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Event-triggered PI control design.

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Abstract: In networked control, event-triggered controllers can provide significant reduction in message traffic as well as device energy consumption, which is potentially important in wireless networks. Motivated by this fact, this work presents a systematic design method for event-triggered PI controllers. The event generation is based on the evaluation of the degradation of a linear quadratic (LQ) performance criterion. Based on the Lyapunov theory, a formal proof of asymptotic stability of the closed-loop under the asynchronous sampling strategy is provided. A simulation example illustrates the main characteristics of the proposed approach.

Keywords: PI control, event-triggered control, networked control, hybrid systems

1. INTRODUCTION

The simple structure of a PID controller, its effectiveness and flexibility to address process control problems have contributed to its popularity in industrial applications (Åström and Hagglund [1995]). Digital communication networks and wireless technology have not only open new possibilities, for this simple and effective controller, but also new challenges in terms of adapting its basic structure and design tools to deal with issues such as unreliable communication channels and the efficient use of the energy (Villanova and Visioli [2012]). In order to make an efficient use of the communication resources, several strategies have been proposed to decrease the burden associated to the transmission of information. The standard uniform sampling has been replaced by sampling approaches based on events and self-triggering schemes (see for instance Mazo and Tabuada [2008], Wang and Lemmon [2010], Fiter et al. [2012]). From these approaches, the plant output or state is sampled only when a certain event or condition related to the system response is fulfilled. The sample is therefore used by the controller to update the plant input in a discrete-time basis. This process configures an asynchronous sampling scheme and leads to stability issues that have to be properly addressed.

Considering PID controllers, one of the first authors to realize the benefits of using event-based controller was Arzen [1999]. He illustrated, by an example, that it is possible to obtain large reductions in CPU utilization with only minor control performance degradation. Later, this approach was improved in Durand and Marchand [2009a] and in Durand and Marchand [2009b], where event-triggering strategies to compute a the new control signal only when the measurement signal sufficiently changes are proposed. Recently, there has been an increased interest in providing insight on stability and performance issues arising from the event-based setting. The stability of event-based controllers can be addressed by using Lyapunov theory. A preliminary analysis based on this theory of a simplified state feedback event-driven control scheme is described by Sandee et al. [2005] . Velasco et al. [2009] provided conditions to ensure the asymptotic stability of the closed-loop system by enforcing sampling, control algorithm computation and actuation, each time the system trajectory reaches a Lyapunov level set. In addition, they also provided conditions to ensure that the sequence of samples is infinite so that the system trajectory will tend to zero as time tends to infinity. In Durand et al. [2011] some relaxations of the Lyapunov sampling condition were proposed to significantly reduce the number of samples. On the other hand, as the plant evolves continuously, but the control signal is updated depending on discrete-time events, event-triggered control systems can be cast as hybrid or impulsive systems. From this point of view many results to asses stability have been proposed considering a hybrid system framework and Lyapunov theory (see for instance Donkers et al. [2011], Seuret and Prieur [2011], Seuret et al. [2013]). In particular, Seuret and Prieur [2011] addresses the stability problem of state feedback controllers. They proposed two new event-triggered algorithms. The first ones makes a Lyapunov-like function decrease whereas the second one is able to ensures the asymptotic stability of the closed-loop system with fewer sampling times. In Seuret et al. [2013], a strategy based on the evaluation of a linear quadratic (LQ) performance criterion is proposed to compute event-triggered saturating state feedback control laws.

These new event-based controllers have already been applied to some relevant industrial processes such as: solar plants (Beschi et al. [2013]), modular crushing plants (Airikka [2012]), room temperature (Hensel et al. [2012]), mine ventilation (Tiberi [2011]) and motion control (Wang et al. [2011]).

