Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Sampled-data disturbance observer for a class of nonlinear systems

Sofiane Ahmed Ali * **Nicolas Langlois** *
and Mohamed Guermouche *

* *IRSEEM Technopôle du Madrillet 76801 Saint Etienne du Rouvray France Tel: +33 (0)2 32 91 58 85; e-mail: {sofiane.ahmedali; nicolas.langlois; mohamed.guermouche}@esigelec.fr*

Abstract: In this paper, a time varying disturbance observer subject to sampled data's measurements is proposed for a class of non-linear systems. The proposed observer combines the advantage of a high gain structure in terms of convergence speed and an output predictor which remains continuous between the sampling times. The exponential convergence of the proposed observer is proved using a Lyapunov function adapted to impulsive systems.

Keywords: Sampled-data systems, Continuous-discrete observer, Extended High Gain Observer (EHGO), Nonlinear Disturbance Observer (NDO).

1. INTRODUCTION

Modern industrial control systems are always affected by uncertainties such as unmodeled dynamics, parametric variations, and external disturbances. Those uncertainties must be taken into account when designing controllers with high precision requirements. A promising way to achieve this task is the use of disturbance rejection technique which consists in a feed-forward control that uses the measurement of the disturbance in order to compensate their effect. Paralytically, it is very difficult to measure these disturbances thus, an observer which estimates these disturbances is particularly needed by the controller. The idea behind is that the disturbance estimation provided by the observer is incorporated then in the control action.

During the past decades, several disturbance observers were proposed in the literature including the unknown input observer (UIO)[Johnson, 1971], the disturbance observer (DOB) [Chen, 2004, Guo and Chen, 2005], the perturbation observer [Kwon and Chung, 2003], the Equivalent Input Disturbance (EID) based estimation [She et al., 2011], the Sliding mode observers (SMO) [Floquet et al., 2006], the Extended State Observers (ESO's) [Han, 1995, 2009, Sun, 2007, Xia et al., 2007] and the extended high gain observer [Freidovich and Khalil, 2008, Serhal et al., 2012] . Note that most of these disturbances observers were designed in continuous time. In fact when it comes to the matter of the implementation of these observers in digital signal processors (DSPs), the measured system outputs are sampled and the observer convergence is affected by this sampling process.

Observer design for continuous-time systems subject to sampled data measurements, depends on the structure of the system. In the case of linear time invariant systems, it is well known that the computation of the exact discretized model of the system is easy. Hence, classical linear observers (such as Luenberger observer, High gain observer) can be employed for the purpose of building a discrete time observer for the system. For linear time variant or non-linear systems the computation of the discretized-time model is usually difficult. To circumvents this difficulty, alternative methodologies have been developed in the literature

Sampled-data observer design can be classified in three different techniques: the first technique is based upon a direct discretization of the model using a discrete-time approximation (such Euler or Runge Kutta approximation) [Assoudi et al., 2002, Laila and Astolfi, 2006]. This technique ignores the inter-sample behaviour of system trajectory during the design process. The second techniques is known as emulation design. The designed observer is performed in continuous time and then discretized for digital implementation. Notice that this technique needs a fast sampling period leading to highly time consuming process which often exceeds the hardware capability. The third technique known as discrete-continuous time observers [Deza et al., 1992, Hammouri et al., 2002, Nadri et al., 2004, Nadri and Hammouri, 2003] which uses the continuous-time model of the plant in the design process. The discrete-continuous time observer design is divided in two steps. The first step occurs between the sampling times where the observer simply copies the dynamic of the system. The second step corrects at the sampling times the estimate state trajectories by using the error between the output of the system and the observer. The exponential convergence of the observer is ensured using Lyapunov function and sufficient conditions on the sampling period are derived in order to guarantee this convergence. The main disadvantage in the design of discrete-continuous time observers is coming from the fact that the estimated state provided by these observer needs to be updated at the sampling times. This leads to increase the complexity of the implementation of these types of observers.

In [Karafyllis and Kravaris, 2009], the authors presents a novel type of discrete-continuous time observer with uses an output predictor. The main advantage of this observer is that the estimated state remains continuous, the update process concerns only the predictor which is re-initialize at the sampling time. Comparing to the classical discrete-continuous time observers the implementation complexity is greatly reduced.

