

Filtered-Covariance Function-Based Subspace Identification with Bound Effects Integration

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Abstract: A novel subspace covariance-based identification method that achieves consistent system estimates in the presence of highly-colored noise without requiring the use of weighting matrices has been derived recently. However, the use of Fourier transform in the computation of correlation functions requires the need to include the estimation of initial and final conditions. In this article we propose a solution to get unbiased estimates regardless of initial or end effects. Moreover, to further improve the model order estimation a filter-based solution is considered. This results in a new covariance-based algorithm that uses frequency weights computed with limit conditions in mind. A simulation example illustrates the algorithm performance.

1. INTRODUCTION

Time domain Subspace-based State-Space Identification methods - referred to as 4SID methods - are a popular class of methods to identify state-space models of Linear Time-Invariant (LTI) systems from experimental input-output data. Compared to the Prediction Error Method (PEM) (Ljung, 1999), the 4SID methods are attractive because they estimate a state-space realization directly from input-output data without requiring canonical parametrizations and non-linear optimizations (Gustafsson, 2002). They are based on robust numerical tools such as the QR-factorization and the Singular Value Decomposition (SVD) (Golub and Van Loan, 1996).

These methods can be stated in the framework of (Van Overschee and De Moor, 1995) which interprets the 4SID methods in terms of the estimation of the range space of a weighted extended observability matrix. Several studies have shown that the most popular 4SID methods only differ from the choice of weighting matrices (see, e.g., (Jansson and Wahlberg, 1998)). In the literature several works have studied the impact of those weightings on the asymptotical performance of the methods (Gustafsson, 2002; Jansson and Wahlberg, 1995; Bauer, 2005). The weighting matrices may also be chosen to specify a certain solving basis. Orthogonal (Peternell, 1995) and balanced basis (Chou and Maciejowski, 1997) or orthogonalized data (Chiuseo and Picci, 2004) can be used to improve the numerical properties of the identification problem. Finally, weighting matrices can influence the estimation of the range of the weighted extended observability matrix and consequently the order of the identified model (Van Overschee and De Moor, 1996a). A different approach, based

on the identification of q-Markov COVariance Equivalent Realizations (COVER), was suggested in (King et al., 1988). This algorithm uses both impulse response and autocorrelation sequences of multivariable linear systems and can be used for a realization, or model reduction.

In (Miller and de Callafon, 2010), a method which uses input-output covariance data was used. The algorithm formulation is similar to popular 4SID methods. However, this one is attractive as it provides consistent estimates without using weighting matrices even in presence of colored noise (Miller and de Callafon, 2010). It is also different from the approach suggested in (King et al., 1988) since this method is not aiming at finding models that cover the q-Markov parameters, but focuses on correlations between signals to minimize the effect of noise conditions. However, this method presents two main drawbacks. First, it uses Fourier transform and inverse Fourier transform to compute the correlation function estimates. Because of this implementation aspect, the identification results are mathematically exact only if the initial and final conditions are estimated during the identification. Second, there is no mean to improve the model order selection. Indeed, without weighting matrices, the range space of the extended observability matrix only depends on the data and cannot be influenced. In this article, we suggest two solutions to circumvent these two limitations. The proposed solutions lead to a new subspace-based algorithm where the estimation is carried out via covariance function where frequency weights are computed with limit conditions in mind, still allowing the use of short data sequences in the covariance based realization algorithm.

In Section 2, we first review the identification problem. Then, it is shown how the bound effects can be considered

with the same algorithm formalism. Taking them into account results in a general formulation of the algorithm and in improved performance when initial and final system states are not zero. In Section 3, it is shown that the same algorithm can be derived with filtered covariance functions. This enables to retain the system dynamics only in a certain frequency band of interest. This results in a model order reduction. In this article, the term *model order reduction* is used instead of *model reduction* often associated to the balanced reduction (Datta, 2003) which yields to a reduced model with different properties. Differences between the two approaches will be discussed in Section 3. The algorithm based on filtered covariance functions is given in Section 4. Finally, Section 5 shows the impact of the two improvements derived in this article through a simulation example.

2. IDENTIFICATION FROM COVARIANCE DATA

2.1 Formulation of the identification problem

Consider a discrete, linear, time-invariant system described in state-space form as

$$x_{k+1} = A x_k + B u_k \quad (1)$$

$$y_k = C x_k + D u_k + v_k, \quad (2)$$

with $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$ and $y_k \in \mathbb{R}^{n_y}$ are the state, input and output vectors, respectively, and where $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $C \in \mathbb{R}^{n_y \times n_x}$, $D \in \mathbb{R}^{n_y \times n_u}$ are the system matrices. The vector $v_k \in \mathbb{R}^{n_y}$ is a possible colored noise and is assumed to be stationary (Ljung, 1999). We assume that the representations given by (2) are minimal descriptions of stable, controllable and observable systems.