In this work, a simple method to design event-based PI controllers is proposed. As in Seuret et al. [2013], the event-triggering strategy is based on the use of a LQ performance criterion. Based on the evaluation of the performance degradation, the decision on when the plant output must be sampled and sent to the controller is taken. Then the controller sends to the plant an updated control signal, which remains constant until the next event. Using Lyapunov theory arguments, a formal proof of stability under the asynchronous sampling strategy is provided.

This paper is organized as follows: Section 2 states the main problem to be addressed. Section 3 describes the PI design strategy based on a LQ performance. Section 4 presents the event-triggering strategy ensuring closed loop stability. Section 5 illustrates the main results by a simple example. Finally, in section 6, some concluding remarks and future work are outlined.

Notations: For a matrix A, A' stands for the transpose of A and $He\{A\} = A + A'$. For a symmetric matrix X, $X < 0$ means that X is a negative definite matrix.

2. PROBLEM STATEMENT

Consider the following continuous-time linear plant:

$$\begin{aligned} \dot{x}_p(t) &= Ax_p(t) + Bu(t) \\ y(t) &= Cx_p(t) \end{aligned} \tag{1}$$

where $x_p \in \Re^n$, $u \in \Re$, $y \in \Re$ are the plant state, input and output, respectively, and A, B and C are constant matrices of appropriate dimensions.

Let now a continuous-time PI controller be given by the following state equation:

$$\begin{aligned} \dot{x}_c(t) &= r - y(t) \\ u(t) &= k_i x_c(t) + k_p(r - y(t)), \end{aligned} \tag{2}$$

where $r \in \Re$ is supposed to be a constant reference and $x_c \in \Re$ the controller state. k_p and k_i are the proportional and the integral gains, respectively.

In this paper we are concerned by the implementation of the control loop through a network. In particular we consider that the controller and the actuator-plant-sensor are in different nodes of the network as depicted in Figure 1. We assume also that the message transmission-reception are costly in terms of energy. This is for example the case in wireless networks where the nodes are feed by batteries.

We are particularly interested in devising an event-triggered strategy to sample and to update the control signal applied to the plant. This means that the output of the plant will be sampled in discrete instants of time and the control action will be supposed to be constant between two subsequent sampling instants t_k and t_{k+1}, $k \in \mathbb{N}$. Note however that, differently from classical digital control approaches, the sampling interval $t_{k+1} - t_k$ is not constant. The system dynamics in the interval $[t_k \quad t_{k+1})$ can be therefore described as follows:

$$\begin{aligned} \dot{x}_p(t) &= Ax_p(t) + Bu(t_k) \\ \dot{x}_c(t) &= r - y(t_k) \\ u(t_k) &= k_i x_c(t_k) + k_p(r - y(t_k)), \end{aligned} \qquad \forall t \in [t_k \quad t_{k+1}) \tag{3}$$

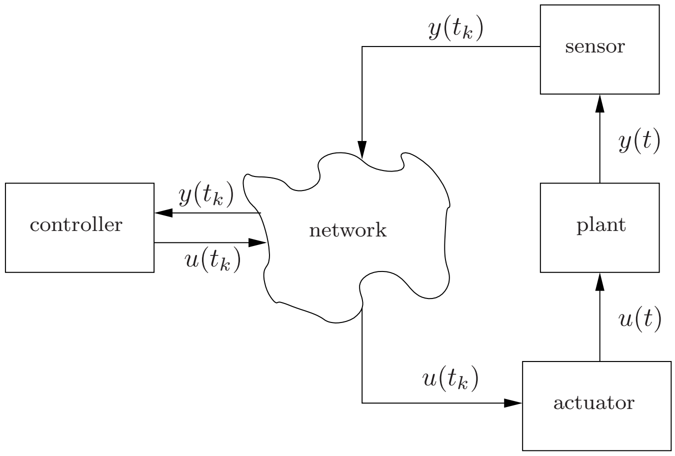

Fig. 1. Networked control loop.

