Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Finite-time Stability Analysis of Switched Linear Singular Systems ⋆

Biao Xia *, **Jie Lian** *, **Cheng Shao** * and **Peng Shi** **

* *Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian, 116024, China (e-mails: xb21209128@mail.dlut.edu.cn, jielian@dlut.edu.cn, cshao@dlut.edu.cn)*
** *School of Electrical and Electronic Engineering, The University of Adelaide, SA 5005, Australia; and College of Engineering and Science, Victoria University, Melbourne VIC 8001, Australia (e-mail: peng.shi@adelaide.edu.au)*

Abstract: This paper investigates the problem of finite-time stability for switched singular systems with state jumps. An effective kind of finite-time stability strategies is proposed under the average dwell time switchings. Based on the multiple Lyapunov functions method, sufficient conditions on switched singular systems to be finite-time stable are presented. A numerical example is given to illustrate the effectiveness of the results obtained.

Keywords: switched singular systems, state jumps, finite-time stability

1. INTRODUCTION

Switched systems are a special subclass of hybrid systems, for which among a family of subsystems, the active subsystem is selected by a switching law at each time instant. In practical applications, such as traffic control (Antsaklis [2000]), network control (Yu et al. [2008]) and multi-agent consensus (Cheng et al. [2007]), etc, switched systems can be used to serve as models to get better control performance. Therefore, switched systems have been studied extensively in the past few decades.

Regarding the stability of switched systems in most of the literatures (Liberzon et al. [1999], Lin et al. [2009], Lu et al. [2009] and Lien et al. [2012], etc), Lyapunov stability theory and its variations or generalizations play a dominant role over an infinite time interval. However, in some practical applications, the system dynamics is mainly considered over a fixed finite time interval, where large values of the state are not acceptable. For instance, in a prescribed time interval, a space vehicle is expected to be controlled in a specified orbit in order to complete a set of experiments; or the state variables (such as temperature, pressure or some other parameters) need be kept within specified bounds in a chemical process (Du et al. [2009]). In these cases, for the purpose of checking that these unacceptable state values are not attained, finite-time stability (FTS) could be used. More specifically, a system is finite-time stable if its state does not exceed prescribed bounds over fixed time interval under bounded initial conditions; hence FTS and Lyapunov asymptotic stability are independent concepts: a system could be finite-time stable but not Lyapunov asymptoticlly stable, and vice versa (Amato et al. [2001]).

The concept of FTS is traced back to the 1960s, when it was introduced in control literature (Dorato [1961]). Since then, there have been many results with regard to finite time stability, especially in recent years. In Natasa et al. [1998], Amato et al. [1999], Amato et al. [2005] and Amato et al. [2006], some criteria or sufficient conditions for finite-time stability and stabilization of continuous-time or discrete-time systems have been obtained. Meanwhile, some results of FTS for different systems are presented in Haimo [1986], Bhat et al. [2000], Orlov [2006], Li et al. [2007] and Lin et al. [2007], but FTS which consists of Lyapunov stability and finite-time convergence is different from that in Natasa et al. [1998], Amato et al. [1999], Amato et al. [2005] and Amato et al. [2006]. In addition, there are some results about FTS of switched systems with normal subsystems (Du et al. [2009], Luan et al. [2011], Xiang et al. [2012], Lin et al. [2013] and Zhao et al. [2013]).

On the other hand, the singular system models are natural representations of dynamic systems and describe a larger class of systems than the normal linear system model. There have been a lot of papers reported on stability analysis and synthesis of singular systems in Dai [1989], Duan [2010] and Xu et al. [2006], and references therein. It should be noticed that the singular system may exhibit finite instantaneous jumps for inconsistent initial conditions in Liu et al. [1995]. Hence, there are some limited results dealing with switched singular systems. At switching instants under arbitrary switching, the states of switched singular systems cannot always keep consistent with the next activated subsystem. As presented in Liberzon et al. [2009] and Trenn [2009], no matter whether all subsystems are regular and impulse-free, it is necessary to allow induced jumps in the solutions for switched singular systems.

⋆ This work is in part supported by the National Natural Science Foundation of China (61104114,61201035,61374070), China Postdoctoral Science Special Foundation (2012T50254), Fundamental Research Funds for the Central Universities (DUT14QY30,DUT14QY31), the Australian Research Council (DP140102180), the National Key Basic Research Program, China (2011CB710706, 2012CB215202), and the 111 Project (B12018).

This is one of the major differences between switched singular systems and switched normal systems. At present, based on the above results, the stability analysis problem for switched singular systems with average dwell time and state-dependent switching have been investigated in Zhou et al. [2013] and Raouf et al. [2010], respectively.

