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Adaptive Leader-following Control of Second-order Multi-agent Systems ⋆

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Abstract: In this paper, a leader-following problem is considered for a group of autonomous agents when a second-order multi-agent system is used to describe the network dynamics. Both the dynamics of the leader and the followers are subjected to unknown disturbances, which are represented by linearly parameterized models. The relative position measurements and the relative velocity measurements are utilized as the new state variables to yield a new distributed system. When both the two relative measurements are available for followers, we design an adaptive tracking control for each follower, which needs no more global information about the bounds of the unknown disturbances. Moreover, analysis is also made of the stability of the tracking error system and the parameter convergence. A numerical example is finally given to illustrate the good performance of the proposed adaptive tracking control.

Keywords: Leader-following, multi-agent systems, adaptive tracking control, parameter convergence.

1. INTRODUCTION

Recent years have witnessed a dramatic increase in the design and analysis of cooperative control of multi-agent systems. In particular, the leader-following coordination is an important control strategy among many others, which involves different forms of multi-agent dynamics, such as homogeneous or heterogeneous dynamics, linear or nonlinear systems. As is well known, the dynamics of multi-agent systems with a first-order integrator or a general linear system is very important in real applications and theoretical studies (see Hong,Hu,&Gao (2006); Qin,Zheng,&Gao (2011); Cao,et al. (2013); Shi,Johansson,&Hong (2013); Yu,et al. (2013), etc.). However, it is also well known that the control design and analysis of a first-order multi-agent system rely directly on the Laplacian matrix while the control gain matrix of a multi-agent system with a general linear system depends heavily on a Riccatti equation or some linear matrix inequality. On the other hand, a specific control design of a second-order multi-agent system is used widely in practical cases, and it can be provided explicitly without solving any matrix equations. But it is often hard to construct a Lyapunov function in the stability analysis, which makes the synthesis of a second-order multi-agent system much challenging.

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Distributed estimation plays a key role in consensus control of multi-agent systems with unknown disturbances or partial measurements. When unknown disturbances are existent in the coupling networks or the agent dynamics, some adaptive control laws have been proposed to deal with the consensus problem effectively. For example, when the global information of the coupling network is unavailable, some adaptive strategies were designed for the coupling strengths between agents in Liu,et al. (2013); Li,et al. (2013). When the agent dynamics suffers from unknown disturbance, decentralized adaptive designs were proposed to reconstruct a prescribed reference velocity in Bai,Arcak,&Wen (2008) and to track the reference velocity by using both relative position and velocity measurements in Bai,Arcak,&Wen (2009). An adaptive synchronization control was designed for a first-order nonlinear leader-follower system having unknown dynamics in Das,&Lewis (2010). An adaptive design was presented to solve a leader-following problem with unknown dynamics and jointly connected network in Yu,&Xia (2012). When both the information of the leader is unavailable and the unknown disturbances exist in the agent dynamics, an adaptive control together with an "observer"-based estimator was designed prudently to ensure the consensus tracking in Hu,&Zheng (2014).

In this paper, we consider an adaptive tracking control design for a second-order leader-follower system with non-identical unknown nonlinear dynamics and switching interconnection topologies. When the agent dynamics is subjected to a bounded unknown disturbance and only the position information of the leader can be measured, a distributed "observer" was firstly designed, in Hong,Hu,&Gao (2006); Hong,Chen,&Bushnell (2008), to estimate the velocity of the leader and then a tracking control was given for each follower by using the relative position measurement and the velocity estimate. However, it was assumed therein that the input of the leader is a common policy known by all followers and at the same time, the velocity of each follower can be measured. This paper extends the works Hong,Hu,&Gao (2006); Hong,Chen,&Bushnell (2008) in two directions. On one hand, assume that the input of the leader and the disturbances of the follower dynamics are unknown, but can be parameterized by some basis functions. Then each follower estimates the unknown parameters by using decentralized adaptive laws, which are built upon the relative measurements in the neighborhood. Based on the adaptive estimation laws, furthermore, if both the relative position measurements and the relative velocity measurements are available, an adaptive tracking controller is designed for every follower. On the other hand, not only are the tracking errors between the followers and the leader guaranteed to converge to zero by employing the proposed adaptive tracking controls, but also the estimation convergence of the unknown parameters can be ensured under a persistent excitation condition on the regressor matrix. Note that the global uniform convergence analyses for both the tracking errors and the parameter estimation errors are much more difficult in the switching topology case than the fixed topology case which was considered in the adaptive control designs in Bai,Arcak,&Wen (2008, 2009); Das,&Lewis (2010).

2. PROBLEM FORMULATION

2.1 Some Preliminaries

Some terminologies in algebraic graph theory are first introduced.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a simple undirected graph with a set of vertices $\mathcal{V} = \{1, \cdots, N\}$ and a set of edges $E \subseteq \mathcal{V} \times \mathcal{V}$. Graph $\mathcal{G}$ is used to describe the interconnection topology of N followers in a leader-follower multi-agent system. In this case, vertex $i \in \mathcal{V}$ represents follower i, and edge (i, j) is in $\mathcal{E}$ if and only if follower i and follower j can exchange information with each other. Meanwhile, follower i and follower j are called neighbors and, accordingly, $\mathcal{N}_i = \{j \mid (i, j) \in \mathcal{E}\}$ denotes the neighbor set of follower i. The weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ of graph $\mathcal{G}$ is a nonnegative matrix with zeros on its diagonal, that is, $a_{ii} = 0$ and $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$. For simplicity, A is assumed to be a $(0, 1)$-matrix in this paper. A path between vertex i and vertex j is a sequence $i_0 \triangleq i, i_1, \cdots, i_{\kappa-1}, i_\kappa \triangleq j$ of distinct vertexes such that $(i_{q-1}, i_q) \in \mathcal{E}$ for $q = 1, \cdots, \kappa$. Graph $\mathcal{G}$ is connected if there exists a path between every two distinct vertices; otherwise, $\mathcal{G}$ has some distinct components, namely, maximal subgraphs of $\mathcal{G}$.