Regarding system (3), we assume that the output of the plant is continuously measured (or, in practice, with a high sampling rate), but it will be sent to the controller, which is supposed to be in another node in the network, only when an event occurs. At this time, i.e. at $t = t_k$, messages between the controller and the sensor-plant node are exchanged. The sensor-plant node sends $y(t_k)$ to the controller and this one sends back the new control value $u(t_k)$ to be applied to the plant until the next event.

The decision on when to sample will be based on the performance degradation of the closed-loop system. The performance criterion we adopt is a classical linear quadratic (LQ) one. The performance degradation will be therefore measured with respect to the nominal performance that it would be achieved by a continuous-time implementation. In particular, since a Lypaunov-based approach is considered, the stability under the event-triggered sampling strategy will be implicitly guaranteed.

3. GUARANTEED COST PI SYNTHESIS

In this section we consider the synthesis of the PI controller in order to guarantee a certain LQ performance for the closed-loop system considering a continuous-time implementation.

Supposing the connection between (1) and (2) is asymptotically stable, the equilibrium point of the closed-loop system is given by:

$$\begin{aligned} \begin{bmatrix} x_{p,eq} \\ x_{c,eq} \end{bmatrix} &= -\begin{bmatrix} A - k_p BC & k_i B \\ -C & 0 \end{bmatrix}^{-1} \begin{bmatrix} k_p B \\ 1 \end{bmatrix} r \\ u_{eq} &= k_i x_{c,eq} + k_p(r - Cx_{p,eq}), \end{aligned} \tag{4}$$

Considering

$$\tilde{x}_p = x_p - x_{p,eq}, \quad \tilde{x}_c = x_c - x_{c,eq}, \quad \tilde{u} = u - u_{eq},$$

the continuous-time closed-loop system can be re-written as:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \mathbb{A}\tilde{x}(t) + \mathbb{B}\tilde{u}(t) \\ \tilde{u}(t) &= [k_p \quad k_i]\mathbb{C}\tilde{x}(t) \end{aligned} \tag{5}$$

with

$$\tilde{x} = \begin{bmatrix} \tilde{x}_p \\ \tilde{x}_c \end{bmatrix}, \mathbb{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \ \mathbb{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ \mathbb{C} = \begin{bmatrix} -C & 0 \\ 0 & 1 \end{bmatrix}$$

Consider now the following linear quadratic criterion

$$\mathcal{J}(\tilde{x},\tilde{u}) = \int_0^\infty \phi(\tilde{x},\tilde{u})dt \tag{6}$$

with

$$\phi(\tilde{x},\tilde{u}) = \tilde{x}'Q\tilde{x} + \tilde{u}'R\tilde{u}, \quad Q = Q' \geq 0, \quad R > 0 \tag{7}$$

Theorem 1. If there exist a matrix $P = P' > 0$ such that the following matrix inequality is verified

$$\begin{bmatrix} He\{P(\mathbb{A} + \mathbb{B}[k_p \;\; k_i]\mathbb{C})\} + Q & \mathbb{C}'[k_p \;\; k_i]' \\ [\,k_p \;\; k_i\,]\,\mathbb{C} & -R^{-1} \end{bmatrix} < 0 \tag{8}$$

then it follows that the closed-loop system is asymptotically stable and

$$\mathcal{J}(\tilde{x},\tilde{u}) < \tilde{x}(0)'P\tilde{x}(0)$$

Proof. Consider a candidate Lyapunov function $V_o(\tilde{x}) = \tilde{x}'P\tilde{x}$.

Applying now Schur's complement to (8) and pre and pos-multiplying the result by $\tilde{x}'$ and $\tilde{x}$ respectively, it follows that:

$$\dot{V}_0(\tilde{x}) + \tilde{x}'Q\tilde{x} + \tilde{u}'R\tilde{u} < 0 \tag{9}$$

Since $Q \geq 0$ and $R > 0$, inequality (9) implies that $\dot{V}_0(\tilde{x}) < 0$ and the asymptotic stability of the closed-loop system follows.