The identification problem considered hereafter consists in estimating first the system order of the system and then the state space matrices (A , B , C , D). But instead of working with the raw input and output data, we consider in this article covariance functions estimates. For any quasi-stationary signals s_k and w_k , the cross-covariance function $R_{sw}(\tau)$ is defined as (Ljung, 1999)

$$R_{sw}(\tau) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=0}^{N-1} E\{s_{k+\tau} w_k^T\} \quad \text{w. p. 1}, \quad (3)$$

where E denotes the expectation.

The input signal u_k is chosen to be a quasi-stationary signal (Ljung, 1999). Since the system is assumed to be stable, the state x_k and the output y_k are also quasi-stationary signals. Thus, the correlation functions $R_{yu}(\tau)$, $R_{xu}(\tau)$, $R_{uu}(\tau)$ and $R_{vu}(\tau)$ exist and are defined as in Eq. (3), respectively. They can be expressed in terms of the state-space matrices (A , B , C , D) as

$$\begin{aligned} R_{xu}(\tau + 1) &= A R_{xu}(\tau) + B R_{uu}(\tau) \\ R_{yu}(\tau) &= C R_{xu}(\tau) + D R_{uu}(\tau) + R_{vu}(\tau) \end{aligned} \quad (4)$$

If the noise v_k is uncorrelated with the input u_k , $R_{vu}(\tau) = 0$. Therefore, using correlation functions instead of raw input and output data offers the advantage of reducing the effect of the noise. This holds even in the presence of colored noise.

2.2 Estimating the correlation function

The theoretical definition for covariance functions given in Eq. (3) implies an infinite number of samples. Practically,

only a finite number of samples are used to estimate covariance functions from real experimental data. A proposed estimate form for $R_{sw}(\tau)$ is (Ljung, 1999)

$$\tilde{R}_{sw}(\tau) = \frac{1}{N} \sum_{t=0}^{N-1} s(t+\tau) w^T(t). \quad (5)$$

This estimate is slightly different from the true covariance function $R_{sw}(\tau)$. Looking at the expected value of this covariance estimate, we would like to have

$$\lim_{N \rightarrow \infty} E\{\tilde{R}_{sw}(\tau)\} = R_{sw}(\tau) \quad \text{w. p. 1}. \quad (6)$$

If this is the case, it is called a ‘‘consistent’’ estimate. Actually, its expected value verifies (Ljung, 1999)

$$E\{\tilde{R}_{sw}(\tau)\} = \frac{N-|\tau|}{N} R_{sw}(\tau) \quad \text{w. p. 1}, \quad (7)$$

where $\frac{N-|\tau|}{N}$ is a weighting factor with the property that $\lim_{N \rightarrow \infty} E\{\tilde{R}_{sw}(\tau)\} = R_{sw}(\tau)$. The proposed estimate is asymptotically unbiased. It can be noticed that the proposed estimate is also unbiased if $|\tau| \ll N$. Moreover, as the time shift τ gets larger, a fewer samples are used when calculating the covariance and the estimate would be biased and exhibit more variance. For these reasons, we will use a slightly different estimate given by

$$\hat{R}_{sw}(\tau) = \omega(\tau) \tilde{R}_{sw}(\tau), \quad (8)$$

where $\omega(\tau)$ is a rectangular window defined by

$$\omega(\tau) = \begin{cases} 1 & \text{if } |\tau| \leq \tau_{max} \\ 0 & \text{if } |\tau| > \tau_{max} \end{cases}. \quad (9)$$

τ_{max} is chosen to verify $\tau_{max} \ll N$ so that the estimate $\hat{R}_{sw}(\tau)$ is an unbiased estimate.

2.3 Initial and end effects

The estimate $\tilde{R}_{sw}(\tau)$ is not directly computed as a convolution product. The following relation is used instead

$$\tilde{R}_{sw}(\tau) = \mathcal{F}^{-1}\{S(\omega)W(\omega)\}, \quad (10)$$

where \mathcal{F}^{-1} denotes the inverse Fourier transform and $S(\omega)$ and $W(\omega)$ are the Fourier transform of the considered signals $s(t)$ and $w(t)$ respectively. It is indeed computationally more efficient to compute first the Fourier transform of the time domain signals and then take the inverse Fourier transform of their product Orfanidis (1996). In the time domain, the Fourier transform of a signal is based on the integration of the signal over the whole (infinite) time scale. Of course, in a practical experiment, the signals are only available on a limited time interval $[t_0, t_1]$. When the signals are non-zero outside this interval or are not periodic, one has to integrate corrective terms in the Fourier analysis.