It should be noted that some results have been provided on the FTS of systems with state jumps, such as the systems with impulsive effects or jumps, singular systems with impulsive effects in Feng et al. [2005], Liu et al. [2008], Li et al. [2013], Liu [2013], Zhao et al. [2008] and Amato et al. [2009]. However, to the best of our knowledge, there are few results on the FTS problem of switched singular systems with state jumps at switching instants. Hence, in this paper, in view of approaches to FTS problem of switched systems in Du et al. [2009], we design an effective kind of FTS strategies for switched singular systems under average dwell time switching case. A numerical examples is given to demonstrate the applicability and validity of the obtained theoretical results.

The rest of this paper is organized as follows. Some notations and problem formulations are described in Section 2. Section 3 provides the main results of two FTS strategies under average dwell time switching case. Finally, A numerical examples is presented in Section 4. Concluding remarks are drawn in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

Throughout the paper, the symbol $\mathbb{N}(\mathbb{N}^+)$ denotes the set of all non-negative (positive) integers, $\mathbb{R}$ denotes the field of real numbers, $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the space of $n \times m$ matrices with real entries. Let $P > 0(\geqslant, <, \leqslant 0)$ denote a symmetric positive definite (positive-semidefinite, negative definite, negative-semidefinite) matrix P. For any symmetric matrix P, $\lambda_M(P)$ and $\lambda_m(P)$ denote the maximum and minimum eigenvalues of matrix P , respectively. For a piecewise-continuous function $f : \mathbb{R} \to \mathbb{R}$, the left-sided evaluation $\lim_{\varepsilon\searrow 0} f(t-\varepsilon)$ at $t \in \mathbb{R}$ is denoted by $f(t^-)$ and the right-sided evaluation $\lim_{\varepsilon\searrow 0} f(t+\varepsilon)$ at $t \in \mathbb{R}$ is denoted by $f(t^+)$. The symbols rank$(\cdot)$ stands for the rank a matrix. The identity matrix of order n is denoted as I_n (or, simply, I if no confusion arises) and $\|\cdot\|$ denotes the Euclidian vector norm.

Consider the systems as follows

$$E_{\sigma(t)}\dot{x}(t) = A_{\sigma(t)}x(t), \quad x(0) = x_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, and $x_0 \in \mathbb{R}^n$ is the initial condition. $\sigma(t) : [0,\infty) \to M=\{1, 2, ..., m\}$ is the switching signal which is a piecewise constant and right continuous function, and m is the number of subsystems. For every $i \in M$, E_i and A_i are constant matrices, and it is assumed that rank$(E_i) = r_i \leqslant n$, where r_i is mode dependent. For simplicity, we use $(E_i,\, A_i)$ to denote the ith subsystem.

Corresponding to the switching signal $\sigma(t)$, we have the following switching sequence
$\{x_0 : (i_0, t_0), (i_1, t_1), \cdots (i_k, t_k), \cdots | i_k \in M, k{=}0, 1, 2, \cdots\}$, which means that i_kth subsystem is activated when $t \in [t_k, t_{k+1})$.

Definition 1 (Luan et al. [2011]). For every $i \in M$, the singular system $(E_i,\, A_i)$ is said to be:

(1) regular if the characteristic polynomial $\det(sE_i - A_i)$ is not identically zero;

(2) free-impulse if $deg(\det(sE_i - A_i)) = rank(E_i)$.

Definition 2 (Orlov [2006]). Given positive constants T, c_1, c_2, with $c_1 < c_2$, a positive definite matrix R and a given switching signal $\sigma(t) \in M$, the switched singular system (1) is said to be finite-time stable with respect to (c_1, c_2, T, R, σ), if

$$x_0^{\mathrm{T}} R x_0 \le c_1 \Rightarrow x^{\mathrm{T}}(t) R x(t) \le c_2, \forall t \in (0, T]. \tag{2}$$

Remark 1: While the extended definition of FTS for singular systems in Feng et al. [2005] and Zhao et al. [2013] as $\forall t \in (0, T]$,

$$x_0^{\mathrm{T}} E^T R E x_0 \le c_1 \Rightarrow x^{\mathrm{T}}(t) E^T R E x(t) \le c_2, \tag{3}$$

in this paper, the concept of FTS is the same as that given first in Du et al. [2009]. Here, through the dynamics decomposition, some norm operations and LMIs, we directly get the result that the state of the switched singular system rises with a bounded exponential rate. Then, Free of irreversible E constraints, we can study the FTS problem of system (1) using Definition 2.

Definition 3 (Liberzon [2003]). For any $T \ge t \ge 0$, let $N_\sigma(t, T)$ denotes the number of switching $\sigma(t)$ over (t, T) if

$$N_\sigma(t, T) \le N_0 + \frac{T - t}{\tau_a}. \tag{4}$$

Note that, in this paper we choose $N_0 = 0$, as commonly used in Sun et al. [2006].