On the other hand, integrating (9) from 0 to ∞ it follows that:

$$\begin{aligned} &\int_0^\infty (\dot{V}_0(\tilde{x}) + \tilde{x}'Q\tilde{x} + \tilde{u}'R\tilde{u})dt = \\ &V_0(\tilde{x}(\infty)) - V_0(\tilde{x}(0)) + \mathcal{J}(\tilde{x},\tilde{u}) < 0 \end{aligned} \tag{10}$$

Hence, since the closed-loop system is asymptotically stable it follows that $V_0(\infty) = 0$ and, therefore

$$\mathcal{J}(\tilde{x},\tilde{u}) < V_0(\tilde{x}(0)) = \tilde{x}(0)'P\tilde{x}(0)$$

■

For a given PI controller, i.e. considering k_p and k_i given, note that the matrix inequality (8) is linear in the variable P, i.e. it is a linear matrix inequality (LMI). Thus, the result of Theorem 1 can be used to certify a certain guaranteed cost performance with respect to criterion (6). This can be done by solving the following optimization problem:

$$\begin{aligned} &\min_P \;\; trace(P) \\ &\text{s.t. } (8) \end{aligned} \tag{11}$$

On the other hand, the PI parameters can be tuned to minimize $J(\tilde{x},\tilde{u})$. This can be accomplished by solving (11) with k_i and k_p as decision variables. Note that in this case constraint (8) is no longer an LMI. However, the optimal solution of the problem can be approximated by considering a grid on the parameters k_p and k_i and iteratively solving (11) for each point of this grid.

4. EVENT-TRIGGERED STRATEGY

Considering variables $\tilde{x}_p$ $\tilde{x}_c$ and $\tilde{u}$, the closed-loop sample-data system (3), for $t \in [t_k \;\; t_{k+1})$, can be rewritten as follows:

$$\dot{\tilde{x}}(t) = \mathbb{A}_1\tilde{x}(t) + \mathbb{B}u_s(t) + \mathbb{E}y_s(t) \tag{12}$$

with $y_s(t) = C\tilde{x}(t_k)$, $u_s(t) = \tilde{u}(t_k)$,

$$\mathbb{A}_1 = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \; \mathbb{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \text{ and } \mathbb{E} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

Define now the vector

$$\xi(t) = [\tilde{x}(t)' \;\; u_s(t)' \;\; y_s(t)']' \tag{13}$$

and the matrix

$$\mathcal{M} = \begin{bmatrix} P\mathbb{A}_1 + \mathbb{A}_1'P + \mu^{-1}Q & P\mathbb{B} & P\mathbb{E} \\ \mathbb{B}'P & \mu^{-1}R & 0 \\ \mathbb{E}'P & 0 & 0 \end{bmatrix} \tag{14}$$

with $\mu > 1$

The event-triggered sampling strategy can therefore be described by the following algorithm:

Algorithm 1.
if $\xi'(t)\mathcal{M}\xi(t) > 0$ then sample, i.e:
$t_{k+1} = t$
$k \leftarrow k + 1$
$y_s(t) = \tilde{y}(t)$
$u_s(t) = \tilde{u}(t)$
otherwise:
$y_s(t) = \tilde{y}(t_k)$
$u_s(t) = \tilde{u}(t_k)$

■

Theorem 2. Consider system (12) with the event-triggered sampling strategy given in Algorithm 1, where $\mathcal{M}$ is defined as in (14) with matrix P verifying (8). Then the following holds:

a) $\lim_{t\to\infty} \tilde{x}(t) = 0$, i.e. the closed-loop system is asymptotically stable.
b) $J(\tilde{x}, u_s) \leq \mu\tilde{x}(0)'P\tilde{x}(0)$

Proof. Let $V(\tilde{x}) = \tilde{x}'P\tilde{x}$ be a candidate Lyapunov function and consider the following notation:

- $\dot{V}_s(\tilde{x}, u_s, y_s)$ for denoting the time derivative of $V(\tilde{x})$ along the trajectories of system (12)
- $\dot{V}(\tilde{x})$ for denoting the time derivative of $V(\tilde{x})$ along the trajectories of system (5)

From the definition of matrix $\mathcal{M}$ and the vector ξ, it follows:

$$\begin{aligned} \xi(t)'\mathcal{M}\xi(t) &= 2\tilde{x}(t)'P(\mathbb{A}_1\tilde{x}(t) + \mathbb{B}u_s(t) + \mathbb{E}y_s(t)) \\ &\quad + \mu^{-1}(\tilde{x}(t)'Q\tilde{x}(t) + u_s(t)'Ru_s(t)) \\ &= \dot{V}_s(\tilde{x}, u_s, y_s) + \mu^{-1}\phi(\tilde{x}, u_s) \end{aligned}$$

with $\phi(\tilde{x}, u_s)$ defined as in (7), with u_s replacing $\tilde{u}$.

Since $Q \geq 0$ and $R > 0$, if $\xi(t)'\mathcal{M}\xi(t) \leq 0$, $\forall t \in [t_{k-1} \;\; t_k)$ we can conclude that

$$\dot{V}_s(\tilde{x}, u_s, y_s) < 0 \quad \forall t \in [t_{k-1} \;\; t_k), \tag{15}$$

Let us consider now the sampling instant $t = t_k$. At this instant, the control and the sensor values are instantaneously updated and it follows that:

$$\begin{aligned} u_s(t_k) = \tilde{u}(t_k) = [k_p \;\; k_i]\mathbb{C}\tilde{x}(t_k) \\ \mathbb{A}_1\tilde{x}(t_k) + \mathbb{B}u_s(t_k) + \mathbb{E}y_s(t_k) = \mathbb{A}\tilde{x}(t_k) + \mathbb{B}\tilde{u}(t_k) \end{aligned}$$

and we can conclude that

$$V_s(\tilde{x}(t_k), u_s(t_k), y_s(t_k)) = V(\tilde{x}(t_k))$$

Supposed now that (8) is verified. From the proof of Theorem 1, it follows that:

$$\dot{V}_s(\tilde{x}(t_k), u_s(t_k), y_s(t_k)) + \phi(\tilde{x}(t_k), u_s(t_k)) < 0 \quad (16)$$

and thus:

$$\dot{V}_s(\tilde{x}(t_k), u_s(t_k), y_s(t_k)) = \dot{V}(\tilde{x}(t_k)) < 0 \quad (17)$$

Note now that (16) can be re-written as follows:

$$\xi(t_k)'\mathcal{M}\xi(t_k) + (1-\mu^{-1})\phi(\tilde{x}(t_k), u_s(t_k)) < 0 \quad (18)$$

Hence, since $\mu > 1$, we conclude that

$$\xi(t_k)'\mathcal{M}\xi(t_k) < 0$$

which means, by continuity, that the interval between two sampling instants is not empty. From this fact and from (15) and (17), it follows that the state converges asymptotically to the origin under the sampling strategy, i.e. item a) is proved.

Let us now prove item b). With this aim consider that $\xi(t)'\mathcal{M}\xi(t) \leq 0$ for $t \in [t_k \quad t_{k+1})$. In this case, $\forall t \in [t_k \quad t_{k+1})$ it follows that:

$$\dot{V}_s(\tilde{x}(t), u_s(t), y_s(t)) + \mu^{-1}\phi(\tilde{x}(t), u_s(t)) \leq 0$$

Integrating the previous expression on the time interval $[t_k \quad t_{k+1})$ leads to

$$\int_{t_k}^{t_{k+1}} (\dot{V}_s(\tilde{x}(t), u_s(t), y_s(t)) + \mu^{-1}\phi(\tilde{x}(t), u_s(t)))dt =$$

$$V(\tilde{x}(t_{k+1})) - V(\tilde{x}(t_k)) + \mu^{-1}\int_{t_k}^{t_{k+1}} \phi(\tilde{x}(t), u_s(t))dt \leq 0$$