Inspired by the solution suggested by Pintelon et al. (1997) for the identification of single-input single-output systems in the frequency domain, we derive in the sequel a solution to integrate these corrective terms in the problem formulation. We note $t_0 = 0$ and $t_1 = (N-1)\Delta t$ with Δt the sampling period. If x_k designates the samples of the state vector at the times $k \Delta t$ and N the number of samples, the Discrete Fourier Transform (DFT) $X(\omega_l)$ for the frequency ω_l ($l = 0, \dots, N-1$) is defined as

$$X(\omega_l) = \sum_{k=0}^{N-1} x_k e^{-j\omega_l k \Delta t} \quad \text{with} \quad \omega_l = \frac{2\pi l}{N \Delta t}. \quad (11)$$

Similarly, the DFT of x_{k+1} is given by

$$\mathcal{F}\{x_{k+1}\} = \sum_{k=0}^{N-1} x_{k+1} e^{-j\omega_l k \Delta t}. \quad (12)$$

Using the change of variable $K = k + 1$, this latter can be rewritten as

$$\begin{aligned} \mathcal{F}\{x_{k+1}\} &= \sum_{K=1}^N x_K e^{-j\omega_l(K-1)\Delta t} \\ &= e^{j\omega_l \Delta t} \left(\sum_{K=0}^{N-1} x_K e^{-j\omega_l K \Delta t} - x_0 + x_N e^{-j\omega_l N \Delta t} \right). \end{aligned} \quad (13)$$

Remarking that the sum is the same as in Eq. (11) and that $e^{-j\omega_l N \Delta t} = 1$, Eq. (13) becomes

$$\mathcal{F}\{x_{k+1}\} = e^{j\omega_l \Delta t} X(\omega_l) - e^{j\omega_l \Delta t} \Delta x, \quad (14)$$

with $\Delta x = x_0 - x_N$. Taking the inverse DFT, the real state x_{k+1} is equal to the calculated state $\tilde{x}_{k+1} = \mathcal{F}^{-1}\{\mathcal{F}\{x_{k+1}\}\}$ plus an additional term

$$x_{k+1} = \tilde{x}_{k+1} + \frac{1}{N} \sum_{l=0}^{N-1} \Delta x e^{j2\pi l(k+1)/N}. \quad (15)$$

Replacing $x(t+1)$ in Eq. (2) by this expression, the correct state-space representation that has to be considered is given by

$$x_{k+1} = A x_k + \bar{B} \bar{u}_k \quad (16)$$

$$y_k = C x_k + \bar{D} \bar{u}_k + v_k, \quad (17)$$

with $\bar{B} = [B \ \Delta x]$, $\bar{D} = [D \ D_x]$ and $\bar{u}_k = [u_k^T \ \nu_k^T]^T$. This shows that initial and end effects can easily be taken into account without changing the identification problem formulation. It can indeed be achieved by only adding to the real system a fictitious input ν_k defined as

$$\nu_k = \frac{1}{N} \sum_{l=0}^{N-1} e^{j2\pi l(k+1)/N} = \begin{cases} 1 & \text{if } k = N - 1 \\ 0 & \text{if } k < N - 1 \end{cases}. \quad (18)$$

The state-space formulation implies to add the vector D_x to the matrix D . This additional term can be seen as the influence of modes that are outside the considered frequency-band upon the bound conditions.

2.4 Subspace identification from covariance data

Using the estimates $\hat{R}_{\bar{u}\bar{u}}$, $\hat{R}_{y\bar{u}}$, $\hat{R}_{x\bar{u}}$ to write the algorithm which was introduced in (Miller and de Callafon, 2010) should improve the identification results since the bias introduced by bound effects is avoided. The main steps of this algorithm are adapted in order to take into account the bound effects and are reviewed in this subsection. In the rest of the article, we note $n_{\bar{u}} = n_u + 1$.