A degree matrix of graph $\mathcal{G}$ is defined by a diagonal matrix $D = \mathrm{diag}(d_1, \cdots, d_n) \in \mathbb{R}^{N \times N}$, and a new matrix

$$L = D - A$$

is called Laplacian matrix. For an undirected graph $\mathcal{G}$, the Laplacian matrix L is symmetric and has the following well-known algebraic properties.

Lemma 1. The Laplacian matrix L of graph $\mathcal{G}$ has at least one zero eigenvalue associated with eigenvector $\mathbf{1} = \mathrm{col}(1, \cdots, 1) \in \mathbb{R}^N$, and all the non-zero eigenvalues are positive. L has a simple zero eigenvalue if and only if graph $\mathcal{G}$ is connected. Furthermore, the number of components of $\mathcal{G}$ equals the multiplicity of the zero eigenvalue of L.

We introduce another graph $\boldsymbol{\mathcal{G}} = (\boldsymbol{\mathcal{V}}, \boldsymbol{\mathcal{E}})$ to describe the interconnection topology of a leader-follower multi-agent system. In this graph, $\boldsymbol{\mathcal{V}} = \{0, 1, \cdots, N\}$, $\boldsymbol{\mathcal{E}} \subseteq \boldsymbol{\mathcal{V}} \times \boldsymbol{\mathcal{V}}$, and $\mathcal{E} \subset \boldsymbol{\mathcal{E}}$. The vertex 0 represents the self-active leader. Obviously, $\mathcal{G}$ is a subgraph of $\boldsymbol{\mathcal{G}}$. The interconnection between followers is undirected. However, since the leader is assumed self-active and hence moving independently, the edge between vertex i and vertex 0 is unidirectional in $\boldsymbol{\mathcal{G}}$, that is, follower i can receive information from the leader while the leader needs no information from any follower. The incidence relationship between follower i and the single leader is denoted by b_i, which is assigned 1 whenever there is an arc starting at the vertex 0 and ending at the vertex i (i.e., the leader is the neighbor of follower i) and 0 otherwise. A leader adjacency matrix B is defined as a diagonal matrix with diagonal elements b_i for $i = 1, \cdots, N$. We say that "$\boldsymbol{\mathcal{G}}$ is connected" if at least one vertex in each component of $\mathcal{G}$ is connected to vertex 0.

In practice, the relationships among neighboring agents may vary over time, and their interconnection topology $\boldsymbol{\mathcal{G}}$ may also be dynamically changing. Suppose that $\boldsymbol{\mathcal{G}}(t)$ is switching according to a piecewise constant signal $\sigma : [t_0, \infty) \to \mathcal{P} = \{1, 2, \cdots, M\}$, where M is the total number of all possible topologies $\boldsymbol{\mathcal{G}}_{\sigma(t)}$ ($\sigma(t) \in \mathcal{P}$). There is an infinite sequence of bounded, non-overlapping, continuous time-intervals $[t_s, t_{s+1})$ $(s = 0, 1, \cdots)$ with $t_0 = 0, t_{s+1} - t_s \geq \tau$ for some constant $\tau > 0$. The topology $\boldsymbol{\mathcal{G}}_{\sigma(t)}$ is time-invariant during each interval $[t_s, t_{s+1})$.

Define a new matrix $H_p = L_p + B_p$ $(p \in \mathcal{P})$ for the switched graph $\boldsymbol{\mathcal{G}}_{\sigma(t)}$. For each p, let $\lambda_{1,p}, \cdots, \lambda_{N,p}$ be the eigenvalues of H_p, and $\lambda_{\min}(H_p)$ and $\lambda_{\max}(H_p)$ denote the smallest and the largest nonzero eigenvalue of H_p, respectively. Then we have the following lemma (Hong,Hu,&Gao (2006)).

Lemma 2. The matrix H_p has nonnegative eigenvalues. H_p is symmetric positive definite if and only if graph $\boldsymbol{\mathcal{G}}_p$ is connected across $[t_s, t_{s+1})$ $(s = 0, 1, \cdots)$.

From Lemma 2 and the fact that the index set $\mathcal{P}$ is finite, the following two numbers

$$\begin{aligned} \mu_{\min} &= \min\{\lambda_{\min}(H_p) \mid p \in \mathcal{P}\}, \\ \mu_{\max} &= \max\{\lambda_{\max}(H_p) \mid p \in \mathcal{P}\}, \end{aligned} \tag{1}$$

are positive constants.

In order to derive the main results of the paper, the following lemmas are needed.

Lemma 3. (Schur Complement Lemma) Let S be a symmetric matrix partitioned into blocks:

$$S = \begin{pmatrix} A & E \\ E^T & C \end{pmatrix},$$

where both A and C are symmetric and square. Then the following properties are equivalent:

- S is positive definite.
- Both A and its the Schur complement $C - E^T A^{-1} E$ are positive definite.

