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Joint Parametric Fault Diagnosis and State Estimation Using KF-ML Method

Zhen Sun * **Zhenyu Yang** **

* *Department of Energy Technology, Aalborg University, 6700 Esbjerg, Denmark (e-mail: zhensunsdu@gmail.com).*
** *Department of Energy Technology, Aalborg University, 6700 Esbjerg, Denmark (e-mail: yang@et.aau.dk).*

Abstract: The paper proposes a new method for a kind of parametric fault online diagnosis with state estimation jointly. The considered fault affects not only the deterministic part of the system but also the random circumstance. The proposed method first applies Kalman Filter (KF) and Maximum Likelihood (ML) technique to identify the fault parameter and employs the result to make fault decision based on the predefined threshold. Then this estimated fault parameter value is substituted into parameterized state estimation of KF to obtain the state estimation. Finally, a robot case study with two different fault scenarios shows this method can lead to a good performance in terms of fast and accurate fault detection and state estimation.

Keywords: Parameter Identification, State Estimation, Kalman Filter, Maximum Likelihood, Fault Detection.

1. INTRODUCTION

During the last two decades, the so called model-based Fault Detection and Diagnosis (FDD) approaches have been received increasing attention from both academic and industry societies (Frank et al. (2000), Isermann (2005)). One important fault category is parametric fault. Generally, for the system with possible parametric fault, if the FDD procedure can identify the parameter related to fault, it can be easily claimed whether the fault happened based on the procedure of parameter identification. Sometimes in order to deal with the fault and maintain the system running, the information of the state is also quite important. For this reason, the fault parameter identification often accompanies with state estimation (Isermann (2005)). Thereby, the Joint Parameter Identification and State Estimation (JPISE) technique is widely applied for the FDD purpose.

In JPISE technique, one popular method is to directly apply Kalman Filter (KF) technique (Zhang et al. (1994)), which can be named as state estimation approach. This kind of approach firstly takes both the state variable and the unknown parameter(s) as an augmented new system state. Then, corresponding KF technique, such as Kalman Filter, Extended Kalman Filter (EKF) or Unscented Kalman Filter (UKF), is used to estimate this new state, thereby the estimation of fault(s) and state can be simultaneously obtained. However, this kind of approach gives rise to explicit multiplication of state by adding the unknown parameter into state variable, meanwhile it is well known that there is no guarantee for global convergency (Hopkins and Vanlandingham (1988)). Moreover, if a fault leads changes not only regrading to the deterministic description of the system but also possibly regarding the random feature, the state estimation approach may not provide a good performance to estimate unknown parameter in random part. From another point of view, the KF itself only applies the means and covariances of the variables in the system, not all the information of system such as the distribution of the noise (in most situation, it is assumed Gaussian distribution). It is reasonable to believe that if both the state and noise information are employed to make the estimation of system, the result could be much more precise. For these reasons, another kind of method based on KF is developed, which is named as "bootstrap" method by Hopkins and Vanlandingham (1988). Within this type of method, the parameter identification and state estimation are carried out sequently. The approach combines KF technique with some statistic methods. The scheme generally consists of two sequential stages. The first stage conducts the state estimation using KF technique, in which the estimated state, including mean and covariance, is a function of unknown parameters. Then, a statistic criterion, such as Maximum Likelihood (ML) or Least Mean Square (LMS), is set up in the second stage based on the estimated parameterized state. Thereby, the fault detection problem can be converted to an optimization of parameterized statistic problem and it can be numerically solved by some optimization algorithms (Dennis and Schnabel (1983)). In this category, the Kalman Filter with Maximum Likelihood (KF-ML) method (Kristensen et al. (2004), Sun and Yang (2010)) is a typical approach, which applies KF technique plus ML method to identify the parameters in stochastic systems. This kind of bootstrap method may be more flexible than the state estimation methods, e.g., being able to directly deal with identification of some nonlinear systems with unknown stochastic characteristics and considering the distribution in the systems.