Adapting traditional subspace identification methods to covariance functions involves the following relation

$$\mathbf{R}_{y\bar{u}} = \Gamma \mathbf{R}_{x\bar{u}} + T \mathbf{R}_{\bar{u}\bar{u}} + \mathbf{R}_{v\bar{u}}, \quad (19)$$

where $\mathbf{R}_{y\bar{u}} \in \mathbb{R}^{n_y \times n_{\bar{u}}}$ is a block-Hankel matrix made of l block-columns of i length sequences of $\hat{R}_{y\bar{u}}(\tau)$ as

$$\mathbf{R}_{y\bar{u}} = \begin{bmatrix} \hat{R}_{y\bar{u}}(0) & \hat{R}_{y\bar{u}}(1) & \dots & \hat{R}_{y\bar{u}}(l-1) \\ \hat{R}_{y\bar{u}}(1) & \hat{R}_{y\bar{u}}(2) & \dots & \hat{R}_{y\bar{u}}(l) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{R}_{y\bar{u}}(i-1) & \hat{R}_{y\bar{u}}(i) & \dots & \hat{R}_{y\bar{u}}(i+l-2) \end{bmatrix}. \quad (20)$$

This block-hankel matrix is expressed as a function of the block-Hankel matrices $\mathbf{R}_{\bar{u}\bar{u}} \in \mathbb{R}^{n_{\bar{u}} \times n_{\bar{u}}}$ and $\mathbf{R}_{v\bar{u}} \in \mathbb{R}^{n_v \times n_{\bar{u}}}$ (defined in a similar way than $\mathbf{R}_{y\bar{u}}$), the extended observability matrix $\Gamma \in \mathbb{R}^{n_y \times n_x}$ and the block-lower-triangular-Toeplitz matrix T composed of the system Markov parameters (Verhaegen and Verdult, 2007).

The goal of 4SID methods is first to estimate the system order from the rank of Γ . In order to estimate the rank of Γ , the propagation of the cross-covariance of the state with the input must be isolated by projecting $\mathbf{R}_{y\bar{u}}$ onto the orthogonal complement of the input auto-correlation block-Hankel matrix $\mathbf{R}_{\bar{u}\bar{u}}$. The $n_{\bar{u}} \times n_{\bar{u}}$ right projection matrix $\Pi_{\bar{u}\bar{u}}$ is defined as in (Miller and de Callafon, 2010). This one preserves the rank of Γ if the input signal is persistently exciting (Katayama, 2005) and if the condition $l \geq (i+1)n_{\bar{u}} + n_x$ is fulfilled (Miller and de Callafon, 2010).

If $\mathbf{R}_{v\bar{u}} = 0$, the order of the system can be determined by examining the rank of $\mathbf{R}_{y\bar{u}} \Pi_{\bar{u}\bar{u}}$. In practical situations, this condition is not strictly verified and $\mathbf{R}_{v\bar{u}} \Pi_{\bar{u}\bar{u}}$ can be a full-rank matrix. However, the effect of the noise on the data is expected to be low compared to the system dynamics influence. A likely value for n_x can therefore be determined by searching for a drop-off in the singular values of $\mathbf{R}_{y\bar{u}} \Pi_{\bar{u}\bar{u}}$. The value of n_x is given by the place of the singular value σ_{n_x} immediately prior to the drop-off. Consequently, the goal is to find the matrix Q that minimizes the least-squares problem

$$\epsilon_{n_x} = \min_{\text{rank}(Q)=n_x} \|Q - \mathbf{R}_{y\bar{u}} \Pi_{\bar{u}\bar{u}}\|_2. \quad (21)$$

The matrix Q_{n_x} that minimizes Eq. (21) is (Golub and Van Loan, 1996)

$$Q_{n_x} = U_{n_x} \Sigma_{n_x} V_{n_x}^T, \quad (22)$$

where $\Sigma_{n_x} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_{n_x})$ contains the first n_x singular values of $\mathbf{R}_{y\bar{u}} \Pi_{\bar{u}\bar{u}}$. An estimate of the system extended observability matrix Γ - denoted $\hat{\Gamma}$ - and an estimate of the state response - denoted $\hat{\mathbf{R}}_{x\bar{u}} \Pi_{\bar{u}\bar{u}}$ - is found from a factorization of Q_{n_x} as

$$Q_{n_x} = \hat{\Gamma} (\hat{\mathbf{R}}_{x\bar{u}} \Pi_{\bar{u}\bar{u}}). \quad (23)$$

A possible and common choice is

$$\hat{\Gamma} = U_{n_x} \Sigma_{n_x}^{1/2}, \quad \hat{\mathbf{R}}_{x\bar{u}} \Pi_{\bar{u}\bar{u}} = \Sigma_{n_x}^{1/2} V_{n_x}^T. \quad (24)$$