By Duan [2010] (Theorem 2.4), there exist nonsingular matrices G_i and H_i such that (E_i, A_i) has the following dynamics decomposition form:

$$\begin{aligned} G_i E_i H_i &= \begin{bmatrix} I_{r_i} & 0 \\ 0 & 0 \end{bmatrix} =: \bar{E}_i, \\ G_i A_i H_i &= \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} =: \bar{A}_i. \end{aligned} \tag{5}$$

Lemma 1 (Dai [1989]). For $i \in M$, the singular system $(E_i,\, A_i)$ is impulse-free if and only if A_{i4} is nonsingular.

We note that decomposition (5) can be obtained via the singular value decomposition on E_i, which is not unique. Furthermore, the state transformation is obtained:

$$\bar{x}(t) = \left[\, \bar{x}_{i_k 1}(t)^T \;\; \bar{x}_{i_k 2}(t)^T \,\right]^T = H_{i_k}^{-1} x(t), \quad t \in [t_k, t_{k+1}) \tag{6}$$

Then the system (E_i, A_i) can be decomposed as:

$$\begin{aligned} \dot{\bar{x}}_{i1}(t) &= A_{i1}\bar{x}_{i1}(t) + A_{i2}\bar{x}_{i2}(t), \\ 0 &= A_{i3}\bar{x}_{i1}(t) + A_{i4}\bar{x}_{i2}(t). \end{aligned} \tag{7}$$

As described in Liu et al. [1995], Liberzon et al. [2009] and Trenn [2009], at switching instants in the system (1), the state jumps (discontinuities) can be evaluated via the action of the consistency projector Γ_{i_k}, mapping $x(t_k^-)$ into a consistent state $x(t_k^+)$ as:

$$x(t_k^+) = \Gamma_{i_k} x(t_k^-), \tag{8}$$

where

$$\Gamma_{i_k} = H_{i_k} \begin{bmatrix} I & 0 \\ -A_{i_k 4}^{-1} A_{i_k 3} & 0 \end{bmatrix} H_{i_k}^{-1}. \tag{9}$$

Remark 2. During $[t_k^+, t_{k+1}^-]$, the $i_k th$ subsystem is activated, and $x(t)$ is continuous. Meanwhile, from (7)–(9), we can obatin $\Gamma_{i_k} x(t) = x(t)$ in $[t_k^+, t_{k+1}^-]$. Hence, when choosing a Lyapunov function $V_{i_k}(x(t)) = x^{\mathrm{T}}(t) E_{i_k}^{\mathrm{T}} P_{i_k} E_{i_k} x(t)$, we also obtain that $V_{i_k}(x(t)) = x^{\mathrm{T}}(t) \Gamma_{i_k}^{\mathrm{T}} E_{i_k}^{\mathrm{T}} P_{i_k} E_{i_k} \Gamma_{i_k} x(t)$. In the following theorem, the conditions that every subsystems $(E_i,\ A_i)$ is regular and free-impulse will be given by showing A_{i4} nonsingular. In addition, unlike in Zhou et al. [2013], condition $\operatorname{rank}(E_i) = r < n$ is not necessary for any subsystems, i.e., $\operatorname{rank}(E_i)$ can be different for any two subsystems.

3. MAIN RESULTS

In this section, we provide the sufficient conditions to guarantee the system (1) finite-time stable under the average dwell time switching.

Theorem 1. For any $i \in M$, suppose that there exist $n \times n$ -matrices $P_i > 0$, Q_i, Y_{i1}, Y_{i2} and $n \times (n - r_i)$-matrices S_i and a constant $\gamma > 0$ such that

$$\Sigma_i = \begin{bmatrix} \Sigma_{i1} & \Sigma_{i2} \\ \Sigma_{i2}^{\mathrm{T}} & -Y_{i2} - Y_{i2}^{\mathrm{T}} \end{bmatrix} < 0, \tag{10}$$

$$\beta\mu < c_2/(c_1 e^{\gamma T}) \ , \tag{11}$$

where $\Sigma_{i1} = Y_{i1}(A_i - E_i) + (A_i - E_i)^{\mathrm{T}} Y_{i1}^{\mathrm{T}} + (2-\gamma) E_i^{\mathrm{T}} P_i E_i$, $\Sigma_{i2} = (A_i - E_i)^{\mathrm{T}} Y_{i2}^{\mathrm{T}} - Y_{i1} + E_i^{\mathrm{T}} P_i + Q_i^{\mathrm{T}} S_i^{\mathrm{T}}$ with S_i satisfying $E_i^{\mathrm{T}} S_i = 0$, $\lambda_1 = \lambda_{\min}(R^{-1})$, $\lambda_2 = \lambda_{\max}(R^{-1})$, $\mu = \frac{\lambda_2}{\lambda_1}$, $\beta = \max\{\alpha, 1\}$, and

$$\alpha = \max_{i \in M} \left\{ \frac{\lambda_M(E_i^{\mathrm{T}} P_i E_i)}{\lambda_m(\bar{P}_{i1})} \left\| H_i \begin{bmatrix} I \\ -A_{i4}^{-1} A_{i3} \end{bmatrix} \right\|^2 \|\Gamma_i\|^2 \right\}. \tag{12}$$