In this paper, we consider the parameterized fault and when the fault happens, the fault related parameters of the system in both deterministic part and random part

will be changed compared with the normal values. The KF technique plus ML method is applied to estimate the fault parameters in an online manner together with using sliding windows technique. Then the fault decision can be made onlinely based on the result of identification. The estimated parameters can be applied to realize the state estimation, and if necessary, Kalman Smoother (KS) technique (Charles and Chen (2009)), can be used in order to get smooth state estimation.

The remainder of the paper is organized as follows: The considered problem is formulated in Section II; The method using joint parameter identification and state estimation for the FDD purpose is given in Section III; Section IV illustrates the proposed algorithm via a robot case; Finally, we conclude the paper in Section V.

2. PROBLEM FORMULATION

In the following, our discussion is restricted to a class of continuous systems with possible abrupt parametric fault. Considering a system which is represented in state-space form using Itô Stochastic Differential Equation (ISDE) model, see Øksendal (2000), as:

$$\begin{cases} \dot{x}_t = [A(\theta)x_t + B(\theta)u_t]dt + E(\theta)dB_t \\ y_k = Cx_k + Du_k + G\omega_k \end{cases} \tag{1}$$

where $x_t \in R^n$ is the system state, $u \in R^m$ is the system input (sometimes can be seen as control variable), $y \in R^r$ is the system output with $r < n$, B_t is an n-dimensional Brown Motion (B.M) and $\omega_t \in R^r$ denotes Gaussian noise in the measurement with zero means and covariance matrix R. θ is the parametric vector of the system. $A(\theta)$, $B(\theta)$, $E(\theta)$ are the system matrices, which can be dependent on the parameter θ. C, D, G are the output matrices and they are assumed to be independent of θ.

The fault in the model is assumed to be a parametric one and located inside the system. When the fault happens, it will affect the value of system matrices by changing parameter θ. It means that

$$\theta = \begin{cases} \theta_0, & Normal\ \ system, \\ \theta_f, & Faulty\ \ system. \end{cases} \tag{2}$$

Note that the coefficient of random part in the system changes from $E(\theta_0)$ to $E(\theta_f)$, i.e., the fault affects both the deterministic part and random part.

The main problem is to estimate both the parameter θ and state x_t from the data of input and measurement in an on-line manner and to make the fault decision based on the estimation.

3. KF-ML METHOD FOR FAULT DETECTION AND STATE ESTIMATION

KF-ML method was firstly applied in Kristensen et al. (2004) for system identification to Stochastic Differential Equation (SDE) models and it was extended in our previous work, see Sun and Yang (2010), to deal with some nonlinear cases. The main procedure using for joint FDD and state estimation can be summarized in the following.

3.1 Parameterized State Estimation

The first stage is to use KF technique to get the parameterized state estimation. It can be referred to Charles and Chen (2009) for general KF theory. Since the measurement is in the discrete time version, the process equation in (1) need to be discretized as:

$$\begin{aligned} x_{t_k} - x_{t_{k-1}} = [A(\theta)x_{t_k} + B(\theta)u_{t_k}](t_k - t_{k-1}) \\ + E(\theta)(B_k - B_{k-1}). \end{aligned}$$

Then the KF is performed to make the state estimation as following:

The initial state can be specified as a random Gaussian vector with mean x_0 and covariance P_0.

Time-updated (Prediction):

$$\begin{aligned} &\hat{x}_k^-(\theta) = \hat{x}_{k-1}(\theta) + [A_k(\theta)\hat{x}_{k-1}(\theta) + B_k(\theta)u_{k-1}](t_k - t_{k-1}), \\ &P_k^-(\theta) = A_k(\theta)P_{k-1}(\theta)A_k^T(\theta) + E_k(\theta)(t_k - t_{k-1})IE_k^T(\theta) \\ &S_k(\theta) = CP_k^-(\theta)C^T + GR_kG^T, \\ &K_k(\theta) = P_k^-(\theta)C^TS_k^{-1}(\theta). \end{aligned} \tag{3}$$

Measurement-updated (Update):

$$\begin{aligned} r_k(\theta) &= y_k - C\hat{x}_k^-(\theta) - Du_k, \\ \hat{x}_k(\theta) &= \hat{x}_k^-(\theta) + K_k(\theta)r_k(\theta), \\ P_k(\theta) &= (I - K_k(\theta)C)P_k^-(\theta). \end{aligned} \tag{4}$$

Note that the subscript k means the corresponding variable is chosen at kth sampling time. θ in the bracket means that the corresponding variable depends on θ.