Once the order n_x and the observability matrix estimate $\hat{\Gamma}$ are estimated, the next step of the algorithm is to estimate the system matrices A and C . The most popular way to estimate these two system matrices uses the shift-invariant structure of the extended observability matrix $\hat{\Gamma}$ (Van Overschee and De Moor, 1996a). In (Miller and de Callafon, 2010) a method that uses the shift-invariant property of the output data, is used to estimate the system matrix A . From Eq. (19), this one starts by remarking that the shifted matrix $\vec{\mathbf{R}}_{y\bar{u}}$ - defined as $\mathbf{R}_{y\bar{u}}$ shifted by a single column to the left - can be expressed as

$$\vec{\mathbf{R}}_{y\bar{u}} = \Gamma A \mathbf{R}_{x\bar{u}} + \vec{T} \vec{\mathbf{R}}_{\bar{u}\bar{u}} + \vec{\mathbf{R}}_{v\bar{u}}, \quad (25)$$

where $\vec{\mathbf{R}}_{\bar{u}\bar{u}}$, $\vec{\mathbf{R}}_{v\bar{u}}$, \vec{T} are defined as $\mathbf{R}_{\bar{u}\bar{u}}$, $\mathbf{R}_{v\bar{u}}$ and T respectively shifted by a single column to the left. The effects of the shifted input matrix $\vec{\mathbf{R}}_{\bar{u}\bar{u}}$ are removed by using the projection $\Pi_{\bar{u}\bar{u}}$ (Miller and de Callafon, 2010). Hence, given $\hat{\Gamma}$ and $\hat{\mathbf{R}}_{x\bar{u}}$ from Eq. (24), the matrix \hat{A} that

best estimates the true system matrix A is found by solving the following least squares problem

$$\min_{\text{rank}(\hat{A})=n_x} \|\hat{\Gamma} \hat{A} \hat{\mathbf{R}}_{x\bar{u}} \Pi_{\bar{u}\bar{u}} - \vec{\mathbf{R}}_{y\bar{u}} \Pi_{\bar{u}\bar{u}}\|_2. \quad (26)$$

The solution of the above problem is given by

$$\hat{A} = \hat{\Gamma}^\dagger \vec{\mathbf{R}}_{y\bar{u}} \Pi_{\bar{u}\bar{u}} (\hat{\mathbf{R}}_{x\bar{u}} \Pi_{\bar{u}\bar{u}})^\dagger, \quad (27)$$

where $\hat{\Gamma}$ and $\hat{\mathbf{R}}_{x\bar{u}} \Pi_{\bar{u}\bar{u}}$ are given by Eq. (24). $(\cdot)^\dagger$ represents the pseudo-inverse. Finally, an estimate of C is then found from the first n_y rows of Γ and the system matrices B and D are estimated via a least squares problem (Verhaegen and Verdult, 2007).

3. FILTERING THE COVARIANCE FUNCTIONS

As described in the previous Section, the model order is chosen from a SVD calculation of $\mathbf{R}_{y\bar{u}} \Pi$. If one tries to select a matrix of rank $\eta = n_x - j$ with $j \geq 1$, the error when solving the least-squares problem becomes (Golub and Van Loan, 1996, Th. 2.5.2)

$$\epsilon_\eta = \|Q_\eta - \mathbf{R}_{y\bar{u}} \Pi\|_2 = \sigma_{\eta+1} > \epsilon_{n_x}, \quad (28)$$

because the position $\eta + 1$ is located before the drop off. For this reason, it is not possible to choose a lower order without obtaining biased results.

Although initially motivated by the desire to pre-whiten the data (Viberg, 1995), the use of weighting matrices in 4SID methods also modifies the range space of the extended observability Γ . Thus, they can be used to influence the model order estimation. In (Van Overschee and De Moor, 1996b, Ch. 5), it was shown that a proper choice of weighting matrices leads to a frequency-weighted balanced state-space basis in which model reductions can be performed. This corresponds to the technique of frequency weighted model reduction given in (Enns, 1984a). Based on the balanced reduction, the frequency weighted reduction consists in introducing input and output frequency weights to enhance certain frequency bands in the balancing procedure.

In our case, as shown in (Miller and de Callafon, 2010), the algorithm does not require the use of weighting matrices to provide consistent estimates. However, similarly to the approach mentioned above, additional weighting matrices could be used to improve the estimate of a reduced order model. Using a filter is another possibility that has the advantage to avoid the choice of such weighting matrices. A non-causal filter (given in terms of the forward time-shift operator q) can indeed be applied to the correlation functions $\tilde{R}_{\bar{u}\bar{u}}(\tau)$ and $\tilde{R}_{y\bar{u}}(\tau)$ as

$$\tilde{R}_{\bar{u}\bar{u},f}(\tau) = F(q) \tilde{R}_{\bar{u}\bar{u}}(\tau) \quad (29)$$

