Cooperative Estimation for Synchronization of Heterogeneous Multi-Agent Systems Using Relative Information ⋆

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Abstract: In this paper, we present a distributed estimation setup where local agents estimate their states from relative measurements received from their neighbours. In the case of heterogeneous multi-agent systems, where only relative measurements are available, this is of high relevance. The objective is to improve the scalability of the existing distributed estimation algorithms by restricting the agents to estimating only their local states and those of immediate neighbours. The presented estimation algorithm also guarantees robust performance against model and measurement disturbances. It is shown that it can be integrated into output synchronization algorithms.

1. INTRODUCTION

Estimator design has been an essential part of controller design ever since the development of state-space based controllers. A milestone was laid by the Kalman Filter in 1960 (Kalman, 1960).

While in the classical estimator design one estimator is used for one system, designing distributed estimators have gained attention since a distributed Kalman Filter was presented in (Olfati-Saber, 2005), (Olfati-Saber et al., 2007), (Carli et al., 2008). In a distributed estimator setup, multiple estimators create an estimate of the system's state, while cooperating with each other. In this setup, even when every single estimator may be able to obtain an estimate of the state on its own, cooperation reduces the effects of model and measurement disturbances (Subbotin and Smith, 2009). Also, the situations are not uncommon where every single estimator is unable to obtain an estimate of the state on its own and cooperation becomes an essential prerequisite (Ugrinovskii and Langbort, 2011), (Ugrinovskii, 2011). The node estimators may even not have a model of the full system, but only know a part of the system (Stankovic et al., 2009).

Apart from distributed estimation, distributed control and coordination of multi-agent system have become an important area in systems control research, with a wide range of possible applications such as vehicle platooning, UAV coordination, and energy networks. Starting from the consensus problem of integrators (Olfati-Saber et al., 2007),

(Ren et al., 2007) the boundaries have been pushed further and further, extending the theory to linear high-order systems (Fax and Murray, 2004), (Tuna, 2008), dynamic controllers (Scardovi and Sepulchre, 2009), heterogeneous systems (Wieland and Allgöwer, 2009), (Wieland et al., 2011), and many more. In these problems, a distributed control scheme is typically applied, i.e. for each agent in the network a separate *local controller* is implemented that can only receive information from the agent and its neighbors. Therefore, an important aspect of this setup is distinguishing between absolute information gained from individual local measurements, $y_k = h_k(x_k)$, and relative information gained through the network, $y_k - y_j$. For instance, in the framework of synchronizing vehicle formations some information, such as the vehicle position, can only be determined from relative measurements, i.e., distances between the vehicles. Some fundamental properties for estimation based on relative information with static agents have been presented in (Barooah and Hespanha, 2007). The case of estimating homogeneous multi-agent systems with relative information was considered in (Acikmese and Mandic, 2011). For heterogeneous multi-agent systems, this problem becomes particularly challenging. In (Wu and Allgöwer, 2012) a solution was presented that was developed from the Internal Model Principle for Synchronization (Wieland and Allgöwer, 2009). However, it is subject to some geometric conditions that restrict the class of systems that can be considered. In (Grip et al., 2012), the same problem is considered and an observer-based synchronization method is presented that solves the problem for leader-follower networks.

In this paper, we tackle the synchronization problem of heterogeneous multi-agent systems from the distributed estimation point-of-view. Our main contribution is the de-

velopment of a new framework for distributed state estimation using relative measurements. In this new framework, cooperation between the local estimators will be crucial due to lack of local detectability, therefore calling it *Cooperative Estimation*. These estimators allow us to estimate absolute information purely based on relative information, similar to (Listmann et al., 2011), but for a more general class of systems. Another purpose of the new framework is to address scalability of the filter network. In multi-agent coordination problems, scalability of the network is important, i.e. the dimension of the local controllers should not increase with the size of the network. However, direct applications of the existing algorithms such as those reported in (Olfati-Saber, 2005), (Olfati-saber, 2006) and (Ugrinovskii and Langbort, 2011), (Ugrinovskii, 2011) result in the order of the estimators growing with the size of the network. Our estimation framework resolves this issue by restricting the agents to perform estimation of the local states and the neighbour states only. In particular, we present an $\mathcal{H}_\infty$ suboptimal design, which can handle model and measurement disturbances.

