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# Adaptive Sliding Backstepping Control of Quadrotor UAV Attitude

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**Abstract:** This paper proposes an adaptive sliding backstepping control law for quadcopter attitude control. By employing adaptive elements in the sliding mode control formulation the proposed control law avoids *a priori* knowledge of the upper bounds on the uncertainty. The controller we propose can be used for systems that are in strict feedback form with matched uncertainties. Numerical simulations show that this control method is capable of guaranteeing global asymptotic tracking of the desired attitude trajectory.

*Keywords:* adaptive control, backstepping control, sliding mode control

## 1. INTRODUCTION

The history of unmanned aerial vehicles(UAVs) goes nearly as far back as the history of manned flight. UAVs can be traced back to 1916 when Elmer Sperry and Peter Hewitt successfully demonstrated their Automatic Plane dubbed "the flying bomb"Lt. Kendra L. B. Cook (2007).Research into unmanned flight continued through out World War 1 and 2 with notable successes of this era being the German V1 and V2 "buzz bombs" which could travel at speeds of 650km/hr Lt. Kendra L. B. Cook (2007). The Vietnam war heralded the large scale use of modern UAVs in combat zones with over 3 000 operations being flown by UAVs during this war Lt. Kendra L. B. Cook (2007). In the last two decades major strides have been made within the area of UAVs as is witnessed by the huge success of Northrop Grumman's Global Hawk drone and General Atomics' Predator drone which have become the weapon of choice for the United States Defense Forces.

From the brief history that has been given it is evident that UAVs have been mostly used in defense related applications. According to Zaka Sarris (2001) by 2000 the the civil UAV market only accounted for 3% of the UAV market. However over the past decade progress in micro electro-mechanical systems(MEMS) and IC miniaturisation has led to a drop in cost of sensors making UAVs economical for civilian use. Thus over the past decade there has been a proliferation of civilian applications of UAVs such as border interdiction, search and rescue, powerline inspection e.t.c.Zaka Sarris (2001). In a majority of these civilian applications rotary UAVs especially quadrotor UAVs(qUAVs) are used. Civilian UAV applications mostly take place in constrained environments(e.g indoors) and hence require UAV platforms that are highly manueverable. qUAVs possess the required manuevarability and thus are perfectly suited for civilian applications. Additionally qUAVs have a very high thrust to weight ratio which translates to lighter platforms, the absence of moving parts in the rotors (i.e.cyclic pitch controls) make the qUAV easier to maintain and control in comparison to other rotary UAVs.

It is known that the state of the qUAV evolves in the Special Euclidean 3 space($SE(3) = \mathbb{R}^3 \times SO(3)$). Thus the qUAV can be split into a translational dynamics subsystems with configuration space $\mathbb{R}^3$ and a rotational subsystem with configuration space $SO(3)$. The focus of the work presented in this paper is the control of the rotational subsystem so as to achieve some desired attitude. It should also be noted that in as much as the focus of the present work is UAV attitude control the theory that is developed in this work can also be applied to other rigid body attitude control problems such as satellites.

Backstepping control is a recursive Lyapunov based control technique for systems in strict feedback form. Backstepping control came about from the concerted efforts of a number on researchers in the 1990s. Cascade integrator backstepping appeared in the work of Saberi, Kokotovic and Sussman A. Saberi et al. (1989) which was developed further by Kanellakopoulos et al I. Kanellakopoulos et al. (1992). Passivity interpretations of the backstepping method were given by Lozano, Brogliato and LandauRogelio Lozano et al. (1992).The backstepping method was extended to cover system with uncertainties for the matched case inM.J. Corless and G. Leitmann (1981). Adaptive control methods and backstepping methods were employed in Kanellakopoulos, Kokotovic and MorseI. Kanellakopoulos et al. (1991) to devise the adaptive backstepping techniques. The important achievement of this techniques was that it could handle the case of extended matching, however this technique had the disadvantage of over parameterization i.e required multiple estimates of the same parameter. The introduction of tuning functions by Kristic, Kanellakopoulo and Kokotovic M.Krstic et al. (1992) managed to remove the over parameterization. Sliding mode control is a powerful technique that ensures robust system performance however it has the drawback of requiring *a priori* knowledge of the uncertainties. Koshkouei and ZinoberA.J. Koshkouei and A.S.I. Zinober (2000) devised a method to combine the adaptive backstepping technique with tuning functions and sliding mode control, this sliding backstepping technique ensures that the tracking error moves along the sliding hyperplane.