As a result, the estimated mean and covariance of the state, both of which are functions of the unknown parameter θ, can be obtained by accomplishing KF process.

3.2 Maximum Likelihood Estimation

The second stage is to make the ML estimation of system parameter θ. Based on the discretized process model and measurement, the ML estimation of the unknown parameter θ can be determined by finding $\hat{\theta}$ that maximizes the likelihood function in the following. This ML function is computed based on the state estimation in one length of time windows. Now suppose $M > N > 0$, where N is the length of sliding windows using for the on-line estimation, introducing the notation

$$\mathscr{Y}_M^N = [y_M, y_{M-1}, \ldots, y_{M-N+2}, y_{M-N+1}],$$

then, the likelihood function becomes the joint probability density, i.e.,

$$L(\theta; \mathscr{Y}_M^N) = p(\mathscr{Y}_M^N \mid \theta).$$

Since the noises in the measurement are independent at each step, it can be obtained that

$$\begin{aligned} L(\theta; \mathscr{Y}_M^N) = \left(\prod_{k=M-N+2}^{M} p(y_k \mid \mathscr{Y}_{k-M+N-1}^{k-1}, \theta) \right) \\ p(\mathscr{Y}_{M-N+1}^{M-N+2} \mid \theta). \end{aligned}$$

In order to carry out the optimization of the likelihood function, the estimated mean and covariance of state are needed, which is solved in the first stage using KF. Since the random part of measurement in (1) is driven by a Gaussian process, and the increment of a Gaussian process still follows normal distribution, it is reasonable to assume that the conditional densities can be approximated by normal densities. Then based on the obtained parameterized state estimation in the first stage, the parameterized likelihood function can be rewritten as:

$$L(\theta; \mathscr{Y}_M^N) = \prod_{k=M-N+2}^{M} \left(\frac{\exp(-\frac{1}{2} r_k^T(\theta) S_k^{-1}(\theta) r_k(\theta))}{\sqrt{\det(S_k(\theta))}(\sqrt{2\pi})^n} \right) p(\mathscr{Y}_{M-N+1}^{M-N+2} \mid \theta). \tag{5}$$

Then, the considered FDD problem via parameter identification need to solve the following optimization problem in an online manner:

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \{ -\ln(L(\theta; \mathscr{Y}_M^N)) \}. \tag{6}$$

Generally, the convex property of the formulated optimization problem (6) need to be explored. But a good initial value could possibly lead to a global optimal solution. Here, for LTI cases, the global optimal point could be obtained. In the case of this paper, the quasi-Newton using BFGS update method, see Dennis and Schnabel (1983), is adopted. After optimization, $\hat{\theta}$ is obtained at the time when measurement y_M is collected. The value $\hat{\theta}$ is considered as the estimated value of the fault parameter at the recent time point.

3.3 Fault Detection Decision

From the last stage, $\hat{\theta}$ is obtained as the estimation of parameter θ which is related to fault. It need to repeat the procedure using a moving window with length N to make an on-line parameter identification of the fault variable. In order to make the fault decision, either the predefined threshold method or some statistical methods (Yang and Akbar Hussain (2007)), may be used to determine whether the fault happened or not. In this paper, the deterministic threshold method is applied. If the value estimated is around the normal value θ_0 within 10% deviation, the system is claimed running well. When the difference between the normal value and the estimated value exceeded this predefined value or the estimated value changed beyond 10% compared with the historical value, it can be claimed that the fault has happened.

3.4 Smoothing State Estimation

If estimated parameter value $\hat{\theta}$ is substituted into the parameterized state estimation (3) and (4), the state estimation $\hat{x}_k(\hat{\theta})$ and $P_k(\hat{\theta})$ can be obtained jointly. But in the state estimation, sometimes Kalman Smoother technique need to be adopted to make the estimation more accurate.