Then system (1) is said to be finite-time stable with respect to $(c_1,\ c_2,\ T,\ R,\ \sigma)$, where $0 < c_1 < c_2$ for any switching signal σ with average dwell-time satisfying

$$\tau_a > \tau_a^* = \frac{T \ln \beta}{\ln(c_2/c_1) - \ln \mu - \gamma T}. \tag{13}$$

Proof. Choose the Lyapunov function candidate as follows

$$V(x(t)) = V_{\sigma(t)}(x(t)) = x^{\mathrm{T}}(t) E_{\sigma(t)}^{\mathrm{T}} P_{\sigma(t)} E_{\sigma(t)} x(t). \tag{14}$$

Next, the proof is divided into two steps. In the first step, the deductive process is showed that every subsystem is regular and impulse-free and state rises with abounded rate. The second step proves that the switched system is finite time stable.

Step 1:

Assume when $t \in [t_k,\ t_{k+1})$, the subsystem $(E_i,\ A_i)$ activates and $V(x(t))$ with respect to t along the trajectory of system (1) yields

$$V_i(x(t)) = x^{\mathrm{T}}(t) E_i^{\mathrm{T}} P_i E_i x(t). \tag{15}$$

Firstly, we show that the system $(E_i,\ A_i)$ is regular and impulse-free. By the decomposition (6),

$$\begin{aligned} \bar{P}_i &= G_i^{-\mathrm{T}} P_i G_i^{-1} = \begin{bmatrix} \bar{P}_{i1} & \bar{P}_{i2} \\ \bar{P}_{i3} & \bar{P}_{i4} \end{bmatrix}, \\ \bar{Q}_i &= H_i^{\mathrm{T}} Q_i = \left[\, \bar{Q}_{i1}^T \ \ \bar{Q}_{i2}^T \,\right]^T, \\ \bar{S}_i &= G_i^{-\mathrm{T}} S_i = \left[\ 0 \ \ \bar{S}_{i1}^T \,\right]^T, \end{aligned} \tag{16}$$

where $\bar{S}_{i1} \in \mathbb{R}^{(n-r_i)\times(n-r_i)}$ is any nonsingular matrix. Pre- and post multiplying $\sum_i$ by $[I \quad (A_i - E_i)^{\mathrm{T}}]$ and $[I \quad (A_i - E_i)^{\mathrm{T}}]^{\mathrm{T}}$, respectively, yields

$$\begin{aligned} \Omega_i = {} & A_i^{\mathrm{T}} P_i E_i + E_i^{\mathrm{T}} P_i A_i - r E_i^{\mathrm{T}} P_i E_i \\ & + A_i^{\mathrm{T}} S_i Q_i + Q_i^{\mathrm{T}} S_i^{\mathrm{T}} A_i < 0, \end{aligned} \tag{17}$$

$$\begin{aligned} \bar{\Omega}_i &= H_i^{\mathrm{T}} \Omega_i H_i = \bar{A}_i^{\mathrm{T}} \bar{P}_i \bar{E}_i + \bar{E}_i \bar{P}_i \bar{A}_i - r \bar{E}_i \bar{P}_i \bar{E}_i \\ &\quad + \bar{A}_i^{\mathrm{T}} \bar{S}_i \bar{Q}_i^{\mathrm{T}} + \bar{Q}_i^{\mathrm{T}} \bar{S}_i^{\mathrm{T}} \bar{A}_i^T \\ &= \begin{bmatrix} * & * \\ * & A_{i4}^{\mathrm{T}} \bar{S}_{i1} \bar{Q}_{i2}^{\mathrm{T}} + \bar{Q}_{i2} \bar{S}_{i1}^{\mathrm{T}} A_{i4} \end{bmatrix} < 0, \end{aligned} \tag{18}$$

where the symbol $*$ represents the matrix block that is not relevant in the following discussion. From (18), we obtain that A_{i4} is nonsingular, which implies that the singular system (E_i, A_i) is regular and free-impulse.

Then, we show that $x(t)$ rises with abounded rate.

Using (5) and (16), it is clear that $V_i(x(t))$ is rewritten as

$$\begin{aligned} V_i(t) &= x^T(t) E_i^{\mathrm{T}} P_i E_i x(t) \\ &= x^{\mathrm{T}}(t) H_i^{-\mathrm{T}} H_i^{\mathrm{T}} E_i^{\mathrm{T}} G_i^{\mathrm{T}} \\ &\quad\ G_i^{-\mathrm{T}} P_i G_i^{-1} G_i E_i H_i H_i^{-1} x(t) \\ &= x^{\mathrm{T}}(t) H_i^{-\mathrm{T}} \bar{E}_i^{\mathrm{T}} \bar{P}_i \bar{E}_i H_i^{-1} x(t) = \bar{x}_{i1}^{\mathrm{T}}(t) \bar{P}_{i1} \bar{x}_{i1}(t). \end{aligned} \tag{19}$$

