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Motion Control Experiments for Identification of Actuator Dynamics

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Abstract: Estimating a dynamic model of a servo actuator often has to be done on the basis of noisy closed-loop experiments in which feedback is controlling the servo actuator. The presence of feedback complicates the estimation procedure due to strong correlation of control signals with disturbances present on the servo system. This paper shows how information on the feedback controller can be used to equalize the effects of disturbances on the control signals and estimate low order models of the actuator dynamics. The approach uniquely relies on a fractional representation of the feedback controller and a coupled least squares optimization problem to estimate an actuator model. Motion control experiments of a servo actuator in a Linear Tape Open drive operating under feedback will be used to illustrate the effectiveness of the procedure and will show the possibility to estimate low order servo actuator models.

Keywords: Identification for control; coprime factorizations; servo systems

1. INTRODUCTION

Understanding actuator dynamics in a servo control system is imperative to design a high bandwidth motion system. It has been recognized that resonance modes that deemed to be insignificant in the open-loop operation of a flexible actuator system, might be detrimental for the performance and robustness in a feedback operation (Hughes, 1987; Skelton, 1989). Closed-loop system identification that uses measured input/output (I/O) data from an actuator operating in feedback operation is an viable and accurate methodology to formulate models suitable for high performance control design (Gevers, 2005) and recent applications in high precision motion control follow that route (Oomen et al., 2013).

Performing identification on the basis of closed-loop data is challenging as the presence of feedback causes strong correlation of both the I/O data with possible disturbances present on the servo system. This challenge manifests itself as a possible error (bias) in the estimation of the actuator dynamics when the estimation of a noise model, modeling the spectral contents of the disturbance, is omitted (Ljung, 1999). The problem of bias in closed-loop identification can be circumvented in a motion control system by using an externally applied reference signal. The reference signal is needed to provide sufficient excitation and can be assumed to be uncorrelated with the disturbance present on the feedback system. An excellent overview of closed-loop identification methods and applications can be found in Perez and Sala (2002) and Gevers (2005). It is shown that the challenge of closed-loop identification should not be circumvented in order to find models suitable for control design (Gevers et al., 2011).

In most of the closed-loop identification methods, an indirect approach is followed to identify actuator dynamics. Common examples include the classical two-stage method (Van den Hof and Schrama, 1993), the bias-elimination least-squares method (Zheng and Feng, 1995), the tailor-made parametrization method in van Donkelaar and Van den Hof (2000) or advanced versions that use (normalized) coprime factorizations (Oomen and Bosgra, 2012). Knowledge of the external reference signal present during the closed-loop experiment can be exploited to formulate de-correlation (projection) methods via identification of non-causal closed-loop transfer functions (Forssell and Ljung, 2000) or formulate Instrumental Variable (IV) estimators (Gilson and Van den Hof, 2005) to deal with unknown or non-linear feedback dynamics. Combining knowledge on both the external reference signal and the possibly non-linear feedback controller is possible in virtual closed loop methods (Agüero et al., 2011) that balancing noise de-correlation against noise modeling in a user-chosen flexible fashion.

Existing closed-loop identification methods are quite powerful in providing the tools to find low order models based on feedback experiments. Most methods only use the fact that an externally applied reference signal is uncorrelated with the disturbance and do not exploit the fact that the dynamics of the noise on the I/O signals is very similar in a linear feedback connection. Furthermore, it may be desirable to exploit linear optimization techniques to allow model estimation to be performed by the firmware of a motion control system without user-chosen interaction. This paper shows how information on linear controller

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dynamics can be used to equalize the noise on the closed-loop I/O data and facilitate an (iterative) Constrained Least Squares (CLS) optimization to find a low order model of an actuator based on motion control experiments.

Noise equalization and the CLS estimation is illustrated on a simulation example to study the variance aspects of the method. The simulation example shows how the combined noise equalization and CLS estimation computes models with a small variance around the bandwidth of the feedback system and estimates the resonance modes of a flexible actuator that are most predominant in the closed-loop operation of the actuator. The paper wraps up a demonstration of the method by an application to the motion control system of a mechanical Linear Tape Open (LTO) drive system.