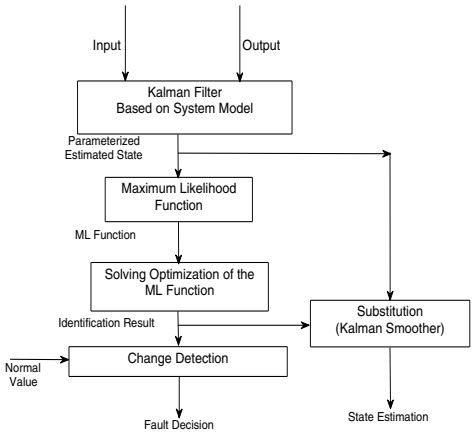

Fig. 1. The scheme using KL-ML method

The Kalman Smoother proceeds backward in time Charles and Chen (2009). It can be summarized as:

Initial with $\hat{x}_M(\hat{\theta})$ and $P_M(\hat{\theta})$, and let $k = M-1, N-2, \ldots, M-N+1$, there is:

$$\begin{aligned} L_k(\hat{\theta}) &= P_k(\hat{\theta}) A_k^T(\hat{\theta}) P_{k+1}^-(\hat{\theta}), \\ \hat{x}_{k|N}(\hat{\theta}) &= \hat{x}_k(\hat{\theta}) + L_k(\hat{\theta})(\hat{x}_{k+1|N}(\hat{\theta}) - \hat{x}_{k+1}^-(\hat{\theta})), \\ P_{k|N}(\hat{\theta}) &= P_k(\hat{\theta}) + L_k(\hat{\theta})(P_{k+1|N}(\hat{\theta}) - P_{k+1}^-(\hat{\theta})) L_k^T(\theta). \end{aligned} \tag{7}$$

3.5 Entire FDD and State Estimation

Summarizing the above stages, the entire scheme for FDD using proposed method can be illustrated in Fig. 1. In the beginning of the process, the length of moving windows N need to be determined. Then based on the input and output data, the estimation procedure can be performed in an on-line manner.

- Employ KF technique to make state estimation (mean and covariance).
- Form the parameterized ML function of fault parameter based on the parameterized state estimation.
- Solve the optimization problem of the ML function, and obtain the value of fault parameter identification.
- Compare the identification result with the historical values and make the fault decision using the predefined deterministic threshold method.
- Substitute the identified parameter into parameterized KF solution, and then obtain the state estimation. If necessary, apply KS for smoothing purpose.
- When a new couple of data arrived, repeat the same procedure as above.

4. CASE STUDY

The space robot system used in Yang et al. (2007) is considered as a case study. The process could be seen in Fig. 2. In the normal situation, system parameters are listed in Table 1, and the dynamic of the normal system is described by:

$$N^2 I_m \ddot{\Omega} + I_{son}(\ddot{\Omega} + \ddot{\epsilon}) + \beta(\dot{\Omega} + \dot{\epsilon}) = T_j^{eff}, \tag{8}$$

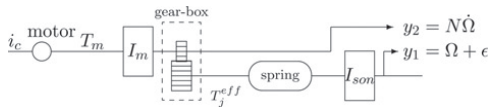

Fig. 2. The process of a robot space referred Yang et al. (2007)

Table 1. System parameters of the space robot system

Symbol	Description	Unit
N=-260.6	Gear-box ratio	–
$I_m = 0.0011$	Inertia of the input axis	kg m^2
Ω	Joint angle of the internal axis	rad
$I_{son} = 400$	Inertia of the output axis	kg m^2
T_j^{eff}	Torque of effective joint input	Nm
ϵ	Joint angle of output axis	rad
$k_t = 0.6$	Motor torque constant	N/%
i_c	Motor current	Am
$\beta = 0.4$	Damping coefficient	N/%
$c = 130\ 000$	Spring coefficient	N/%
T_{def}	Deformation torque of gear box	Nm
T_m	Motor torque	Nm

$$I_{son}(\ddot{\Omega} + \ddot{\epsilon}) + \beta(\dot{\Omega} + \dot{\epsilon}) = -T_{def}. \tag{9}$$