$$\tilde{R}_{y\bar{u},f}(\tau) = F(q) \tilde{R}_{y\bar{u}}(\tau). \quad (30)$$

Noting $U(\omega_l)$ and $Y(\omega_l)$ the Fourier transform of $u(t)$ and $y(t)$ respectively, the filtered estimates $\tilde{R}_{\bar{u}\bar{u},f}(\tau)$ and $\tilde{R}_{y\bar{u},f}(\tau)$ are given by

$$\tilde{R}_{\bar{u}\bar{u},f}(\tau) = \mathcal{F}^{-1}\{\tilde{\phi}_{uu,f}(\omega_l)\} \quad (31)$$

$$\tilde{R}_{y\bar{u},f}(\tau) = \mathcal{F}^{-1}\{\tilde{\phi}_{yu,f}(\omega_l)\}, \quad (32)$$

where the operator \mathcal{F}^{-1} is the inverse Fourier transform and the filtered spectral density estimates $\tilde{\phi}_{uu,f}(\omega_l)$ and $\tilde{\phi}_{yu,f}(\omega_l)$ are given by

$$\tilde{\phi}_{uu,f}(\omega_l) = F(\omega_l) U(\omega_l) U(\omega_l)^* \quad (33)$$

$$\tilde{\phi}_{yu,f}(\omega_l) = F(\omega_l) Y(\omega_l) U(\omega_l)^*, \quad (34)$$

with $F(\omega_l)$ defining the frequency response of the chosen filter.

Choosing a non-causal filter $F(q)$ such that its frequency response $F(\omega_l)$ is equal to 1 if $\omega_l \in [\omega_{min}, \omega_{max}]$ and equal to 0 elsewhere yields to apply a frequency band selection. Using such a filter, the influence on the data of poles located outside the selected frequency band is reduced. The corresponding singular values of $\mathbf{R}_{y\bar{u}} \Pi$ are consequently considerably decreased. Hence, the drop-off appears after a smaller number of singular values and a model of lower order can be identified. If the system dynamics over the entire frequency range is not of interest, this filter is a way to focus on a frequency band of interest. It enables to identify a model that will fit with the system response in this frequency band regardless of the system dynamics at other frequencies.

Compared to a balanced reduction which is a common way to perform a model reduction, the approach presented here is slightly different. The balanced reduction indeed truncates the less controllable and observable states. The obtained results are therefore not necessarily similar since the truncation operation is done regardless the frequency location of poles. The frequency weighted generalization of the balanced truncation introduced in (Enns, 1984b) includes frequency weighting to emphasize states in a certain frequency band. If the weighted matrices are chosen to emphasize the same frequency band, then similar results should be obtained but via a multi-step procedure. Equivalent results with 4SID methods can be obtained with a particular choice of the weighting matrices as shown in (Van Overschee and De Moor, 1996a) in a one-step procedure. But in these methods only causal filters are used and frequency weighting approaches suffer from the difficulty arising when one wants to obtain an accurate frequency band selection. Methods based on the frequency-limited grammians calculation (Gawronski and Juang, 1990) circumvent this problem. But this approach requires the knowledge of the full system. The algorithm proposed in this article aims at providing the same simplicity of use with the advantage to get a model of reduced order in a one step procedure from measurement data.

The use of the filter $F(q)$ offers a second advantage for the identification algorithm. It reduces the noise so $\tilde{R}_{y\bar{u},f}(\tau)$ can more easily be truncated using the temporal window $\omega(\tau)$. Hence, less data points can be considered.

4. ALGORITHM BASED ON FILTERED COVARIANCE FUNCTIONS

Let $\hat{R}_{y\bar{u},f}(\tau)$ and $\hat{R}_{\bar{u}\bar{u},f}(\tau)$ be the filtered estimates of the cross covariance function $R_{y\bar{u},f}(\tau)$ and $R_{\bar{u}\bar{u},f}(\tau)$. They are calculated from the measured input-output data $\bar{u}(t)$ and $y(t)$ and can be expressed in terms of the state-space matrices the same way as the non filtered covariance functions in (4).

