

Course of lab activities on control theory based on the Lego NXT^{*}

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Abstract: This article presents practical course content of a control theory subject used in the Control Systems and Informatics department of the ITMO University. There are many different examples of the empirical development and model based control algorithms. Course begins with the simple line tracer robot and ends with the inverted pendulum on a cart. During the course, students are offered to learn the problems related with identification, modelling and programming, based on the Lego NXT platform.

Keywords: LEGO Mindstorms NXT, mobile robots, Ziegler-Nichols' ultimate gain method, modal control, Ackermann's formula.

INTRODUCTION

One of the important problem is decreasing of the students' education level. The interest in the natural sciences falls because the students do not know how to apply the theoretical knowledge. Is it useful for me in my life? – question that students often asked a teachers. But if the students go straight from theory to practice it is a very strong motivation to learn fundamental science.

Last year we introduce to students a new kind of learning methodology based on Lego NXT platform. Lego used as educational platform and had many different implementations A. Valera et al. (2011) and Baum et.al. (2000). But we did it for first year students without complex things. Positive feedback were received from students. Since now they have more interest in control theory applications. Maybe, they don't understand complex mathematics, but will be interested in explanation in future.

First lab activity includes Ziegler-Nichols' application to the PID controller for the line tracer robot. Second activity is the identification NXT motor mechanical time constant and back EMF constant. Third activity is modelling and developing motor angle proportional controller. Fourth activity is measuring NXT motor torque constant. The last activity is modelling and developing NXT Inverted pendulum.

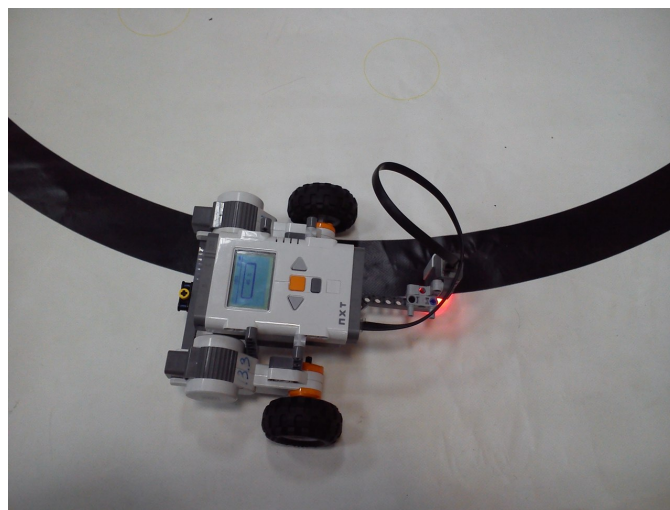


Fig. 1. Line tracer robot.

LAB ACTIVITY I

At first, we show to students simple robotic systems, and teach them methods, that do not require profound knowledge in physics and mathematics. For this deep we decide to use Ziegler-Nichols' ultimate gain method (closed-loop method). During this work, students have time to learn NXC program language for NXT brick, and try to use some abilities of Scilab, open source software for numer-

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ical computation <http://www.scilab.org>. Students can use open source software not only at university, but can also work at home on their own projects. Ziegler-Nichols' method is used for PID-controller, one of the most popular algorithms, and good example in control theory J.G. Ziegler and N. B. Nichols (1942).

For this task students construct simple line tracer robot 1 with one light sensor. The track looks like number eight. We begin from the P-control law. The robot is able to pass this line, we increase the proportional gain, until the robot starts to make stable oscillations, moving along the line. Maximum of the workable proportional coefficient is the ultimate gain. After we get measurements of the light sensor from robot, we commit the period of the signal oscillations.

