no. 2 and 6 (in parameter a_2 and a_6) and faults in damper no. 2 and 6 (in b_2 and b_6) are considered here.

Based on the system setup, an SVD analysis of $\bar{S}_i$ is then developed giving the maximal gain of $\bar{S}_i$, the frequency ω_i where the maximal gain is obtained and the associated input and output directions. The result of this analysis is given in Table 3.

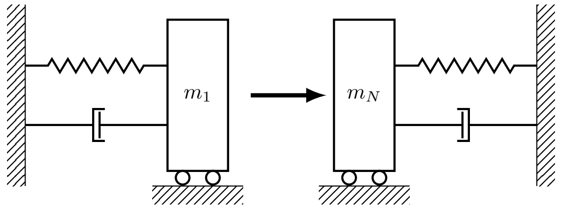

Fig. 2. Spring mass system with N masses.

Table 3. Data for analysis of $\bar{S}_i$ for fault in the springs and in the dampers.

Fault	$\sigma_{max}(\bar{S}_i)$	ω_i, [rad/s]	Input direction v_i	Output direction u_i
a_2	0.4630	0.5174	$\begin{pmatrix} 0.8581 \\ 0.5135 \end{pmatrix}$	$(\,0.9993\ 0.0376\,)$
a_6	0.1082	0.2306	$\begin{pmatrix} 0.3520 \\ 0.9360 \end{pmatrix}$	$(\,0.0328\ 0.9995\,)$
b_2	0.0108	0.5091	$\begin{pmatrix} 0.8588 \\ 0.5122 \end{pmatrix}$	$(\,0.9993\ 0.0384\,)$
b_6	0.0024	0.2296	$\begin{pmatrix} 0.3514 \\ 0.9362 \end{pmatrix}$	$(\,0.0325\ 0.9995\,)$

Table 3 gives maximal singular value of $\bar{S}_i$ and the associated input and output directions with respect to a given fault.

As a result of of the analysis given in Table 3, we can see that two sets of $(\bar{\eta}, \bar{\varepsilon})$ need to be applied for optimal detection of the four possible parametric faults. The two sets are given by:

- Optimal auxiliary input and residual vector for detection of faults on a_2 and b_2:
$$\eta_2 = \begin{pmatrix} 0.8581 \\ 0.5135 \end{pmatrix} \sin(0.5174t) = \begin{pmatrix} 0.8581 \\ 0.5135 \end{pmatrix} \bar{\eta}_2$$
$$\bar{\varepsilon}_2 = (\,0.9993\ 0.0376\,)\,\varepsilon$$
- Optimal auxiliary input and residual vector for detection of faults on a_6 and b_6:
$$\eta_6 = \begin{pmatrix} 0.3514 \\ 0.9362 \end{pmatrix} \sin(0.2296t) = \begin{pmatrix} 0.3514 \\ 0.9362 \end{pmatrix} \bar{\eta}_6$$
$$\bar{\varepsilon}_6 = (\,0.0325\ 0.9995\,)\,\varepsilon$$

The faults considered in in the four parameters are a change of 5% change of a_2, a_6 and a 20% change of b_2, b_6, respectively.

The first step is group-wise fault isolation. Let's consider a faulty in a_2. The two illustration signals δ_2 and δ_6 based on the two residual signal $\bar{\varepsilon}_2$ and $\bar{\varepsilon}_2$ given by (21) are shown in Fig. 3.

It is clear from Fig. 3 that δ_2 is very sensitive to a fault in a_2 (and in b_2) and that δ_6 is insensitive to a fault in a_2 (and in b_2). Considering a fault in a_6 (and b_6), δ_2 will be insensitive to the fault and δ_6 will be very sensitive to a fault in a_6 (and b_6). This gives directly a group-wise isolation of the four possible faults. By using the sets of signals $(\bar{\eta}_2, \bar{\varepsilon}_2)$ and $(\bar{\eta}_6, \bar{\varepsilon}_6)$, we can isolate the two faults a_2 and b_2 from a_6 and b_6.