The work presented in this paper is an extension of the approach proposed by the authors in [Karafyllis and Kravaris, 2009] to a class of non-linear system subject to time varying disturbance and sampled data's measurements. The proposed observer combines the well known technique of state augmentation and the output predictor in [Karafyllis and Kravaris, 2009] in order to estimate simultaneously the state and the disturbance. The exponential convergence of the observation and the time varying disturbances term. errors are derived by using a Lyapunov approach adapted to impulsive systems.

The paper is organized as follows: Section 2 gives some mathematical definitions which will be used in the paper. Section 3 describes the class of the system considered in this paper. Section 4 reviews the framework of the extended high gain observers. A class of hybrid observers for state systems subject to time varying disturbances with sampled measurements is designed in Section 5. The exponential convergence of the observation error is proved using the Lyapunov approach. In Section 6, an academic example is presented in order to show the effectiveness of our approach.

2. PRELIMINARIES

Throughout this paper, the following mathematical notations are adopted. Let $\mathbf{R} = (-\infty, +\infty)$, $\mathbf{R}_+ = (0, +\infty)$, $\mathbf{R}_0^+ = [0, +\infty)$. The Euclidian norm is The euclidian norm on $\mathbb{R}^n$ is noted by $\|.\|$. For p, q, n, $m \in \mathbf{N}$, $\mathbf{R}^{p\times q}$ represents the set of real matrices of order $p \times q$ and. If $P \in \mathbf{R}^{p\times p}$, $P > 0$ means that P is positive definite. $\lambda_{min}(P)$ (resp.$\lambda_{max}(P)$ for $P \in \mathbf{R}^{p\times p}$ are the minimum and maximum eigenvalues of P. The notation $(t_k)_{k\geq 0}$ represents a strictly increasing sequence, such that $\lim\limits_{k\to+\infty} t_k = \infty$ which models the sampling times.We denote by τ the maximum allowable sampling period: $\tau = max_{k\in N}(t_{k+1} - t_k)$.

3. SYSTEM DESCRIPTION

We consider the following non-linear system with unknown disturbances described as follows:

$$\begin{cases} \dot{x} = A\,x + \varphi(x,u) + B_d d(t) \\ y = C\,x = x_1 \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ and $y \in \mathbf{R}$ represent respectively the state vector and the output. The vector $u \in \mathbf{R}^m$ describes the set of admissible inputs (bounded and measurable). $d(t)$ denote the the matched or the mismatched disturbances. B_d with dimension $n \times 1$. The matrices A and C have the following structure :

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \vdots \\ \vdots & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \dots & \dots & 1 \\ 0 & \dots & \dots & \dots & 0 \end{pmatrix} \tag{2}$$

$$B_d = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \tag{3}$$

$$C = (\,1\ 0\ \dots\ 0\,)\,. \tag{4}$$

and the vector the function $\phi(x,u)$ has a triangular structure with respect to x, i.e.

$$\phi(x,u) = \begin{pmatrix} \phi_1(x_1) \\ \phi_2(x_1,x_2) \\ \vdots \\ \phi_n(x) \end{pmatrix} \tag{5}$$

Throughout the paper we assume that the following hypotheses hold :

Assumption 1. The state is x belongs to a compact set $\chi \in \mathbf{R}^n$ and the input $||u||$ is supposed bounded.

Assumption 2. The functions $\phi_i(x,u)$ are globally Lipschitz and of class $\mathbf{C^1}$ on a compact χ with respect to x, uniformly in u, i.e:

$$\exists \kappa_\phi > 0 \quad \text{such that} \quad \forall (x_1, x_2) \in \mathbf{R^n} \times \mathbf{R^n}, \quad \forall u \in U$$
$$\|\phi_i(x_1,u) - \phi_i(x_2,u)\| \leq \kappa_\phi \|x_1 - x_2\| \tag{6}$$

4. EXTENDED HIGH GAIN OBSERVER FOR CONTINUOUS TIME SYSTEM

Following the general framework of the Extended State Observer (ESO) [Han, 1995, 2009], we add an extended variable $x_{n+1} = d$ to system (1). The augmented state system is obtained.

