Global Sensitivity Analysis of the Optimal Capacity of Intersections

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Abstract: While the problem of optimizing traffic light phases dates back to the fifties, in the daily practice the optimal solutions are often still determined by means of deterministic approaches that assume to know exactly the incoming traffic flows and the other intersection parameters. In this framework, the problem of parameter uncertainty is normally neglected, and the flow variability is only taken into account by the responsive traffic light plans. In this paper, a hybrid model of traffic light dynamics, and a global sensitivity analysis approach are proposed with the aim of providing a methodology for evaluating the performance robustness of different traffic light optimization approaches with respect to parameters uncertainty and variability. To this end, two well known solution approaches are compared and discussed by means of a real world case study.

Keywords: Modeling and simulation of transportation systems; Automatic control, optimization, real-time operations in transportation

1. INTRODUCTION

The problem of optimizing traffic light phases dates back to the fifties (Webster [1958]). Since there, different solutions and strategies (Papageorgiou et al. [2003]), have been proposed for large scale networks. Valuable examples are TRANSYT (Robertson [1969]) and SCOOT (Hunt et al. [1982]), which were characterized by limited traffic-responsive capabilities, or OPAC Gartner [1983], PRO-DYN (Henry et al. [1984]) and RHODES (Mirchandani and Head [2001]) which, on the contrary, implemented traffic-response strategies.

However, in the daily practice the optimal solutions are often still determined by means of deterministic approaches that neglect the problem of parameter uncertainty. In addition, in the common practice, the deterministic design approaches proposed by Webster [1958], Allsop [1971, 1976], Improta and Cantarella [1984] for isolated intersections, are still widely used.

In parallel, it has been developed the general theory of hybrid systems, i.e., of those systems characterized by two kinds of states: *normal states* whose variation is governed by a set of differential equations, and *macro-states* whose change is driven by the occurrence of particular conditions or external events (Lygeros et al. [2003]). In this framework, urban transportation networks and intersections, together with the relevant traffic lights, can be suitably modeled as hybrid systems with macro-states consisting of different set of flows enabled to cross the intersection, and different traffic light signals. The macro-state transitions

are driven by the changes of traffic light signals (Febbraro and Sacco [2014], Basile et al. [2012], Kim et al. [2008].

In this paper, a framework for evaluating the sensitivity of the performances of two common optimization approaches is defined, based on the simulations of a hybrid model of signalized intersections. In particular, the basic sensitivity analysis performed in Febbraro and Sacco [2014] is enhanced via the so called *global sensitivity analysis* proposed in Saltelli et al. [2010]. As regards the signal settings optimization models considered in this paper, they consists of the SIGCAP model (Allsop [1976]) and the Phases' Lengths and Sequence (PLS) model (Improta and Cantarella [1984]).

The main result of this paper results to be the definition of a framework for evaluating the effects of the uncertainty of intersection parameters on the optimal performance. In doing so, it is worth saying that the considered approach is not *local*, being able, on the contrary, to consider all the variability range of the uncertain parameters, without approximations (Wainwright et al. [2014]). By means of the results of this sensitivity analysis, it is possible to identify those parameter whose variability should be explicitly taken into account in the statement of traffic light optimization problems.

The paper is organized as follows: after a brief recall of the considered traffic light optimization problems (Sec. 2) and of the traffic light hybrid model (Sec. 3), the global sensitivity analysis approach is introduced (Sec. 4). Then, the proposed sensitivity analysis is applied to a real world
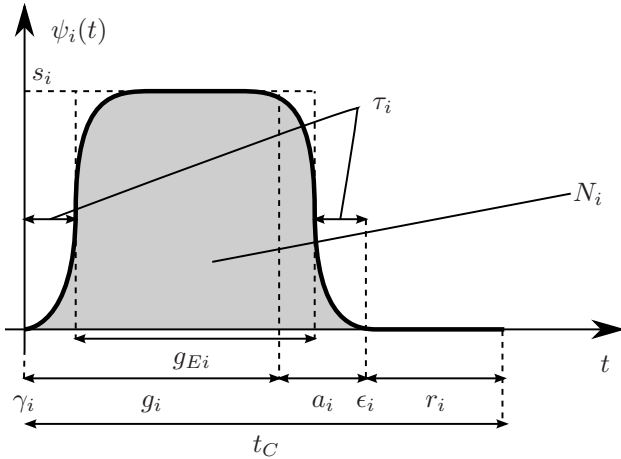
Fig. 1. Shape of the flow departing from the queue at the access i.

case study consisting of a real world intersection in the Italian city of Benevento (Sec. 5).