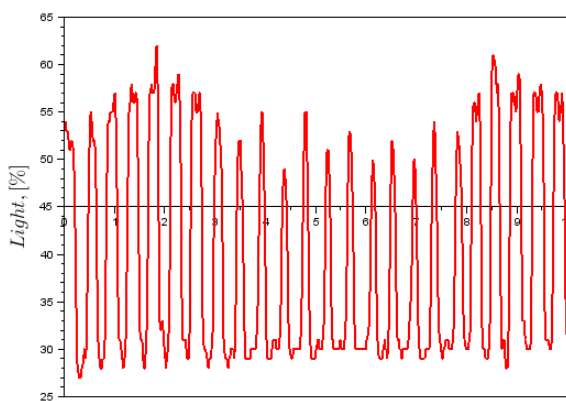


Fig. 2. Measurements of light sensor in time.

In fig. 2 for our line tracer ultimate gain $K_u = 7$ and the period of oscillation $T_u = 0.43 \text{ sec}$. The PID-controller gains can be calculated. Fig. 3 shows the system with different control laws.

LAB ACTIVITY II

In the second lab activity we start series of works that help us to describe the pendulum system in state space. For beginning students should get constants which associate the values of control voltage and output torque. The first necessary value is mechanical time constant. We can determine it with assignment constant control signal, and measuring the value of turning angle of the motor. We can calculate the mechanical constant using 1.

$$\phi = \omega_{nls}t - T_m\omega_{nls} + T_m\omega_{nls} \exp(-t/T_m), \quad (1)$$

where t – time, ϕ – turning angle, ω_{nls} – no load speed, T_m – mechanical time constant. Using this equation students can process data from motor in a Scilab application. The least squares method gives no load speed and mechanical time constant for NXT motor (fig. 4). In our case we got $\omega_{nls} = 14.2 \frac{\text{rad}}{\text{sec}}$ and $T_m = 0.079 \text{ sec}$. Control voltage value is $U = 7V$. Supposing that there is no viscous friction and other disturbances in motor, we can use formulas for ideal

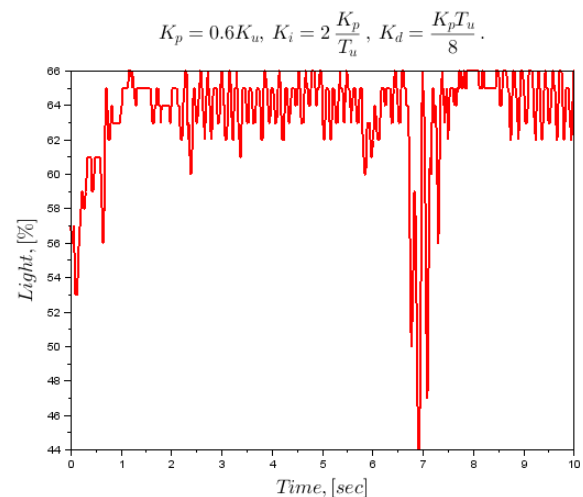
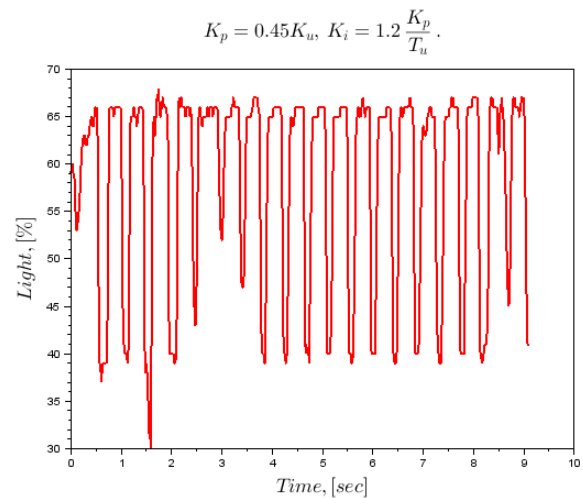
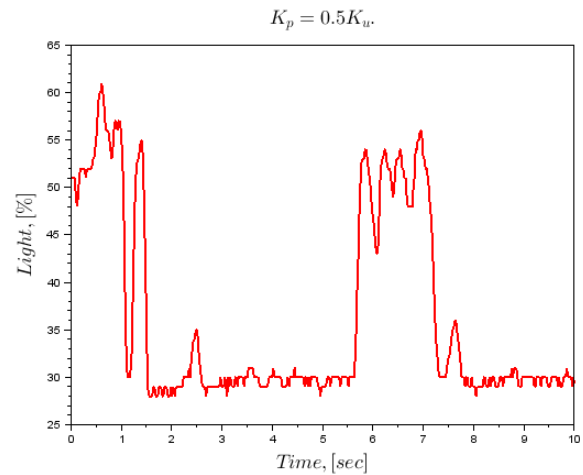