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \overline{\varphi(x,u)} + Dh(t) \\ y = \bar{C}\bar{x} = x_1 \end{cases} \tag{7}$$

Where:

$$\bar{x} = \begin{bmatrix} x \\ d \end{bmatrix}_{(n+1)\times 1} \tag{8}$$

$$\bar{A} = \begin{bmatrix} A_{(n\times n)} & (B_d)_{(n\times 1)} \\ 0_{(1\times 1)} & 0_{(1\times 1)} \end{bmatrix}_{(n+1)\times(n+1)}$$

$$\varphi(x,u) = \begin{bmatrix} \varphi(x,u)_{(n\times 1)} \\ 0_{(1\times 1)} \end{bmatrix}_{(n+1)\times 1}$$

$$D = \begin{bmatrix} 0_{(n\times 1)} \\ 1_{(1\times 1)} \end{bmatrix}_{(n+1)\times 1} \quad \bar{C} = \begin{bmatrix} C_{(1\times n)}, 0_{(1\times 1)} \end{bmatrix}_{1\times(n+1)}$$

and

$$\dot{d} = h(t) \tag{9}$$

Assumption 3. The pair $(\bar{A}, \bar{C})$ state is observable.

This means that their exist a pair of symmetric positive matrices $(P, Q) \in \mathbb{R}^{n+1\times n+1}$ such that:

$$(\bar{A}-K\bar{C})^T P + P(\bar{A}-K\bar{C}) = -Q \tag{10}$$

Assumption 4. The function $h(t)$ is bounded by a real constant μ such that $(|h(t)| < \mu)$.

For system (7) an Extended High Gain Observer (EHGO) [Freidovich and Khalil, 2008, Serhal et al., 2012] can be designed as follows:

$$\begin{cases} \dot{\hat{\bar{x}}} = \bar{A}\hat{\bar{x}} + \overline{\varphi(\hat{x},u)} - \theta\triangle_\theta^{-1}K(\bar{C}\hat{\bar{x}} - y(t)) \\ y = \bar{C}\bar{x} = x_1 \end{cases} \tag{11}$$

Where $\triangle_\theta$ is a diagonal matrix $(n+1)\times(n+1)$ defined by :

$$\triangle_\theta = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{\theta^n} \end{pmatrix}$$

5. EXTENDED HIGH GAIN OBSERVER WITH OUTPUT PREDICTOR

Assume now that the output data are available for measurements at each t_k such that system (1) is:

$$\begin{cases} \dot{\bar{x}} &= \bar{A}\bar{x} + \overline{\varphi(x,u)} + Dh(t) \\ y(t_k) &= \bar{C}\bar{x}(t_k) = x_1(t_k) \end{cases} \tag{12}$$

A candidate observer for system (12) is:

$$\begin{cases} \dot{\hat{\bar{x}}} &= \bar{A}\hat{\bar{x}} + \overline{\varphi(\hat{x},u)} - \theta\triangle_\theta^{-1}K(\bar{C}\hat{\bar{x}} - w(t)) \\ \dot{w}(t) &= \bar{C}\left(\bar{A}\hat{\bar{x}} + \varphi(\hat{x},u)\right) \qquad t\in[t_k,\, t_{k+1}) \;\; k\in\mathbb{N} \\ w(t_k) &= y(t_k) \end{cases} \tag{13}$$

The vector $\hat{\bar{x}}$ is the continuous-time estimate of the system state $\bar{x}$. The vector $w(t)$ represents the prediction of the output between two sampling times. The prediction $w(t)$ is updated (re-initialised) at each sampling instant t_k.

Now we are able to state the main result of this paper.

Theorem 1. Under assumptions (1-4), system (13) is a sampled data observer for system (12) with the following property: For sufficiently large values of parameters θ and $k_{i=1,..,n+1}$, there exists a real positive bounded τ_{MASP} such for all $\tau \in (0, \tau_{MASP})$, the observation error is ultimately bounded and the corresponding ultimate bound can be made as small as desired by choosing values of θ high enough.