After designing the local estimators, we analyze the synchronizing behavior of the controller presented in (Wieland et al., 2011) where these cooperating observers are employed. It turns out that in addition to synchronization, we can show guaranteed $\mathcal{H}_\infty$-performance for the closed loop system.

The rest of the paper is organized as follows: In Section 2, we present some mathematical preliminaries and the system class which is considered. Section 3 presents the cooperative estimation design with suboptimal $\mathcal{H}_\infty$-performance. The loop is closed in Section 4, where we analyze the performance, when the estimates are used in an synchronization setup. Section 5 depicts our results with a simulation example. The proofs of the results presented in Section 3 and 4 will be given in a full version of the paper.

2. PRELIMINARIES

Throughout the paper the following notation is used: I_n denotes the $n \times n$ identity matrix. Let A be a quadratic matrix. If A is positive definite, it is denoted $A > 0$, and $A < 0$, if A is negative definite. The weighted norm of a vector $\|x\|_W$ is defined as $\sqrt{x^\top W x}$ and the unweighted norm $\|x\|$ is defined as $\sqrt{x^\top x}$, respectively. $A \otimes B$ denotes the Kronecker product of two matrices A and B.

2.1 Communication graphs

In this section we briefly review some definitions on communication graphs. We use a directed, unweighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ to describe the interconnections between the individual agents. $\mathcal{V} = \{v_1, ..., v_N\}$ is the set of vertices, where $v_k \in \mathcal{V}$ represents the k-th agent. $\mathcal{E} \subset V \times V$ is the set of edges, which models information flow, i.e. the k-th agent receives information from the j-th agent if and only if $(v_j, v_k) \in \mathcal{E}$. $\mathcal{A}$ is the adjacency matrix, which encodes the edges, where $a_{kj} = 1$ if $(v_j, v_k) \in \mathcal{E}$ and $a_{kj} = 0$ otherwise. A *path* from vertex v_{i_1} to vertex v_{i_l} is a sequence of vertices $\{v_{i_1}, ..., v_{i_l}\}$ such that $\{v_{i_j}, v_{i_{(j+1)}}\} \in \mathcal{E}, j = 1, ..., l-1$. Moreover, the incidence matrix is the $|\mathcal{E}| \times N$ matrix encoding the edges, where $E_{ij} = 1$, if vertex

j is the tail of edge $\mathcal{E}_i$ and $E_{ij} = -1$, if vertex j is the head of edge $\mathcal{E}_i$.

Definition 1. A directed graph $\mathcal{G}$ is *strongly connected*, if for every pair of vertices $v_k, v_j, k, j = 1, ..., N, j \neq k$, there exists a path from v_k to v_j.

Moreover, we use the definition of independent strongly connected components defined in (Wieland, 2010).

Definition 2. An *independent strongly connected component (iSCC)* of a directed graph $\mathcal{G}$ is a subgraph $\widetilde{\mathcal{G}} = (\widetilde{\mathcal{V}}, \widetilde{\mathcal{E}}, \widetilde{\mathcal{A}})$, which is strongly connected and satisfies $(v, \widetilde{v}) \notin \mathcal{E}$ for any $v \in \mathcal{V} \backslash \widetilde{V}$ and $\widetilde{v} \in \widetilde{V}$.