2. PROBLEM FORMULATION

Consider a linear (discrete-time) feedback control or servo system described by the equations

$$\begin{aligned} y(t) &= G_0(q)u(t) + d(t) \\ u(t) &= r(t) + C(q)y(t) \end{aligned} \tag{1}$$

where $|t| \in \mathbb{N}$ denotes a discrete-time dependency. In the closed-loop equations given in (1), a known and persistently exciting reference signal $r(t)$ is added to the output of a known feedback controller $C(q)$ and both the input/ouput (I/O) signals $\{u(t), y(t)\}$ are available for identification of the unknown actuator dynamics $G_0(q)$. The objective is to estimate a (low order) model approximation of $G_0(q)$ based on the available information in the closed-loop signals $\{u(t), y(t)\}$, the reference signal $r(t)$ and the feedback controller $C(q)$. Although this objective is similar to many other existing closed-loop estimation techniques (Gevers, 2005), in this paper an additional requirement is imposed: to only use (recursive) Least Squares (LS) or Instrumental Variable (IV) estimation techniques to facilitate the computational implementation of parameter estimation in the firmware of a motion control system.

For the sake of simplicity, the I/O signals $\{u(t), y(t)\}$ are assumed to be scalar (single input, single output actuator) and rewritten as

$$\begin{aligned} y(t) &= \frac{G_0(q)}{1 - C(q)G_0(q)} r(t) + \frac{1}{1 - C(q)G_0(q)} d(t) \\ u(t) &= \frac{1}{1 - C(q)G_0(q)} r(t) + \frac{C(q)}{1 - C(q)G_0(q)} d(t) \end{aligned} \tag{2}$$

indicating that presence of feedback complicates the estimation procedure to obtain a (low order) model for $G_0(q)$ due to strong correlation of both I/O signals $\{u(t), y(t)\}$ with the disturbance $d(t)$ present on the servo system. The problem of finding a low order model for $G_0(q)$ is especially challenging when the estimation of a noise model, modeling the spectral contents of the disturbance $d(t) = H_0(q)e(t)$ as a filtered white noise $e(t)$, is omitted. Using only the I/O signals $\{u(t), y(t)\}$ and ignoring or approximating the noise dynamics $H_0(q)$ in the general prediction error framework will induce bias in the estimation of a model for $G_0(q)$ (Ljung, 1999).

Most of the closed-loop identification methods summarized in the introduction of this paper will provide solutions to the problem of finding a low order model based on feedback experiments, but do not exploit the fact that the dynamics from $r(t)$ to the I/O signals $\{u(t), y(t)\}$ *and* the noise on the I/O signals $\{u(t), y(t)\}$ is similar. Moreover, scaling of the gain of the system G_0 and the controller C at the cross-over frequency ω_c for which $|G_0(e^{j\omega_c})C(e^{j\omega_c})| = 1$ can influence the quality of the closed-loop identification. It can be observed from (2) that in a poorly scaled feedback systems where the actuator gain $|G_0(e^{j\omega_c})|$ is 'small' and the controller dynamics $|C(e^{j\omega_c})|$ is 'large', the noise on the input $u(t)$ will be significantly larger than the noise on the output $y(t)$. In that case, estimating the map from the reference signal $r(t)$ to the input $u(t)$ in the first step of an indirect identification method will lead to (closed-loop) models with a large variance. It is evident that the common noise dynamics can be exploited and the results will be beneficial in reducing the variance on the models being estimated, especially in motion control system where a large noise might be present on the I/O signals.

3. CLOSED-LOOP NOISE EQUALIZATION

To exploit the common noise dynamics and allow gain adjustment in case of a poorly scaled feedback system, information on the feedback controller $C(q)$ in the form of its transfer function

$$C(q) = N_c(q)/D_c(q)$$

can be used to define the filtered signals

$$y_f(t) = N_c(q)y(t), \;\; u_f(t) = D_c(q)u(t) \tag{3}$$

The filtering in (3) rewrites the closed loop signal in (2) into the closed-loop noise equalized signals

$$\begin{aligned} y_f(t) &= \frac{G_0(q)N_c(q)}{1 - C(q)G_0(q)} r(t) + \frac{N_c(q)}{1 - C(q)G_0(q)} d(t) \\ u_f(t) &= \frac{D_c(q)}{1 - C(q)G_0(q)} r(t) + \frac{N_c(q)}{1 - C(q)G_0(q)} d(t) \end{aligned} \tag{4}$$