The actuator part including a DC-motor and a gear box is simplified as $T_j^{eff} = NT_m$ and $T_m = k_t i_c$. Torque T_{def} due to the deformed spring is described by $T_{def} = c\epsilon$. In the actual system, the controllable input is the motor current i_c, and the measured signals are encoder output $\Theta = \Omega + \epsilon$ and tachometer output $N\dot{\Omega}$. The original system was a SIMO system. The fault scenario is assumed to disturb the motor constant. It will have an influence on both the value of the motor torque constant and the system random circumstance. Fault parameter is assumed as θ and motor toque constant is taken as θk_t. If the system runs normally, the motor toque constant is k_t with $\theta_0 = 1$. But when the fault happens and the motor toque constant changes to $\theta_f k_t$. The fault also affects the random part of system by changing the value of σ.

Define state vector $x_s = [\Omega, \dot{\Omega}, \epsilon, \dot{\epsilon}]^T$, output vector $y = [\Omega + \epsilon, N\dot{\Omega}]^T$, and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{c}{N^2 I_m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{\beta}{I_{son}} & -(\frac{c}{N^2 I_m} + \frac{\beta}{I_{son}}) & -\frac{\beta}{I_{son}} \end{bmatrix},$$

$$E = \begin{bmatrix} 0 \\ \frac{k_t}{NI_m} \\ 0 \\ -\frac{k_t}{NI_m} \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & N & 0 & 0 \end{bmatrix},$$

Then the normal/faulty system could be described as follows:

$$\begin{cases} dx(t) = [Ax(t) + E(\theta)u(t)]dt + a\sigma dB_t \\ y(t) = Cx(t) + \omega_t \end{cases} \tag{10}$$

where $E(\theta) = [0 \ \ \frac{\theta k_t}{NI_m} \ \ 0 \ \ -\frac{\theta k_t}{NI_m}]^T$, B_t is one two dimensional Brown Motion, the noises ω_t is the Gaussian noise which denotes by $\omega_t = [0 \ \ 1 \ \ 0 \ \ 1]^T v_1$, where v_1 is one dimensional Gaussian processes with means $\mathbf{0}$, and covariances R. $a = 0.001$ and σ is the unknown parameter in the random item. In the test, it is assumed that $R = \begin{bmatrix} 0.001^2 & 0 \\ 0 & 0.001^2 \end{bmatrix}$.

The system runs according to the following rules.

$$\Theta = [\theta \ \ \sigma]^T = \begin{cases} [\theta_0 \ \ \sigma_0]^T = [1 \ \ 1]^T, & Normal \ \ system, \\ [\theta_f \ \ \sigma_f]^T, & Faulty \ \ system. \end{cases} \tag{11}$$

In the following simulation tests, the whole time the system running is 31.4 seconds and at the 10th second the fault happened. The multi-step input u_1 and sinusoid input u_2 are concerned as the system inputs.

$$u_1(t) = \begin{cases} 0.1, & for \ \ t < 5s \\ -0.5, & for \ \ 5s \le t < 20s \\ 0.2, & for \ \ others \end{cases},$$

$$u_2(t) = 0.5sin(0.8t).$$

The identification of the fault parameter need to wait for the first N outputs in the beginning. During this waiting period, the fault parameter estimated is set as the initial value of the optimization and state estimation is based on all the sample points before we get the N-th point. As soon as N sample points are collected, a moving window with length N is used to on-linely update the estimation. The initial condition for the system is $x(0) = [0.01, 0, 0, 0]^T$. The sampled interval is set as 0.1 second. The initial estimation values are assumed as $\tilde{\theta} = 0.3$ and $\tilde{\sigma} = 0.9$.

In order to test the effect of different conditions, two scenarios are considered as following:

- **Case a:** input signal u_1, if fault happens, parameters change to $\theta_f = 1.5$, $\sigma_f = 10$ and the length of moving windows is set as 30.
- **Case b:** input signal u_2, if fault happens, parameters change to $\theta_f = 0.5$, $\sigma_f = 1.5$ and the length of moving windows is set as 10.