Fig. 3. P,PI,PID control laws and coefficients calculation formulas.

engine. It means that a back EMF U_{emf} value equals to the control voltage, and in the rotating motor current is 0 A (the real value $I = 0,05 \text{ A}$).

$$K_e = \frac{U_{emf}}{\omega_{nls}}. \quad (2)$$

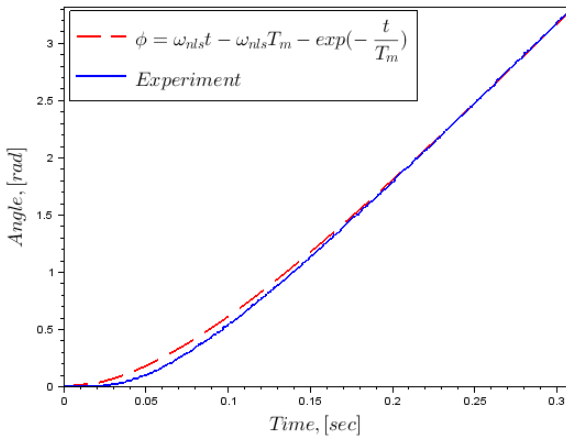


Fig. 4. Identification of model' parameters.

For our motor $K_e = 0.49 \text{ V} \cdot \text{sec}$. The next step in this lab work, is to check parameters in the open-loop model. Students have to synthesize control law $U = 7 \cdot \sin(\pi \cdot t)$, where U – the control signal. It should be done for different frequencies.

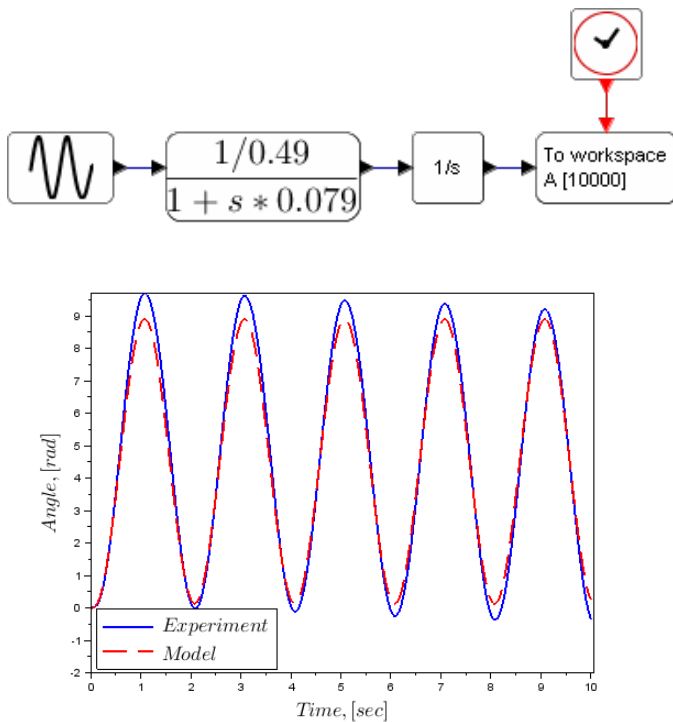


Fig. 5. Checking model' parameters.