This simple operation ensures that the noise on the filtered I/O signals $\{u_f(t), y_f(t)\}$ has been equalized and contains the same dynamics. Realizing that the (unknown) system $G_0(q)$ can also be written in a coprime representation

$$G_0(q) = N_0(q)/D_0(q)$$

the filtered I/O signals $\{u_f(t), y_f(t)\}$ are written as

$$\begin{aligned} A_0(q)y_f(t) &= N_0(q)r_y(t) + C_0(q)d(t) \\ A_0(q)u_f(t) &= D_0(q)r_u(t) + C_0(q)d(t) \end{aligned} \tag{5}$$

where the filtered reference signals $r_y(t)$ and $r_u(t)$ are defined according to

$$r_y(t) = N_c(q)D_c(q)r(t), \;\; r_u(t) = D_c(q)^2 r(t) \tag{6}$$

and the polynomials $A_0(q)$ and $C_0(q)$ are given by

$$\begin{aligned} A_0(q) &= D_c(q)D_0(q) - N_c(q)N_0(q) \\ C_0(q) &= N_c(q)D_c(q)D_0(q) \end{aligned}$$

It should be noted that all the filter operations given in (3) and (6) only require information on the controller $C(q)$ and are simple Finite Impulse Response (FIR) filter operations that yield bounded output signals. Furthermore, the closed-loop noise equalized I/O signals $\{y_f(t), u_f(t)\}$ in (5) satisfy three important properties.

- First of all, if $d(t)$ is a white noise, both the map from $r_y(t)$ to $y_f(t)$ and the map from $r_u(t)$ to $u_f(t)$

exhibit an ARMAX structure (Ljung, 1999). The noise equalization ensures that the $A_0(q)$ and $C_0(q)$ polynomials are the same for both maps. It should be noted that this also holds in case the dynamics of the noise $d(t)$ can be represented by a (high) order FIR model

$$d(t) = H_0(q)e(t),\ H_0(q) = \sum_{k=0}^{n} h_0(k)q^{-k}$$

where $e(t)$ is a white noise. In that case, the noise $d(t)$ is still equalized on the filtered I/O signals $\{y_f(t), u)f(t)\}$ but the $C_0(q)$ polynomial is expanded to $C_0(q)H_0(q)$ to maintain the ARMAX structure.

- The transfer function of the map from $r_y(t)$ to $y_f(t)$ and the map from $r_u(t)$ to $u_f(t)$ respectively given by $N_0(q)/A_0(q)$ and $D_0(q)/A_0(q)$ constitutes the dynamics of the unknown and possibly unstable system $G_0(q)$, but each map individually is stable. The additional filtering in (6) ensures that the stable coprime factors of $G_0(q)$ can be estimated instead of the possibly unstable $G_0(q)$ itself.
- Finally, the order and/or parameters of $A_0(q)$ are irrelevant when computing a model for $G_0(q)$ via the ratio of the map from $r_y(t)$ to $y_f(t)$ and the map from $r_u(t)$ to $u_f(t)$. This property can be exploited during the identification of a (low order) model for G_0 while trying to approximate the closed-loop dynamics and the noise dynamics with a (high order) parametrization for $A_0(q)$.

It should be noted that the last property can only be exploited when both maps are used during the identification. Approximations of the noise and closed-loop dynamics can be made by simply ignoring the estimation of the C_0 polynomial and increasing the order of the A_0 polynomial to get a better approximation of the noise dynamics, while keeping the order of N_0 and D_0 polynomials bounded to find a (low order) model for G_0. The idea of using a Least Squares optimization with a unique and easy computable global minimum, while increasing the order of the A_0 polynomial to allow for an approximation of the ARMA noise dynamics and reduce bias on the model estimation, has been exploited in closed-loop identification as early as (Zhy and Stoorvogel, 1992) and in Verhaegen and Verdult (2007), Badwe et al. (2011). However, in these methods the order of the A_0 polynomial influences the order of the model being estimated, requiring additional model reduction. With the proposed closed-loop estimation with noise equalization, the freedom in choosing a higher order model for A_0 will not influence the order of the model being estimated. More details on the closed-loop identification procedure that exploits the favorable properties of the noise equalization follows.