Lemma 4. (Barbalat's Lemma (Popov (1973))) If a function $f(t)$ is uniformly continuous and $\lim_{t\to\infty}\int_0^t f(s)ds$ exists and is finite, then $\lim_{t\to\infty} f(t)=0$.

2.2 Leader-following with Unknown Disturbances

In this paper, a leader-follower system consists of a self-active leader with double integrator

$$\begin{cases}\dot{x}_0(t)=v_0(t),\\ \dot{v}_0(t)=a_0(t),\end{cases} \tag{2}$$

and N followers also with double integrators

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t)+\delta_i(t),\end{cases} \quad i=1,\cdots,N, \tag{3}$$

where $x_i(t), v_i(t)\in\mathbb{R}$ represent, respectively, the position and velocity of agent i ($i=0,1,\cdots,N$), $a_0(t)$ is an unknown acceleration of the leader, $u_i(t)$ is the control input to follower i, $\delta_i(t)$ is an unknown time-varying disturbance of follower i and generally depends on both $x_i(t)$ and $v_i(t)$, which are omitted in the disturbance for notational simplicity.

In the design of consensus tracking control, all followers are largely dependent upon some kind of relative measurements from the neighbors, and such relative measurements are usually divided into two classes:

- The relative position measurement
$$\xi_i(t)=\sum_{j\in\mathcal{N}_i(t)} a_{ij}\big(x_i(t)-x_j(t)\big)+b_i\big(x_i(t)-x_0(t)\big), \tag{4}$$
where $i=1,\cdots,N$, and $\mathcal{N}_i(t)$ denotes the index set of the neighbors of follower i at time t.
- The relative velocity measurement
$$\zeta_i(t)=\sum_{j\in\mathcal{N}_i(t)} a_{ij}\big(v_i(t)-v_j(t)\big)+b_i\big(v_i(t)-v_0(t)\big), \tag{5}$$
for $i=1,\cdots,N$.

The objective of this paper is to design a consensus tracking control together with adaptive laws for each follower by means of both the relative measurements (4) and (5). For the leader-follower system (2)–(3), the leader-following problem is called resolved, provided that for each follower $i\in\{1,\cdots,N\}$,

$$\lim_{t\to\infty}\big(x_i(t)-x_0(t)\big)=0, \quad \lim_{t\to\infty}\big(v_i(t)-v_0(t)\big)=0.$$

3. MAIN RESULTS

In this section, we first consider designing an adaptive tracking control for the second-order leader-follower system (2)–(3) by using relative measurements, and then analyze the convergence of consensus tracking errors with the help of a common Lyapunov function method.

3.1 Adaptive Tracking Control Design

Since the acceleration $a_0(t)$ and the disturbances $\delta_i(t)$ ($i=1,\cdots,N$) are unknown, the techniques in classical adaptive control (Marino,&Tomei (1995)) and multi-agent systems (Bai,Arcak,&Wen (2008); Yu,&Xia (2012)) are utilized to parameterize them as follows

$$a_0(t)=\phi_0^T(t)\omega_0, \tag{6}$$

and

$$\delta_i(t)=\phi_i^T(t)\omega_i, \tag{7}$$

where $\phi_0(t),\phi_i(t)\in\mathbb{R}^m$ are basis function vectors and $\omega_0,\omega_i\in\mathbb{R}^m$ are unknown constant parameter vectors to be estimated.

Let the parameter vectors ω_0,ω_i be estimated by follower i as $\hat{\omega}_{i0}(t),\hat{\omega}_i(t)\in\mathbb{R}^m$ and the functions $a_0(t),\delta_i(t)$ be estimated by follower i as $a_i(t),\hat{\delta}_i(t)$. This gives rise to

$$a_i(t)=\phi_0^T(t)\hat{\omega}_{i0}(t), \tag{8}$$

and

$$\hat{\delta}_i(t)=\phi_i^T(t)\hat{\omega}_i(t), \tag{9}$$

for $i=1,\cdots,N$.

The following transformed system with disturbances results from differentiating the two relative measurements $\xi_i(t)$ and $\zeta_i(t)$:

$$\begin{cases}\dot{\xi}_i(t)=\zeta_i(t),\\ \dot{\zeta}_i(t)=\sum\limits_{j\in\mathcal{N}_i(t)} a_{ij}\big(u_i(t)-u_j(t)+\delta_i(t)-\delta_j(t)\big)\\ \qquad\qquad +b_i\big(u_i(t)+\delta_i(t)-a_0(t)\big).\end{cases} \tag{10}$$

Introduce two tracking error vectors $\bar{x}(t)=x(t)-x_0(t)\mathbf{1}$ and $\bar{v}(t)=v(t)-v_0(t)\mathbf{1}$, where $x(t)$ and $v(t)$ are the concatenations $\mathrm{col}\{x_1(t),\cdots,x_N(t)\}$ and $\mathrm{col}\{v_1(t),\cdots,v_N(t)\}$, respectively. Then the two relative measurements (4) and (5) are representable respectively in the following vector form as

$$\xi(t)=H_p\bar{x}(t), \tag{11}$$

and

$$\zeta(t)=H_p\bar{v}(t). \tag{12}$$

So the system (10) takes the following compact form

$$\begin{cases}\dot{\xi}(t)=\zeta(t),\\ \dot{\zeta}(t)=H_p[u(t)-a_0(t)\mathbf{1}+\delta(t)],\end{cases} \tag{13}$$

where $H_p=L_p+B_p$ ($p\in\mathcal{P}$), $u(t)=\mathrm{col}\{u_1(t),\cdots,u_N(t)\}\in\mathbb{R}^N$, $\delta(t)=\mathrm{col}\{\delta_1(t),\cdots,\delta_N(t)\}\in\mathbb{R}^N$.