Some of the theoretical advances outlined in the preceding paragraph have been investigated with regards to qUAV control. Madani and BenallegueT. Madani and A. Benallegue (2006) presented a backstepping based controller for qUAV trajectory tracking. In S. Bouabdallah and R. Seigwart (2007) the authors improved on the backstepping controller by adding an integral term into the controls to improve steady state errors, A.A Mian and W. Daobo (2008) further employed a full PID backstepping method for qUAV control. Frazzoli et alE. Frazzoli et al. (2000) used adaptive backstepping control to design a trajectory tracking controller, their results showed that the controller could even perform aggressive maneuvers(e.g paths were the UAV is initially upside down).

In this paper we are going to present an adaptive sliding backstepping scheme for qUAV attitude tracking. The next section outlines the mathematical model for the qUAV attitude. In section 3 the main result of this paper is developed, this section will present the general adaptive sliding backstepping scheme. Section 4 presents the simulation results of the adaptive sliding backstepping attitude controller and finally conclusions and final remarks are given in section 5.

## 2. MODELING

In describing the attitude of the quadrotor UAV we shall use the Z-Y-X Euler angle notation, where the Euler angle vector $\Theta = [\phi,\ \theta,\ \psi]$ denotes the roll, pitch and yaw respectively. The angular velocity is $\omega^B = [p,\ q,\ r]$ where the superscript B denotes that the angular velocity is a body frame vector.

### *2.1 Kinematics*

We state without proof the kinematic equations of the rigid body however the interested reader can consult I.Raptis and K. Valavanis (2011) for a detailed derivation.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & sin\phi tan\theta & cos\phi tan\theta \\ 0 & cos\phi & -sin\phi \\ 0 & \frac{sin\phi}{cos\theta} & \frac{cos\phi}{cos\theta} \end{bmatrix}}_{\mathbf{\Psi}(\mathbf{\Theta})} \begin{bmatrix} \mathrm{p} \\ \mathrm{q} \\ \mathrm{r} \end{bmatrix} \tag{1}$$

### *2.2 Dynamics*

For rotational motion Newton's $2^{nd}$ law of motion states that the rate of change of angular momentum is equal to the net torque acting on the body. This can be expressed as :

$$\frac{d\mathbf{H}^B}{dt_I} = \tau \tag{2}$$

The angular momentum $\mathbf{H}^B = \mathbf{I}\omega^B$. With $\mathbf{I}$ being the $3\times3$ inertia matrix gien by :

$$\mathbf{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \tag{3}$$

If we assume the quadrotor to be perfectly symmetrical about all of its three axis we now have $I_{xy} = I_{xz} = I_{yz} = 0$ and the inertia matrix becomes $\mathbf{I} = \mathrm{diag}(I_{xx}\ I_{yy}\ I_{zz})$. In equation 2 we are differentiating a body frame vector in the inertia frame. Using the equation of Coriolis we have Grant R. Fowles and George L. Cassiday (1999):

$$\frac{d\mathbf{H}^B}{dt^I} = \frac{d\mathbf{H}^B}{dt^B} + \omega_{b/i} \times \mathbf{H}^B \tag{4}$$