4. CONSTRAINED LEAST-SQUARES MODEL ESTIMATION

The closed-loop identification procedure in this paper exploits the favorable properties of the noise equalization and formulates a Coupled Least Squares (CLS) optimization problem with a unique and easy computable global minimum to estimate a (low order) model for G_0. To introduce the main idea behind the proposed closed-loop identification procedure, a model for the closed-loop noise equalized signals in (5) is parametrized as follows

$$\mathcal{A}(q,\theta)\begin{bmatrix} y_f(t) \\ u_f(t) \end{bmatrix} = \mathcal{B}(q,\theta)\begin{bmatrix} r_y(t) \\ r_u(t) \end{bmatrix} + \begin{bmatrix} \varepsilon_y(t,\theta) \\ \varepsilon_u(t,\theta) \end{bmatrix} \tag{7}$$

where $\mathcal{A}(q,\theta)$ and $\mathcal{B}(q,\theta)$ are *diagonal* matrix polynomial

$$\begin{aligned} \mathcal{A}(q,\theta) &= \begin{bmatrix} A(q,\theta_A) & 0 \\ 0 & A(q,\theta_A) \end{bmatrix} \\ \mathcal{B}(q,\theta) &= \begin{bmatrix} N(q,\theta_N) & 0 \\ 0 & D(q,\theta_D) \end{bmatrix} \end{aligned}$$

parametrized according to

$$\begin{aligned} A(q,\theta_A) &= 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \\ N(q,\theta_N) &= q^{-n_k}(n_0 + n_1 q^{-1} + \cdots + n_{n_n} q^{-n_n}) \\ D(q,\theta_D) &= d_0 + d_1 q^{-1} + \cdots + d_{n_d} q^{-n_d} \end{aligned} \tag{8}$$

where n_k is the number of delays desired in the model and

$$\begin{aligned} \theta_A &= \begin{bmatrix} a_1 & \cdots & a_{n_a} \end{bmatrix} \\ \theta_N &= \begin{bmatrix} n_0 & \cdots & n_{n_n} \end{bmatrix} \\ \theta_D &= \begin{bmatrix} d_0 & \cdots & d_{n_d} \end{bmatrix} \end{aligned}$$

Although the closed-loop noise equalized signals in (5) exhibit an ARMAX structure, the reason for choosing the parametrization in (7) is obvious: the prediction error $[\varepsilon_y(t,\theta)\ \varepsilon_u(t,\theta)]^T$ will be linear in the parameters θ and Least Squares (LS) minimization of the prediction error has a unique global minimum. Special attention has to be given to the coupled estimation of the polynomial coefficients of $\mathcal{A}(q,\theta)$, as both outputs $y_f(t,\theta)$ and $u_f(t,\theta)$ are weighted by the same coefficients in the $A(q,\theta_A)$ polynomial, creating a Coupled Least Squares (CLS) problem.

The main reason for the repetition of $A(q,\theta_A)$ on the diagonal of $\mathcal{A}(q,\theta)$ is to allow the computation of an (open-loop) I/O model that will be independent of the order of the polynomial $A(q,\theta_A)$ or the parameter values θ_A. In fact, if $N(q,\theta_N)$ and $D(q,\theta_D)$ are coprime and do not share common zeros, the order and dynamics of

$$G(q,\theta) = N(q,\theta_N)/D(q,\theta_D) \tag{9}$$

only depends on the choice of the size and the numerical values of θ_N and θ_D. Although $A(q,\theta_A)$ and its parameter θ_A is irrelevant for the open-loop model $G(q,\theta)$ in (9), it should be stressed that the polynomial $A(q,\theta_A)$ does serve an important role in the closed-loop identification. The order of the polynomial $A(q,\theta_A)$ and its parameters θ_A can be used to compute a better approximation of the closed-loop ARMA noise dynamics in (5) and reduce bias on the model estimate due to the linear regression parametrization.