2.2 System model

We consider a group of N agents described by the differential equation

$$\dot{x}_k = A_k x_k + B_k u_k + \widetilde{B}_k \xi_k \qquad (1)$$

where $x_k \in \mathbb{R}^{n_k}$ is the agents state variable, $u_k \in \mathbb{R}^{m_k}$ is the agents control input, $\xi_k(t) \in \mathcal{L}_2[0, \infty)$ is a $\mathcal{L}_2$-integrable disturbance function. The agents are interconnected and the topology is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. It is assumed that the output $y_k = C_k x_k \in \mathbb{R}^r$ is not available to the agents as controller input. Instead, disturbed relative information of the form

$$z_{kj} = y_j - y_k + \omega \eta_{kj} \in \mathbb{R}^r$$

is available to the controller at node k, if and only if there is an edge (v_j, v_k), i.e. $a_{kj} = 1$. Here, $\eta_{kj}(t) \in \mathcal{L}_2[0, \infty)$ is also a $\mathcal{L}_2$-integrable disturbance function and $\omega \in \mathbb{R}$ is a positive weight. Let p_k, q_k denote the in-degree and out-degree of vertex v_k, respectively. For all $k = 1, ..., N$, $z^{(k)\top} = \begin{bmatrix} z_{kj_1}^\top & ... & z_{kj_{p_k}}^\top \end{bmatrix}$ is defined as the vector of all available measurements at node k and $\eta^{(k)\top} = \begin{bmatrix} \eta_{kj_1}^\top & ... & \eta_{kj_{p_k}}^\top \end{bmatrix}$ the corresponding measurement disturbance vector.

In this paper, we assume that the communication topology of the network is identical to the measurement topology, i.e., agent k is able to receive information from agent j through communication, if and only if it measures the relative output z_{kj}.

With the stacked vector $x = [x_1^\top, ..., x_N^\top]^\top$, the global system can be written as

$$\dot{x} = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} B_1 & & \\ & \ddots & \\ & & B_N \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}$$
$$+ \begin{bmatrix} \widetilde{B}_1 & & \\ & \ddots & \\ & & \widetilde{B}_N \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_N \end{bmatrix}. \qquad (2)$$

The global output is the vector of stacked $z^{(k)}$, which is

$$z = (E \otimes I_r) \begin{bmatrix} C_1 & & \\ & \ddots & \\ & & C_N \end{bmatrix} x + \omega \eta, \qquad (3)$$

where E is the incidence matrix corresponding to the graph $\mathcal{G}$ and η is the vector of stacked $\eta^{(k)}$.
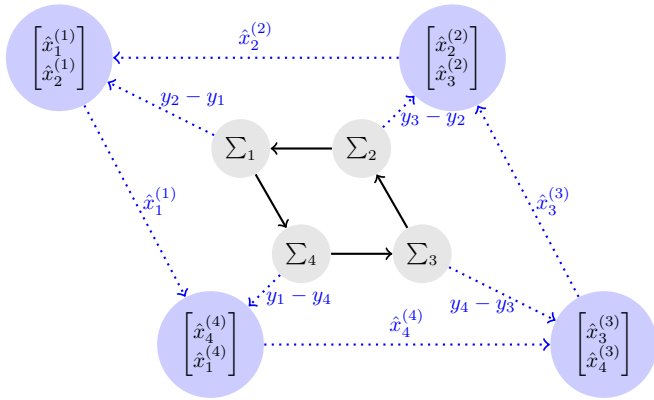
Fig. 1. Example for the estimator structure. The inner graph represents the agents and the measurement topology, the outer graph represents the estimators and the communication topology.

3. COOPERATIVE ESTIMATION DESIGN

For the sake of estimator design, in this section, it is assumed that $u_k = 0$ for all $k = 1, ..., N$.