Hence, the algorithm described in Section 2.4 can be formulated using the filtered covariance functions by first writing

$$\mathbf{R}_{y\bar{u},f} = \Gamma \mathbf{R}_{x\bar{u},f} + T \mathbf{R}_{\bar{u}\bar{u},f} + \mathbf{R}_{v\bar{u},f} \quad (35)$$

$$\vec{\mathbf{R}}_{y\bar{u},f} = \Gamma A \mathbf{R}_{x\bar{u},f} + \vec{T} \vec{\mathbf{R}}_{\bar{u}\bar{u},f} + \vec{\mathbf{R}}_{v\bar{u},f}. \quad (36)$$

The matrices $\mathbf{R}_{y\bar{u},f}$, $\mathbf{R}_{x\bar{u},f}$, $\mathbf{R}_{\bar{u}\bar{u},f}$, $\mathbf{R}_{v\bar{u},f}$, $\vec{\mathbf{R}}_{y\bar{u},f}$, $\mathbf{R}_{x\bar{u},f}$, $\vec{\mathbf{R}}_{\bar{u}\bar{u},f}$, $\vec{\mathbf{R}}_{v\bar{u},f}$ are defined as in Eq. (20) using the filtered covariance functions. The projection $\Pi_{\bar{u}\bar{u},f}$ onto the orthogonal complement of the filtered block-Hankel matrices $\mathbf{R}_{\bar{u}\bar{u},f}$ and $\vec{\mathbf{R}}_{\bar{u}\bar{u},f}$ is defined similarly to the non filtered projection. Depending on the applied filter and regarding to the system mode frequencies, the influence of some modes on the filtered data may not be seen or may be very low. In this situation, a significant drop-off in the singular values of $\mathbf{R}_{y\bar{u},f} \Pi_{\bar{u}\bar{u},f}$ is after the first η singular values. The position η is given by

$$\eta = n_x - (2c + r), \quad (37)$$

where c is the number of complex modes and r the number of real modes whose influence are not significant in the filtered data. Hence, the matrix Q_ν that minimizes the least-squares problem

$$\epsilon_\eta = \min_{\text{rank}(Q)=\nu} \|Q - \mathbf{R}_{y\bar{u},f} \Pi_{\bar{u}\bar{u},f}\|_2, \quad (38)$$

is obtained from the SVD of $\mathbf{R}_{y\bar{u},f} \Pi_{\bar{u}\bar{u},f}$ and given by

$$Q_\eta = U_\eta \Sigma_\eta V_\eta^T. \quad (39)$$

In this situation the residual error ϵ_η is small and the estimation of a model of reduced order η does not lead to undesirable results.

As a consequence, the objective which is not to find the best captures ($\hat{A}, \hat{B}, \hat{C}, \hat{D}$) but to identify the reduced system (A_f, B_f, C_f, D_f) of order η that best estimates the system dynamics in the selected frequency band can directly be achieved from the data matrix $\hat{R}_{y\bar{u},f} \Pi_{\bar{u}\bar{u},f}$. An estimate of the extended observability matrix of the reduced system - denoted $\hat{\Gamma}_f$ - and of the filtered state response - denoted $\mathbf{R}_{x\bar{u},f}$ are indeed first given by a factorization of Q_η as

$$Q_\eta = \hat{\Gamma}_f (\mathbf{R}_{x\bar{u},f} \Pi_{\bar{u}\bar{u},f}), \quad (40)$$

where a possible choice is

$$\hat{\Gamma}_f = U_\eta \Sigma_\eta^{1/2}, \quad \mathbf{R}_{x\bar{u},f} \Pi_{\bar{u}\bar{u},f} = \Sigma_\eta^{1/2} V_\eta^T. \quad (41)$$

Then, using the shift-invariant property as shown in Section 2 leads to the reduced state-matrix estimate \hat{A}_f . The matrix \hat{C}_f is found from the first n_y rows of $\hat{\Gamma}_f$. Finally, the input matrices \hat{B}_f, \hat{D}_f and the initial state-input correlation $\hat{R}_{x\bar{u},f}(\tau_0)$ are retrieved from a sequence of input-output correlation by solving a least squares problem.

5. ILLUSTRATIVE EXAMPLE

The covariance-based algorithm is applied to the identification of a flexible aircraft structure during flight tests. The reduced model used to illustrate the improvements of the algorithm presents the characteristics of an aeroelastic model of aircraft structure. It is a 6th order model with 1 input and 4 outputs. The 3 modes are low damped modes with the specificity that the second and third one have very closed frequency and damping values. The results shown in this Section are obtained with short excitations used during flight tests. Typically, the used excitations are 0.2 s pulse. The outputs are corrupted by a non-white noise obtained from an appropriate simulation of the

background noise that affects flight tests. The accuracy of the identification is evaluated by the criterion

$$J(\theta) = \frac{1}{N} \sum_{k=1}^N (y_{0k} - \hat{y}_k(\theta))^2, \quad (42)$$

where θ is the vector of parameters, y_{0k} the output vector of the true system and $\hat{y}_k(\theta)$ the output vector of the identified system.