It can be shown that the CLS problem can still be written in a standard LS problem by the appropriate definition of a parameter vector and a regressor. Without loss of generality and to simplify notations, it is assumed that $n_n = n_d$. In that case, the parametrization in (7) can be written in a linear regression form

$$\begin{bmatrix} \varepsilon_y(t,\theta) \\ \varepsilon_u(t,\theta) \end{bmatrix} = \begin{bmatrix} y_f(t) \\ u_f(t) \end{bmatrix} - \theta^T \phi(t) \tag{10}$$

where the parameter vector θ has the special form

$$\theta^T = \left[\begin{bmatrix} n_0 & 0 \\ 0 & d_0 \end{bmatrix} \cdots \begin{bmatrix} n_{n_n} & 0 \\ 0 & d_{n_d} \end{bmatrix} \begin{bmatrix} a_1 & 0 \\ 0 & a_1 \end{bmatrix} \cdots \begin{bmatrix} a_{n_a} & 0 \\ 0 & a_{n_a} \end{bmatrix}\right]$$

and where the regressor is given by

$$\phi(t) = \begin{bmatrix} r_y(t-n_k) \\ r_u(t) \\ \vdots \\ r_y(t-n_k-n_n) \\ r_u(t-n_d) \\ y_f(t-1) \\ u_f(t-1) \\ \vdots \\ y_f(t-n_a) \\ u_f(t-n_a) \end{bmatrix}$$

The special structure of the parameter θ in (10) due to the diagonal matrix polynomial $\mathcal{A}(q,\theta)$ and $\mathcal{B}(q,\theta)$ is easily enforced by vectorizing θ via Kronecker calculus (Bellman, 1970) to obtain

$$\begin{aligned} \varepsilon(t,\theta) &= y(t) - \bar{\phi}^T(t)\bar{\theta}, \\ \bar{\phi}^T(t) &= [\phi^T(t) \otimes I_{2\times 2}] \\ \bar{\theta} &= \mathrm{vec}(\theta^T) \end{aligned} \tag{11}$$

where $\otimes$ denotes the Kronecker product and $\mathrm{vec}(\cdot)$ the Kronecker vector operation. Eliminating those columns in $\bar{\phi}^T(t)$ for which $\bar{\theta}$ has zero entries and realizing that $\bar{\theta}$ has repeated entries for the diagonal matrix polynomial $\mathcal{A}(q,\theta)$ allows (11) to be reduced to the final format

$$\begin{bmatrix} \varepsilon_y(t,\theta) \\ \varepsilon_u(t,\theta) \end{bmatrix} = \begin{bmatrix} y_f(t) \\ u_f(t) \end{bmatrix} - \tilde{\phi}^T(t)\tilde{\theta} \tag{12}$$

where the modified regressor $\tilde{\phi}(t)$ is given by

$$\tilde{\phi}(t) = \begin{bmatrix} \begin{bmatrix} r_y(t-n_k) & 0 \\ 0 & r_u(t) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} r_y(t-n_k-n_n) & 0 \\ 0 & r_u(t-n_d) \end{bmatrix} \\ [y_f(t-1)\ u_f(t-1)] \\ \vdots \\ [y_f(t-n_a)\ u_f(t-n_a)] \end{bmatrix}$$

The modified parameter vector $\tilde{\theta}$ in (12) has no zero or repeating entries and given by

$$\tilde{\theta}^T = \left[[n_0\ d_0] \ \cdots \ [n_{n_n}\ d_{n_d}]\ a_1 \ \cdots \ a_{na} \right]$$

