Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Accuracy of Fridman's estimates for sampling interval: nonlinear system case study

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Abstract: An attempt to evaluate accuracy of Fridman's sampling interval estimates for nonlinear discrete-continuous systems where the controlled plant belongs to a class of cascade passifiable Lurie systems. Numerical results obtained for master-slave configuration of two mobile robots demonstrate good accuracy of Fridman's estimates: error of the sampling interval estimate is less than 25% of the value obtained from extensive simulation. In contrast, the error obtained by conventional method from quadratic Lyapunov function is more than 75% of the value obtained from simulation.

Keywords: Stability of nonlinear systems, Passivity-based control, Networked systems, LMI

1. INTRODUCTION

Recently there was observed a strong interest in an approach to the sampling time evaluation based on transformation of discrete-continuous system models to continuous delayed system with time-varying (seesaw) delay. Efficiency of an approach has increased since it was combined with the descriptor method of delayed systems analysis proposed by Emilia Fridman in 2001 (Fridman (2001)). The idea has become equipped with a powerful calculation tools based on LMI and has become a powerful design method allowing one to estimate maximum sampling interval providing stability of the closed loop system. It allows designer to seriously reduce conservativeness of the sampling interval estimates (Fridman et al. (2004); Fridman (2010)). However the Fridman's method was previously developed for only to linear systems. It was extended to a class of nonlinear Lurie system just recently Seifullaev and Fradkov (2013). However conservativeness of the estimates has not been evaluated.

In this paper an attempt to evaluate accuracy of Fridman's estimates for a class of nonlinear systems is made. The problem of measuring conservativeness of Fridman's method for nonlinear system is in that there are no tight bounds for sampling interval for general nonlinear systems that could be used to compare with Fridman's estimates in order to evaluate their accuracy. Therefore in this paper a class of cascade nonlinear systems is chosen (namely, passifiable systems Andrievsky and Fradkov (2006); Fradkov et al. (1999); Polushin et al. (2006)) for which conventional type bounds for sampling interval can be evaluated efficiently by means of quadratic Lyapunov functions. The control is chosen in such a way that the derivative of the Lyapunov function for the system with the integrator should be strictly negative for nonzero values of the system state vector, and so by the Lyapunov theorem the asymptotic stability of the entire model follows. For the master-slave system where the param- eters are nonidentical, the robust synchronization problems have also been considered Ji et al. (2010); Balasubramaniam and Theesar (2014); Ji et al. (2014). Such analytic bounds were evaluated in Usik (2012). In this paper the bounds of Usik (2012) are improved and used to calculate the sampling interval bounds numerically for an example system (networks of three mobile robots). An alternative estimate of sampling interval for mobile robots is made by a 'nonlinear' version of Fridman's method developed in Seifullaev and Fradkov (2013). The obtained numerical results are compared.

In Section II and III the results of Usik (2012) and Seifullaev and Fradkov (2013), correspondingly are briefly exposed for completeness. In Section IV conventional and Friedman's estimates for example nonlinear system are evaluated numerically and compared.

2. CONVENTIONAL ESTIMATES OF SAMPLING INTERVAL FOR CASCADE PASSIFIABLE SYSTEMS

Consider n cascade dynamical systems of the Lurie type with nonlinear input cascades

$$\dot{z}_i(t) = Az_i(t) + B\varphi(y_i) + Bu_i(t) + \quad (1)$$

$$+ \sum_{j=1}^{n} \alpha_{ij}\varphi_{ij}(z_i(t) - z_j(t)), \quad (2)$$

$$\dot{u}_i(t) = \psi(u_i, t) + w_i(t),\ y_i(t) = Cz_i(t), \quad (3)$$

where $\varphi_{ij}(x), i = 1, ..., n, j = 1, ..., d$ – functions that describe the relationship between the systems, $\alpha_{ij} \in \mathbb{R}^1$. Also consider the master system:

$$\dot{z}_0(t) = Az_0(t) + B\varphi(y_0),\ y_0(t) = Cz_0(t). \quad (4)$$

where $z_i(t), z_0(t)$ are the n-dimensional vectors of the object state. $y_i(t), y_0(t)$ are scalar outputs. A is the $n \times n$ matrix, B is the $n \times 1$ matrix, C is the $1 \times n$ matrix, $\varphi(y)$, $\psi(u, t)$ are the continuous nonlinearities lying in the sector.

Our goal is to achieve zero asymptotic state error: $z_i(t) - z_0(t) \to 0$ where $t \to \infty, i = 1, ..., n$.