Let $E_i \dot{x}(t) - E_i x(t) = y(t)$, then one can obtain the following equation:

$$\begin{aligned} &\dot{V}_i(t) - \gamma V_i(t) \\ &\quad = 2\dot{x}^{\mathrm{T}}(t) E_i^{\mathrm{T}} P_i E_i x(t) - \gamma x^T(t) E_i^{\mathrm{T}} P_i E_i x(t) \\ &\quad = [E_i x(t) + y(t)]^{\mathrm{T}} P_i E_i x(t) \\ &\qquad + x(t)^{\mathrm{T}} E_i^{\mathrm{T}} P_i [E_i x(t) + y(t)] \\ &\qquad - \gamma x^T(t) E_i^{\mathrm{T}} P_i E_i x(t). \end{aligned} \tag{20}$$

Using the equations $E_i\dot{x}(t) - E_i x(t) = y(t)$ and $E_i\dot{x}(t) = A_i x(t)$, it follows that

$$(A_i - E_i)x(t) = y(t). \tag{21}$$

Hence for any matrices Y_{i1} and Y_{i2} satisfying (10), the following equation always holds:

$$2[x^{\mathrm{T}}(t)Y_{i1} + y^{\mathrm{T}}(t)Y_{i2}][(A_i - E_i)x(t) - y(t)] = 0. \tag{22}$$

For any matrix $S_i \in R^{n\times(n-r_i)}$ satisfying $E_i^{\mathrm{T}}S_i = 0$ and $E_i\dot{x}(t) - E_i x(t) = y(t)$, it is not difficult to see that the following equation holds:

$$2y^{\mathrm{T}}(t)S_i Q_i x(t) = 0. \tag{23}$$

Adding the left sides of (22) and (23) to $\dot{V}_i(t) - \gamma V_i(t)$ yields:

$$\dot{V}_i(t) - \gamma V_i(t) = \xi^{\mathrm{T}}(t)\Sigma_i\xi(t), \tag{24}$$

where $\xi(t) = [x(t) \quad y(t)]$ and Σ_i is defined in (10). Using (10), it follows from above inequality that

$$V_i(x(t)) \le e^{\gamma(t-t_k)}V_i(x(t_k)). \tag{25}$$

Noticing (19), then one can obtain from (25) that

$$\|\bar{x}_{i1}(t)\|^2 \le \frac{\lambda_M(E_i^{\mathrm{T}}P_iE_i)}{\lambda_m(\bar{P}_{i1})}e^{\gamma(t-t_k)}\|x(t_k)\|^2. \tag{26}$$

In view of (6) and (7), it is seen for $t \in [t_k, t_{k+1})$,

$$\begin{aligned}\|x(t)\|^2 &= \left\|H_i\begin{bmatrix}\bar{x}_{i1}(t)\\ \bar{x}_{i2}(t)\end{bmatrix}\right\|^2\\ &= \left\|H_i\begin{bmatrix}I\\ -A_{i4}^{-1}A_{i3}\end{bmatrix}\bar{x}_{i1}(t)\right\|^2\\ &\le \left\|H_i\begin{bmatrix}I\\ -A_{i4}^{-1}A_{i3}\end{bmatrix}\right\|^2\|\bar{x}_{i1}(t)\|^2.\end{aligned} \tag{27}$$

Step 2:

Since the state jump exists at switching instant t_k from the jth subsystem to the ith subsystem due to inconsistent initial condition, based on (8), (12), (26) and (27), the following inequality holds:

$$\|x(t)\|^2 \le \beta e^{\gamma(t-t_k^-)}\left\|x(t_k^-)\right\|^2,\ \forall t\in[t_k^-, t_{k+1}^-]. \tag{28}$$

For any $t \in (0, T)$, let N be the number of switching of $\sigma(t)$ over $(0, T)$, which implies that $N_\sigma(0, T) \le N$. Noticing $\beta \ge 1$ from (13) and using the relation $N \le \frac{T}{\tau_a}$, so

$$\begin{aligned}&\lambda_{\min}(R^{-1})x^{\mathrm{T}}(t)Rx(t)\\ &\quad\le \|x(t)\|^2 \le \beta e^{\gamma(t-t_k^-)}\left\|x(t_k^-)\right\|^2\\ &\quad\le \beta^2 e^{\gamma(t-t_{k-1}^-)}\left\|x(t_{k-1}^-)\right\|^2\\ &\quad\le \cdots \le \beta^N e^{\gamma(t-t_0)}\|x(t_0)\|^2\\ &\quad\le \lambda_{\max}(R^{-1})\beta^N e^{\gamma(t-t_0)}x^{\mathrm{T}}(t_0)Rx(t_0)\\ &\quad\le \lambda_{\max}(R^{-1})\beta^{\frac{T}{\tau_a}}e^{\gamma(t-t_0)}c_1.\end{aligned} \tag{29}$$