Clearly, LS minimization of the prediction error

$$\hat{\theta}_{LS}^N = \arg\min_\theta \frac{1}{N}\sum_{t=1}^N [\varepsilon_y(t,\theta)\ \varepsilon_u(t,\theta)] \begin{bmatrix} \varepsilon_y(t,\theta) \\ \varepsilon_u(t,\theta) \end{bmatrix}$$

now follows immediately from the linear regression expression in (12) and given by

$$\hat{\theta}_{LS}^N = \left[\frac{1}{N}\sum_{t=1}^N \tilde{\phi}(t)\tilde{\phi}^T(t) \right]^{-1} \left[\frac{1}{N}\sum_{t=1}^N \tilde{\phi}(t) \begin{bmatrix} \varepsilon_y(t,\theta) \\ \varepsilon_u(t,\theta) \end{bmatrix} \right] \tag{13}$$

As a final remark, it is reiterated here that the degree n_a of the $A(q,\theta_A)$ polynomial can be chosen larger than the specified order of the model computed by (9) to allow for a better approximation of both the closed-loop dynamics and the dynamics of the noise $d(t)$ and reduce the bias on the model estimation. The method does not preclude the use of Steiglitz-McBride iterations (Stoica and Söderström, 1981) or iterative Instrumental Variable (IV) estimators (Gilson and Van den Hof, 2005) to further improve estimation results by reducing the estimation bias caused by the colored regression noise in (5).

5. SIMULATION EXAMPLE

Consider the simulation example of a discrete-time system $G_0(q)$ defined as the Zero Order Hold (ZOH) discrete-time equivalent of a continuous-time actuator

$$G(s) = K\frac{s^2 + 2\beta_1\omega_1 s + \omega_1^2}{(s^2 + 2\beta_2\omega_2 s + \omega_2^2)(s^2 + 2\beta_3\omega_3 s + \omega_3^2)s^2}$$

sampled at the normalized frequency of 1Hz. In this example, $K = 2.5 \cdot 10^{-4}$, the resonance frequencies are given by $\omega_2 = 0.1$rad/s and $\omega_3 = 0.5$rad/s and a complex zero at the frequency of $\omega_1 = 0.15$rad/s. The ZOH discrete-time equivalent is controlled by a digital PD controller

$$C(q) = \frac{-0.5 + 0.49q^{-1}}{1 - 0.95q^{-1}} = \frac{N_c(q)}{D_c(q)} \tag{14}$$

providing a phase margin of only 24 degrees due to the (unknown) resonance modes present in the actuator dynamics. The closed-loop signals in the feedback connection are described by (2) and closed-loop experiments with $N = 1000$ points are generated. The reference signal $r(t)$ is simply changed randomly between the values of $-1, 1$ for the motion control experiments and a closed-loop white-noise $d(t)$ with a variance of 0.1 is added to the actuator output $y(t)$ to challenge the closed-loop system identification.

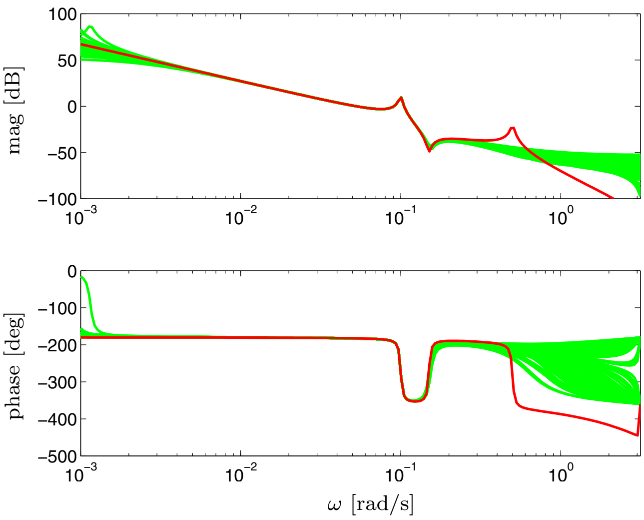

Fig. 1. Bode plot of sixth order actuator dynamics $G_0(q)$ (red/dark line) and estimated 4th order models $G(q,\hat{\theta}_{LS}^N)$ with the CLS optimization using 100 Monte-Carlo simulations (green/light lines).

It is clear that $G_0(q)$ is a sixth order system and is marginally stable due to the double integrator that is typical for a dynamic relation between a force input and a position output in a free moving inertial actuator. Based on the closed-loop data, estimating a full order sixth order model would be an obvious choice, but the objective in this example is to find a lower 4th order model that can capture the double integrator behavior and provide an accurate estimate of the main resonance mode that is most predominant in the closed-loop data.