2.1 Evaluation of control

Let us introduce state synchronization error $e(t) = z_i(t) - z_0(t)$, and output synchronization error $\sigma_i(t) = y_i(t) - y_0(t) = Ce_i(t)$. And the error system

$$\dot{e}_i(t) = Ae_i(t) + B\xi(\sigma_i, t) - Bu_i(t) + \quad (5)$$

$$+ \sum_{j=1}^{n} \alpha_{ij}\varphi_{ij}(e_i(t) - e_j(t)), \quad (6)$$

$$\sigma_i(t) = Ce_i(t), \quad (7)$$

$$\dot{u}_i(t) = \psi(u_i, t) + KCAe_i(t) + KCB\xi(\sigma_i, t) + v_i(t). \quad (8)$$

where $\xi(\sigma_i, t) = \varphi(\sigma_i + y_0(t)) - \varphi(y_0(t))$ is a new nonlinearity and $v_i(t) = (-\gamma - KCB)u_i + \gamma K\sigma_i$. The control goal will take the form $\lim_{t\to\infty} e_i(t) = 0$,

For synthesis of the control we will used the method of backstepping Fradkov et al. (1999).

2.2 Conditions of Passification and Asymptotic Stabilization

To obtain the conditions for achieving the goal, the following assumptions are made:

(1) linear system $\dot{e}(t) = Ae(t) - Bu(t)$, $\sigma(t) = Ce(t)$ is the hyperminimum-phase, i.e., the matrix function $\Gamma(\lambda) = \lim_{\lambda\to\infty} \lambda W(\lambda)$ is nondegenerate and positive definite Fradkov et al. (1999), where $W(\lambda) = C(\lambda I - A)^{-1}B = \beta(\lambda)/\alpha(\lambda)$ is the transfer function of the system. For the case with the scalar output, this means: the degree of the denominator $\alpha(\lambda)$ is equal to n. The numerator $\beta(\lambda)$ is of the Hurwitz degree $n - 1$ with positive coefficients;
(2) $\xi(\sigma, t)$ lies in the sector, i.e., $a\sigma^2 \leqslant \xi(\sigma, t)\sigma \leqslant b\sigma^2$, where a, b are the sector parameters;
(3) $\psi(u, t)$ also lies in the sector, i.e., $cu^2 \leqslant \psi(u, t)u \leqslant du^2$, where c, d are the sector parameters;
(4) functions $\varphi_{ij}(x), i = 1, ..., n, j = 1, ..., d$ are Lipschitz:
$$\varphi_{ij}(x) : \|\varphi_{ij}(x) - \varphi_{ij}(x')\| \leqslant L_{ij}\|x - x'\|, L_{ij} > 0.$$

From the hyperminimum-phase property and the passification theorem Andrievsky and Fradkov (2006) it follows that the minimum distance η_0 between the roots of the numerator of a transfer function and the imaginary axis will be positive. We will select the parameters η and K in such a way that $0 < \eta < \eta_0$, $2\|\tilde{D}\|\|P\|\|C\| \max(|a|, |b|) + 2\|P\| \max(|c|, |d|) < \eta\lambda_{min}$, where $\tilde{D} = \begin{pmatrix} B \\ KCB \end{pmatrix}$, P is the positive definite matrix in the Lyapunov quadratic function $V(x) = x^T Px$, λ_{min} is the least eigenvalue of the symmetric matrix P.

The following result holds Usik (2012):

Theorem 1. Let the assumptions (1)-(4) be fulfilled and the inequality

$$\begin{aligned} &- \eta_i\lambda_{\min}(P_i) + 2\|\tilde{D}\|\lambda_{\max}(P_i) \max(|a|, |b|)\|\tilde{C}\| + \\ &+ 2\lambda_{\max}(P_i) \max(|c|, |d|) + \\ &+ 2\lambda_{\max}(P_i) \sum_{j=1}^{n} (2|L_{ij}\alpha_{ij}| + |L_{ji}\alpha_{ji}|) < 0, \\ &\qquad i = 1, ..., n, \end{aligned} \quad (9)$$

holds where $\tilde{D} = \begin{pmatrix} B \\ KCB \end{pmatrix}$, P is the positive definite matrix in the Lyapunov quadratic function $V(x) = x^T Px$, $\lambda_{\min}, \lambda_{\max}$ – are the least and the largest eigenvalues of the given matrix. Then there exist numbers K, γ, such that the system (2), (3) will be passive with the quadratic storage function, while the closed system with the control $v_i(t) = (-\gamma - KCB)u_i + \gamma K\sigma_i$ will be asymptotically stable.

2.3 The Discrete Controller and Conditions of Exponential Synchronization

Consider the discrete controller $v_i(t) = (-\gamma - KCB)u_i(t_k) + \gamma K\sigma_i(t_k)$, $t_k \leqslant t \leqslant t_{k+1}$, where $t_k = kh$ are the instants of time with the discretization step h.