Relying on Definition 2, (13), (29) and for $t > 0$ with $t_0 = 0$,

$$\begin{aligned}x^{\mathrm{T}}(t)Rx(t) &\le \frac{\lambda_{\max}(R^{-1})}{\lambda_{\min}(R^{-1})}\beta^{\frac{T}{\tau_a}}e^{\gamma T}c_1\\ &\le \mu\beta^{\frac{T}{\tau_a}}e^{\gamma T}c_1 \le c_2.\end{aligned} \tag{30}$$

According to Definition 2, the switched singular system (1) is finite-time stable with respect to $(c_1,\ c_2,\ T,\ R,\ \sigma)$. This completes the proof. □

Remark 3. Let $N = 0$, which implies that only a subsystem activates over $(0, T]$, then Theorem 1 is still feasible to study the FTS problem of the singular system. When $\beta = 1$, we can conclude from Theorem 1 that the condition (12) is sufficient for the FTS of the switched singular system (1) under arbitrary switching. Compared with (17), (10) obtained by adding relaxation factors (22) and (23) reduces some conservativeness. Furthermore, since β takes the maximum value over all possible switching modes, the system may be still finite time stable for the average dwell time less than the one calculated here. Therefore, a future research topic for a tighter bound on the average dwell time deserves further investigation.

However, because of respective properties of modes, the switched system (1) may be continuous at some switching instants.

Assumption 1. For all $i, j \in M$, assume that $\mathrm{rank}(E_i) = r < n$, the conditions are feasible as follow:

$$H_i = H, \tag{31}$$

$$G_iA_iH = \begin{bmatrix}A_{i1} & A_{i2}\\ 0 & A_{i4}\end{bmatrix}. \tag{32}$$

Here, combining (9) with (31) and (32), one can obtain

$$\Gamma_i = \Gamma_j, \quad \forall i, j \in M. \tag{33}$$

Then from (8), *Remark 2* and (33), it is easily seen that the switched system (1) is continuous at any switching instant.

In the following, aimed at this case, the FTS conditions of system (1) is presented under the average dwell time switching.

Corollary 1. For any $i \in M$, suppose that there exist $n \times n$ -matrices $P_i > 0$, Q_i, Y_{i1}, Y_{i2} and $n \times (n - r)$-matrices S_i and a constant $\gamma > 0$ such that *Assumption 1*, the LMIs (10) and (11) are feasible, where

$$\beta = \max_{i\in M}\left\{\frac{\lambda_M(E_i^{\mathrm{T}}P_iE_i)}{\lambda_m(\bar{P}_{i1})}\left\|H\begin{bmatrix}I\\ 0\end{bmatrix}\right\|^2\right\}.$$

Then the switched singular system (1) is said to be finite-time stable with respect to $(c_1,\ c_2,\ T,\ R,\ \sigma)$, where $0 < c_1 < c_2$ for any switching signal σ with average dwell-time satisfying (13).

4. NUMERICAL EXAMPLES

Example 1. Consider the switched system consisting of two subsystems described by:

Table 1. the ratios of $\|x(t_k^+)\|^2$ to $\|x(t_k^-)\|^2$

t	1s	2s	3s	4s	5s	6s	7s	8s	9s
Calculated value	0.8915	0.9412	0.8915	0.9412	0.8915	0.9412	0.8915	0.9412	0.8915
Theoretical value					1.0811				

$$E_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ -1 & -1 & 4 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix}.$$

Let

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.25 & 0.25 & 1 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.25 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}.$$

Then we get

$$\bar{A}_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

For the given values $c_1 = 1$, $c_2 = 10$, $\gamma = 0.145$ and matrix $R = I$, let us apply Theorem 1 and solve corresponding matrix inequalities, leading to feasible solutions

$$P_1 = \begin{bmatrix} 40.7886 & -40.4624 & 0 \\ -40.4624 & 81.3588 & 0 \\ 0 & 0 & 37.7974 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 80.9754 & -40.3874 & 0 \\ -40.3874 & 40.7701 & 0 \\ 0 & 0 & 37.7974 \end{bmatrix},$$

and $\mu = 1$. Using (13), $\beta = 1.0811$. For $T = 10s$ the average dwell time

$$\tau_a = \frac{10 \times \ln 1.0811}{\ln 10 - \ln 1 - 0.145 \times 10} = 0.9149s.$$