The system comes back to its equilibrium state 50 seconds after the excitation. To show the impact of bound effects on the results, identification procedures with different record duration - noted d - are launched and no filter is applied to the data. For each value of d , 100 Monte-Carlo simulations of the noise were realized. Figure 1 shows the mean values of the identification results obtained with and without bound conditions integration as a function of the duration d . We can see that when the bound

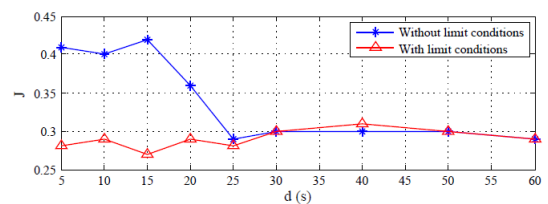


Fig. 1. Influence of bound effects on identification results.

effects are explicitly taken into account, a shorter duration can be considered without deteriorating the identification accuracy. In the case simulated here, it leads to even better results because the output signal energy is decreasing with time.

Now we consider the case $d = 15$ s for which the mean signal to noise ratio is 19 dB. We want a 4th order model that represents the dynamic behaviour of the two close modes only. A filter that selects the frequency between $f_{min} = 2.5$ Hz and $f_{max} = 5$ Hz is applied to the covariance functions. Results are compared with mode values estimated by a two step model reduction: first a 6th order model is identified without filter and is reduced by using a frequency limited balanced reduction performed with the Model REduction Toolbox (Poussot-Vassal and Vuillemin, 2012). Estimated modes in these two cases are shown on Fig. 2. Without

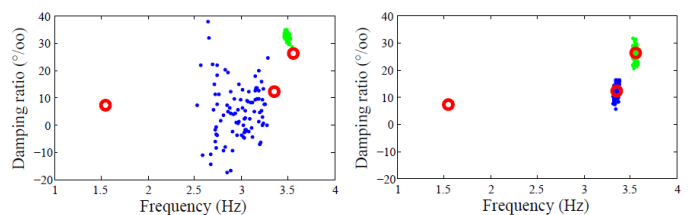


Fig. 2. Locations of the two identified modes with the frequency-limited balanced reduction (left) and with the filtered covariance-based 4SID identification (right). The system modes are represented by the red circles.

filter, the system is not well-identified. Particularly, the third mode dynamic is hardly captured by the algorithm. Consequently, the third mode is still not well estimated after the reduction step. Conversely the filtered covariance-based subspace identification provides better results in a one step procedure thanks to the frequency band selection.

Moreover, it can be seen in Fig 3 that the reduced order models identified fit the system dynamics in the selected frequency band. Fig 4 shows the impact of the filter on

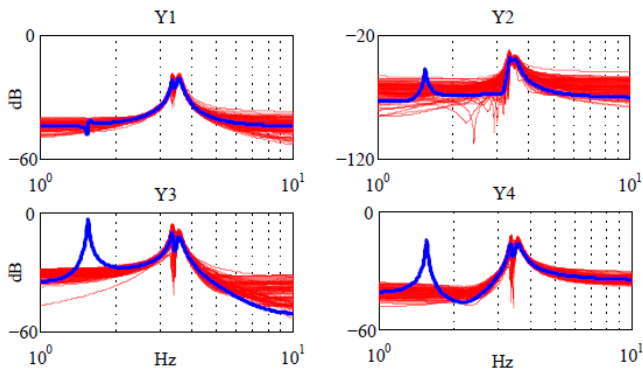


Fig. 3. Bode-plot of the reduced order models (red) and of the high order system (blue).

singular values for one of the 100 simulations. When the filter is used, the drop-off is clearly seen after the fourth singular value.

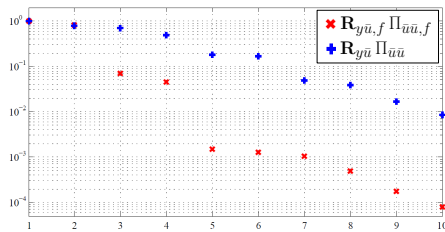


Fig. 4. Singular values of $R_{y\bar{u}} \Pi$ and $R_{y\bar{u},f} \Pi_{\bar{u},f}$.

6. CONCLUSION

In (Miller and de Callafon, 2010), a subspace identification algorithm based on correlation functions has been introduced. This algorithm provides consistent estimates even when data are corrupted by colored noise. In this article, we have presented two improvements for this method. First, we have integrated bound effects in the algorithm formulation. This improves the identification accuracy when the system is not at an equilibrium state at the beginning and final time considered for the identification. Second, we have shown that the model order selection can be improved by making use of non-causal filters. This results in a new covariance based algorithm that uses frequency weights computed by taking into account initial and final conditions. Both solutions respect the initial algorithm formalism and are therefore easy to implement.

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