Theorem 2. Consider the system (7) - (8) with the discrete-time controller. And the inequality

$$\begin{aligned} &- \eta_i\lambda_{\min}(P_i) + 2\|\tilde{D}\|\lambda_{\max}(P_i) \max(|a|, |b|)\|\tilde{C}\| + \\ &+ 2\lambda_{\max}(P_i) \max(|c|, |d|) + \\ &+ 2\lambda_{\max}(P_i) \sum_{j=1}^{n} (2|L_{ij}\alpha_{ij}| + |L_{ji}\alpha_{ji}|) < 0, \\ &\qquad i = 1, ..., n, \end{aligned} \quad (10)$$

holds where $\tilde{D} = \begin{pmatrix} B \\ KCB \end{pmatrix}$, P – is the positive definite matrix in the Lyapunov quadratic function $V(x) = x^T Px$, $\lambda_{\min}, \lambda_{\max}$ – are the least and the largest eigenvalues of the given matrix.

Select the discretization step satisfying the inequalities:

$$\|\tilde{C}\|\varkappa_i\|\tilde{K}\|e^{L_G h} + L_G e^{-\eta h} \leqslant \|\tilde{C}\|\varkappa_i\|\tilde{K}\| + L_G, \quad (11)$$

for $i = 1..n$, where $\varkappa_i$ is the coefficient of the estimate of the system output in terms of the Lyapunov function:

$|\tilde{\sigma_i}| \leqslant \varkappa_i\sqrt{V}$, L_G is the Lipschitz constant of the right side of the system (7) - (8).

Then the system under consideration is exponentially stable, i.e. the synchronization error exponentially tends to zero.

Proof.

The initial system (7) - (8) can be represented in the form

$$\dot{x}_i(t) = \tilde{A}x_i(t) + \tilde{B}v_i(t) + \tilde{B}\psi(u_i,t) + \tilde{D}\xi(\sigma_i,t), \quad (12)$$
$$\tilde{\sigma_i}(t) = \tilde{C}x_i(t),\ v_i(t) = \tilde{K}\tilde{\sigma_i}(t_k). \quad (13)$$

Let's define $G_i(x,t) = \tilde{A}x(t)+\tilde{B}v(t)+\tilde{B}\psi(u,t)+\tilde{D}\xi(\sigma_i,t)$. This is a Lipschitz function with constant L_{G_i}: $|G_i(x_1,t)-G_i(x_2,t)| \leqslant (\|\tilde{A}\| + \|\tilde{B}\|\|\tilde{K}\|\|\tilde{C}\| + \|\tilde{D}\|\max(|a|,|b|)\|\tilde{C}\| + \|\tilde{B}\|\max(|c|,|d|) + \sum_{j=1}^{n}(2|\alpha_{ij}L_{ij}|+|\alpha_{ji}L_{ji}|))\|x_1-x_2\| = L_{G_i}\|x_1 - x_2\|$. The controller $v_i(t)$ represented in the following form $v_i(t) = \tilde{K}\tilde{\sigma_i}(t) - \tilde{K}\delta_i(t)$, where $\delta_i(t) = \tilde{\sigma_i}(t) - \tilde{\sigma_i}(t_k)$ is the discretization error. The $\delta_i(t)$ satisfies the inequality $|\delta_i(t)| = \|\tilde{C}\int_t^{t_k} G_i(x,t)\,dt\| \leqslant (t-t_k)\|\tilde{C}G_i(x_k,t_k)\| + \int_{t_k}^{t} L_{G_i}\|\delta_i(t)\|\,dt$. We obtain the estimation on $\delta_i(t)$ by applying Gronwall's inequality:

$$\|\delta_i(t_{k+1})\| \leqslant \|\tilde{C}G_i(x_k,t_k)\|\frac{e^{L_{G_i}h}-1}{L_{G_i}}. \quad (14)$$

Denote $C_h = \frac{e^{L_G h}-1}{L_G}$. Then $\|\delta_i(t_{k+1})\| \leqslant \sqrt{V_{i_k}}C_h$.

For the system (7) - (8) we choose a Lyapunov function $V = \sum_{i=1}^{n} x_i^T P_i x_i$. Calculate the derivative of $V(x)$.

$$\dot{V} < -\eta V - \sum_{i=1}^{n} e_i^T P\tilde{B}\tilde{K}\delta_i \leqslant$$
$$\leqslant -\eta V + \sum_{i=1}^{n} |\tilde{\sigma_i}(t)||\tilde{K}||\delta_i(t)|. \quad (15)$$

Denote $\gamma_i = |\tilde{\sigma_i}(t)||\tilde{K}||\delta_i(t)|$. Thus inequality (15) can be rewritten as $\dot{V} \leqslant -\eta_1 V + \sum_{i=1}^{n}\gamma_i$. Fixed i: $\dot{V_i} \leqslant -\eta V_i + \gamma_i$. Integrating it on the interval (t_k, t_{k+1}) and considering $|\tilde{\sigma_i}(t_k)| \leqslant \varkappa_i\sqrt{V_{i_k}}$, we obtain the following inequality:

$$V_{k+1} \leqslant e^{-\eta_1 h}V_k + \frac{\gamma_{k+1}}{\eta_1} - \frac{\gamma_k}{\eta_1}e^{-\eta_1 h} \leqslant$$
$$\leqslant e^{-\eta_1 h}V_k + C_h K_1\sqrt{V_{k+1}V_k}. \quad (16)$$

Let's find the conditions when the inequality holds $V_{k+1} \leqslant V_k$.