Consider $t_0 = 0$ and, since $\lim_{t\to\infty}\tilde{x}(t) = 0$, there exists an integer $\bar{k} \in (0 \quad \infty)$ such that $V(x(t_{\bar{k}+1})) = 0$. Hence, we can conclude that:

$$\sum_{k=0}^{\bar{k}}(V(\tilde{x}(t_{k+1})) - V(\tilde{x}(t_k)) + \mu^{-1}\int_{t_k}^{t_{k+1}} \phi(\tilde{x}(t), u_s(t))dt) =$$

$$-V(\tilde{x}(0)) + \mu^{-1}\int_0^{\infty} \phi(\tilde{x}(t), u_s(t))dt \leq 0$$

which leads to

$$\mathcal{J}(\tilde{x}, u_s) = \int_0^{\infty} \phi(\tilde{x}(t), u_s(t))dt \leq \mu V(\tilde{x}(0)) = \mu\tilde{x}(0)'P\tilde{x}(0)$$

■

The event-triggering strategy defined in Algorithm 1 is supposed to run in the actuator-plant-sensor node, where the sensor device is assumed to have a processor (i.e. it is a smart sensor). In this case, note that both the plant and control states should be available to perform the test $\xi'\mathcal{M}\xi$. Of course, in general applications only the plant output is measured. Moreover, assuming that the controller is running in a different node and that the communication is done only when an event occurs, the current value of its state is not available in the actuator-sensor-plant node.

However, if we assume that the controller sends also the value of its state at time t_k, the controller state can be directly obtained as follows in the actuator-plant-sensor node:

$$\begin{aligned} x_c(t) &= \int_{t_k}^{t}(r - y(t_k))dt + x_c(t_k) \\ &= (t - t_k)(r - y(t_k)) + x_c(t_k) \end{aligned}$$

On the other hand, since $y(t)$ is supposed to be continuously available at the sensor node, the value of $x_p(t)$ can be recovered through a classical Luenberger observer:

$$\dot{\hat{x}}_p(t) = A\hat{x}_p(t) + Bu_s(t) + L(C\hat{x}_p(t) - y(t))$$

where $\hat{x}_p(t)$ denotes the estimate of $x_p(t)$.

Thus, both $x_c(t)$ and $x_p(t)$ can be recovered at the actuator-plant-sensor node in order to implement the event-triggered strategy.

Remark 3. The proposed framework and event-triggered strategy apply also to the case where we have a controller-actuator-plant node and a sensor node. This is the case, for instance, when the plant and the actuator are geographically separated from the local where the output measures are effectively taken by the sensor.

5. NUMERICAL EXAMPLE

In this section, an event-triggered PI controller as described in previous sections is used to control a simple first order plant. The simplicity of the plant makes possible to have some insights on why sampling is triggered at specific instants of time.

The plant is described by:

$$\dot{x}_p(t) = -x_p(t) + u(t) \quad (19)$$

$$y(t) = x_p(t) \quad (20)$$

The design parameters are the weighting matrices $Q = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ and $R = 0.15$. The controller gains have been determined by minimizing the trace of P subject to (8) leading to $k_p = 4.4747$ and $k_i = 4.4740$. For $x_p(0) = 0$, $x_c(0) = 0$ and $r = 1$ the output of the continuous plant $y(t)$ and the sampled output $y(t_k)$ considering Algorithm 1 with $\mu = 40$ is depicted in Figure 2 . Details of the event-triggered sampling of the output can be seen in Figure 3.

The discrete-time control signal, $u(t_k)$ is shown in figure 4. Again, note the effect of the event-triggered sampling.

In this experiment, a total of 12 samples were performed to drive the system to its reference. Figure 5 shows the temporal distribution of the events. At each moment, a value of 0 means no event and a value of 1 means a event. Although not regularly spaced in time, events tend to happen in bursts. Note that the sampling is more frequent when the output is changing fast and is close to the reference.