Applying this to equation 2 we have for the rotational dynamics:

$$\mathbf{I}\dot{\omega}^B = -\omega \times \left(\mathbf{I}\omega^B\right) + \tau^B \tag{5}$$

where $\tau^B = \left[\tau_\phi^B\ \tau_\theta^B\ \tau_\psi^B\right]^T$ is the torque acting on the quadrotor expressed in the vehicle frame. In expanded form the rotational dynamics are given by the equations:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{J_{yy}-J_{zz}}{J_{xx}} qr \\ \frac{J_{zz}-J_{xx}}{J_{yy}} pr \\ \frac{J_{xx}-J_{yy}}{J_{zz}} pq \end{bmatrix} + \begin{bmatrix} \frac{1}{J_{xx}}\tau_\phi \\ \frac{1}{J_{yy}}\tau_\theta \\ \frac{1}{J_{zz}}\tau_\psi \end{bmatrix} \tag{6}$$

### *2.3 Full Attitude Dynamics*

The full attitude dynamics are given by equations 1 and 5 which are repeated here in a more compact form.

$$\dot{\mathbf{\Theta}} = \mathbf{\Psi}\left(\mathbf{\Theta}\right)\omega^B \tag{7}$$

$$\mathbf{I}\dot{\omega}^B = -\omega \times (\mathbf{I}\omega) + \tau \tag{8}$$

From this one can clearly see that the attitude dynamics are in strict feedback form which makes them amenable to backstepping control.

## 3. CONTROLLER DESIGN

In outlining the proposed adaptive sliding backstepping method we shall make use of the general system given by:

$$\dot{x}_1 = x_2 \tag{9}$$

$$\dot{x}_2 = f_1\left(x_1, x_2\right) + g_1\left(x_1, x_2\right)x_3 \tag{10}$$

$$\dot{x}_3 = f_2\left(x_1, x_2, x_3\right) + g_2\left(x_1, x_2, x_3\right)u \tag{11}$$

where $g_1(0,0) \neq 0$, $g_2(0,0,0) \neq 0$ and $f_2(x_1,x_2,x_3)$ and $g_2(x_1,x_2,x_3)$ are unknown but bounded functions. The control task is to ensure that $x_1 = 0$ is asymptotically stable.

### *3.1 Backstepping Control*

Let us now apply the backstepping procedure to our general system. If we take $x_2$ as a pseudo-control for the first equation of our system and if there exists a positive definite unbounded function $V_1(x_1)$ then we can find a function $\pi_1(x_1)$ such that the following inequality is satisfied.

$$\frac{\partial V_1}{\partial x_1}\pi_1\left(x_1\right) \leq -W\left(x_1\right) \tag{12}$$

where $W(x_1)$ is a positive definite function. This implies that if $x_2$ was an actual control then $x_2 = \pi(x_1)$ would make equation 9 asymptotically stable. However as this is not the case we can define an error variable $z_1 = x_2 - \pi_1(x_1)$ such that we have the following dynamics:

$$\dot{x}_1 = \pi_1(x_1) + z_1 \tag{13}$$

$$\dot{z}_1 = f_1 - z_1 \frac{\pi_1}{\partial x_1} - \pi \frac{\partial \pi_1}{\partial x_1} + g_1 x_3 \tag{14}$$

If we construct a new augmented Lyapunov function $V_2(x_1, z_1)$:

$$V_2(x_1, z_1) = V_1(x_1) + \frac{z_1^2}{2} \tag{15}$$

Now if we take $x_3$ as the pseudo-control for the new dynamics given by equations (13) and (14) we choose a function $\pi_2(x_1, z_1)$ such that $x_3 = \pi_2(x_1, z_1)$ will make the time derivative of the augmented Lyapunov function(15) negative definite. Such a function $\pi_2(x_1, z_1)$ is given by:

$$\pi_2(x_1, z_1) = \frac{1}{g_1(x_1, z_1)} \left[ -f_1 + z_1 \frac{\partial \pi_1}{\partial x_1} + \pi_1 \frac{\partial \pi_1}{\partial x_1} - \frac{\partial V_1}{\partial x_1} - \lambda_2 z_1 \right] \tag{16}$$