Proof. Consider the observer $e_{\bar{x}}$ and the output $e_w(t)$ errors defined as following:

$$\begin{cases} e_{\bar{x}}(t) = \hat{\bar{x}} - \bar{x} \\ e_w(t) = w(t) - y(t) = w(t) - \bar{C}\bar{x} \end{cases} \tag{14}$$

Combining (12) and (13) the dynamics of the state and the output errors are given by:

$$\begin{cases} \dot{e}_{\bar{x}} &= \left(\bar{A} - \theta\triangle^{-1}K\bar{C}\right)e_{\bar{x}} + \left(\overline{\varphi(\hat{x},u)} - \overline{\varphi(x,u)}\right) \\ &\quad +\theta\triangle^{-1}Ke_w - Dh(t) \\ \dot{e}_w &= \bar{C}\bar{A}e_{\bar{x}} + \bar{C}\left(\overline{\varphi(\hat{x},u)} - \overline{\varphi(x,u)}\right) \end{cases} \tag{15}$$

Using the following well known properties: $\theta\triangle^{-1}\bar{A}\triangle = \bar{A}$ and $\triangle^{-1}K\bar{C} = \triangle^{-1}K\bar{C}\triangle$.

The dynamic of equation (15) becomes:

$$\begin{cases} \dot{e}_{\bar{x}} &= \theta\triangle^{-1}\left(\bar{A} - K\bar{C}\right)\triangle e_{\bar{x}} + \left(\overline{\varphi(\hat{x},u)} - \overline{\varphi(x,u)}\right) \\ &\quad +\theta\triangle^{-1}Ke_w - Dh(t) \\ \dot{e}_w &= \bar{C}\bar{A}e_{\bar{x}} + \bar{C}\left(\overline{\varphi(\hat{x},u)} - \overline{\varphi(x,u)}\right) \end{cases} \tag{16}$$

Using the well-known change of coordinate in the high gain literature $\bar{e}_{\bar{x}} = \triangle e_{\bar{x}}$ yields to:

$$\begin{cases} \dot{\bar{e}}_{\bar{x}} &= \theta\left(\bar{A} - K\bar{C}\right)\bar{e}_{\bar{x}} + \triangle\left(\overline{\varphi(\hat{x},u)} - \overline{\varphi(x,u)}\right) + \\ &\quad \theta Ke_w - \triangle Dh(t) \\ \dot{e}_w &= \theta\bar{e}_{\bar{x}2} + \left(\overline{\varphi_1(\hat{x},u)} - \overline{\varphi_1(x,u)}\right) \end{cases} \tag{17}$$

Inspired by the works in [Farza et al., 2004][Ahmed-Ali, 2012], Let us choose the following Lyapunov function:

$$V = \alpha_1\bar{e}_{\bar{x}}^T P\bar{e}_{\bar{x}} + \alpha_2\psi(t)\left|e_w(t)\right|^2 \tag{18}$$

Where α_1, α_2 are positive constants,$\psi(t)$ is bounded positive function designed for the purpose of correcting the error between the predictor and the output. $\psi(t)$ satisfies the following conditions:

$$\begin{cases} \dot{\psi}(t) &< 0 \qquad t\in[t_k,\, t_{k+1}) \\ \psi(t_k) &= \gamma,\ \forall k\in\mathbb{N} \\ \psi(t_k+\tau) &= \gamma^{-1}, \gamma > 1 \end{cases} \tag{19}$$

To prove the exponential stability of the observation and the disturbance errors, it is sufficient to find a condition involving α_1, α_2 and the maximum sampling period τ_{MASP} so that the following equalities hold:

$$\dot{V} \le -\beta_1 V + \beta_2\sqrt{V} \qquad t\in[\,t_k,\, t_{k+1}\,) \tag{20}$$

where β_1 and β_2 are real positives constants.