As depicted in figure (fig. 5), these curves are too close to each other. The motor parameters were identified correctly. But the initial point of oscillation is moving in the bottom part of graph, the maximum voltage of PWM in different direction has difference 0.2V . This value disperses on the control switch.

LAB ACTIVITY III

In this lab activity students check obtained result in a closed-loop system. For this task we synthesize control law

for steering the angle of the motor with the proportional controller. This work includes programming and modelling closed-loop system, it helps students to associate mathematical formulas and text of the program.

Our control algorithm compares the current angle with a given one, in our case it is $\pi \text{ [rad]}$, and based on the difference generates a control signal for the motor.

$$U = Kp \cdot (\pi - \phi). \quad (3)$$

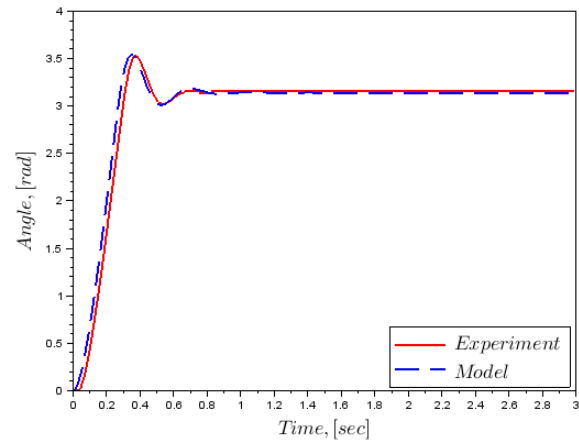
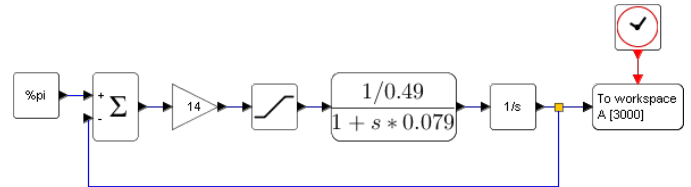


Fig. 6. Model of the P-controller and real motor.

From modeling the closed-loop system, using our findings, we can get that the experimental curve have error not more than 2%. All our calculations were made correctly, and we can proceed to the calculation of torque constant.

LAB ACTIVITY IV

We need a mathematical description of our robotic system, for this we control output torque of the motor. And we should get the torque constant K_t for this task. Consider the equation of the motor without the viscous friction:

$$\begin{cases} \dot{\omega} = \frac{K_t}{J} I, \\ \dot{I} = \frac{1}{L} U - \frac{K_e}{L} \omega - \frac{R}{L} I. \end{cases} \quad (4)$$

where $J = 10^{-6} \text{ kg} \cdot \text{m}^2$ – moment of inertia of the rotor, $R = 6 \text{ Ohm}$ – electric resistance, $L = 0.0047 \text{ H}$ – inductance of the armature, ω – rotor speed, I – the rotor current. Equation 5 calculate the value of the torque constant.

$$K_t = \frac{M_{lrt} i^2}{I_{lrc}}, M_{lrt} = \frac{\omega_{nls} J}{T_m}, \quad (5)$$

where $M_{l_{rt}}$ – the locked rotor torque, $I_{l_{rt}}$ – the locked rotor current, $i = 48$ – the NXT motor’s gear ratio. Substitute values of the parameters of our motor. Torque constant and back EMF constant values should be close, for NXT these constants are $K_t = 0.42 \frac{Nm \cdot m}{Ah}$ and $K_e = 0.49 V \cdot sec$. For input/output model we also need electrical time constant $T_e = \frac{L}{R} = 0.008 sec$. Lego NXT motor is PWM (Pulse Width Modulation) controlled. But in our equation the continuous control signal is used. Now students can check, that using PWM and DC control give the same results. In the figure 7 it can be seen, NXT motor with a reduction gear.