Those samples are decided based on the value of $\xi'(t)\mathcal{M}\xi(t)$, which is shown in Figure 6. Note that a sampling event and a consequent change in the control value is performed as soon as $\xi'(t)\mathcal{M}\xi(t) > 0$. Thus the value of $\xi'(t)\mathcal{M}\xi(t)$ above 0 tends to be very small. In Figure 7 it can be seen that $\xi'(t)\mathcal{M}\xi(t)$ actually becomes positive in the instants corresponding to the sampling events.

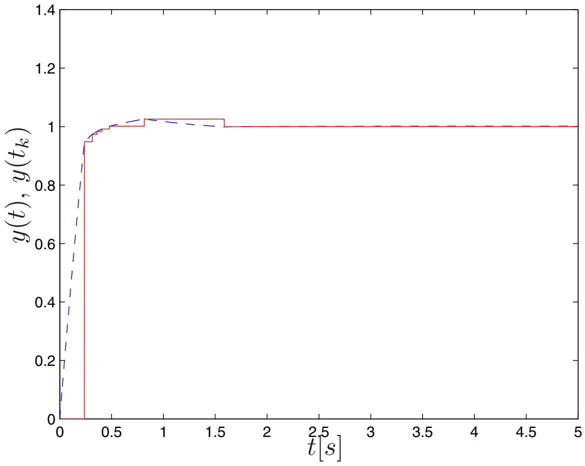

Fig. 2. Output of the continuous plant $y(t)$ (dashed) and the sampled output $y(t_k)$ (solid).

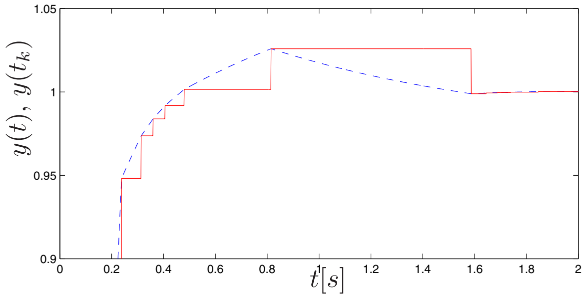

Fig. 3. Details of the event-triggered sampling. Output of the continuous plant $y(t)$ (dashed) and the sampled output $y(t_k)$ (solid).

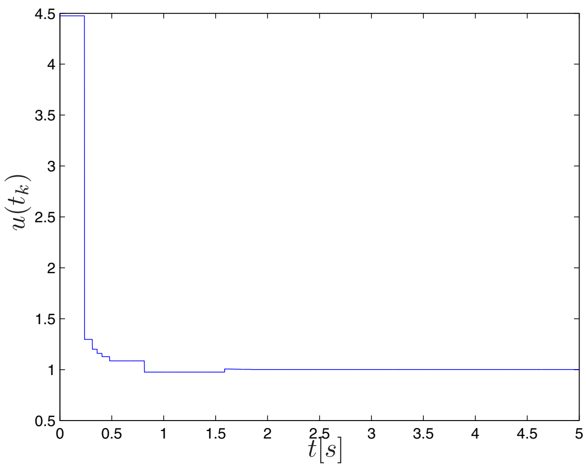

Fig. 4. Control effort.

The evolution of the Lyapunov function is shown in Figure 8. As expected, its value decreases as the time increases and its derivative is always negative.