According to (21), for any switching signal $\sigma(t)$ with average dwell-time $\tau_a \geq \tau_a^* = 0.9149s$, system (1) is finite time stable with respect to $(1, 10, 10, I, \sigma(t))$. Fig. 1(a) shows the state trajectory over $0 \sim 10s$ under a periodic switching signal with interval time $\nabla T = 1s$ from the initial state $x(0) = [1 \ \ 0 \ \ 0]^{\mathrm{T}}$. In Fig.1 (c), though it is easy to see that system (1) is finite-time stable under the switching signal $\sigma(t)$, the bound c_2 is large compared to the maximum value of the $x^{\mathrm{T}}(t)Rx(t)$. There is a dominant factor behind the phenomenon. From Table 1, Through the simulation calculation, we can find that the ratios of $\|x(t_k^+)\|^2$ to $\|x(t_k^-)\|^2$ at switching instants are smaller than β. Moreover, from $1.0881^9 \times 0.8915^{-5} \times 0.9412^{-4} = 4.5652$, we can easily seen that the bound c_2 is more than 4 times larger than $x^{\mathrm{T}}(t)Rx(t)$ as Fig.1 (c) shows.

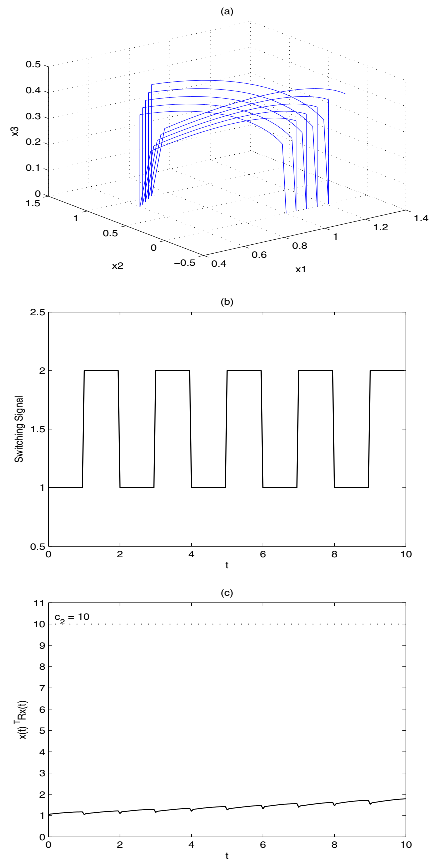

Fig. 1. The system response under switching signal $\sigma(t)$: (a) State trajectory; (b) The switching signal; (c) $x^{\mathrm{T}}(t)Rx(t)$

5. CONCLUSIONS

The finite-time stability of switched singular systems with state jumps at switching instants has been studied. Finite-time stability strategies are derived based on the multiple Lyapunov functions method. Under average dwell time switchings, the sufficient condition for the switched singular system to be finite-time stable is presented. A numerical example is given to illustrate the effectiveness of the obtained theoretical results. Some other interesting topics not mentioned in this paper include the problems of the finite-time stabilization and the finite-time boundedness analysis and synthesis for switched linear continuous-time singular systems. These topics will be studied in the future.

REFERENCES

F. Amato, M. Ariola, and P. Dorato. Finite-time control of linear systems subject to parametric uncertainties and disturbances. *Automatica*, 37(9), pp. 1459–1463, 2001.

F. Amato, M. Ariola, and P. Dorato. Robust finite-time stabilization of Linear uncertain systems via gainscheduled output fFeedback. *Proceedings of the 14th IFAC World Congress*, pp. 85–90, 1999.

F. Amato, and M. Ariola. Finite-time control of discrete-time linear systems. *IEEE Transactions on Automatic Control*, 50(5), pp. 724–729, 2005.

F. Amato, M. Ariola, and C. Cosentino. Finite-time stabilization via dynamic output feedback. *Automatica*, 42(2), pp. 337–342, 2006.

F. Amato, R. Ambrosino, M. Ariola, and C. Cosentino. Finite-time stability of linear time-varying systems with jumps. *Automatica*, 45, pp. 1354–1358, 2009.

P. S. Antsaklis. Pecial issue on hybrid systems: Theory and applications a brief introduction to the theory and applications of hybrid systems. *Proc.IEEE*, 88(7):887–897, 2000.

S. P. Bhat and D. S. Bernstein. Finite-time stability of continuous autonomous systems. *SIAM Journal on Control and Optimization*, 38(3), pp. 751–766, 2000.

D. Cheng, J. Wang, and X. Hu. An extension of LaSalles invariance principle and its application to multi-agent consensus. *IEEE Transaction on Automatic Control*, 53(7), pp. 1765–1770, 2007.

L. Dai. Singular control systems. *Springer-Verlag*, Berlin, Germany, 1989.

P. Dorato. Short time stability in linear time-varying systems. *Proceedings of the IRE International Convention Record Part 4*, pp. 83–87, 1961.

H. B. Du, X. Z. Lin, and S. H. Li. Finite-time stability and stabilization of switched linear systems. *Proceedings of the 48th IEEE Conference on Decision and Control*, Shanghai, pp.1938–1943, 2009.