Denote $x = \sqrt{\frac{V_{k+1}}{V_k}}, \alpha = \exp^{-\eta h}, \beta = C_h K_1$. In this notation, the inequality (15) can be rewritten as $x \leqslant \alpha\frac{1}{x} + \beta$. If $\beta < 2, \alpha < 1-\beta$ then inequalities $x \leqslant 1$ and $x \leqslant \alpha\frac{1}{x} + \beta$ will be met. It is clear that that this is a condition on the sampling step h (11) formulated in the theorem. Therefore, $\|x(t_k)\| \to 0$ exponentially.

We write estimation of the error rate $\delta(t)$: $\|x(t) - x(t_k)\| \leqslant C_h\|x(t_k)\|$, which is equivalent the following inequality $\|x(t)\| \leqslant (C_h+1)\|x(t_k)\|$. This implies the exponential stability of solutions of $x(t) \to 0$, $\forall t \in (t_k, t_{k+1})$.

3. SAMPLING INTERVAL ESTIMATION BASED ON FRIDMAN'S METHOD AND LMI

Let us describe an alternative approach to estimation of the sampling interval based on the results proposed in Seifullaev and Fradkov (2013).

Consider a nonlinear system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} q_i\xi_i(t) + Bu(t),$$
$$\sigma_i(t) = r_i^T x(t), \quad \xi_i(t) = \varphi_i(\sigma_i(t),t), \quad i = 1,\dots,N, \quad (17)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control function, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are constant matrices, $q_i \in \mathbb{R}^n, r_i \in \mathbb{R}^n$ are constant vectors.

Assume that $\xi_i(t) = \varphi_i(\sigma_i(t),t)$ are nonlinear functions satisfying

$$\mu_{1_i}\sigma_i^2 \leqslant \sigma_i\xi_i \leqslant \mu_{2_i}\sigma_i^2, \quad i = 1,\dots,N \quad (18)$$

for all $t \geqslant 0$, where $\mu_{1_i} < \mu_{2_i}$ are real numbers.

Given a sequence of sampling times $0 = t_0 < t_1 < \dots < t_k < \dots$ and a piecewise constant control function

$$u(t) = u_d(t_k), \quad t_k \leqslant t < t_{k+1}, \quad (19)$$

where $\lim_{k\to\infty} t_k = \infty$.

Assume that $h \in \mathbb{R}\,(h > 0)$ and

$$t_{k+1} - t_k \leqslant h, \quad \forall k \geqslant 0 \quad (20)$$

and consider a sampled-time control law

$$u(t) = Kx(t_k),\ t_k \leqslant t < t_{k+1}, \quad (21)$$

where $K \in \mathbb{R}^{m\times n}$. The law (21) can be rewritten as follows:

$$u(t) = Kx(t-\tau(t)), \quad (22)$$

where $\tau(t) = t - t_k$, $t_k \leqslant t < t_{k+1}$.

It is required to analyze the influence of the upper bound h of sampling intervals on the closed-loop system exponential stability:

$$\begin{aligned}&\dot{x}(t) = Ax(t) + BKx(t-\tau(t)) + \sum_{i=1}^{N} q_i\xi_i(t),\\&\sigma_i(t) = r_i^T x(t), \quad \xi_i(t) = \varphi_i(\sigma(t),t), \quad i = 1,\dots,N,\\&\tau(t) = t - t_k, \quad t \in [t_k, t_{k+1}).\end{aligned} \quad (23)$$

Thus, instead of the traditional reduction to discrete-time system an alternative method was used: the effect of sampling is considered as delay followed by the construction and use of Lyapunov-Krasovskii functional Fridman (2010). With S-procedure Yakubovich et al. (2004) the estimation of sampling step is reduced to feasibility analysis of linear matrix inequalities (LMI). The following result is obtained based on the results obtained in Seifullaev and Fradkov (2013).