6. CONCLUDING REMARKS

A systematic design method for event-triggered PI controller has been proposed. The method is based on a

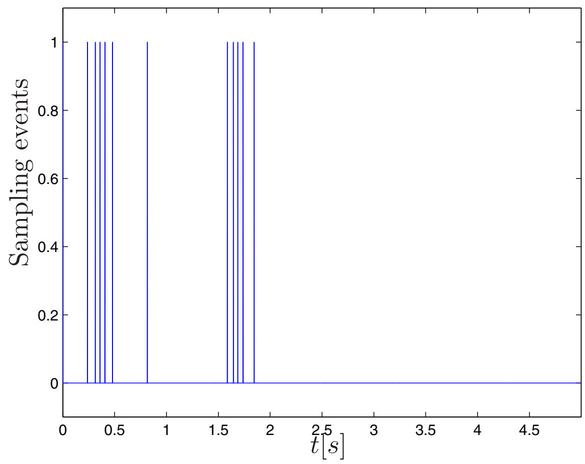

Fig. 5. Sampling events. 0 means no event, 1 means event.

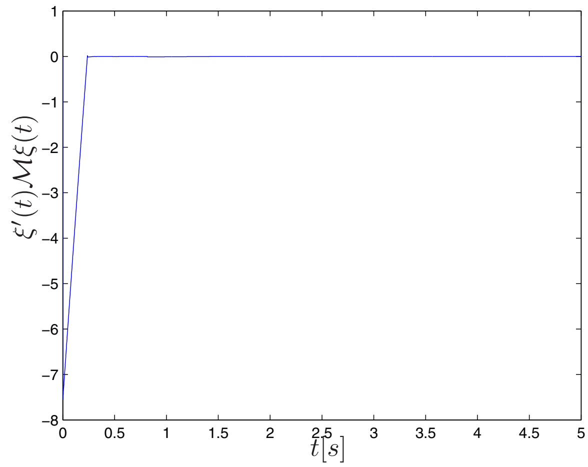

Fig. 6. Evolution of $\xi'(t)\mathcal{M}\xi(t)$.

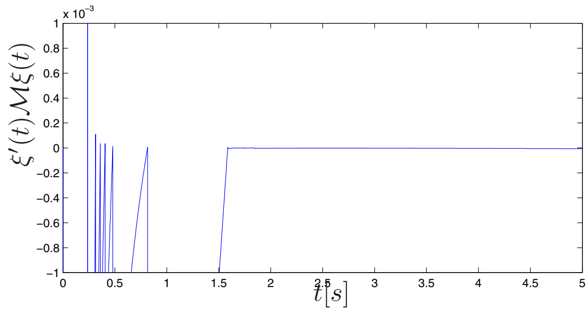

Fig. 7. Detail of the evolution of $\xi'(t)\mathcal{M}\xi(t)$.

LQ performance criterion evaluation which leads to an event-triggered strategy ensuring closed loop stability. The simulation results have shown the flexibility of the design method to cope with different design requirements. In particular, the application of the proposed methodology in networked control, mainly in wireless networks, where the battery consumption is in general critical and transmissions should be minimized, seems to be promising. In this context further developments should be carried out to consider package losses, communication delay effects and

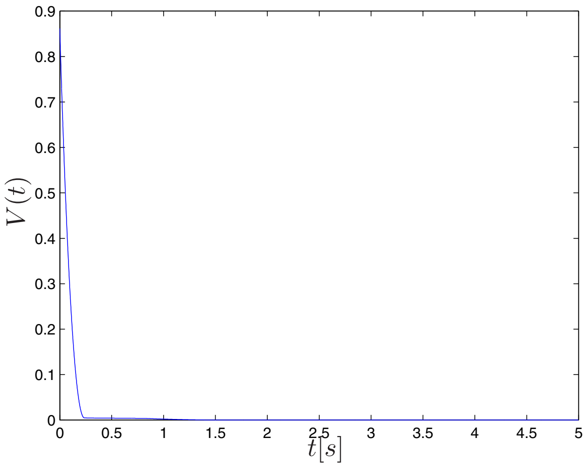

Fig. 8. Evolution of the Lyapunov function.

noisy measurements in the dynamics of the closed-loop system.

On the other hand, the extension of the approach to cope with saturation effects and incorporate an anti-windup compensation to the PI controller is an ongoing work. Also, it seems to be possible the extension of the approach to deal with plants presenting time-delays in order to consider classical models arising in process control.

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