The output data is plotted in Fig. 3 and Fig. 6.

From the results, it can be obtained that

- *Fault detection:*

  As shown in Fig. 4 and Fig. 7, the procedure need to wait 3 second and 1 second to collect enough points to make the estimation. During this period, the estimated parameter remains at the initial value. When the data is enough to make the estimation, there are some differences for the two scenarios. In **Case a**, for parameter identification in deterministic part, before 10th second, the estimated value is much close to 1 which is true value of normal system. At 5th second, the estimated value has a small deviation to the true value. It is due to the effect of input signal which is converted to the different direction at that time. When the fault happened at 10th second, the estimated value has a large jump or deviation.

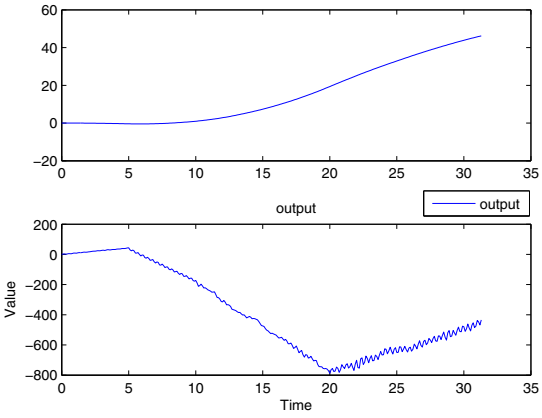

Fig. 3. System output

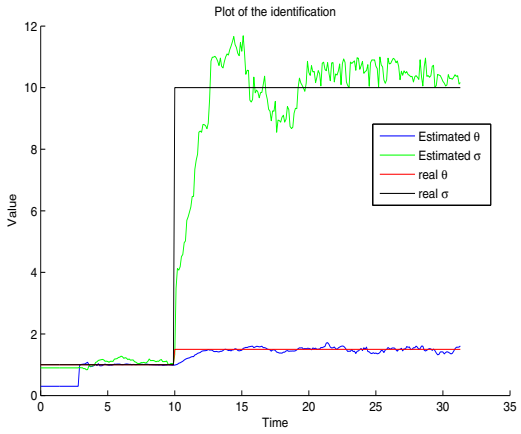

Fig. 4. Parameter identification

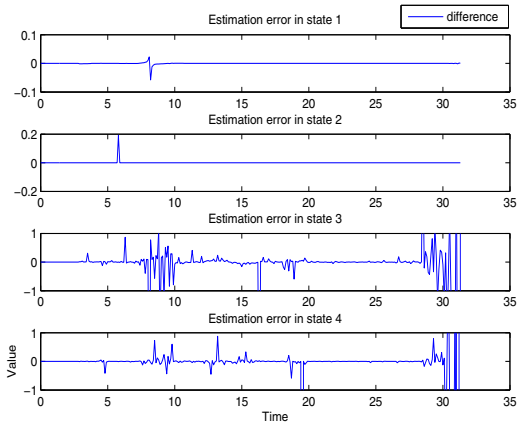

Fig. 5. State estimation error

After a while, the estimated value changes to be close to a new steady-state value which is nearly the true value of parameter in the fault system. During almost all period, the error for fault related parameter identification of deterministic part is within 4%. If the fault criterion is set as 10% deviation, it is obvious that the fault happened at 10th second in the deterministic part of system and its magnitude can be obtained accurately as well. For the random part