Theorem 3. Given $\alpha > 0$, let there exist matrices $P \in \mathbb{R}^{n\times n}$ $(P > 0)$, $Q \in \mathbb{R}^{n\times n}$ $(Q > 0)$, $P_2 \in \mathbb{R}^{n\times n}$, $P_3 \in \mathbb{R}^{n\times n}$, $X \in \mathbb{R}^{n\times n}$, $X_1 \in \mathbb{R}^{n\times n}$, $T \in \mathbb{R}^{n\times n}$, $Y_1 \in \mathbb{R}^{n\times n}$, $Y_2 \in \mathbb{R}^{n\times n}$, $Y_3^{(i)} \in \mathbb{R}^{n\times n}$ $(i = 1,\dots,N)$, and positive numbers $\{\varkappa_{0i}\}_{i=1}^{N}$, $\{\varkappa_{1i}\}_{i=1}^{N}$ such that following LMIs are feasible:

$$\begin{bmatrix} P+h\dfrac{X+X^T}{2} & hX_1-hX \\ * & -hX_1-hX_1^T+h\dfrac{X+X^T}{2} \end{bmatrix} > 0, \quad (24)$$

$$\begin{bmatrix} \Phi_{11}^- & \Phi_{12}^- & \Phi_{13}^- & \Phi_{14}^{-(1)} & \dots & \Phi_{14}^{-(N)} \\ * & \Phi_{22}^- & \Phi_{23}^- & \Phi_{24}^{(1)} & \dots & \Phi_{24}^{(N)} \\ * & * & \Phi_{33}^- & \Phi_{34}^{(1)} & \dots & \Phi_{34}^{(N)} \\ * & * & * & \Phi_{44}^{-(1)} & \dots & 0 \\ * & * & * & \vdots & \ddots & \vdots \\ * & * & * & 0 & \dots & \Phi_{44}^{-(N)} \end{bmatrix} < 0, \quad (25)$$

$$\begin{bmatrix} \Phi_{11}^+ & \Phi_{12}^+ & \Phi_{13}^+ & \Phi_{14}^{+(1)} & \dots & \Phi_{14}^{+(N)} & hY_1^T \\ * & \Phi_{22}^+ & \Phi_{23}^+ & \Phi_{24}^{(1)} & \dots & \Phi_{24}^{(N)} & hY_2^T \\ * & * & \Phi_{33}^+ & \Phi_{34}^{(1)} & \dots & \Phi_{34}^{(N)} & hT^T \\ * & * & * & \Phi_{44}^{+(1)} & \dots & 0 & hq_1^T {Y_3^{(1)}}^T \\ * & * & * & \vdots & \ddots & \vdots & \vdots \\ * & * & * & 0 & \dots & \Phi_{44}^{+(N)} & hq_N^T {Y_3^{(N)}}^T \\ * & * & * & * & * & * & -hQe^{-2\alpha h} \end{bmatrix} < 0, \quad (26)$$

where

$$\Phi_{11}^- = A^T P_2 + P_2^T A + 2\alpha P - Y_1 - Y_1^T - \\ -(1-2\alpha h)\,\frac{X+X^T}{2} - \sum_{i=1}^{N} \varkappa_{0i}\mu_{1i}\mu_{2i} r_i r_i^T,$$

$$\Phi_{11}^+ = A^T P_2 + P_2^T A + 2\alpha P - Y_1 - Y_1^T - \\ -\frac{X+X^T}{2} - \sum_{i=1}^{N} \varkappa_{1i}\mu_{1i}\mu_{2i} r_i r_i^T,$$

$$\Phi_{12}^- = P - P_2^T + A^T P_3 - Y_2 + h\,\frac{X+X^T}{2},$$

$$\Phi_{12}^+ = P - P_2^T + A^T P_3 - Y_2,$$

$$\Phi_{13}^- = Y_1^T + P_2^T BK - T + (1-2\alpha h)(X - X_1),$$

$$\Phi_{13}^+ = Y_1^T + P_2^T BK - T + (X - X_1),$$

$$\Phi_{14}^{-(i)} = P_2^T + \frac{1}{2}\varkappa_{0i}(\mu_{1i}+\mu_{2i}) r_i,$$

$$\Phi_{14}^{+(i)} = P_2^T + \frac{1}{2}\varkappa_1{}^-{}_i(\mu_{1i}+\mu_{2i}) r_i,$$

$$\Phi_{22}^- = -P_3 - P_3^T + hQ, \quad \Phi_{22}^+ = -P_3 - P_3^T,$$

$$\Phi_{23}^- = Y_2^T + P_3^T BK - h(X - X_1),$$

$$\Phi_{23}^+ = Y_2^T + P_3^T BK,$$

$$\Phi_{24}^{(i)} = P_3^T q_i, \quad \Phi_{34}^{(i)} = Y_3^{(i)} q_i,$$

$$\Phi_{33}^- = T + T^T - (1-2\alpha h)\,\frac{X+X^T-2X_1-2X_1^T}{2},$$

$$\Phi_{33}^+ = T + T^T - \frac{X+X^T-2X_1-2X_1^T}{2},$$

$$\Phi_{44}^{-(i)} = -\varkappa_{0i}, \quad \Phi_{44}^{+(i)} = -\varkappa_{1i}.$$

Then system (23) is exponentially stable with decay rate α.

4. EXAMPLE. THREE MOBILE ROBOTS

We will compare two methods of estimation of sampling step by the example of a model nonlinear system consisting of three mobile three-wheeled robots in master-slave configuration.