Splitting V in two terms V_1, V_2:

$$\begin{cases} V_1 = \alpha_1\bar{e}_{\bar{x}}^T P\bar{e}_{\bar{x}} \\ V_2 = \alpha_2\psi(t)\left|e_w(t)\right|^2 \end{cases} \tag{21}$$

Computing the time derivative of V_1 yields to:

$$\dot{V}_1 = \alpha_1(\dot{\bar{e}}_{\bar{x}}^T P\bar{e}_{\bar{x}} + \bar{e}_{\bar{x}}^T P\dot{\bar{e}}_{\bar{x}}) \tag{22}$$

Considering (17),(10) and after a simple computations we get:

$$\begin{aligned} \dot{V}_1 = &-\alpha_1\theta\bar{e}_{\bar{x}}^T Q\bar{e}_{\bar{x}} + 2\alpha_1\bar{e}_{\bar{x}}^T P\triangle\left(\overline{\varphi(\hat{x},u)} - \overline{\varphi(x,u)}\right) \\ &+2\alpha_1\theta\bar{e}_{\bar{x}}^T PKe_w - 2h(t)\alpha_1\bar{e}_{\bar{x}}^T P\triangle D \end{aligned} \tag{23}$$

Using the following well-known property:

$$\lambda_{min}(Q)\left\|\bar{e}_{\bar{x}}\right\|^2 \le \bar{e}_{\bar{x}}^T Q\bar{e}_{\bar{x}} \le \lambda_{max}(Q)\left\|\bar{e}_{\bar{x}}\right\|^2 \tag{24}$$

We get:

$$\begin{aligned} \dot{V}_1 \le &-\alpha_1\theta\lambda_{max}(Q)\left\|\bar{e}_{\bar{x}}\right\|^2 + 2\alpha_1\bar{e}_{\bar{x}}^T P\triangle\left(\overline{\varphi(\hat{x},u)} - \overline{\varphi(x,u)}\right) \\ &+2\alpha_1\theta\bar{e}_{\bar{x}}^T PKe_w - 2h(t)\alpha_1\bar{e}_{\bar{x}}^T P\triangle D \end{aligned} \tag{25}$$

Using the Schwartz inequality, we have the following upper bounds:

$$|2\alpha_1\bar{e}_{\bar{x}}^T P\triangle\left(\overline{\varphi(\hat{x},u)}-\overline{\varphi(x,u)}\right)| \leq 2\alpha_1\xi_1\lambda_{max}(P)\left\|\bar{e}_{\bar{x}}\right\|^2 \quad (26)$$
$$|2h(t)\alpha_1\bar{e}_{\bar{x}}^T P\triangle D| \leq 2\mu\alpha_1\xi_2\lambda_{max}(P)\left\|\bar{e}_{\bar{x}}\right\|$$

In the other hands we have:

$$\begin{aligned}|2\alpha_1\theta\bar{e}_{\bar{x}}^T PKe_w| &\leq 2|\alpha_1\theta e_w||\bar{e}_{\bar{x}}^T PK| \quad (27)\\ &\leq 2|\alpha_1\theta e_w|\left\|\bar{e}_{\bar{x}}\right\|\left\|PK\right\|\end{aligned}$$

which leads to:

$$|2\alpha_1\theta\bar{e}_{\bar{x}}^T PKe_w| \leq 2|\alpha_1\theta e_w|\left\|\bar{e}_{\bar{x}}\right\|\lambda_{max}(P)\left\|K\right\| \quad (28)$$

Using the Young inequality we have:

$$|2\alpha_1\theta\bar{e}_{\bar{x}}^T PKe_w| \leq (\alpha_1\theta)^2|e_w|^2 + \lambda_{max}^2(P)\left\|K\right\|^2\left\|\bar{e}_{\bar{x}}\right\|^2 \quad (29)$$

Combining (25-29):

$$\dot{V}_1 \leq -\alpha_1\left(\theta\lambda_{max}(Q) - 2\lambda_{max}(P)\xi_1 - \frac{\lambda_{max}^2(P)\left\|K\right\|^2}{\alpha_1}\right)\left\|\bar{e}_{\bar{x}}\right\|^2 + 2\alpha_1\mu\lambda_{max}(P)\xi_2\left\|\bar{e}_{\bar{x}}\right\| + (\alpha_1\theta)^2|e_w|^2 \quad (30)$$

Let us now compute time derivative of V_2.

$$\dot{V}_2 = \alpha_2\dot{\psi}(t)\left|e_w(t)\right|^2 + 2\alpha_2\psi(t)\left|e_w(t)\right|\left|\dot{e}_w(t)\right| \quad (31)$$