2. TRAFFIC LIGHT OPTIMIZATION MODELS

In this section, the considered optimization models for isolated intersection are briefly recalled. To this aim, consider the shape of a generic flow $\psi_i(t)$ departing from the queue at the access i depicted in Fig. 1. With reference to such a figure, let:

- s_i be the saturation flow of the access i, that is, the maximum number of vehicles that can cross the intersection per unit time;
- g_i, a_i and r_i be the durations of the green, amber, and red signals for the access i;
- $t_C = g_i + a_i + r_i$ be the traffic light cycle time;
- γ_i and ϵ_i be the starting time of the green signal and the ending time of the amber signal for the flow departing from the access i, respectively;
- N_i be the total number of vehicles leaving the queue at the access i, during a whole traffic light cycle;
- τ_i be the so-called *lost time* at the access i, representing the departing inertia of vehicles which do not move instantaneously at the beginning of the green signal, and the stopping inertia of vehicles which do not stop instantaneously at the beginning of the amber signal;
- $g_{Ei} = g_i + a_i - \tau_i$ is the so-called effective green.

In addition, f_i is the is the instantaneous input flow arriving at the access i, p_k, $k = 1, 2, \ldots, K$, is the length of the kth phase of the traffic light, and Δ is the *phase matrix*, whose generic element $\delta_{i,k}$ is equal 1 if the flow at access i is enabled to cross the intersection during phase k, and 0 otherwise. With this definition, it is possible to state

$$g_i + a_i = \sum_{k=1}^{K} \delta_{i,k} p_k, \quad \forall i. \tag{1}$$

Finally, let $\mathbf{M}$ be the *compatibility matrix*, whose generic element $m_{i,j}$ is equal 1 if both the flows at accesses i and j can safely cross the intersection at the same time, and 0 otherwise.

As regards the intersection performances, consider the *intersection capacity* defined as

$$\xi = \min_i \{\xi_i\} = \min_i \left\{ \frac{s_i g_{Ei}}{f_i t_C} \right\}, \tag{2}$$

that is, the minimum among the ratios between the number of vehicles departing from access i during the green/amber phase and the number of vehicles arriving at i during a cycle, computed for all the accesses.

In the following, two particular capacity maximization problems are described. In doing so, for the sake of simplicity, only the fixed cycle time formulation are considered, although it is easy to extend the proposed sensitivity analysis to other more general optimization problems.

To conclude, it is worth underlining that the optimal timings are computed by setting the incoming flows, the saturation flows, and the lost times to their nominal value. Therefore, these models provide nominal values of the optimal capacities.

2.1 Capacity Maximization (SIGCAP)

The first considered optimization model consists of the maximization of the intersection capacity given the phase matrix Δ. The Linear Programming (LP) formulation of this problem is

$$\max_{p_k} \min_i \xi_i$$
$$s.t. \ t_C = \sum_k p_k$$
$$g_{Ei} = \sum_k \delta_{i,k} p_k - \tau_i \ \forall i \tag{3}$$
$$\xi_i = \frac{s_i g_{Ei}}{f_i t_C} \qquad \forall i$$
$$g_{Ei} \geq 0 \qquad \forall i$$

where the phase lengths p_k, $k = 1, 2, \ldots, K$ are the optimization variables. Such a problem, known as SIGCAP, has been proposed in Allsop [1976].