Considering that the robots move at low speed, one can restrict the discussion to the kinematic model of the driving and the driven vehicles. The model can be represented in the following way Latombe (1991):

$$\begin{aligned} &\dot{x}_1(t) = v\cos(\varphi_1(t)), && \dot{y}_1(t) = v\cos(\varphi_2(t)), \\ &\dot{x}_2(t) = v\sin(\varphi_1(t)), && \dot{y}_2(t) = v\sin(\varphi_2(t)), \\ &\dot{\varphi}_1(t) = \omega, && \dot{\varphi}_2(t) = u(t), \\ \\ &\dot{z}_1(t) = v\cos(\varphi_3(t)), \\ &\dot{z}_2(t) = v\sin(\varphi_3(t)), \\ &\dot{\varphi}_3(t) = r(t), \end{aligned} \quad (27)$$

where $u(t), r(t)$ are the control functions, ω is the fixed angular velocity, v is the fixed linear velocity.

The system (27) can be represented in the form

$$\begin{aligned} &\dot{x}_1(t) = v + v\,(\cos(\varphi_1(t)) - 1), \\ &\dot{x}_2(t) = v\,\varphi_1(t) + v\,(\sin(\varphi_1(t)) - \varphi_1(t)), \\ \\ &\dot{y}_1(t) = v + v\,(\cos(\varphi_2(t)) - 1), \\ &\dot{y}_2(t) = v\,\varphi_2(t) + v\,(\sin(\varphi_2(t)) - \varphi_2(t)), \\ &\dot{\varphi}_2(t) = u(t). \\ \\ &\dot{z}_1(t) = v + v\,(\cos(\varphi_3(t)) - 1), \\ &\dot{z}_2(t) = v\,\varphi_3(t) + v\,(\sin(\varphi_3(t)) - \varphi_3(t)), \\ &\dot{\varphi}_3(t) = r(t), \end{aligned} \quad (28)$$

Thus, at small values of angle $\varphi_i(t), i = 1, 2, 3$, the motion along axes $x_1(t), y_1(t), z_1(t)$ can be neglected.

Introduce the following notation:

$$e_1(t) = x_2(t) - y_2(t), \quad e_2(t) = x_2(t) - z_2(t), \quad (29)$$

$$\varepsilon_1(t) = \varphi_1(t) - \varphi_3(t), \quad \varepsilon_2(t) = \varphi_1(t) - \varphi_2(t), \quad (30)$$

$$\xi_1(t) = \sin(\varphi_1(t)) - \sin(\varphi_2(t)) + \varphi_2(t) - \varphi_1(t), \quad (31)$$

$$\xi_2(t) = \sin(\varphi_1(t)) - \sin(\varphi_3(t)) + \varphi_3(t) - \varphi_1(t). \quad (32)$$

Using these notation, we can rewrite the system model as follows:

$$\begin{aligned} &\dot{e}_1(t) = v\varepsilon_1(t) + v\xi_1(\varepsilon_1, t), && \dot{e}_2(t) = v\varepsilon_2(t) + v\xi_2(\varepsilon_2, t) \\ &\dot{\varepsilon}_1(t) = w_1(t), && \dot{\varepsilon}_2(t) = w_2(t), \end{aligned} \quad (33)$$

where

$$\xi_i(\varepsilon_i(t), t) = 2\cos\frac{\varphi_1(t) + \varphi_{i+1}(t)}{2}\sin\frac{\varepsilon_i(t)}{2} - \varepsilon_i(t),\ i = 1, 2. \quad (34)$$

Denote $\alpha_i(t) = \cos\frac{\varphi_1(t)+\varphi_{i+1}(t)}{2}$ and rewrite (34) as follows

$$\xi_i(\varepsilon(t), t) = 2\alpha_i(t)\sin\frac{\varepsilon_i(t)}{2} - \varepsilon_i(t),\ i = 1, 2. \quad (35)$$

Nonlinearities (35) satisfy

$$-2\,\varepsilon_i^2 \leqslant \varepsilon_i\,\xi_i \leqslant 0,\ i = 1, 2 \quad (36)$$

for all $t \geqslant 0$ (see Fig.1).