3.1 Proposed estimation algorithm

At every agent k, a local estimator is implemented that estimates its own state x_k and the states of its neighbors, i.e., x_j with $a_{kj} = 1$. For this purpose, it is assumed that the estimator at agent k knows the models of its neighbors. The vector of local estimates is defined as $\hat{x}^{(k)\top} = \begin{bmatrix} \hat{x}_k^{(k)\top} & \hat{x}_{j_1}^{(k)\top} & ... & \hat{x}_{j_{p_k}}^{(k)\top} \end{bmatrix} \in \mathbb{R}^{\sigma_k}$ and the corresponding error vector is

$$e^{(k)} = x^{(k)} - \hat{x}^{(k)} = \begin{bmatrix} x_k - \hat{x}_k^{(k)} \\ x_{j_1} - \hat{x}_{j_1}^{(k)} \\ \vdots \\ x_{j_{p_k}} - \hat{x}_{j_{p_k}}^{(k)} \end{bmatrix} \in \mathbb{R}^{\sigma_k},$$

where σ_k is the dimension of the local estimator state. The estimator architecture can be depicted as shown in Figure 3.1, with a cyclic topology as an example.

The estimator dynamics are proposed as

$$\begin{aligned} \dot{\hat{x}}^{(k)} =& A^{(k)}\hat{x}^{(k)} + L^{(k)}(z^{(k)} - C^{(k)}\hat{x}^{(k)}) \\ &+ K^{(k)} \sum_{j=1}^{N} a_{kj}(M_j^{(k)}\hat{x}_j^{(j)} - N_j^{(k)}\hat{x}^{(k)}) \end{aligned} \quad (4)$$

with initial condition $\hat{x}_0^{(k)} = 0$. The filter gains to be designed are $L^{(k)} \in \mathbb{R}^{\sigma_k \times rp_k}$ and $K^{(k)} \in \mathbb{R}^{\sigma_k \times \sigma_k}$. $A^{(k)}$ is the block-diagonal matrix with $A_k, A_{k_1}, ..., A_{k_{p_k}}$ on its diagonal and $C^{(k)}$ is the output matrix

$$C^{(k)} = \begin{bmatrix} -C_k & C_{j_1} & ... & 0 \\ -C_k & 0 & \ddots & 0 \\ -C_k & 0 & 0 & C_{j_{p_k}} \end{bmatrix}.$$

$M_j^{(k)} \in \mathbb{R}^{\sigma_k \times n_j}$ is a matrix of the form $M_j^{(k)\top} = \begin{bmatrix} 0 & I_{n_j} & 0 \end{bmatrix}$ and $N_j^{(k)} \in \mathbb{R}^{\sigma_k \times \sigma_k}$ is a diagonal matrix such that

$$M_j^{(k)}\hat{x}_j^{(j)} - N_j^{(k)}\hat{x}^{(k)} = \begin{bmatrix} 0 \\ \hat{x}_j^{(j)} - \hat{x}_j^{(k)} \\ 0 \end{bmatrix}.$$

We can now formulate the design problem of this section.

Problem 1: Determine estimator gains $L^{(k)}$, $K^{(k)}$ in (4) such that the following two properties are satisfied simultaneously.

(i) In the absence of model and measurement disturbances (i.e., when $\xi_k = 0$, $\eta = 0$), the estimation errors decay so that $e^{(k)} \to 0$ exponentially for all $k = 1, ..., N$.

(ii) The estimators (4) provide guaranteed $\mathcal{H}_\infty$ performance in the sense that

$$\begin{aligned} &\sum_{k=1}^{N} \int_0^\infty e^{(k)\top} W^{(k)} e^{(k)} dt \\ &\leq \gamma^2 \sum_{k=1}^{N} \int_0^\infty (\|\xi^{(k)}\|^2 + \|\eta^{(k)}\|^2)dt + I_0, \end{aligned} \quad (5)$$

for $\xi^{(k)\top} = \begin{bmatrix} \xi_k^\top & \xi_{j_1}^\top & ... & \xi_{j_{p_k}}^\top \end{bmatrix}$ and a positive definite matrix $P^{(k)}$. In (5), $W^{(k)}$ is a positive semi-definite weighting matrix and $I_0 = \sum_{k=1}^{N} x_0^{(k)\top} P^{(k)} x_0^{(k)}$ is the cost due to the observer's uncertainty about the initial conditions of the agents.