2.2 Phases' Lengths and Sequence (PLS) Optimization

The second considered problem consists of the generalization of the problem in Eqs. (3), by means of the further optimization of the sequence of the phases (Improta and Cantarella [1984]). Considering two consecutive traffic light cycles are considered at a time, the Linear Integer Programming (LIP) formulation of this problem results to be

$$\max_{p_k} \ \min_i \xi_i$$

$$s.t. \ g_{Ei} = \epsilon_i - \gamma_i - \tau_i \qquad \forall i$$

$$\begin{cases} \epsilon_j \le \gamma_i + M\omega_{i,j} \\ \epsilon_i \le \gamma_j + M(1 - \omega_{i,j}) \end{cases} \quad \forall i,j | m_{i,j} = 0$$

$$\xi_i = \frac{s_i g_{Ei}}{f_i t_C} \qquad \forall i \qquad (4)$$

$$g_{Ei} \ge 0 \qquad \forall i$$

$$\epsilon_i \ge \gamma_i \qquad \forall i$$

$$\epsilon_i \ge 0 \qquad \forall i$$

$$\omega_{i,j} \in \{0,1\} \qquad \forall i$$

$$-t_c \le \gamma_i \le t_C \qquad \forall i$$

where the optimization variables are the starting time (γ_i) and ending time (ϵ_i) of the interval during which the access i can cross the intersection, $\forall i$. In this formulation, M is an high constant representing infinite, and $\omega_{i,j}$ are binary "switching" variables associated to the null elements of the compatibility matrix. If $\omega_{i,j} = 1$ (resp., $\omega_{i,j} = 0$), the access i is enabled before (resp., after) the incompatible access j.

3. THE SIMULATION MODEL

In this section, after a brief introduction on hybrid systems, the model of the hybrid dynamics of a generic intersection is described.

3.1 Basics on Discrete Event and Hybrid Systems

Hybrid Systems (HS) can be thought of as the most general systems, gathering ordinary Time Driven Systems (TDS), and Discrete Event Systems (DES) at a time.

In this framework, on one hand, TDS are the well-known systems whose state variables assume real numeric values, and whose dynamics is described by differential equations. The dynamics of flows along road stretches is a valuable example of TDS.

On the other hand, DES can be defined as discrete-state event-driven systems where the evolution of the state variables, which are not necessarily numeric, depends entirely on the occurrence of asynchronous, sometimes stochastically predictable, events. In DES, events can be identified with specific actions (for instance, somebody presses a button), with spontaneous occurrences dictated by nature (for instance, a computer goes down for whatever reason too difficult to figure out), or with the results of several conditions which are suddenly all met (for instance, the fluid in a tank exceeds a given value).

Traffic lights are valuable examples of DES, being characterized by discrete states corresponding to the different "colors" shown to queues at the accesses of the signalized intersection. The transitions among these states are instantaneous and, in general, asynchronous, especially when traffic lights perform responsive plans, i.e., when the lengths of the phases are changed continuously to optimize the traffic behavior, based on real time traffic measures.
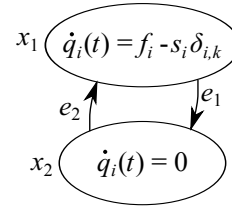

Fig. 2. State diagram with time-driven dynamics of a single access.

In conclusion, HS are those systems gathering different time-driven dynamics, each represented by a different set of differential equations associated with a single state macro-state; when the macro-state changes due to the occurrence of an event, the time-driven dynamics results to be represented by a different set of equations. In the following section, it will be shown the representation of intersection dynamics via a HS model.

3.2 Hybrid Model of an Isolated Intersection

The queue dynamics at a generic access i of an intersection, during the generic phase k of the traffic light, is modeled by the equation

$$\dot{q}_i(t) = \begin{cases} f_i - s_i \delta_{i,k} & \text{if } q_i(t) > 0 \\ 0 & \text{if } q_i(t) = 0 \end{cases} \quad \forall i \qquad (5)$$

and can be represented by means of the state diagram in Fig. 2, where::

- e_1 is the event representing the reaching of the condition $q(t) = 0$, when $\delta_{i,k} = 1$;
- e_2 is the green-to-red switch event of the signal of the access i, when $q_i(t) = 0$.