Verify, that the assumptions of Theorem 1 are fulfilled. Transfer functions of each system are equal v/λ. The

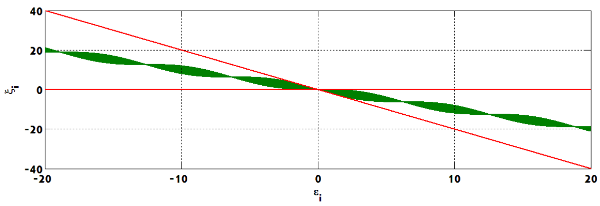

Fig. 1. Sector nonlinearities,i=1,2.

degree of denominator is 1, the degree of numerator is 0, $v > 0$. Indeed, there exists feedbacks in the form $\varepsilon_i = Ke_i$, $K < 0$, such that the linear system is asymptotically stable. Nonlinearties ξ_i are in sector, as it noted in (38). Consider the Lyapunov quadratic functions $V_i(e_i) = e_i^T H_i e_i$ and compare the assumption 4) with following inequality:

$$\dot{V}_i = 2e_iPv(\varepsilon_i + \xi_i) \leqslant 2(e_i)^2 HvK(1 + \max(|a|, |b|)).$$

This inequality hold with $K < 0$. Using the backstepping method, we synthesize the discrete controllers

$$w_i(t) = -\gamma(\varepsilon_i(t_k) - Ke(t_k)) + Kv\varepsilon_i(t_k), i = 1, 2. \quad (37)$$

Represent systems (33), (37) in the following way:

$$\begin{aligned} \dot{X}_i(t) &= AX_i(t) + Du_i(t) + F\xi_i(t, \sigma(t)), \\ u(t) &= \tilde{K}X_i(t_k), \end{aligned} \quad (38)$$

where $X_i = \begin{bmatrix} e_i \\ \varepsilon_i \end{bmatrix}$, $A = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $F = \begin{bmatrix} v \\ 0 \end{bmatrix}$, $\tilde{K} = [\gamma K \;\; Kv - \gamma]$, Consider the Lyapunov quadratic functions $V_i(X_i) = X_i^T P_i X_i$. For applying Theorem 2, calculate the parameter $\eta_i < 0$:

$$\begin{aligned} \dot{V}_i &= ((A + B\tilde{K})X_i + F\xi_i(\varepsilon_i, t))^T PX_i + \\ &+ X_i^T P_i((A + B\tilde{K})X_i + F\xi_i(\varepsilon_i, t)) \leqslant \\ &\leqslant 2(\max Re(A + B\tilde{K}) + \max(|0|, |-2|)v)V_i \\ &= \eta_i V_i. \end{aligned} \quad (39)$$

Define the vehicle motion velocity $v = 0.1$ m/s. Then

$$\eta_i = 2(\max(-\gamma, 0.1K) + 0.2) < 0. \quad (40)$$

Select γ, K such that (40) holds:$K = -5, \gamma = 0.6$, then $\eta_i = -0.6$. Evaluate in Matlab P_i, $\|P_i\| = 60.9306$, so we can estimate parameter $\varkappa_i$:

$$\|X_i\| \leqslant \varkappa_i\sqrt{V_i} \leqslant \varkappa\sqrt{\|P_i\|}\|X_i\|. \quad (41)$$

Assume, that $\varkappa_i = 1/\sqrt{\|P_i\|} = 0.1281$. Fig. 7 illustrate the results of Theorem 2.

For applying Theorem 3, represent systems (33), (37) with defined parameters in the following way:

$$\begin{aligned} \dot{X}(t) &= AX(t) + Du(t) + F_1\xi_1(t, \sigma(t)) + F_2\xi_2(t, \sigma(t)), \\ u(t) &= \tilde{K}X(t_k), \end{aligned} \quad (42)$$

where $X = \begin{bmatrix} e_1 \\ \varepsilon_1 \\ e_2 \\ \varepsilon_2 \end{bmatrix}$, $A = \left[\begin{array}{cc|cc} 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0 \end{array}\right]$, $D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$,

$F_1 = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $F_2 = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 0 \end{bmatrix}$, $\tilde{K} = \begin{bmatrix} -3 & -1.1 & 0 & 0 \\ 0 & 0 & -3 & -1.1 \end{bmatrix}$,

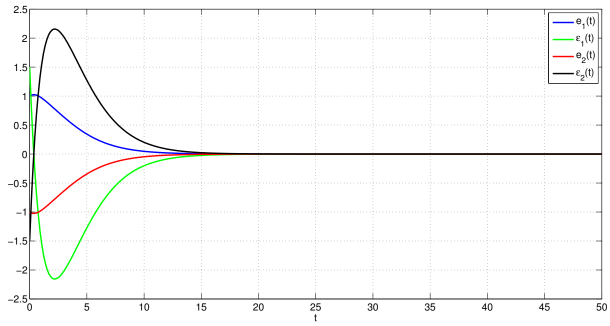

Fig. 2. Errors e_i and ε_i, $i = 1, 2$ in system (33), (37). with sampling interval $h = 0.15$

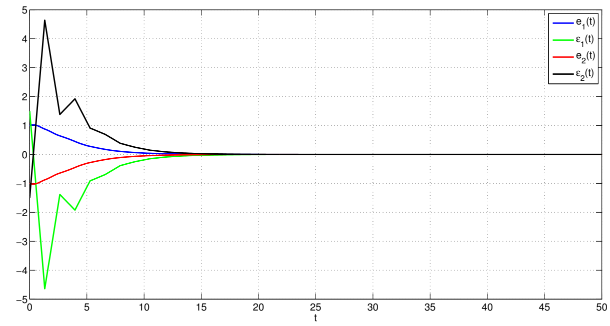

Fig. 3. Errors e_i and ε_i, $i = 1, 2$ in system (33), (37). with sampling interval $h = 1.32$