Taking into account that$|\bar{e}_{\bar{x}2}| \leq \left\|\bar{e}_{\bar{x}}\right\|$ we can deduce from (17)

$$|\dot{e}_w| \leq (\theta+\xi_1)\left\|\bar{e}_{\bar{x}}\right\| \quad (32)$$

Hence we have:

$$\dot{V}_2 \leq \alpha_2\dot{\psi}(t)\left|e_w(t)\right|^2 + 2\alpha_2\psi(t)\left|e_w(t)\right|(\theta+\xi_1)\left\|\bar{e}_{\bar{x}}\right\| \quad (33)$$

Using again the young inequality one has:

$$\dot{V}_2 \leq (\alpha_2\dot{\psi}(t) + \alpha_2^2\psi(t)^2)\left|e_w(t)\right|^2 + (\theta+\xi_1)^2\left\|\bar{e}_{\bar{x}}\right\|^2 \quad (34)$$

combining (30) and (34) we finally have:

$$\dot{V} \leq -\alpha_1\left(\theta\lambda_{max}(Q) - 2\lambda_{max}(P)\xi_1 - \frac{\lambda_{max}^2(P)\left\|K\right\|^2 + (\theta+\xi_1)^2}{\alpha_1}\right)\left\|\bar{e}_{\bar{x}}\right\|^2 + 2\alpha_1\mu\lambda_{max}(P)\xi_2\left\|\bar{e}_{\bar{x}}\right\| + (\alpha_2\dot{\psi}(t) + \alpha_2^2\psi(t)^2 + (\alpha_1\theta)^2)|e_w|^2 \quad (35)$$

Choosing

$$\dot{\psi}(t) = -\alpha_2\left(\psi(t)^2+1\right) \qquad t\in[\,t_k,\ \tau\,)\, and \tau \in [0,\ \tau_{MASP}) \quad (36)$$

leads to:

$$\dot{V} \leq -\alpha_1\left(\theta\lambda_{max}(Q) - 2\lambda_{max}(P)\xi_1 - \frac{\lambda_{max}^2(P)\left\|K\right\|^2 + (\theta+\xi_1)^2}{\alpha_1}\right)\left\|\bar{e}_{\bar{x}}\right\|^2 + 2\alpha_1\mu\lambda_{max}(P)\xi_2\left\|\bar{e}_{\bar{x}}\right\| + (-\alpha_2^2 + (\alpha_1\theta)^2)|e_w|^2 \quad (37)$$

Thus, we can say from (37) that if we choose α_1, α_2 such that:

$$\begin{cases}\alpha_1 = \frac{\lambda_{max}^2(P)\|K\|^2+(\theta+\xi_1)^2}{\theta\lambda_{max}(Q)-2\lambda_{max}(P)\xi_1-\beta_1\lambda_{min}(P)}\\ \alpha_2 = \sqrt{\alpha_1^2\theta^2}\end{cases} \quad (38)$$

With $\theta > \frac{2\lambda_{max}(P)\xi_1+\beta_1\lambda_{min}(P)}{\lambda_{max}(Q)}$ and $\beta_1 > 0$

We have:

$$\dot{V} \leq -\beta_1\,\alpha_1\lambda_{min}(P)\left\|\bar{e}_{\bar{x}}\right\|^2 + 2\alpha_1\mu\lambda_{max}(P)\xi_2\left\|\bar{e}_{\bar{x}}\right\| \quad (39)$$

Using again the well known property for the Lyapunov function defined in (18)

$$\left\|\bar{e}_{\bar{x}}\right\| \leq \frac{1}{\sqrt{\alpha_1\lambda_{min}(P)}}\sqrt{V} \quad (40)$$

We derive:

$$\dot{V} \leq -\beta_1 V + \beta_2\sqrt{V} \quad (41)$$

with $\beta_2 = \frac{2\alpha_1\mu\lambda_{max}(P)\xi_2}{\sqrt{\alpha_1\lambda_{min}(P)}}$

Integrating (41)between t_k and t yields to:

$$V(t) \leq V(t_k)\exp^{-\beta_1(t-t_k)} + \left(\frac{\beta_2}{\beta_1}\right)^2 \quad (42)$$