G. Duan. Analysis and design of descriptor linear systems. *Springer-Verlag*, Berlin, Germany, 2010.

J. Feng, Z. Wu and J. Sun. Finite-time control of linear singular systems with parametric uncertainties and disturbances. *Acta Automatica Sinica*, 31(4), pp. 634–637, 2005.

V. T. Haimo. Finite-time controllers . *SIAM Journal on Control and Optimization*, 24(4), pp. 760–770, 1986.

S. Li, and Y. Tian. Finite-time stability of cascaded time-varying systems. *International Journal of Control*, 80(4), pp. 646–657, 2007.

D. Liberzon, and A. S. Morse. Basic problems in stability and design of switched systems. *IEEE Control Systems Magazine*, 19, pp. 59–70, 1999.

D. Liberzon. Switching in systems and control. *Brikhauser*, Boston, 2003.

D. Liberzon, and S. Trenn. On stability of linear switched differential algebraic equations. *Proceedings of the 48th IEEE conference on decision and control*, pp. 2156–2161, 2009.

C. H. Lien, K. W. Yu, H. C. Chang, L. Y. Chung, and J. D. Chen. Switching signal design for exponential stability of discrete switched systems with interval time-varying delay. *Journal of the Franklin Institute*, 349, 2182–2192, 2012.

X. Lin and S. Li. Finite time set stabilization of Chuas chaotic system. *IEEE International Conference on Control and Automation*, pp. 2890–2893, 2007.

X. Lin, H. Du, S. Li, Y. Zou. Finite-time stability and finite-time weighted L_2-gain analysis for switched systems with time-varying delay. *IET Control Theory & Applications*, 7,(7), pp. 1058-1069, 2013.

H. Lin, and P. J. Antsaklis. Stability and stabilizability of switched linear systems: a survey on recent results. *IEEE Transactions on Automatic Control*, 45, pp. 965–972, 2009.

W. Liu, W. Yan, and K. Teo. On initial instantaneous jumps of singular systems. *IEEE Transactions on Automatic Control*, 40, pp. 1650–1655, 1995.

L. Liu, and J. Sun. Finite-time stabilization of linear systems via impulsive control. *International Journal of Control*, 81(6), pp. 905–909, 2008.

L. Lu, Z. L. Lin, and H. J. Fang. L_2-gain analysis for a class of switched systems. *Automatica*, 45, pp. 965–972, 2009.

X. L. Luan, F. Liu, and P. Shi. Robust finite-time control for a class of extended stochastic switching systems. *International Journal of Systems Science*, 42, pp. 1197–1205, 2011.

A. K. Natasa, and K. Viseslava. Finite-time stability of time-varying linear singular systems. *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, Florida, USA, pp. 3831–3836, 1998.

Y. Orlov. Finite time stability and robust control synthesis of uncertain switched systems. *SIAM Journal on Control and Optimization*, 43(4), pp. 1253–1271, 2006.

J. Raouf, and H. Michalska. Exponential stabilization of singular systems by controlled switching. *Proceedings of the 49th IEEE conference on decision and control*, pp. 414–419, 2010.

X. M. Sun, J. Zhao, and J. H. David. Stability and gain analysis for switched delay systems: A delay-dependent method. *Automatica*, 42(10), pp. 1769–1774, 2006.

S. Trenn. Distributional differential algebraic equations. *PhD, Dissertation, Institute fr Mathematik, Technische Universit?t Ilmenau*Ilmenau, Germany, 2009.

Z. R. Xiang, C. H. Qiao, and M. S. Mahmoud. Finite-time analysis and HN control for switched stochastic systems. *Journal of the Franklin Institute*, 349, pp. 915–927, 2012.

S. Xu, and J. Lam. Robust control and filtering of singular systems. *Springer-Verlag*, Berlin, Germany, 2006.

J. Yu, L. Wang, M. Yu, Y. Jia, and J. Chen. Packet-loss dependent controller design for networked control systems via switched system approach. *Proceedings of 47th IEEE Conference on Decision and Control*, pp. 3354–3359, 2008.

S. Zhao, L. Liu, and J. Sun. Finite-time stability of linear time-varying singular systems with impulsive effects. *International Journal of Control*, 81(11), pp. 1824–1829, 2008.

G. Zhao, J. C. Wang,. Finite time stability and L_2-gain analysis for switched linear systems with state-dependent switching. *Journal of the Franklin Institute*, 350, pp. 1075–1092, 2013.

L. Zhou, D. W. C. Ho, and G. S. Zhai. Stability analysis of switched linear singular systems. *Automatica*, vol. 49, no. 5, pp. 1481–1487, 2013.

F. Li and X. Zhang, Delay-range-dependent robust filtering for singular lpv systems with time variant delay, *International Journal of Innovative Computing, Information and Control*, vol.9, no.1, pp.339-353, 2013.

P. Liu, Improved delay-dependent robust exponential stabilization criteria for uncertain delay singular systems, *International Journal of Innovative Computing, Information and Control*, vol.9, no.1, pp.165-178, 2013.