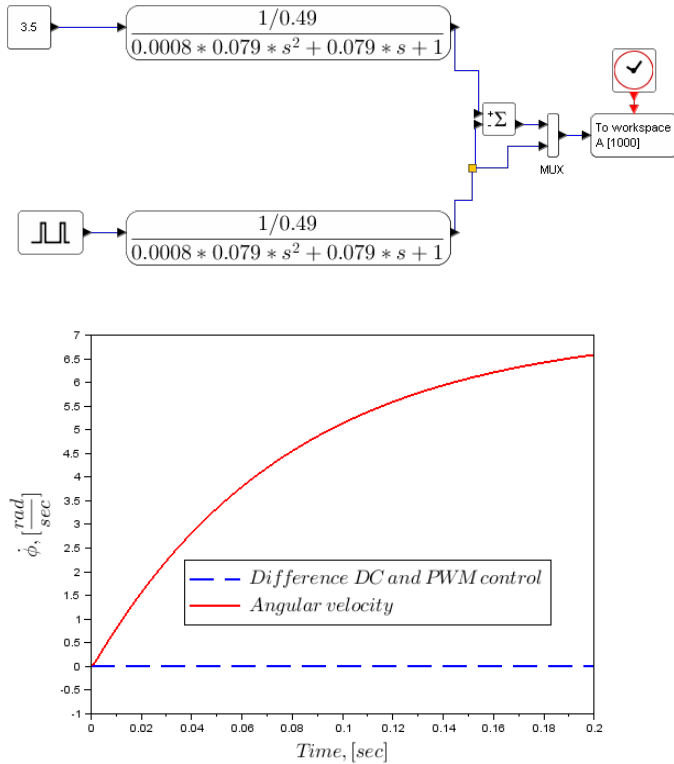


Fig. 7. Comparison of DC and PWM control.

It should be noted, that PWM signal works only with high frequency. If it is more than electrical time constant, system will become unworkable. An oscilloscope should be used for demonstration of the PWM signal. After this students should be sure, that the continuous control can be used.

LAB ACTIVITY V

Now we have all data for equations of our robotic system. To demonstrate the main results of these activities, we chose a well-known problem, inverted pendulum on a cart. We take the HiTechnic Angle Sensor for measuring the angle of the pendulum.

From the Euler-Lagrange’ equation we get mathematical formulas describing the inverted pendulum on a cart. Students use Maxima, computer algebra system for calculations.