Note that not all these events are feasible in all the states: in particular, the event e_1 can occur only when the access i is enabled and $q_i(t) > 0$, whereas e_2 can occur only if $q_i(t) = 0$. Moreover, e_2 can occur only at fixed instants determined by the signal plan, whereas the event e_1 can occur at a variable time depending on the values of f_i, s_i, and $q_i(t)$.

As regards the model of the whole intersection in Fig. 3, the relevant model has been already introduced in Febbraro and Sacco [2014], where the complete state diagram is also reported and discussed.

By means of this model, it is possible to compute the samples that feed the sensitivity indexes computation described in the following section.

4. SENSITIVITY ANALYSIS

As mentioned, the problems described in Sec. 2.1 and Sec. 2.2 assume that all the intersection parameters are perfectly known and constant. Nevertheless, in real world, they are not. In fact:

- the incoming flows f_i are estimated by measures or by assignment processes. In any case, while the mean values are generally considered in the traffic light optimization, such flows vary continuously;
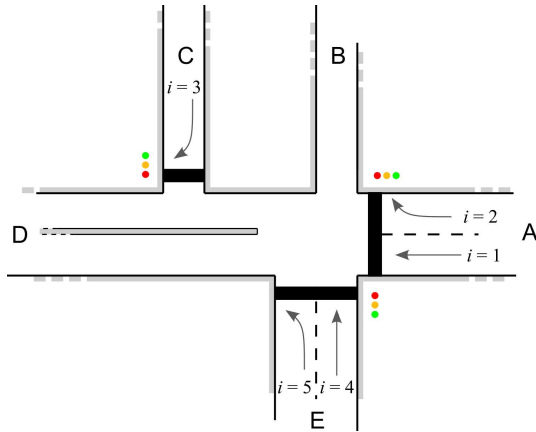
Fig. 3. An example of intersection in the city of Benevento, Italy.

- the saturation flows s_i mainly depend on the geometrical layout of the intersection and on the composition of the queues (i.e., on the kind of vehicles), that can not be perfectly forecast;
- the lost times τ_i depend on the geometrical layout of the intersection and on the "aggressiveness" of drivers. Hence they are difficult to determine and are, in general, variable.

The target of the following analysis is to asses the importance of the incoming flows, the saturation flows, and the lost times (hereafter generally indicated as *input parameters*), on the intersection capacity. In this analysis, it is assumed that the traffic light timing is a-priori determined via the optimization problems in Sec. 2.1 and Sec. 2.2, whose solution has been computed by considering the nominal values of the input parameters.

In the next section, the definition of the considered indexes are briefly recalled. For a further comprehension of such indexes, the reader may refer to Saltelli et al. [2010], where the theoretical background is defined, or to Wainwright et al. [2014] where a comparison between different indexes is performed.

4.1 Indexes definition

Consider the intersection capacity ξ in Eq. (2). The following analysis aims at understanding what happens to ξ when an input variable is set to a specific value, whereas the others are free to vary randomly. To fix the ideas, assume that the incoming flow at access i is fixed, that is, $f_i = \overline{f}_i$, whereas the other incoming parameters are considered to be stochastic variables.

Then, let $\sigma^2_{\sim f_i}(\xi|\overline{f}_i)$ be the *residual variance* of ξ computed with respect to the variability of all the parameters except f_i. Obviously, such a variance is conditioned by the particular value $\overline{f}_i$ assumed by the flow f_i.

Moreover, let $\sigma^2(\xi)$ be the *total variance* of ξ, computed when all the inputs can vary. With these definitions, $\sigma^2_{\sim f_i}(\xi|\overline{f}_i)$ is expected to be as smaller than $\sigma^2(\xi)$ as bigger is the contribution of variability of f_i on the variability of ξ. In other words, fixing the parameter that affects the capacity the most, greatly reduces the variance of the capacity.