Again we know that $x_3$ is not an actually control so we define the error variable $z_2 = x_3 - \pi_2(x_1, z_1)$ such that now the whole system can be transformed from the $(x_1, x_2, x_3)$ space to the $(x_1, z_1, z_2)$ space where the transformed system dynamics are:

$$\dot{x_1} = \pi_1(x_1) + z_1 \tag{17}$$

$$\dot{z_1} = -\lambda_1 z_1 + z_2 \tag{18}$$

$$\dot{z_2} = -(z_1 + \pi_1) \frac{\partial \pi_2}{\partial x_1} + \left( \lambda_1 z_1 + \frac{\partial V_1}{\partial x_1} - z_2 \right) \frac{\partial \pi_2}{\partial z_1} + f_2(x_1, x_2, x_3) + g_2(x_1, x_2, x_3) u \tag{19}$$

### *3.2 Sliding Backstepping*

Up until now the procedure we have followed has been no different from the usual backstepping but now consider the $z_2$ equation in which $f_2(x_1, x_2, x_3)$ and $g_2(x_1, x_2.x_3)$ are unknown. If we can make $z_2 = 0$ that would mean that $x_3 = \pi_2(x_1, z_1)$ which has been shown in the preceding section makes the $(x_1, z_1)$ dynamics asymptotically stable. Now we can apply sliding mode techniques to ensure that we arrive at the $z_2 = 0$ manifold within a finite time and stay there. The control task now becomes finding a control($u$) such that the following condition is met:

$$\frac{1}{2} \frac{dz_2^2}{dt} \leq -\eta |z_2| \tag{20}$$

This is the sliding mode condition. Before proceeding we state an important assumption.

*Assumption 1.* $g_2(x_1, x_2, x_3)$ can be expressed as $g_2(x_1, x_2, x_3) = g_{20}(x_1, x_2, x_3) + \hat{g}_2(x_1, x_2, x_3)$ where $g_{20}(x_1, x_2, x_3)$ is the nominal part and $\hat{g}_2(x_1, x_2, x_3)$ is the uncertain part.

Using the sliding mode technique we divide the control $u$ into an equivalent control($u_{eq}$) and a switching control ($u_{sw}$). The equivalent control is the control that ensures that for the nominal $z_2$ dynamics $\dot{z}_2$ is always zero. Now the nominal $z_2$ dynamics are given by :

$$\dot{z}_2 = -(z_1 + \pi) \frac{\partial \pi_2}{\partial x_1} + \left( \lambda_1 z_1 + \frac{\partial V_1}{\partial x_1} - z_2 \right) \frac{\partial \pi_2}{\partial z_1} + g_{20}(x_1, x_2, x_3) u_{eq} \tag{21}$$

Thus the equivalent control($u_{eq}$) is :

$$u_{eq} = \frac{1}{g_{20}(x_1, x_2, x_3)} \left[ (z_1 + \pi_1) \frac{\partial \pi_2}{\partial x_1} - \left( \lambda_1 z_1 + \frac{\partial V_1}{\partial x_1} - z_1 \right) \right] \tag{22}$$

Before we move on to the design of the switching controller let us state another important assumption;

*Assumption 1.* There exists a function $\beta(x_1, z_1, z_2)$ such that the following inequality is satisfied:

$$\beta(x_1, z_1, z_2) > \left| \frac{f_2 g_2 - \hat{g}_2 u_{eq}}{g_{20} g_2} \right| + \epsilon \tag{23}$$

Now if we reconsider the sliding condition:

$$\frac{1}{2} \frac{dz_2^2}{dt} = z_2 \left[ -(z_1 + \pi) \frac{\partial \pi_2}{\partial x_1} + \left( \lambda_1 z_1 + \frac{\partial V_1}{\partial x_1} - z_2 \right) \frac{\partial \pi_2}{\partial z_1} + f_2(x_1, x_2, x_3) + g_2(x_1, x_2, x_3)(u_{eq} + u_{sw}) \right] \tag{24}$$