The weighting matrix $W^{(k)}$ is a design parameter and can be chosen as needed. As it will turn out in Chapter 4, a specific choice of $W^{(k)}$ is needed in order to guarantee synchronization performance.

3.2 Filter gains design

We define the matrices

$$\begin{aligned} Q^{(k)} =& P^{(k)}A + A^\top P^{(k)} - G^{(k)}C^{(k)} - (G^{(k)}C^{(k)})^\top \\ &- F^{(k)} \sum_{j=1}^{N} a_{kj} N_j^{(k)} - (F^{(k)} \sum_{j=1}^{N} a_{kj} N_j^{(k)})^\top \\ &+ \alpha P^{(k)} + q_k \pi_k \begin{bmatrix} P_{11}^{(k)} & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\widetilde{B}^{(k)} = \begin{bmatrix} \widetilde{B}_k & 0 & 0 & 0 \\ 0 & \widetilde{B}_{j_1} & ... & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \widetilde{B}_{j_{p_k}} \end{bmatrix},$$

where $P^{(k)} \in \mathbb{R}^{\sigma_k \times \sigma_k}$ is a symmetric, positive definite matrix and $P_{11}^{(k)} \in \mathbb{R}^{n_k \times n_k}$ is the top-left submatrix of $P^{(k)}$. π_k and α are positive constants which will later play the role of design parameters. With these definitions, we are ready to present our main result.

Theorem 1. Consider a group of N interconnected agents described by (1), (2), (3). Let a collection of matrices $F^{(k)}$, $G^{(k)}$ and $P^{(k)}$, $k = 1, ..., N$, be a solution of the LMIs

$$
\begin{bmatrix}
Q^{(k)}+W^{(k)} & -\omega G^{(k)} & P^{(k)}\widetilde{B}^{(k)} & F^{(k)}M_{j_1}^{(k)} & \cdots & F^{(k)}M_{j_{p_k}}^{(k)} \\
-\omega G^{(k)\top} & -\gamma^2 I & 0 & 0 & 0 & 0 \\
(P^{(k)}\widetilde{B}^{(k)})^\top & 0 & -\gamma^2 I & 0 & 0 & 0 \\
(F^{(k)}M_{j_1}^{(k)})^\top & 0 & 0 & -\pi_{j_1}P_{11}^{(j_1)} & 0 & 0 \\
\vdots & 0 & 0 & 0 & \ddots & 0 \\
(F^{(k)}M_{j_{p_k}}^{(k)})^\top & 0 & 0 & 0 & 0 & -\pi_{j_{p_k}}P_{11}^{(j_{p_k})}
\end{bmatrix}
$$
$$
<0 \tag{6}
$$

for all $k = 1, ..., N$, then Problem 1 admits a solution of the form

$$
\begin{aligned}
L^{(k)} &= (P^{(k)})^{-1}G^{(k)} \\
K^{(k)} &= (P^{(k)})^{-1}F^{(k)}.
\end{aligned} \tag{7}
$$

Remark 1. The choice of α determines the convergence speed of the estimators, where a larger α enforces faster convergence of the estimates. □

Remark 2. In the case when $u_k \not\equiv 0$, the control inputs also have to be added to the estimator dynamics, i.e. equation (4) has to be modified as follows

$$
\begin{aligned}
\dot{\hat{x}}^{(k)} =& A^{(k)}\hat{x}^{(k)} + B^{(k)}u^{(k)} + L^{(k)}(z^{(k)} - C^{(k)}\hat{x}^{(k)}) \\
&+ K^{(k)}\sum_{j=1}^{N} a_{kj}(M_j^{(k)}\hat{x}_j^{(j)} - N_j^{(k)}\hat{x}^{(k)}),
\end{aligned}
$$

where

$$
B^{(k)} = \begin{bmatrix}
B_k & 0 & 0 & 0 \\
0 & B_{j_1} & \cdots & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & B_{j_{p_k}}
\end{bmatrix}, u^{(k)} = \begin{bmatrix}
u_k \\
u_{j1} \\
\vdots \\
u_{j_{p_k}}
\end{bmatrix}.
$$
□