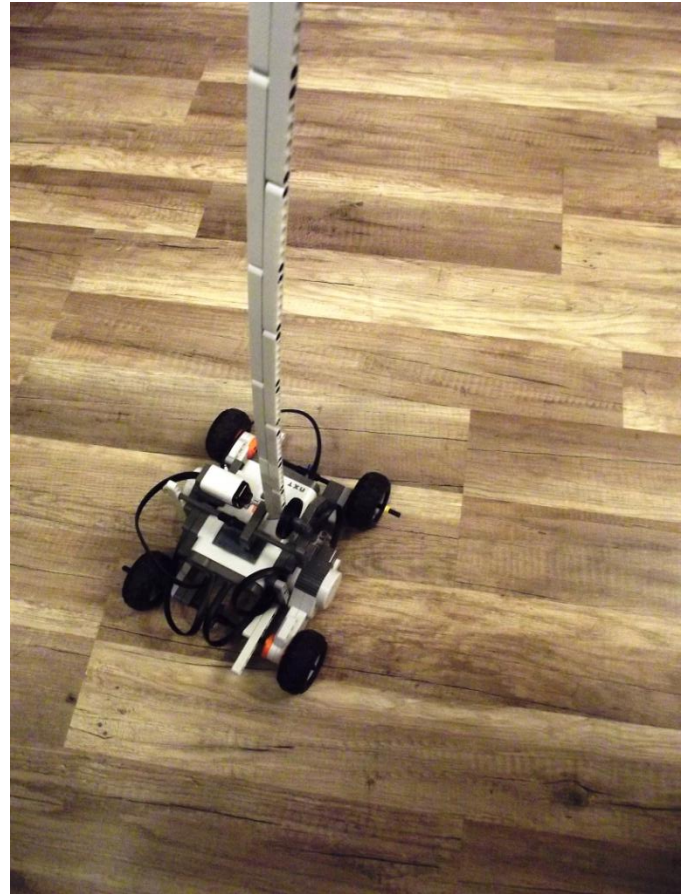


Fig. 8. Inverted pendulum on a cart based on the Lego NXT.

$$\begin{cases} m_p l r \ddot{\theta} + (J_p + m_p l^2) \ddot{\psi} - m_p g l \psi = 0, \\ k_1 r^2 \ddot{\theta} + m_p l r \ddot{\psi} + 2 \frac{K_t K_e}{R} \dot{\theta} = 2 \frac{K_t}{R} U, \end{cases} \quad (6)$$

where $k_1 = (\frac{4J_w}{r^2} + m_c + 4m_w + m_p)$, m_p – mass of the pendulum, l – distance to the center of mass of the pendulum, r – radius of the wheel, θ – turning angle of the wheel, J_p – inertia moment of the pendulum, ψ – turning angle of the pendulum about the center, J_w – inertia moment of the wheel about the center, m_c – mass of the cart, m_w – mass of the wheel.

Rewrite this system in matrix form

$$E \ddot{X} + F \dot{X} + G X = H U. \quad (7)$$

For our pendulum $X^T = [\theta, \psi]$. Divide the equation by E :

$$\ddot{X} = -E^{-1} F \dot{X} - E^{-1} G X + E^{-1} H U. \quad (8)$$

We need to present this equation in state space form for calculation of the feedback gains. Rewrite this equation in the extended form $\dot{X} = A X + B U$, where $X^T = [\psi, \dot{\theta}, \dot{\psi}]$:

$$A = \begin{vmatrix} 0 & 0 & 1 \\ -E^{-1} G[1, 2] & -E^{-1} F[1, 1] & -E^{-1} F[1, 2] \\ -E^{-1} G[2, 2] & -E^{-1} F[2, 1] & -E^{-1} F[2, 2] \end{vmatrix} \quad (9)$$

$$B = \begin{vmatrix} 0 \\ E^{-1}H[1,1] \\ E^{-1}H[2,1] \end{vmatrix}. \quad (10)$$

To obtain an etalon model from the binom of Newton $\lambda^3 + 3a\lambda^2 + 3a^2\lambda + a^3$. $a = \frac{\tau_0}{\tau}$, where τ_0 – an etalon transient time, τ – a desired transient time. For our inverted pendulum $\tau = 0.31 \text{ sec}$. Also we need to calculate the roots of the etalon model α_n .

Now we can use Ackermann’s formula to calculate the state variable feedback matrix Richard C. Dorf et. al. (2012).

$$K = [0 \ 0 \ \dots \ 0 \ 1]P_c^{-1}q(A), \quad (11)$$

$$q(A) = A^n + \alpha_{n-1}A^{n-1} + \dots + \alpha_1A + \alpha_0I, \quad (12)$$

where P_c – controllability matrix. We get the raw state variable feedback matrix, because we don’t impose limits on the value of the control signal. For determine feedback matrix we fix first coefficient and add step for the second, in our case 0.3. Simulate system four times and take monotone trajectory according to input limits. Simulation results are presented by the figures 9,10.