The following lemma is needed for describing the relationships between the states $\xi(t),\zeta(t)$ and $\bar{x}(t),\bar{v}(t)$, and its proof follows straightforwardly from Lemma 2.

Lemma 5. If the switched interconnection topologies $\boldsymbol{\mathcal{G}}_p$ are connected during each time-interval $[t_s,t_{s+1})$, then the following statements are equivalent:

- $\lim_{t\to\infty}\xi(t)=0$, $\lim_{t\to\infty}\zeta(t)=0$.
- Consensus tracking is achieved, i.e., $\lim_{t\to\infty}\bar{x}(t)=0$, $\lim_{t\to\infty}\bar{v}(t)=0$.

Assume that the two relative measurements $\xi(t)$ and $\zeta(t)$, given by (4) and (5) respectively, are available for all followers. Then, for the transformed system (10) or (13), adaptive laws and a tracking control can be designed with the relative measurements $\xi(t)$ and $\zeta(t)$.

Since the parameter vectors ω_0,ω_i are unknown in the linearly parameterized models of $a_0(t)$ and $\delta_i(t)$ ($i=1,\cdots,N$), we develop two decentralized adaptive laws below:

$$\begin{aligned}\dot{\hat{\omega}}_{i0}(t) =& -\frac{k}{l}\phi_0(t)\Big[\sum_{j\in\mathcal{N}_i(t)} a_{ij}\big(\xi_i(t)-\xi_j(t)\big)+b_i\xi_i(t)\Big]\\ &-\phi_0(t)\Big[\sum_{j\in\mathcal{N}_i(t)} a_{ij}\big(\zeta_i(t)-\zeta_j(t)\big)+b_i\zeta_i(t)\Big],\end{aligned} \tag{14}$$

and

$$\begin{aligned}\dot{\hat{\omega}}_{i}(t) =& \frac{k}{l}\phi_i(t)\Big[\sum_{j\in\mathcal{N}_i(t)} a_{ij}\big(\xi_i(t)-\xi_j(t)\big)+b_i\xi_i(t)\Big]\\ &+\phi_i(t)\Big[\sum_{j\in\mathcal{N}_i(t)} a_{ij}\big(\zeta_i(t)-\zeta_j(t)\big)+b_i\zeta_i(t)\Big],\end{aligned} \tag{15}$$

where $i = 1, \cdots, N$, and the constants l and k are two positive gains to be designed.

Finally, based on the adaptive laws (14) and (15), a tracking control is designed for each follower i as

$$u_i(t) = -k\xi_i(t) - l\zeta_i(t) + \phi_0^T(t)\hat{\omega}_{0i}(t) - \phi_i^T(t)\hat{\omega}_i(t). \tag{16}$$

3.2 Convergence Analysis

For the basis functions $\phi_0(t), \phi_i(t)$, we define two matrices $\Phi_0(t) = I_N \otimes \phi_0(t) \in \mathbb{R}^{mN\times N}$, $\Phi(t) = \mathrm{diag}\{\phi_1(t), \cdots, \phi_N(t)\} \in \mathbb{R}^{mN\times N}$. Let $\bar{\Omega}_0 = \mathrm{col}\{\hat{\omega}_{10}-\omega_0, \cdots, \hat{\omega}_{N0}-\omega_0\} \in \mathbb{R}^{mN}$ and $\bar{\Omega} = \mathrm{col}\{\hat{\omega}_1-\omega_1, \cdots, \hat{\omega}_N-\omega_N\} \in \mathbb{R}^{mN}$. Applying the control (16) together with the adaptive laws (14) and (15) to the system (10) yields the closed-loop system

$$\begin{cases}\dot{\xi}(t) = \zeta(t),\\ \dot{\zeta}(t) = -kH_p\xi(t) - lH_p\zeta(t) + H_p\Phi_0^T(t)\bar{\Omega}_0(t)\\ \qquad -H_p\Phi^T(t)\bar{\Omega}(t),\\ \dot{\bar{\Omega}}_0(t) = -\dfrac{k}{l}\Phi_0(t)H_p\xi(t) - \Phi_0(t)H_p\zeta(t),\\ \dot{\bar{\Omega}}(t) = \dfrac{k}{l}\Phi H_p(t)\xi(t) + \Phi(t)H_p\zeta(t).\end{cases}$$

Let $\epsilon(t) = \mathrm{col}\{\xi(t), \zeta(t)\}$. We have

$$\dot{\epsilon}(t) = F_p\epsilon(t) + \Delta_1(t), \tag{17}$$

where $F_p = \begin{pmatrix} 0 & I_N\\ -kH_p & -lH_p\end{pmatrix}$, $\Delta_1(t) = \mathrm{col}\Big(0, H_p\Phi_0^T(t)\bar{\Omega}_0(t) - H_p\Phi^T(t)\bar{\Omega}(t)\Big)$, I_N is an N-dimensional identity matrix, and 0 denotes a zero matrix of appropriate dimension.