As it can be seen from the LMIs (6), the design problem has to be solved in a centralized manner. However, this calculation is only done a priori. The resulting cooperative estimators (4) are local and their complexity only increases with the number of neighbors, not with the total size of the network. In contrast, a direct application of the algorithms developed in (Olfati-Saber, 2005), (Olfati-saber, 2006) and (Ugrinovskii and Langbort, 2011), (Ugrinovskii, 2011), to the problem considered here would result in the order of the estimators growing with the size of the network. Therefore, the method presented in this paper is scalable and guarantees $\mathcal{H}_\infty$ performance.

3.3 Detectability conditions

Feasibility of the LMIs (6) is a sufficient condition for Problem 1 to have a solution, but it is not a necessary condition. However, simulations show that infeasibilty of (6) often relates to detectability issues. Clearly, in order for the estimation algorithm to work, the global system (2) with the output matrix (3) has to be detectable. Moreover, as a stricter necessary condition, we have the following proposition:

Proposition 2. Let $\widetilde{\mathcal{G}}$ be an iSCC of $\mathcal{G}$ and let the vertices in $\widetilde{\mathcal{G}}$ be $v_1, ..., v_\rho$, $\rho \leq N$, without loss of generality. Let $\widetilde{E}$ be the component incidence matrix, which only considers the edge set $\widetilde{\mathcal{E}}$.

Then detectability of the system

$$
\begin{aligned}
\dot{\widetilde{x}} &= \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_\rho \end{bmatrix} \widetilde{x} \\
y &= \left(\widetilde{E} \otimes I_r\right) \begin{bmatrix} C_1 & & \\ & \ddots & \\ & & C_\rho \end{bmatrix} \widetilde{x},
\end{aligned} \tag{8}
$$

with $\widetilde{x} = [x_1^\top, ..., x_\rho^\top]^\top$ is a necessary condition for the existence of a solution to Problem 1.

Remark 3. Detectability of the iSCC can be well analyzed using the theory presented in Zelazo and Mesbahi (2008).

4. SYNCHRONIZATION USING RELATIVE IMPERFECT MEASUREMENTS

The cooperative estimator design introduced above can be combined with various synchronization methods. In this section we study the combination of a synchronization algorithm presented in (Wieland et al., 2011) with the observer design from Section 3. This results in a synchronization scheme in which local controllers rely on state estimates obtained from imperfect relative output information. We also analyze disturbance attenuation properties of such controllers.

The controller is proposed in the following form:

$$
\dot{\zeta}_k = S\zeta_k + \sum_{j=1}^{N} a_{kj}(\zeta_j - \zeta_k) \tag{9}
$$

$$
u_k = \Lambda_k\zeta_k + H_k(\hat{x}_k^{(k)} - \Pi_k\zeta_k), \tag{10}
$$

where the matrices $S \in \mathbb{R}^{\nu \times \nu}, R \in \mathbb{R}^{r \times \nu}, \Pi_k \in \mathbb{R}^{n_k \times \nu}$ and $\Lambda_k \in \mathbb{R}^{m_k \times \nu}$ are obtained from the Francis equations

$$
\begin{aligned}
A_k\Pi_k + B_k\Lambda_k &= \Pi_k S \\
C_k\Pi_k &= \Gamma,
\end{aligned} \tag{11}
$$

where $k = 1, ..., N$ and ν is a positive integer which determines the dimension of the internal model for each controller. It is shown in (Wieland, 2010) that under mild assumptions on the closed loop systems, solvability of (11) with (S, Γ) being observable is a necessary condition for synchronization of the unperturbed systems (1). Therefore the assumption of the existence of a solution to (11) is not a conservative assumption.