After substituting the expression of $u_{eq}$ into equation 24 we get

$$\frac{1}{2} \frac{dz_2^2}{dt} = z_2 \left[ -\frac{\hat{g}_2 u_{eq}}{g_{20}} + f_2 + g_2 u_{sw} \right] \tag{25}$$

If we choose $u_{sw}$ as:

$$u_{sw} = -\beta(x_1, z_1, z_2)\, sign(z_2) \tag{26}$$

Thus we now have:

$$\frac{1}{2} \frac{dz_2^2}{dt} \leq -\epsilon |z_2|$$

which satisfies the sliding condition. This means if we start off the $z_2 = 0$ manifold we will reach this manifold after some finite time. The full sliding backstepping control is:

$$u = \frac{1}{g(x_1, x_2, x_3)} \left[ (z_1 + \pi_1) \frac{\partial \pi_2}{\partial x_1} - \left( \lambda 1 z_1 + \frac{\partial V_1}{\partial x_1} - z_1 \right) \right] - \beta(x_1, z_1, z_2)\, sign(z_2) \tag{27}$$

### *3.3 Adaptive Control*

The control described by equation (26) requires the knowledge of the bounds of the functions $f_2(x_1, x_2, x_3)$ and $g_2(x_1, x_2, x_3)$. We are thus going to modify this control structure making it adaptive such that the need to know the bounds of $f_2(x_1, x_2, x_3)$ and $g_2(x_1, x_2, x_3)$ is removed. Now if we assume that there exists some positive constant $K_d$ such that $\beta(x_1, z_1, z_2) < K_d$ for all time, then we can define $\hat{K}_d$ as the estimate for this constant and the estimation error $\tilde{K}_d = K_d - \hat{K}_d$.

Recall that in the previous section we had chosen our control as $u = u_{eq} + u_{sw}$,for the switching control ($u_{sw}$) let us replace $\beta(x_1, z_1, z_2)$ with the estimate $\hat{K}_d$ this gives the new switching control as :

$$u_{sw} = -\hat{K}_d sign(z_2) \tag{28}$$

Consider the candidate Lyapunov function given by

$$V = \frac{1}{2} z_2^2 + \frac{\tilde{K}_d^2}{2\gamma} \tag{29}$$

where $\gamma$ is a positive constant. The time derivative of the candidate Lyapunov function becomes:

$$\dot{V} = z_2 \left[ -(z_1+\pi)\frac{\partial \pi_2}{\partial x_1} + \left(\lambda_1 z_1 + \frac{\partial V_1}{\partial x_1} - z_2\right)\frac{\partial \pi_2}{\partial z_1} + f_2(x_1,x_2,x_3) + g_2(x_1,x_2,x_3)(u_{eq}+u_{sw}) \right] + \frac{\tilde{K}_d \dot{\hat{K}}_d}{\gamma} \quad (30)$$

Substituting for $u_{eq}$ and $u_{sw}$ we are left with:

$$\dot{V} = z_2\left[f_2(x_1,x_2,x_3) - \hat{K}_d sign(z_2)\right] + \frac{\tilde{K}_d \dot{\hat{K}}_d}{\gamma} \quad (31)$$

Noting that $\hat{K}_d = K_d - \tilde{K}_d$ we can rewrite the expression for $\dot{V}$ as:

$$\begin{aligned}\dot{V} &= z_2\left[f_2(x_1,x_2,x_3) - \left(K_d - \tilde{K}_d\right) sign(z_2)\right] + \frac{\tilde{K}_d \dot{\hat{K}}_d}{\gamma} \\ &= z_2\left[f_2(x_1,x_2,x-3) - K_d sign(z_2)\right] \\ &+ \tilde{K}_d\left(|z_2| + \frac{\dot{\hat{K}}_d}{\gamma}\right) \qquad (32)\end{aligned}$$

If we choose the adaptation law given by :

$$\dot{\hat{K}}_d = \gamma|z_2| \quad (33)$$

The time derivative of the Lyapunov function will be negative definite and thus guaranteeing asymptotic convergence to the sliding manifold $z_2 = 0$. From the adaptation law we see that the estimated gain $\hat{K}_d$ is always increasing and the rate of increase is proportional to the "distance" from the sliding surface. The presented adaptation law tends to overestimate the gain since the estimate does not decrease even if $z_2 = 0$.