To proceed further, we make an additional assumption that the regressor matrix $\Phi_r(t) = \begin{pmatrix}\Phi_0(t)\\ \Phi(t)\end{pmatrix} \in \mathbb{R}^{2mN\times N}$ is persistently exciting (PE) (Marino,&Tomei (1995)) in the sense that there exist two positive reals T and α, such that

$$\int_t^{t+T} \Phi_r(s)\Phi_r^T(s)ds \geq \alpha I_N, \quad \forall t. \tag{18}$$

The PE condition is used to ensure the parameter convergence, i.e., $\lim_{t\to\infty}\big(\hat{\omega}_{i0}(t)-\omega_0(t)\big) = 0$, $\lim_{t\to\infty}\big(\hat{\omega}_i(t)-\omega_i(t)\big) = 0$, for any initial condition $\hat{\omega}_{i0}(0), \hat{\omega}_i(0)$, $i = 1, \cdots, N$.

We are now ready to state the main result about the stability of system (17).

Theorem 6. Consider the leader-follower system (2)–(3). Assume that the interconnection topology $\boldsymbol{\mathcal{G}}_{\sigma(t)}$ is connected across each time-interval $[t_s, t_{s+1})$ $(s = 0, 1, \cdots)$ and $\phi_i(t)$ $(i = 0, 1, \cdots, N)$ are uniformly bounded. Then the leader-following problem is resolved under the adaptive tracking control (16). Furthermore, if the PE condition (18) is satisfied and $\dot{\phi}_i(t)$ $(i = 0, 1, \cdots, N)$ are uniformly bounded, then the parameter estimation errors converge to zero.

Proof. A common Lyapunov candidate is defined for the system (17) as

$$V_1(t) = \epsilon^T(t)P_1\epsilon(t) + \bar{\Omega}_0^T(t)\bar{\Omega}_0(t) + \bar{\Omega}^T(t)\bar{\Omega}^T(t), \tag{19}$$

where $P_1 = \begin{pmatrix}\frac{2k^2}{l^2}I_N & \frac{k}{l}I_N\\ \frac{k}{l}I_N & I_N\end{pmatrix}$ is positive definite for positive constants l and k.

Firstly we show that $\dot{V}_1(t) \leq 0$ at any non-switching instants. Assume that the subsystem $p \in \mathcal{P}$ is active. The time derivative of $V_1(t)$ along the trajectory of the system (17) is given by

$$\begin{aligned}\dot{V}_1(t) =& \epsilon^T(t)[P_1F_p + F_p^TP_1]\epsilon(t) + 2\epsilon^T(t)P_1\Delta_1(t)\\ &+ 2\bar{\Omega}_0^T(t)\dot{\bar{\Omega}}_0(t) + 2\bar{\Omega}^T(t)\dot{\bar{\Omega}}^T(t).\end{aligned}$$

Applying the adaptive laws about $\bar{\Omega}_0(t)$ and $\bar{\Omega}(t)$, we have

$$\dot{V}_1(t) = -\epsilon^T(t)Q_p\epsilon(t),$$

where

$$\begin{aligned}Q_p =& -[P_1F_p + F_p^TP_1]\\ =& \begin{pmatrix}\frac{2k^2}{l}H_p & 2kH_p - \frac{2k^2}{l^2}I_N\\ 2kH_p - \frac{2k^2}{l^2}I_N & 2lH_p - \frac{2k}{l}I_N)\end{pmatrix}.\end{aligned}$$

By Lemma 2 and Lemma 3, one can see that the matrix H_p is positive definite and Q_p is also positive definite for $l > \sqrt{\frac{\max\{k,1\}}{\mu_{\min}}}$ and $k > 0$. Since the set $\mathcal{P}$ is finite, $\{\lambda_{\min}(Q_p) \mid p \in \mathcal{P}\}$ is also finite. Define $\beta_1 = \min\{\lambda_{\min}(Q_p) \mid p \in \mathcal{P}\}$. Thus, we have

$$\dot{V}_1(t) \leq -\beta_1\epsilon^T(t)\epsilon(t) \leq 0. \tag{20}$$

Therefore, $\lim_{t\to\infty} V_1(t) = V_1(\infty) \geq 0$ exists.

Secondly, we shall show that $\lim_{t\to\infty}\epsilon(t) = 0$. Consider the infinite sequence $\{V_1(t_s) \mid s = 0, 1, \cdots, \}$. From Cauchy convergence criteria, we have that, for any $c > 0$, there exists a positive number T_c such that $|V_1(t_{s+1}) - V_1(t_s)| < c$, $\forall s \geq T_c$. Equivalently, we have $|\int_{t_s}^{t_{s+1}}\dot{V}_1(t)dt| < c$ or $\int_{t_s}^{t_{s+1}}\dot{V}_1(t)dt > -c$. From (20), it follows that $\int_{t_s}^{t_s+\tau}\epsilon^T(t)\epsilon(t)dt < \int_{t_s}^{t_{s+1}}\epsilon^T(t)\epsilon(t)dt < \frac{c}{\beta_1}$, which implies that $\lim_{t\to\infty}\int_t^{t+\tau}\epsilon^T(t)\epsilon(t)dt = 0$. Moreover, from (20), $\epsilon(t), \bar{\Omega}_0(t)$ and $\bar{\Omega}(t)$ are uniformly bounded for any $t \geq 0$ and so is $\dot{\epsilon}(t)$ due to (17) and the assumption that $\phi_0(t), \phi_i(t)$ are uniformly bounded. Therefore, $\epsilon^T(t)\epsilon(t)$ is uniformly continuous. Using Barbalat's Lemma, we obtain $\lim_{t\to\infty}\epsilon^T(t)\epsilon(t) = 0$, then $\lim_{t\to\infty}\xi(t) = 0, \lim_{t\to\infty}\zeta(t) = 0$. Thus, from Lemma 5 it follows that $\lim_{t\to\infty}\big(x_i(t) - x_0(t)\big) = 0, \lim_{t\to\infty}\big(v_i(t) - v_0(t)\big) = 0$. This means that consensus tracking is reached.