In the following, the term $\epsilon_k = x_k - \Pi_k\zeta_k$ will be called the local regulation error. As this error is the quantity which (10) regulates to zero, our performance analysis will focus on this characteristic of the system.

Problem 2: Find $H_k, k = 1, ..., N$, such that the following two properties are satisfied simultaneously.

(i) Exact synchronization is achieved in the absence of disturbance and measurement disturbances, i.e. when $\xi_k = 0$, $\eta = 0$, then $y_j - y_k \to 0$ for all $j, k = 1, ..., N$.

(ii) $\mathcal{H}_\infty$ performance with respect to the regulation error is achieved in the sense that

$$\sum_{k=1}^{N} \int_0^\infty \epsilon_k^\top R_k \epsilon_k dt \le \sum_{k=1}^{N} \int_0^\infty (\kappa^2 \|\xi_k\|^2 + \theta^2 \|\eta^{(k)}\|^2) dt$$

$$+ \sum_{k=1}^{N} \epsilon_{k,0}^\top X_k \epsilon_{k,0} + I_0 + \mu^2 \sum_{k=1}^{N} \int_0^\infty \|\sum_{j=1}^{N} a_{kj}(\zeta_j - \zeta_k)\|^2 dt$$

$$(12)$$

with positive definite weighting matrices R_k and X_k, for $k = 1, ..., N$, and real parameters $\mu, \kappa, \theta > 0$ to be defined later.

The last term of (12) characterizes the effect of the uncertainty that the controller for agent k has in relation to the initial states of the internal models of its neighbours. This term has to be included into the performance inequality because the internal system (9) is neither affected by the estimators, nor by the controllers (10). Therefore, large initial mismatches between ζ_k, $k = 1, ..., N$ will generate significant transient dynamics which will affect output regulation error and controller performance. In the case when ζ_k, $k = 1, ..., N$, are initialized identically, this term vanishes.

Theorem 3. Consider an interconnected system consisting of N LTI systems (1), with the communication topology represented by a connected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ and the dynamic local controllers given as (9), (10). Suppose the following conditions hold:

(1) The Francis equations (11) have a solution (S, Γ) which is an observable pair, with $\sigma(S) \subset j\mathbb{R}$.
(2) There exist constants $\mu > 0$ and $\lambda > 0$ such that for each $k = 1, \ldots, N$, the algebraic Riccati equation

$$X_k A_k + A_k^\top X_k + R_k$$
$$- X_k \left(\frac{1}{\lambda^2} B_k B_k^\top - \frac{1}{\mu^2} \widetilde{B}_k \widetilde{B}_k^\top - \frac{1}{\mu^2} \Pi_k \Pi_k^\top \right) X_k = 0$$

$$(13)$$

has a positive definite symmetric solution X_k such that $A_k - \frac{1}{\lambda^2} B_k B_k' X_k$ is a Hurwitz matrix.
(3) For a given $\gamma > 0$, Problem 1 with the weighting matrices

$$W^{(k)} = \begin{bmatrix} \frac{1}{\lambda^2} X_k B_k B_k^\top X_k & 0 \\ 0 & 0 \end{bmatrix} \quad (14)$$

has a solution.

Then the collection of matrices H_k defined as

$$H_k = -\frac{1}{\lambda^2} B_k^\top X_k \quad (15)$$

solves Problem 2.

Performance bound (12) may be conservative since in this paper we did not optimize performance of the closed loop system. Improving this bound as well as analysis of other synchronization control schemes will be the subject of our future research.