Using again (40), we get finally:

$$\left\|\bar{e}_{\bar{x}}\right\| \leq \frac{1}{\sqrt{\alpha_1\lambda_{min}(P)}}\left(\sqrt{V(t_k)}\exp^{\frac{-\beta_1}{2}(t-t_k)} + \left(\frac{\beta_2}{\beta_1}\right)\right) \quad (43)$$

To compute the value of the maximum sampling period τ_{MASP} we shall integrate equation (36) between t_k and $t_k + \tau_{MASP}$. From (36) we have:

$$\int_{t_k}^{t_k+\tau_{MASP}} \frac{\partial\psi}{\psi^2+1} = -\alpha_2\int_{t_k}^{t_k+\tau_{MASP}} \partial t \quad (44)$$

Which leads to:

$$\tau_{MASP} = \left(\frac{1}{\alpha_2}\left(\arctan(\gamma) - \arctan(\gamma^{-1})\right)\right) \quad (45)$$

6. APPLICATION

This section is dedicated to the illustration of the proposed observer by means of a second order system which belongs to the class of system studied above.

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -2x_1 - x_2 - exp(x_1) + u + d\end{cases} \quad (46)$$

Where $x = \begin{pmatrix}x_1\\x_2\end{pmatrix}$ and d are respectively the vector of the states of the system and the disturbance term. The system input is $u = 10\sin(t)$ and the disturbance term $d = 150sin(0.1\pi t) + 100sin(0.12\pi t)$. the initial conditions of the system and the observer are respectively are chosen: $\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}5\\20\end{pmatrix}$ and $\begin{pmatrix}\hat{x}_1\\\hat{x}_2\end{pmatrix} = \begin{pmatrix}1\\9\end{pmatrix}$.

the Observer gains was chosen such that: $K = \begin{pmatrix}6\\11\\-6\end{pmatrix}$.

The sampled time of the output measurements is $\tau = 10^{-2}s$

Remark 2. Since system (46) is not globally Lipschitz, we will define a bounded compact set χ inside which our states observer will be initialized.