Finally, we use a contradiction method to show the parameter convergence. Assume that for any $c > 0$, there exists $T_1 > 0$ such that

$$\|\bar{\Omega}_0\| > c, \|\bar{\Omega}\| > c, \ \forall t \geq T_1. \tag{21}$$

Without loss of generality, for the infinite sequence of time-intervals $[t_s, t_{s+1})$ $(s = 0, 1, \cdots)$, consider an infinite subsequence of time-intervals $[t_{s_n}, t_{s_{n+1}})$ $(n = 0, 1, \cdots)$ with identical length T. Define $\bar{\Omega}_E(t) = \mathrm{col}\{\bar{\Omega}_0(t), \bar{\Omega}(t)\}$ and a function

$$\Psi(t) = \frac{1}{2}\big[\bar{\Omega}_E^T(t+T)\bar{\Omega}_E(t+T) - \bar{\Omega}_E^T(t)\bar{\Omega}_E(t)\big]. \tag{22}$$

Because $\lim_{t\to\infty} \epsilon^T(t)\epsilon(t) = 0$, and $\lim_{t\to\infty} V_1(t) = V_1(\infty)$ exists, from $V_1(t) = \epsilon^T(t)P_1\epsilon(t) + \bar{\Omega}_E^T(t)\bar{\Omega}_E(t)$, we have $\lim_{t\to\infty} \bar{\Omega}_E^T(t)\bar{\Omega}_E(t) = V_1(\infty)$ and then $\lim_{t\to\infty} \Psi(t) = 0$ due to (22). So, $\forall c_\psi > 0$, there exists $T_{c_\psi} > 0$ such that

$$\|\Psi(t) - \Psi(t')\| < c_\psi, \ \forall t, t' > T_{c_\psi}. \tag{23}$$

Differentiating $\Psi(t)$ at time instant t_{s_n} yields

$$\begin{aligned}
\dot{\Psi}(t_{s_n}) &= \int_{t_{s_n}}^{t_{s_n}+T} \frac{d}{ds}\big[\bar{\Omega}_E^T(s)\dot{\bar{\Omega}}_E(s)\big]ds \\
&= \int_{t_{s_n}}^{t_{s_n}+T} \frac{d}{ds}\Big\{\big[-\bar{\Omega}_0^T(s)\Phi_0(s) + \bar{\Omega}^T(s)\Phi(s)\big]H_pR^T\epsilon(s)\Big\}ds \\
&= \int_{t_{s_n}}^{t_{s_n}+T} \Big\{\big[\epsilon^T RH_p\Phi_r^T\Phi_r - \bar{\Omega}_r^T\dot{\Phi}_r\big]H_pR^T \\
&\quad - \bar{\Omega}_r^T\Phi_rH_pR^TF_p\Big\}\epsilon ds - \int_{t_{s_n}}^{t_{s_n}+T} \bar{\Omega}_r^T\Phi_rH_p^2\Phi_r^T\bar{\Omega}_r ds \\
&= \rho_1 - \rho_2,
\end{aligned} \tag{24}$$

where ρ_1 and ρ_2 respectively denote the first and the second integrals at the righthand side of (24), $R = \mathrm{col}\{\frac{k}{l}, 1\} \otimes I_N$, $\bar{\Omega}_r = \mathrm{col}\{\bar{\Omega}_0, -\bar{\Omega}\}$.

The boundedness of $V_1(t)$ implies that $\epsilon(t), \bar{\Omega}_E(t)$ are also bounded. Thus, there exist $\mathcal{M}_e, \mathcal{M}_\Omega > 0$ such that $\|\epsilon(t)\| \leq \mathcal{M}_e, \|\bar{\Omega}_E(t)\| \leq M_\Omega$. Furthermore, as $\Phi_r(t)$ and $\dot{\Phi}_r(t)$ are assumed to be bounded by $\mathcal{M}_\Phi$ and $\mathcal{M}_{\dot{\Phi}}$, respectively, we get

$$\rho_1 \leq \mathcal{M}\int_{t_{s_n}}^{t_{s_n}+T} \|\epsilon(s)\|ds,$$

where $\mathcal{M} = \frac{\sqrt{l^2+k^2}}{l}\mu_{\max}\big[\mu_{\max}\mathcal{M}_e\mathcal{M}_\Phi^2 + \mathcal{M}_\Omega\mathcal{M}_{\dot{\Phi}} + \sqrt{(k^2+l^2)\mu_{\max}+1}\mathcal{M}_\Omega\mathcal{M}_\Phi\big]$. Since $\lim_{t\to\infty} e(t) = 0$, we obtain that $\forall c > 0$, there exists $T_2 > 0$ such that

$$\rho_1 \leq \frac{1}{2}\alpha\mu_{\min}^2 c^2, \ \forall t_{s_n} \geq T_2.$$

Now let us consider ρ_2. From the contradiction conclusion (21), we have $\|\bar{\Omega}_r\| > c$. From the PE condition (18), it follows that

$$\begin{aligned}
\rho_2 \geq& \mu_{\min}^2 \int_{t_{s_n}}^{t_{s_n}+T} \bar{\Omega}_r^T\Phi_r\Phi_r^T\bar{\Omega}_r ds \\
\geq& \mu_{\min}^2 c^2 \int_{t_{s_n}}^{t_{s_n}+T} \frac{\bar{\Omega}_r^T}{\|\bar{\Omega}_r\|}\Phi_r\Phi_r^T\frac{\bar{\Omega}_r}{\|\bar{\Omega}_r\|}ds \\
\geq& \alpha\mu_{\min}^2 c^2, \ \forall t_{s_n} \geq T_1.
\end{aligned}$$