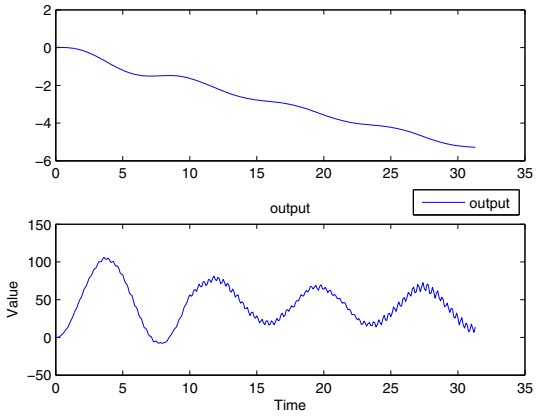

Fig. 6. System output

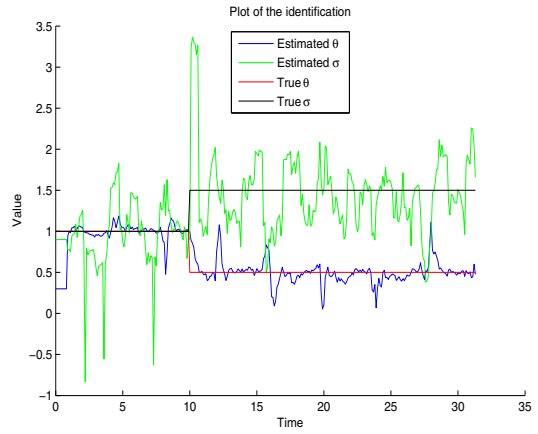

Fig. 7. Parameter identification

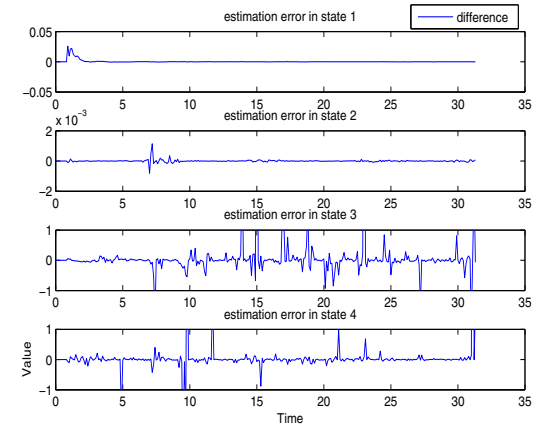

Fig. 8. State estimation error

estimation, it is not so good as the estimation for the deterministic part. It can be seen that in the whole period, the estimated value has some deviation all the time. It is due to the fact that only tens of data can not catch all the information of the stochastic process. If it is to get the precise estimation for the parameter in the random part, it need thousands of data or more. But that is laborious and in many situations for FDD, it is unnecessary. However, it can still detected that

more than 10% deviation emerged at 10th second, i.e., it can be claimed the fault happened at 10th second in the random part as well. In **Case b**, the performance is not so good as in **Case a**. It has more fluctuations for the estimation than that in **Case a**. This is because the length of moving windows is much shorter than **Case a**. However, it can still provide us the information of the fault parameter and the right fault decision can be made based on the estimation. In the whole, from these two cases, it is believed that the fault happened at 10th second in both deterministic part and random part.

- *State estimation:*

  Fig. 5 and Fig. 8 show the error in the state estimation and it is expressed in percentage by the ratio between the error and true state. The results illustrate that most of state estimation errors are within 1% for both of two cases. But as the same phenomenon observed in the parameter identification, at those intervals and times when the condition of the system/input is changed, the estimations would have relatively large temporal oscillations.

5. CONCLUSIONS AND FUTURE WORKS

An online approach using jointly parameter identification and state estimation for fault detection and diagnosis is proposed for systems where fault can affect both the deterministic part and random circumstance. This approach firstly applies the KF technique to get the parameterized state estimation. Then, ML function is formulated based on the result of parameterized state estimation and noise distribution knowledge. An optimization problem of ML function is solved using numerical algorithm and the optimal value is taken as the estimated fault parameter. The fault detection decision can be made based on the predefined threshold method. Moreover, this estimated fault value can be substituted into the parameterized state estimation and thereby the corresponding state estimation is finally obtained. Sometimes, KS need to be used to make the state estimation more smooth and accurate.

The simulation result based on a robot system showed that this approach can provide an accurate estimation and fast detection time in terms of fault parameter identification and state estimation. However, the performance can be slightly affected by the length of moving windows and input signals using for the estimation. Moreover, the parameter identification in the random part calls for a large number of data.

To extend this method to the recursive manner is part of our future work.

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