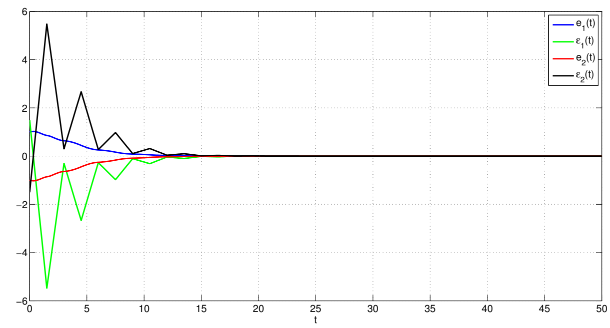

Fig. 4. Errors e_i and ε_i, $i = 1, 2$ in system (33), (37). with sampling interval $h = 1.5$

In Table 1 there are values of maximum upper bound h when (42) is exponentially stable with a small enough decay rate.

Theorem 2	Theorem 3	Simulation
$h = 0.16$	$h = 1.32$	$h = h_*$, $1.80 < h_* < 1.82$

Table 1. Maximum upper bound on the variable sampling

In figures (2) - (6) we can see the system (33) for various sampling interval.

5. CONCLUSIONS

An attempt to evaluate accuracy of Fridman's sampling interval estimates for a class of nonlinear discrete-continuous

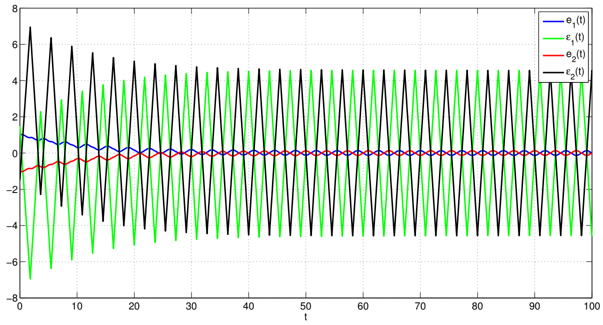

Fig. 5. Errors e_i and ε_i, $i = 1, 2$ in system (33), (37). with sampling interval $h = 1.81$

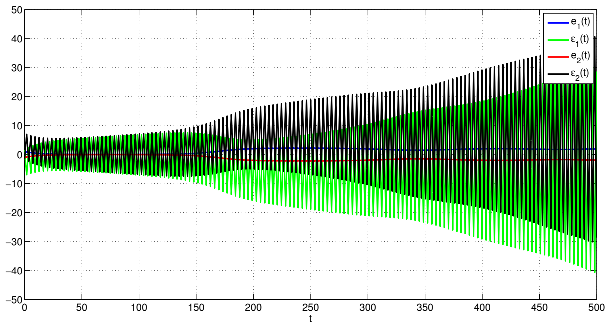

Fig. 6. Errors e_i and ε_i, $i = 1, 2$ in system (33), (37). with sampling interval $h = 1.82$

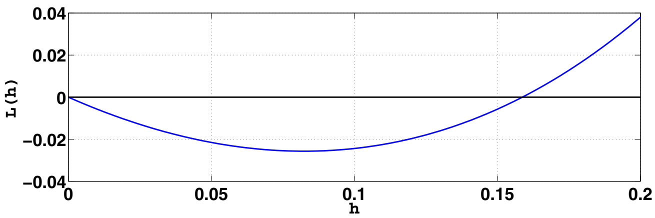

Fig. 7. Function $L(h) = \|\tilde{C}\|\varkappa_i\|\tilde{K}\|e^{L_G h} + L_G e^{-\eta h} \leqslant \|\tilde{C}\|\varkappa_i\|\tilde{K}\| + L_G$, for estimation discretization step h.

systems is made. The proposed approach is applicable to the cascade passifiable Lurie systems. Numerical results obtained for master-slave configuration of two mobile robots demonstrate good accuracy of Fridman's estimates: error of the sampling interval estimate is less than 25% of the value obtained from extensive simulation. In contrast, the error obtained by conventional method from quadratic Lyapunov function is more than 75% of the value obtained by simulation.

Future study will be devoted to evaluation of accuracy of sampling interval estimates for other classes of nonlinear systems. Also we will apply our approach to event triggered systems (Xie et al. (2013); Yu and Antsaklis (2013)), by introducing deadband according to the values some "triggered" function that can be considered as a goal function in a version of the speed-gradient method.

ACKNOWLEDGEMENTS

This work was supported by Russian Foundation for Basic Research (project 11-08-01218), and Russian Federal Program "Cadres" (agreements 8846, 8855)

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