Thus, $\dot{\Psi}(t_{s_n}) = \rho_1 - \rho_2 \leq -\frac{1}{2}\alpha\mu_{\min}^2 c^2$ for any $t_{s_n} \geq T_3$ with $T_3 = \max\{T_1, T_{c_\psi}, T_2\}$. Then from the sign-preserving theorem of continuous functions, there exists a time-interval $[t_{s_n}, t_{s_n} + \Delta T)$ with $t_{s_n} \geq T_3, \Delta T > 0$ such that

$$\dot{\Psi}(t) \leq -\frac{1}{2}\alpha\mu_{\min}^2 c^2, \quad \forall t \in [t_{s_n}, t_{s_n} + \Delta T).$$

Integrating $\dot{\Psi}(t)$ from t_{s_n} to $t_{s_n} + \Delta T$ and selecting $c_\psi = \frac{1}{2}\alpha\mu_{\min}^2 c^2\Delta T$ yields

$$\Psi(t_{s_n}) - \Psi(t_{s_n} + \Delta T) > c_\psi,$$

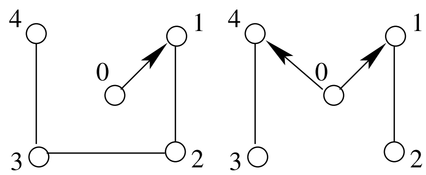
which contradicts (23). Therefore, the conjecture (21) is not true and the parameter convergence is guaranteed. Hence, the proof is complete. ∎

Remark 7. The results of Theorem 6 show that consensus tracking can be achieved without PE condition and some boundedness assumptions, which are introduced to ensure the parameter convergence. Meanwhile, much more prudence should be taken in the parameter convergence analysis when the interconnection topology is switching.

Remark 8. From Theorem 6, we see that, even though the acceleration of the leader is unknown, the tracking error system (17) is still guaranteed to be globally uniformly asymptotically stable through designing the decentralized adaptive estimation laws and the distributed tracking control. This represents a major improvement when comparing with the existing works (Hong,Hu,&Gao (2006); Hong,Chen,&Bushnell (2008)).

4. A NUMERICAL EXAMPLE

This section presents a numerical example to demonstrate the usefulness of the adaptive tracking control developed for the leader-follower system. Assume that there are four followers and one leader in the example. Consider two possible interconnection topologies shown in Figure 1. Note



(a) $\mathcal{G}_1$ (b) $\mathcal{G}_2$

Fig. 1. Switching networks

that the topology $\mathcal{G}_2$ associated with the four followers in Figure 1(b) is not connected, even though the two networks $\mathcal{G}_1$ and $\mathcal{G}_2$ are connected. For simplicity of computer simulations, assume that the switching signal $\sigma(t)$ is periodic, that is, the interconnection topology of the leader-follower system switches according to $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_1, \mathcal{G}_2, \cdots$, with a period $T_\sigma = 5$ time steps. Assume that the basis functions are defined by $\phi_0(t) = \mathrm{col}\{\sin(t), \cos(t)\}$ and $\phi_i(t) = \mathrm{col}\{\cos((i+1)t), \sin((i+1)t)\}$ $(i = 1, 2, 3, 4)$ for the unknown acceleration $a_0(t)$ and the disturbances $\delta_i(t)$, respectively. The initial positions and velocities of all agents are distributed uniformly in the interval $[-5, 5]$. The initial estimation error vectors $\bar{\Omega}_0(t) \in \mathbb{R}^8$ and $\bar{\Omega}(t) \in \mathbb{R}^8$ are distributed uniformly in the interval $[-1, 1]$.

In this example, the gains are given by $k = 2, l = 6$. Figure 2 shows that the position tracking errors and the velocity tracking errors of the four followers converge to zero under the adaptive tracking control. At the same

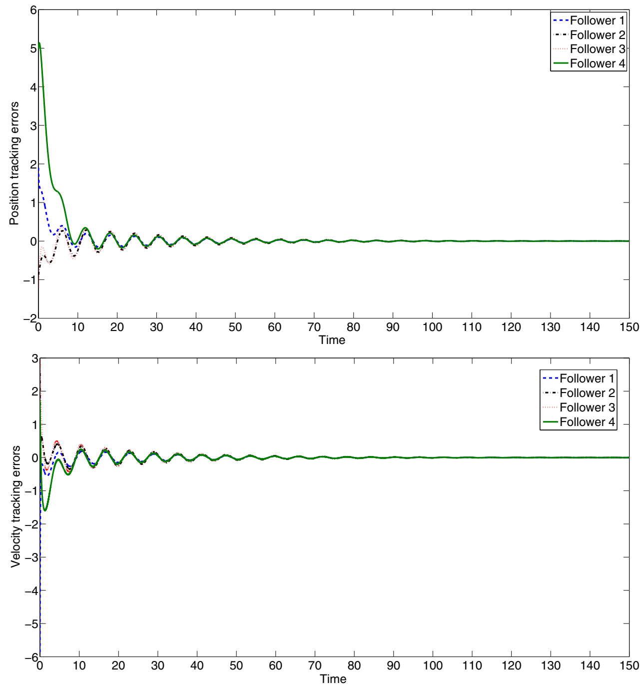


Fig. 2. Evolution of the tracking errors $x_i(t) - x_0(t)$, $v_i(t) - v_0(t)$ $(i = 1, 2, 3, 4)$ under the adaptive tracking control (16)