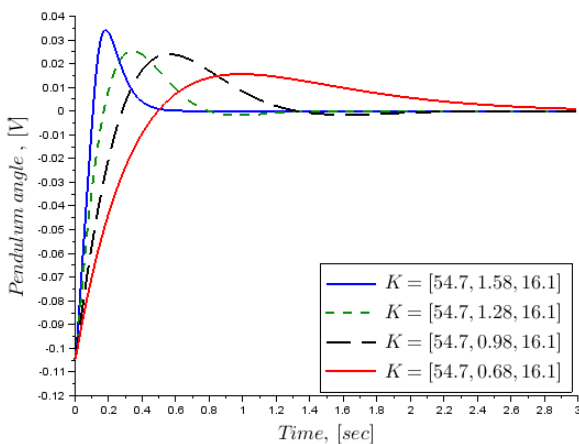
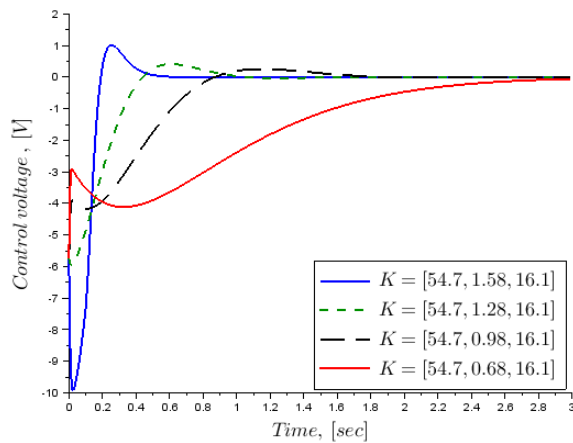


Fig. 9. Transients in the system at various of the feedback gains.

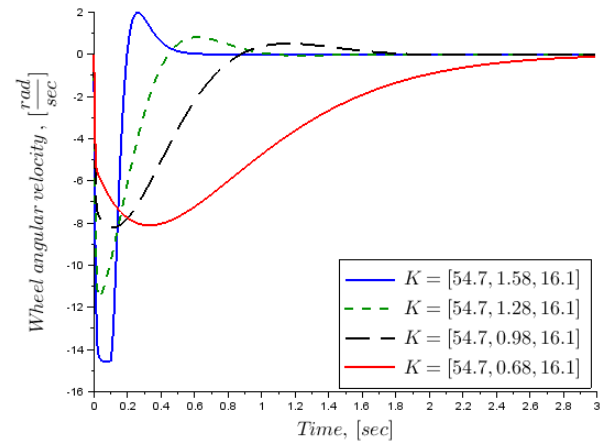
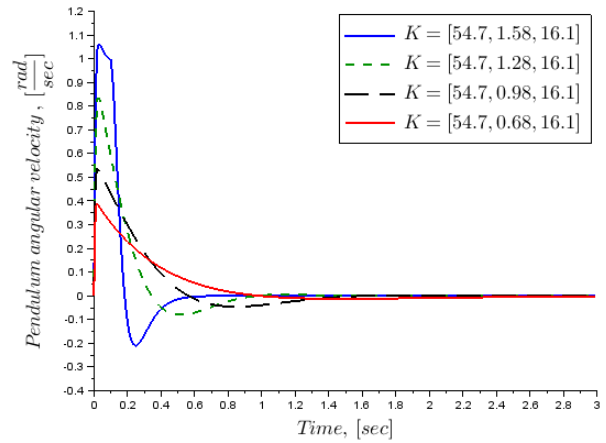


Fig. 10. Angular velocity of the pendulum and wheels.

CONCLUSION

The article presents empirical and model based control methods, implemented on the basis of Lego NXT. Control of the pendulum system is popular task of the control theory. Many different pendulum systems like SegWay, Furuta pendulum, Kapitza pendulum, inverted pendulum on a cart We made with the Lego NXT. We got good feedback from students, and some first year students have started working with our group. All projects of the Control Systems and Informatics department of the ITMO University are available on our youtube channel <http://www.youtube.com/itmo4robots>. Video lectures of this course <http://www.courses.ifmo.ru/>.

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