5. SIMULATION EXAMPLE

To illustrate the cooperative estimators (4) we take four agents connected in a cyclic topology, corresponding to Figure 3.1. The four agents' dynamics are described by (1), where the matrices are

(a) Estimator of agent 1   (b) Estimator of agent 2
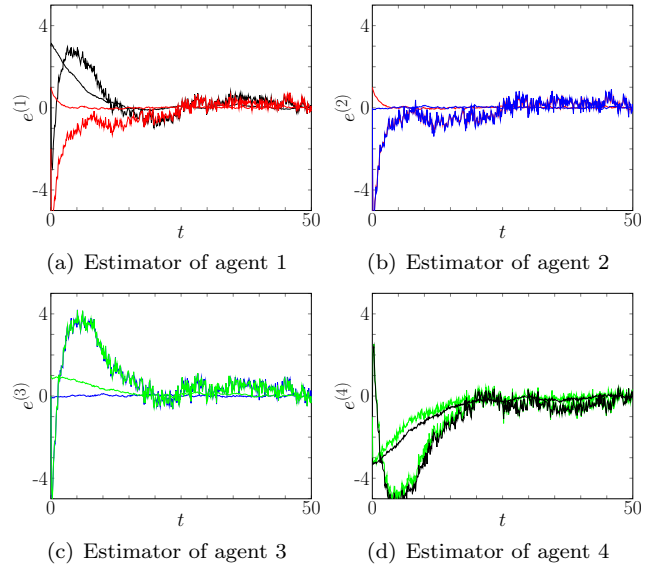
(c) Estimator of agent 3   (d) Estimator of agent 4

Fig. 2. Plots of the local estimation errors. Black lines represent the estimation error of agent 1. Red, blue, and green lines represent the estimation error of agent 2, 3 and 4, respectively.

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0.1 & 1 \\ 0 & -1 \end{bmatrix} \quad A_4 = \begin{bmatrix} 0.1 & 1 \\ 0 & 0 \end{bmatrix}.$$

The input and output matrices are given as

$$B_k = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \widetilde{B}_k = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \quad C_k = [1 \ 0] \quad \omega = 0.1$$

for all $k = 1, 2, 3, 4$. Note that for all k, the pair $(A^{(k)}, C^{(k)})$ is not detectable. When we consider the estimation problem only, i.e. we design the local estimators (4) by solving the LMIs (6), we achieve a performance bound of $\gamma = 5.61$. The parameters are chosen as

$$W_k = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} \quad \alpha = 0.1 \quad \pi_k = 0.025.$$

Simulations of the local estimators are shown in Figure 5.

To put the performance into perspective: The centralized H_∞-estimator for system (2) with output (3) achieves a performance of $\gamma = 5.19$.

Next, we consider the synchronization problem. For the given systems, the Francis Equations (11) are solvable with the matrices

$$S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \Gamma = [1 \ 0]$$

$$\Pi_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ \Pi_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ \Pi_3 = \begin{bmatrix} 1 & 0 \\ -0.1 & 1 \end{bmatrix} \ \Pi_4 = \begin{bmatrix} 1 & 0 \\ -0.1 & 1 \end{bmatrix}$$
$$\Lambda_1 = [0 \ 0] \quad \Lambda_2 = [0 \ 1] \quad \Lambda_3 = [-0.1 \ 0.9] \quad \Lambda_4 = [0 \ -0.1].$$

The algebraic Ricatti-equation (13) with $R_k = I_2$ for $k = 1, 2, 3, 4$ has a solution for $\mu = 1.2$ and $\lambda = 0.1$. Then, we take the resulting weights W_k given by (14) and solve the LMIs (6) for the estimators. Using this design method, we achieve performance bounds of $\kappa = 11.50$ and $\theta = 11.44$.

6. CONCLUSION

In this paper, we presented an LMI-based solution to the design problem of local estimators, which estimate local state variables within an heterogeneous multi-agent system. The dimension of the local estimators do not grow with the number of agents in the network. In particular, every local estimator may deal with a local system that is even not detectable, as cooperation between the local estimators establishes detectability. Moreover, the presented algorithm guarantees $\mathcal{H}_\infty$-performance for both the estimators and for synchronization. Further work will include the application to other synchronization algorithms, time-varying topologies and separate measurement and communication topologies.

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