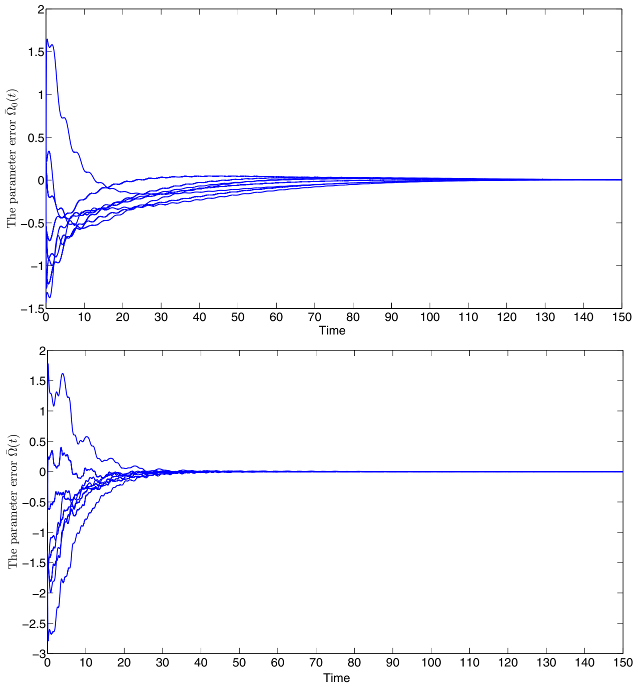


Fig. 3. Evolution of the parameter estimation errors $\hat{\omega}_{i0}(t) - \omega_0 \in \mathbb{R}^2$, $\hat{\omega}_i(t) - \omega_i \in \mathbb{R}^2$ $(i = 1, 2, 3, 4)$ under the adaptive laws (14) - (15)

time, the parameter convergence of $\bar{\Omega}_0(t)$ and $\bar{\Omega}(t)$ is also guaranteed by applying the decentralized adaptive laws, as illustrated in Figure 3.

5. CONCLUSION

In this paper, a leader following problem has been investigated for a multi-agent system with unknown disturbances and switching networks. A linearly parameterized modeling approach has been used to model the unknown disturbances in the agent dynamics. Then the decentralized adaptive laws and the tracking control have been developed for a transformed system with available relative position and velocity measurements. Moreover, the tracking stability and the parameter convergence have been proved by the virtue of an appropriate common Lyapunov function and a PE condition. Finally, a numerical example has been given to demonstrate the effectiveness of the developed adaptive tracking control.

REFERENCES

Bai, H., Arcak, M., & Wen, J. (2008). Adaptive design for reference velocity recovery in motion coordination. *Systtems and Control Letters*, 57(8), 602–610.

Bai, H., Arcak, M., & Wen, J. (2009). Adaptive motion coordination: using relative velocity feedback to track a reference velocity. *Automatica*, 45(4), 1020–1025.

Cao, Y., Yu, W., Ren, W., & Chen, G. (2013). An Overview of Recent Progress in the Study of Distributed Multi-agent Coordination. *IEEE Transactions on Industrial Informatics*, 9(1), 427–438.

Das, A., & Lewis, F.L. (2010). Distributed adaptive control for synchronization of unknown nonlinear networked systems. *Automatica*, 46(12), 2014–2021.

Hong, Y., Chen, G., & Bushnell, L. (2008). Distributed observers design for leader-following control of multi-agent networks. *Automatica*, 44(3), 846–850.

Hong, Y., Hu, J., & Gao, L. (2006). Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica*, 42(7), 1177–1182.

Hu, J., & Zheng, W. X. (2014). Adaptive tracking control of leader-follower systems with unknown dynamics and partial measurements, *Automatica*, http://dx.doi.org/10.1016/j.automatica.2014.02.037.

Liu, B., Wang, X., Su, H., Gao, Y, & Wang, L. (2013). Adaptive second-order consensus of multi-agent systems with heterogeneous nonlinear dynamics and time-varying delays, *Neurocomputing*, 118, 289–300.

Li, Z., Ren, W., Liu, X., & Fu, M. (2013). Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols. *IEEE Transactions on Automatic Control*, 58(7), 1786–1791.

Marino, R., & Tomei, P. (1995). *Nonlinear Control Design: Geometric, Adaptive and Robust.* Prentice Hall, London, UK.

Popov, V. M. (1973). *Hyperstability of Control Systems.* Springer-Verlag, New York.

Qin, J., Zheng, W. X., & Gao, H. (2011). Consensus of multiple second-order vehicles with a time-varying reference signal under directed topology. *Automatica*, 47(9), 1983–1991.

Shi, G., Johansson, K. H., & Hong, Y. (2013). Reaching an optimal consensus: dynamical systems that compute intersections of convex sets. *IEEE Transactions on Automatic Control*, 58(3), 610–622.

Yu, H., & Xia, X. (2012). Adaptive consensus of multi-agents in networks with jointly connected topologies. *Automatica*, 48(8), 1783–1790.

Yu, W., Ren, W., Zheng, W. X., Chen, G., & Lü, J. (2013). Distributed control gains design for consensus in multi-agent systems with second-order nonlinear dynamics. *Automatica*, 49(7), 2107–2115.