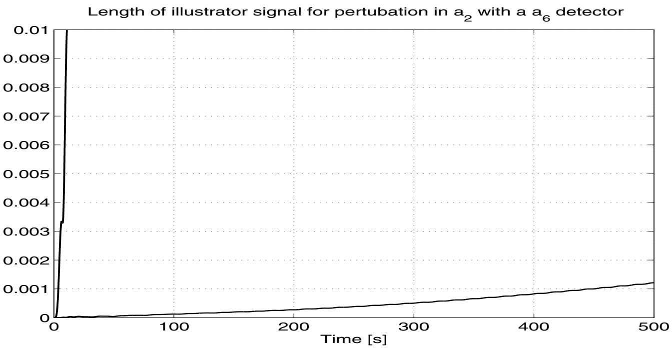

Fig. 3. Group-wise fault isolation. Illustration signals for detection of a_2 based on $(\bar{\eta}_2, \bar{\varepsilon}_2)$ (upper curve) and by using $(\bar{\eta}_6, \bar{\varepsilon}_6)$ (lower curve).

The last step is an isolation of the faults in the two groups. In this example, it is possible to isolate the faults in the two groups by using the phase shift through the fault signature matrix. We will use the integrals of the two signals s_i and c_i given by (22) as illustration signals.

Using these two illustrations signals, the isolation of the two faults a_2 and b_2 can now be done as shown in Fig. 4.

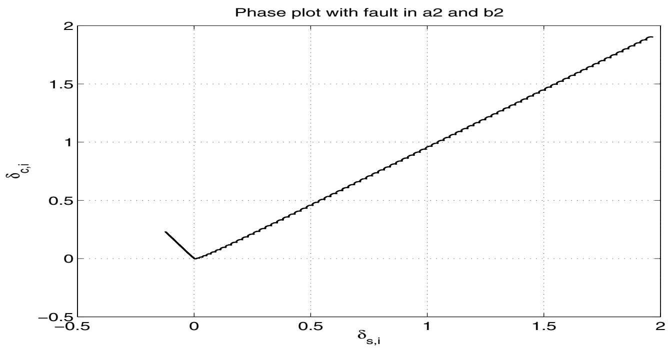

Fig. 4. Individual fault isolation of faults in a_2 and b_2. The two curves $(\delta_{s_i}, \delta_{c_i})$ are given for a_2 (the right curve) and for b_2 (the left curve).

It is clear from Fig. 4 it is possible to isolate a_2 from b_2 due to the large difference in phase shift through the fault signature matrix. In this case, the two curves are almost orthogonal on each other which give optimal condition for a distinction of the effects from the two possible faults. The same result is obtained for isolation of a_6 and b_6. This is shown in Fig. 5.

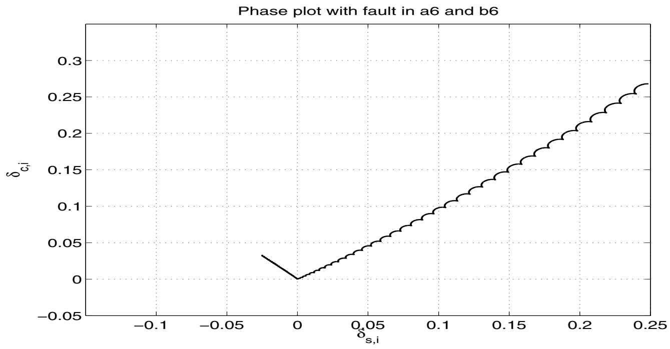

Fig. 5. Individual fault isolation of faults in a_6 and b_6. The two curves $(\delta_{s_i}, \delta_{c_i})$ are given for a_6 (the right curve) and for b_6 (the left curve). Zoom of the central part of the curves.

6. CONCLUSION

In the presented approach for fault isolation in MIMO systems, the isolation step consists of two parts, a group-wise isolation followed by an individual isolation of the single faults in the fault groups. The group-wise fault isolation is based directly on the applied input and output signals for fault detection. The individual fault isolation is based on the phase shift through the fault signature matrix, that can be used for fault isolation.

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