The noise equalization summarized in Section 3 is used and parameters are estimated via CLS optimization outlined in Section 4. With $n_k = 0$, $n_n = 4$, $n_d = 4$ and an order $n_a = 5$ for the common $A(q, \theta_A)$ polynomial in the CLS problem, the LS parameter estimate $\hat{\theta}^N_{LS}$ in (13) is computed using a Steiglitz-McBride iteration (Stoica and Söderström, 1981). To illustrate the effectiveness in estimating the main resonance mode that is most predominant in the closed-loop data, 100 Monte Carlo simulations where performed to inspect the variance of the estimated model. The result of this simulation example is summarized in Fig. 1 where it can be seen that an excellent estimate of the resonance mode at $\omega_2 = 0.1$rad/s is obtained. It can be observed that the variance of the model estimates is very low around the cross-over frequency of the closed-loop system – a feature that is due to the (noise equalized) closed-loop data used in the identification.

It should be noted that this example illustrates two important features. Firstly, the ability to estimate a model of a marginally stable actuator dynamics operating under a stabilizing feedback using only a simple Least Squares (LS) optimization. Secondly, the possibility to find a lower order model that emphasizes the predominant behavior of the actuator dynamics operating under closed-loop conditions.

6. APPLICATION TO LTO ACTUATOR

For the application example in this paper, closed-loop experimental data is used from the data tracking servo control system in a Linear Tape Open (LTO) drive available at the System Identification and Control Laboratory (SICL) at the University of California, San Diego.

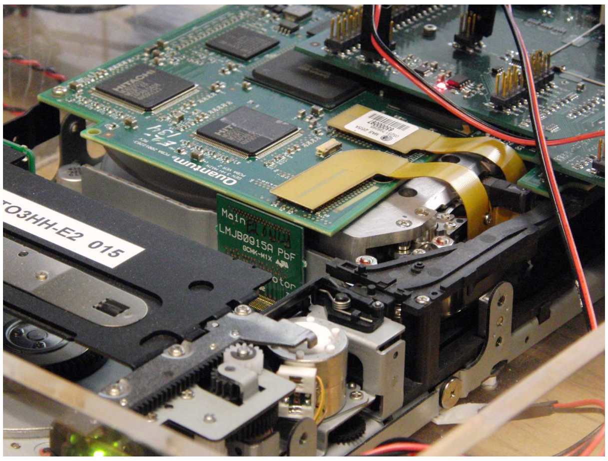

Fig. 2. Photograph of LTO actuator in a magnetic tape drive for motion control and tape track following.

In the LTO (version 3) drive depicted in Fig. 2, a flexible tape runs along a magnetic read/write head and decodes the digital Position Error Signal (PES) $y(t)$ (Pantazi et al., 2012; Lantz et al., 2012). During tape transport, Lateral Tape Motion (LTM) is one of the main disturbance $d(t)$ on the PES $y(t)$ and a second order embedded digital servo controller $C(q)$ is used to provide a control signal $u(t)$ to the LTO servo actuator for motion control and track following with a cross-over frequency of approximately 1kHz. The closed-loop signals in the feedback connection are again described by (2) and closed-loop experiments with $N = 10^5$ are used for the identification of a low order model of the LTO servo actuator dynamics. As the lateral movement of the tape during normal servo operation of the tape generates quite some noise in the LTO motion control system, at least 5 seconds of data sampled at 20kHz is needed to average the effect of the LTM and find a reliable model of the servo actuator.