### *3.4 Attitude Controller*

Having developed an adaptive sliding backstepping controller for a general system we can now use this result to formulate a controller for the quadcopter attitude. For brevity sake we shall develop a controller for only the pitch dynamics however this can be easily adapted to the roll and yaw dynamics as they do not differ much from the pitch dynamics.

We desire the pitch($\phi$) to track a time varying reference signal $\phi_d$, thus we define the tracking error $e_\phi = \phi - \phi_d$ and the integral of the tracking error $\chi = \int e_\phi dt$. Thus the error dynamics are describe by the differential equations:

$$\dot{\chi} = e_\phi \quad (34)$$

$$\dot{e}_\phi = -\dot{\phi}_d + (sin\phi tan\theta)\, q + (cos\phi tan\theta)\, r + p \quad (35)$$

$$\dot{p} = \frac{J_{yy}-J_{zz}}{J_{xx}} qr + \frac{1}{J_{xx}}\tau_\phi \quad (36)$$

From equations 33-35 we can see that the pitch dynamics are similar in form to the general $3^{rd}$ order system presented in the previous section, the following correspondences should be evident between the two systems.

$$\begin{array}{ll} x_1 = \chi & f_1 = -\dot{\phi}_d + (sin\phi tan\theta)\, q + (cos\phi tan\theta)\, r \\ x_2 = e_\phi & f_2 = \frac{J_{yy}-J_{zz}}{J_{xx}} qr \\ x_3 = p & g_1 = 1 \\ g_2 = \frac{1}{J_{xx}} & \end{array}$$

If we choose the Lyapunov function $V_1 = \frac{1}{2}\chi^2$ following the backstepping procedure as outlined in the previous section gives the following pseudo-controls and the respective "tracking" errors.

$$\begin{aligned}\pi_1 &= -\lambda_1\chi \\ z_1 &= e_\phi + \lambda_1\chi \\ \pi_2 &= \dot{\phi}_d - (sin\phi tan\theta)\, q - (cos\phi tan\theta)\, r \\ &\quad - (\lambda_1+\lambda_1)\, e_\phi - (\lambda_1\lambda_2+1)\,\chi \\ z_2 &= p - \dot{\phi}_d + (sin\phi tan\theta)\, q + (cos\phi tan\theta)\, r \\ &\quad + (\lambda_1+\lambda_2)\, e_\phi + (\lambda_1\lambda_2+1)\,\chi\end{aligned}$$

Thus the control torque in the $\phi$ direction is given by:

$$\tau_\phi = (z_1+\pi_1)\frac{\partial \pi_2}{\partial \chi} - \left(\lambda_1 z_1 + \frac{\partial V_1}{\partial \chi} - z_1\right) - \hat{K}_d sign(z_2) \quad (37)$$

$$\dot{\hat{K}}_d = \gamma|z_2| \quad (38)$$

## 4. RESULTS

The controller designed in the previous section was simulated in MATLAB/SIMULINK environment to see its performance. The first simulation results show the unit step response of the closed loop system. It should be noted that in implementing the switching function instead of using the signum function we used the hyperbolic tangent function so as to eliminate the chattering caused by the signum function.