To avoid the dependence of the variance $\sigma^2_{\sim f_i}(\xi|\overline{f}_i)$ on the specific value $\overline{f}_i$, the relevant mean $\mathrm{E}_{f_i}[\sigma^2_{\sim f_i}(\xi|\overline{f}_i)]$, computed over all the admissible values of f_i, is considered in place of it. Again, the more significant is contribution of the variability of the flow f_i on the variability of the capacity, the smaller is the expectation $\mathrm{E}_{f_i}[\sigma^2_{\sim f_i}(\xi|\overline{f}_i)]$, since it results to be the mean of only small (positive) values.

Furthermore, since

$$\sigma^2(\xi) = \mathrm{E}_{f_i}[\sigma^2_{\sim f_i}(\xi|\overline{f}_i)] + \sigma^2_{f_i}(\mathrm{E}_{\sim f_i}[\xi|\overline{f}_i]), \qquad (6)$$

it turns out that for f_i to be an important factor on the variability of the capacity, the term $\mathrm{E}_{f_i}[\sigma^2_{\sim i}(\xi|f_i = \overline{f}_i)]$ has to be small and, consequently, $\sigma^2_{f_i}(\mathrm{E}_{\sim f_i}[\xi|\overline{f}_i])$ tends to coincide with the total variance $\sigma^2(\xi)$.

Therefore, is it possible to define the first-order sensitivity index of f_i as

$$S_{f_i} = \frac{\sigma^2_{f_i}(\mathrm{E}_{\sim f_i}[\xi|\overline{f}_i])}{\sigma^2(\xi)} \in [0,1]. \qquad (7)$$

which is always positive and as closer to 1 as greater is the influence of f_i on the capacity ξ.

Then, while the index in Eq. (7) considers only the first order influence of the variable f_i, in many cases the effects of the variability of such a parameter may be *hidden* in other parameters, thus resulting into second, or higher, order effects. In other words, a low value of S_i does not necessarily imply that the corresponding parameter has scarce effect on the output variance. In fact, since it might considerably contribute to the total output variance, by means of its combination with the other parameters.

To tackle with this problem, consider the *total* sensitivity index

$$ST_{f_i} = 1 - \frac{\sigma^2_{\sim f_i}(\mathrm{E}_{f_i}[\xi| \sim f_i])}{\sigma^2(\xi)} \qquad (8)$$

which provides the sum of the effects of any order in which the factor f_i is involved, is also considered. In Eq.(8), the term $\mathrm{E}_{f_i}[\xi| \sim f_i]$ is the mean (over all the values of f_i) of the values of the capacity obtained fixing f_i and allowing all the input parameters to vary, and $\sigma^2_{\sim f_i}(\mathrm{E}_{f_i}[\xi| \sim f_i])$ is the relevant variance.

The more ξ is sensitive to f_i, the more $\mathrm{E}_{f_i}[\xi| \sim f_i]$ and $\sigma^2_{\sim f_i}(\mathrm{E}_{f_i}[\xi| \sim f_i])$ are small. Hence, in this case, the total sensitivity index ST_{f_i} tends to 1.

To conclude, it is worth saying that the numerical values of these indexes can be computed by means of approximated formulas based on N repeated simulations that suitably provide samples for applying the sample mean and sample variance expressions, as explained in Saltelli et al. [2010].

5. CASE STUDY

In this section, the application of the sensitivity analysis to a real world case study is described. To this end, consider the scheme depicted in Fig. 3 representing a real world

Table 1. Parameters characterizing the intersection in Fig. 3.