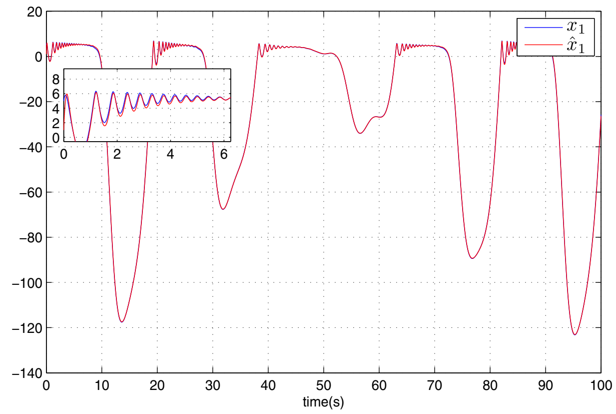

Fig. 1. Evolution of the estimated state $\hat{x}_1$ for $\theta = 3$

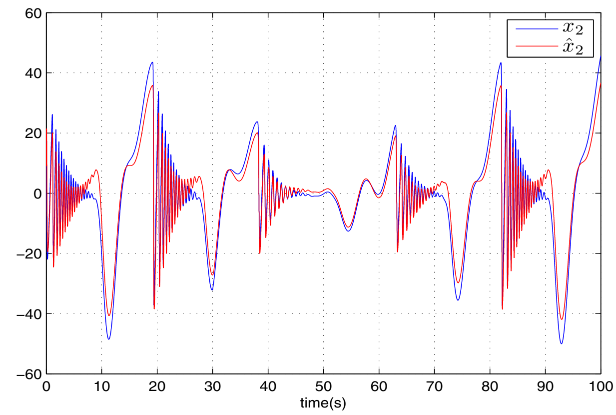

Fig. 2. Evolution of the estimated state $\hat{x}_2$ for $\theta = 3$

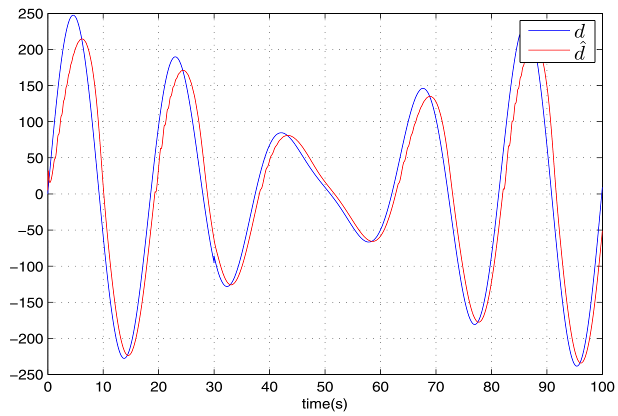

Fig. 3. Evolution of the estimated disturbance $\hat{d}$ for $\theta = 3$

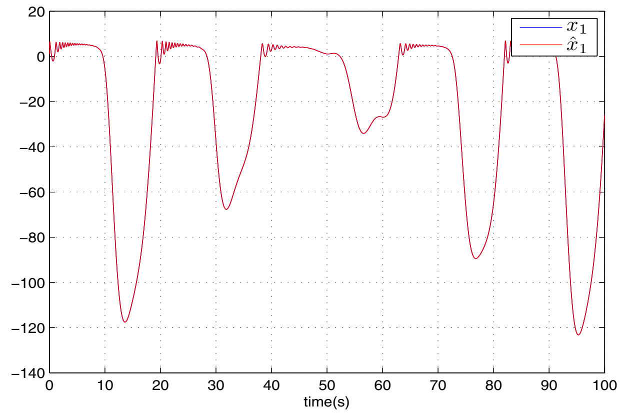

Fig. 4. Evolution of the estimated state $\hat{x}_1$ for $\theta = 10$

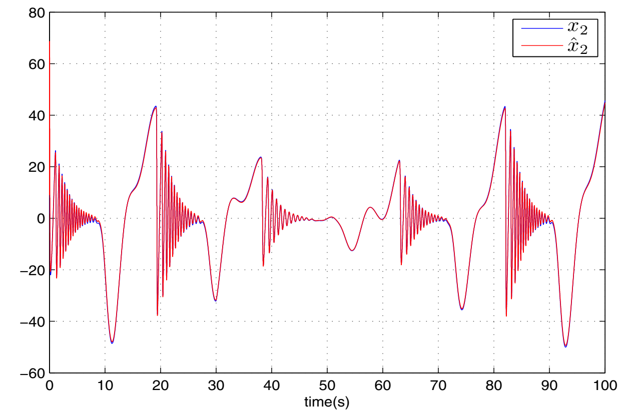

Fig. 5. Evolution of the estimated state $\hat{x}_2$ for $\theta = 10$

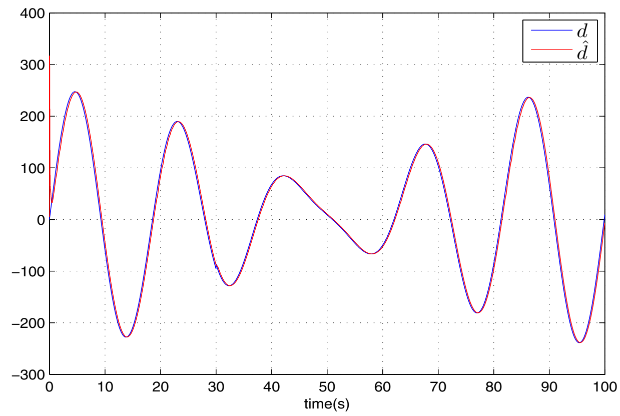

Fig. 6. Evolution of the estimated disturbance $\hat{d}$ for $\theta = 10$

Figs 1-6 shows the simulation results of observer (13) for two different values of parameter θ. One can observe that when we increase the value of θ, the convergence of the estimated states $\hat{x}_1, \hat{x}_2$ and the estimated disturbance $\hat{d}$ is improved . It should be also noted that the estimated states and disturbance converge exponentially to the states of the system.

7. CONCLUSION

In this paper, we designed a discrete-continuous observer for a class of continuous sampled data systems subject to time-varying disturbance. the exponential convergence of the proposed observer is proven using a Lypunov function adapted to hybrid systems. Sufficient conditions on the sampled time constant values and the parameters of the system is derived in order to guarantee the exponential convergence of the observer. An academic simulation is presented in order to show the benefits of our approach.

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