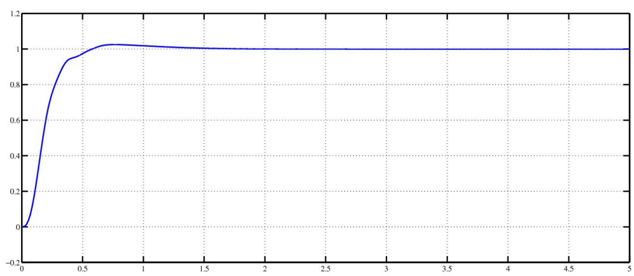


Fig. 1. $\phi$ angle in radians

Figure 1-3 show that the angles have a settling time of about 0.5 second with very little overshoot however this performance is achieved at the cost of large controls. As was stated earlier the adaptation law tends to overestimate the sliding mode gain, this characteristic of this kind of adaptive sliding mode control is also highlighted in F. Plestan et al. (2010). The next set of simulation results show the system performance when the reference signal for all three signals is a sinusoid of amplitude 1.

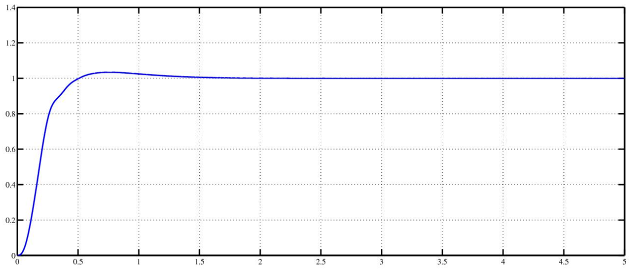

Fig. 2. $\theta$ angle in radians

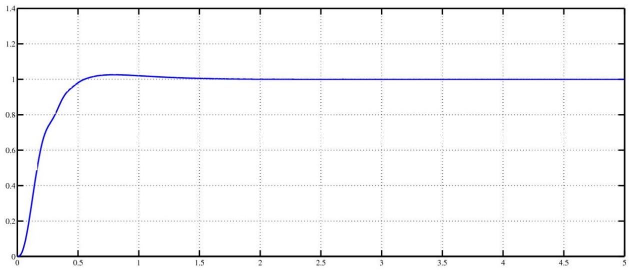

Fig. 3. $\psi$ angle in radians

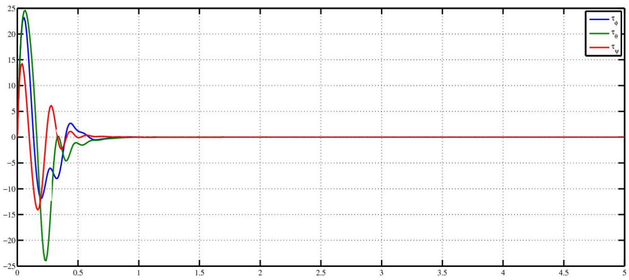

Fig. 4. Control torques

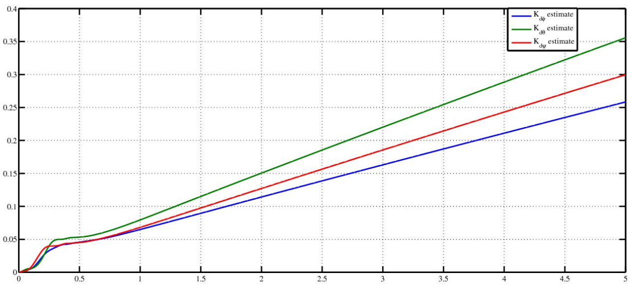

Fig. 5. Sliding gain estimates (blue = $\hat{K}_{d\phi}$, green = $\hat{K}_{d\theta}$, red = $\hat{K}_{d\psi}$)

## 5. CONCLUSIONS

We have presented an adaptive sliding backstepping control scheme for attitude tracking for quadrotor UAV. The sliding mode aspect of the controller ensures that the controller is robust against uncertainties however in conventional sliding mode control there is need to know the bound of the uncertainty which is difficult to determine in real life. As such to try to alleviate this problem we