O/D pairs	A/B	A/D	C/D	E/B	E/D
Index	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
$\overline{f}_i$ (veh/hour)	127	142	13	391	440
$\overline{s}_i$ (veh/hour)	1200	1200	1200	1200	1200
τ_i (s)	3	3	3	3	3

Table 2. Green time for the accesses of the intersection in Fig. 3.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
SIGCAP	11.29 s	11.29 s	28.71	28.71 s	28.71 s
PLS	11.29 s	10.42 s	28.71 s	25.84 s	28.71 s

Table 3. Capacities of the accesses of the intersection in Fig. 3. The whole intersection capacity is put into evidence in bold.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
SIGCAP	**1.69**	1.9596	59.32	1.97	**1.69**
PLS	**1.75**	**1.75**	59.3181	**1.75**	**1.75**

intersection in the city of Benevento, Italy. The relevant nominal incoming flows $\overline{f}_i$ have been determined by means of an assignment process on the whole transportation network of Benevento, and are reported in Tab. 1. Moreover, the nominal saturation flows $\overline{s}_i$ have been determined by ad-hoc measures. As regards the traffic light plans, they have been computed via the aforementioned SIGCAP and PLS optimization models. Then, the relevant phase and compatibility matrices are

$$\Delta_{fix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{M} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}. \quad (9)$$

In this framework, Δ_{fix} is the fixed matrix considered in the implementation of the SIGCAP optimization problem, whereas the optimal phase matrix determined with the PLS problem results to be

$$\Delta_{opt} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad (10)$$

Then, for what concerns the optimal timings, the solutions of the problems in Sec. 2.1 and Sec. 2.2, computed assuming $t_C = 40$ s, i reported in Tab. 2 and Tab. 3.

As regards the input data for the sensitivity analysis, it is worth saying that:

- the traffic flows f_i arriving at the incoming direction i, $\forall i$, are assumed to be stochastic variables with uniform distribution $\mathcal{U}_i(0.7\overline{f}_i, 1.3\overline{f}_i)$, $\forall i$;
- the saturation flows s_i of the incoming direction i, are assumed to be stochastic variables with uniform distribution $\mathcal{U}_i(0.7\overline{s}_i, 1.3\overline{s}_i)$, $\forall i$;
- the lost times τ_i of the incoming direction i, are assumed to be stochastic variables with uniform distribution $\mathcal{U}_i(0.7\overline{\tau}_i, 1.3\overline{\tau}_i)$, $\forall i$.
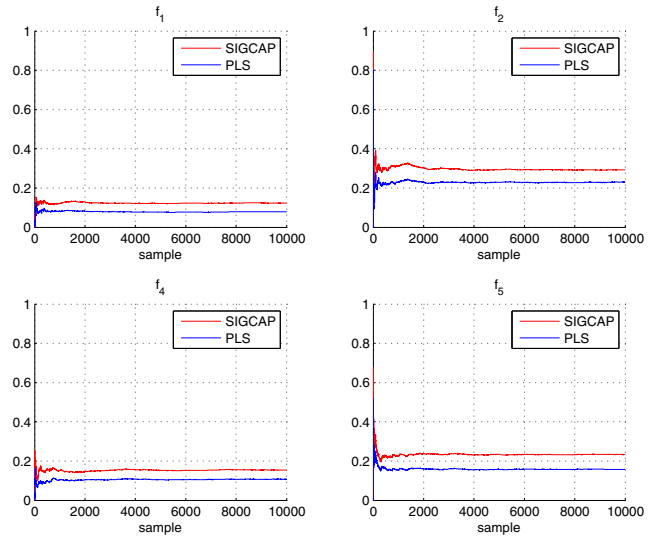

Fig. 4. Total sensitivity of incoming flows.

5.1 Numerical Results

As regards in numerical results, consider the graph depicted in Fig. 4 and Fig. 5, and the values reported Tab. 4 and Tab. 5. Note that, due to the negligible incoming flow at the access $i = 3$, the relevant indexes results to be null. Such a phenomenon depends of the fact that the capacity of the whole intersection never coincides with the one of access $i = 3$, which is very high (Tab. 3). In other words, the intersection capacity is insensitive to the uncertainty of the parameters characterizing such an access. For this reason, the relevant graphs are not reported.