Aiming for a sixth order model, the algorithm summarized in Section 4 is initiated with $n_k = 0$, $n_n = 7$, $n_d = 6$ and an order $n_a = 12$ for the common $A(q, \theta_A)$ polynomial in the CLS problem. The LS parameter estimate $\hat{\theta}^N_{LS}$ in (13) is again computed using a Steiglitz-McBride iteration. The resulting sixth order model of the LTO actuator is summarized in the Bode plot of Fig. 3. The Bode plot of the model is compared with the estimated frequency response of the servo actuator obtained by standard closed-loop spectral analysis (Wang and de Callafon, 2012). Using the notation $\Phi yr(j\omega)$ to indicate the cross-spectral density function between $r(t)$ and $y(t)$, the estimate

$$\hat{\Phi}yr(j\omega) = \frac{\sum_{k=1}^{p} Y_k(\omega)R_k^*(\omega)}{\sum_{k=1}^{p} R_k(\omega)R_k^*(\omega)}$$

is found via the Welch method of averaging an N-point Fourier transforms $Y_k(\omega) = \sum_{t=1}^{N} y_k(t)e^{-j\omega t}$ and $R_k(\omega) = \sum_{t=1}^{N} r_k(t)e^{-j\omega t}$ Ljung (1999) of the signals $y_k(t)$ and $r_k(t)$ for different experiments k. Based on this estimate, frequency domain data $\mathcal{G}(j\omega)$ of the servo actuator $G_0(e^{j\omega})$ is computed via

$$\mathcal{G}(j\omega) = \frac{\hat{\Phi}yr(j\omega)}{\hat{\Phi}ur(j\omega)} \tag{15}$$

and the Bode response of the estimate $\mathcal{G}_i(j\omega)$ in (15) has been included in Fig. 3 for comparison purposes.

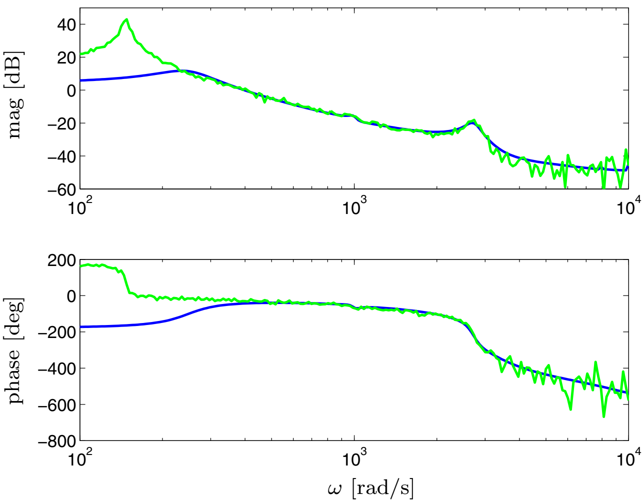

Fig. 3. Bode plot of estimated sixth order model $G(q, \hat{\theta}^N_{LS})$ with the CLS optimization (blue/dark lines) compared to frequency domain estimate $\mathcal{G}(j\omega)$ obtained by closed-loop spectral analysis (green/light lines)

It can be observed from Fig. 3 that the sixth order model obtained with the CLS estimation using only 5 seconds of closed-loop data measured at 20kHz captures some of the small resonance modes and their accompanying phase shift close to the cross-over frequency of 1kHz and the main resonance mode close to 3kHz. The large low frequency resonance mode around 140Hz is omitted in the low order model approximation. The results indicate that the low order model has emphasized the resonance modes that are most dominant during the closed-loop operation of the servo actuator, as expected from a closed-loop identification method. The resulting modeling results can be used to re-calibrate the embedded servo controller to account for unmodelled dynamics or production variations in the LTO servo actuator.

7. CONCLUSIONS

Information on the dynamics of a linear feedback controller can be used to equalize the effects of disturbances on the input/ouput signals during a closed-loop experiment. Noise equalization is a finite impulse response (FIR) filtering and ensures that the noise equalized actuator input signal and output signal have the same noise dynamics. In turn, the noise equalization can be exploited in the system identification to estimate a (low order) model of the actuator dynamics. When approximating the closed-loop data in a simple linear regression, the parameter estimation problem can be written as coupled least squares (CLS) optimization. In the CLS optimization, the closed-loop transfer functions and the noise on the equalized input/ouput data are restricted to have the same pole locations. The CLS can be written as standard linear regression estimation problem for which a unique LS minimum can be computed. Motion control experiments of a servo actuator in a Linear Tape Open drive operating under feedback illustrate the effectiveness of the procedure. Low order models approximate the predominant dynamic behavior of the actuator dynamics operating under closed-loop conditions.

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