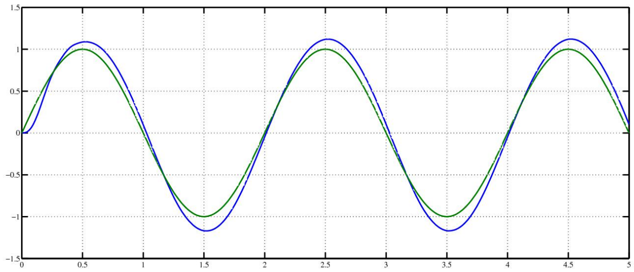

Fig. 6. $\phi$ angle in radians( green = reference signal, blue = actual angle)

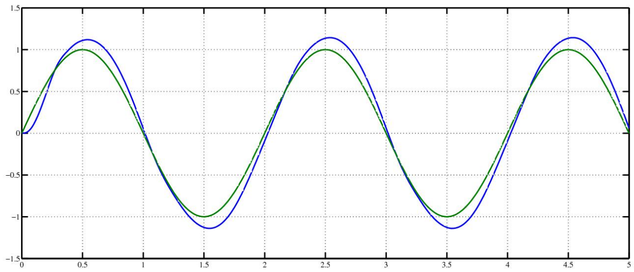

Fig. 7. $\theta$ angle in radians( green = reference signal, blue = actual angle)

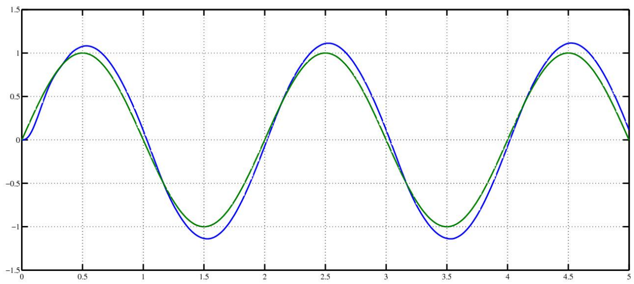

Fig. 8. $\psi$ angle in radians( green = reference signal, blue = actual angle)

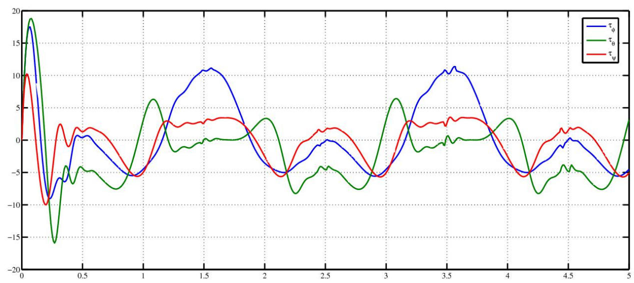

Fig. 9. Control torques

couple the sliding backstepping controller with an adaptive estimator for the sliding gain which removes the need to know the upper bounds of the uncertainties. The presented methodology has the disadvantage of overestimating the sliding gain which results in unnecessarily large controls. Simulations of the adaptive sliding backstepping controller

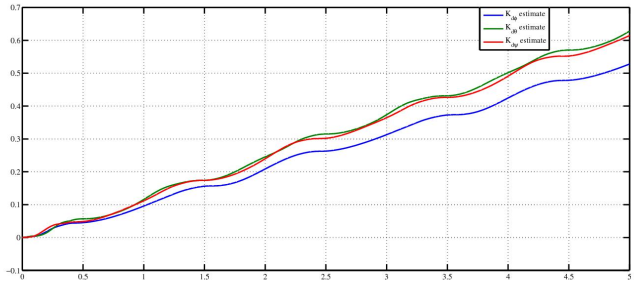


Fig. 10. Sliding gain estimates (blue = $\hat{K}_{d\phi}$, green = $\hat{K}_{d\theta}$, red = $\hat{K}_{d\psi}$)

showed that the controller is able to track constant and time varying signals almost perfectly.

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