As regards the depicted graphs, they shows the shapes of the total sensitivity indexes of the incoming flows, computed with respect of the number of simulated samples. It is easy to note that, three thousand samples can be enough for computing stable indexes, allowing to consider the Nth sample as representative of the relevant indexes. Note that similar results are obtained for the all the computed sensitivity indexes.

For what concerns the index values, in Tab. 4 and Tab. 5 it is possible to note that:

- the variability of the lost time is negligible in all the cases. This is the reason why the relevant graphs are not reported;
- in general, the sensitivity indexes computed with the PLS optimal solution are better than those computed with the SIGCAP optimal solution;
- the access $i = 2$ is characterized by the highest indexed in all the cases;
- the indexes are greater for the accesses with higher traffic flows ($i = 2$, $i = 5$).

As a general comment, it is worth saying that the total sensitivity indexes of incoming and saturation flows are relatively high, meaning that the relevant variability should be explicitly considered in the optimization problem statement. On the contrary, the possibility of neglecting the lost times can allow a significant reduction of the optimization variables in stochastic programming formulations of capacity optimization.

Table 4. Sensitivity index (in bold the highest index of each parameter).

	f_1	f_2	f_3	f_4	f_5	s_1	s_2	s_3	s_4	s_5	τ_1	τ_2	τ_3	τ_4	τ_5
SIGCAP	0.012	**0.148**	0	0.048	0.07	0.058	**0.154**	0	0.056	0.119	0.008	**0.011**	0	0.003	0.003
PLS	0.014	**0.095** (-35%)	0	0.027	0.027	0.032	**0.097** (-37%)	0	0.031	0.063	0.007	**0.0009** (-18%)	0	0.002	0.00

Table 5. Total sensitivity index (in bold the highest index of each parameter).

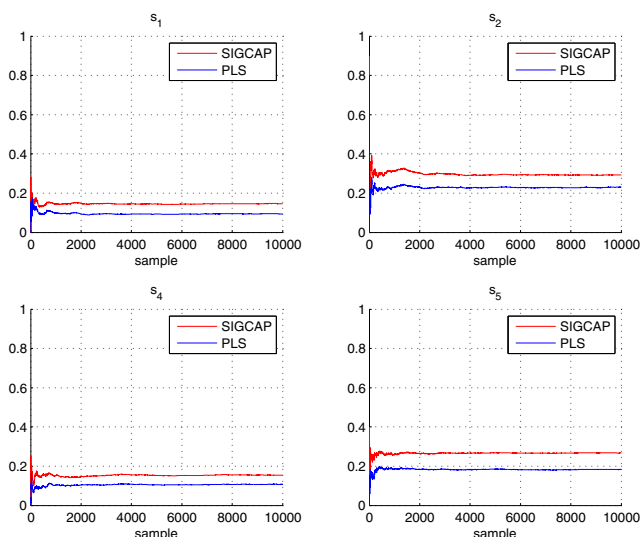|	f_1	f_2	f_3	f_4	f_5	s_1	s_2	s_3	s_4	s_5	τ_1	τ_2	τ_3	τ_4	τ_5
SIGCAP	0.124	**0.293**	0	0.154	0.234	0.145	**0.342**	0	0.185	0.268	0.022	**0.057**	0	0.004	0.004
PLS	0.078	**0.231** (-21%)	0	0.108	0.158	0.094	**0.279** (-18%)	0	0.137	0.183	0.015	**0.039** (-31%)	0	0.002	0.003

Fig. 5. Total sensitivity of saturation flows

6. CONCLUSIONS

In this paper, a hybrid model of signalized intersection has been introduced and used for evaluating, via simulation, the performances and the relevant sensitivity of two well-known optimization approaches. The performed analysis shows that the PLS optimization approach provides, in general, the best results in terms of both performance and robustness. In addition, it has been shown how the proposed modeling and analysis framework can be suitably used for identifying those parameters that have to be determined with high accuracy or explicitly considered in the optimization problems, and those whose uncertainty is negligible. In this connection, to tackle with the parameter uncertainty and variability, work is in progress to reformulate the considered optimization problems in a suitable stochastic stochastic programming framework.

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