Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Set-based Fault-Tolerant Control of Convex Polytopic LPV Systems using a Bank of Virtual Actuators

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Abstract:
We propose a robust actuator fault tolerant control strategy for systems with linear parameter varying (LPV) uncertainty model description. The scheme employs a set-based robust fault detection and identification (FDI) approach and a bank of *virtual actuators* (VA). An interesting feature is that the virtual actuators are used both for FDI and controller reconfiguration (CR) tasks. The robust FDI method is based on the separation of relevant sets defined for measurable residual signals, which are computed based on the virtual actuator signals and taking into account model uncertainty, noises and process disturbances. For CR, each VA is designed to operate adequately in combination with a nominal controller (designed for the fault-free plant) to achieve correct reconfiguration for a particular fault situation in a considered range of fault scenarios. The resulting robust fault tolerant control scheme ensures boundedness of the closed-loop system trajectories under a wide range of actuator fault scenarios. The performance of the scheme is illustrated through a simulation example.

Keywords: Fault tolerant control, fault detection and isolation, LPV systems, convex polytopic uncertainty, actuator faults, controller reconfiguration, virtual actuators, invariant sets.

1. INTRODUCTION

Fault tolerant control (FTC) systems integrate fault detection and identification (FDI) with controller reconfiguration (CR) in an overall strategy aimed at maintaining closed-loop stability and designated performance levels under a range of fault situations. Detailed treatments of FDI, FTC and CR can be found in the books [1]–[3]. Recently, *virtual actuators* (VA) have been introduced as a reconfiguration approach after actuator faults for linear systems, see [1] and [3]. In [4], some of the current authors proposed the use of a *bank of virtual actuators* in an FTC scheme for LTI systems, with the new feature that the VAs perform both FDI and CR tasks, without the need of additional FDI observers. Each VA is designed to operate in combination with a nominal controller to achieve correct CR for a particular fault situation within a finite range of scenarios. A residual signal is defined for each VA with distinctive behaviour when its model "matches" the actual fault situation and when changes to a "non-matching" fault situation occur. The FDI principle relies on monitoring the residual signals to assess which VA matches the current fault situation and should be engaged.

In [5], the authors extended for the case of sensor faults the 'dual' virtual-sensor based FTC methodology to a class of systems having polytopic model uncertainties with a linear parameter varying (LPV) description. The LPV modelling approach has received major attention from the control community in recent years as a tractable framework to deal with nonlinear systems. A motivation behind this interest is the connection between LPV models and gain-scheduling control, which is an effective technique applicable to a large class of nonlinear systems [6]. LPV systems typically employ *self-scheduling* control, where the parameters of the control system are scheduled in real time according to the current value of the varying parameter. The self-scheduling idea was employed for CR after sensor faults in [7]. Our work in [5] extended the approach of [7] to consider the design of both, the FDI unit and the CR module, in an integrated fashion.

In the present paper, the scheme of [4] based on a *bank of virtual actuators* performing both FDI and CR tasks is extended to discrete-time systems with convex LPV model uncertainty. A schematic of the proposed FTC scheme is shown in Figure 1. In this scheme, a bank of self-scheduled VAs operates in closed-loop with a self-scheduled observer-based tracking controller designed for the nominal (fault free) plant. A suitable residual signal is associated to each VA. Correct FDI is guaranteed if a residual "matching" set (which characterises the fault free situation) has no intersection with "non-matching" sets (which characterise all other fault situations within a finite range of considered scenarios). A switching logic monitors these residual signals to determine which set they belong to, and engages in the loop the VA that matches the currently diagnosed fault situation. Closed-loop stability under both healthy and faulty conditions is guaranteed under the proposed robust FTC scheme. The performance of the scheme is illustrated by a numerical example.

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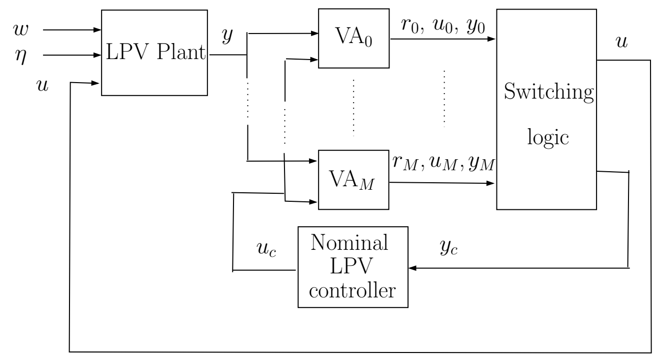

Fig. 1. FTC scheme with LPV plant, bank of VAS: VA$_0$ to VA$_M$, switching logic and nominal LPV controller.

2. CONVEX POLYTOPIC LPV PLANT AND ACTUATOR FAULT MODELS

We consider a discrete-time LPV system given by [2]

$$x^+ = A(\rho)x + B(\rho)Fu + Ew, \tag{1a}$$

$$y = C(\rho)x + \eta, \tag{1b}$$

$$v = C_v x, \tag{1c}$$

where x and $x^+ \in \mathbb{R}^n$ are the current and successor system states, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^r$ is a bounded process disturbance, $y \in \mathbb{R}^p$ is the plant measured output, $v \in \mathbb{R}^q$ is a performance output and $\eta \in \mathbb{R}^p$ is a bounded measurement noise. $\rho \in \mathbb{R}^L$ is an *a priori* unknown time-varying parameter whose measurement is available at each sample time, and $A(\rho) \in \mathbb{R}^{n\times n}$, $B(\rho) \in \mathbb{R}^{n\times m}$, $C(\rho) \in \mathbb{R}^{p\times n}$, for each ρ (the results can be easily extended to the case where the matrix E also depends on ρ).

The "fault matrix" $F \in \mathbb{R}^{m\times m}$ in (1a) is used to model actuator faults. We consider a finite range of fault situations represented by the matrix F taking $M+1$ different values $F \in \{F_0, F_1, \ldots, F_M\}$. In particular, $F_0 = I$ (the identity matrix) represents the "healthy" situation, that is, no actuator fault. Typically, the most critical faults for the process performance are considered, for example, total outage of actuators. We will say that an abrupt change in the actuator fault situation occurs if F changes from $F = F_i$ to $F = F_j$, $i, j \in \{0, \ldots, M\}$, $j \neq i$, at some time $k_F \geq 0$. The parameter ρ is assumed to lie in some bounded set $\Gamma \subset \mathbb{R}^L$ and we assume that the system matrices can be written as a convex combination $A(\rho) = \sum_{i=1}^N \alpha_i(\rho)A_i$, $B(\rho) = \sum_{i=1}^N \alpha_i(\rho)B_i$ and $C(\rho) = \sum_{i=1}^N \alpha_i(\rho)C_i$ for certain constant matrices $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$, and continuous functions $\alpha_i : \Gamma \to \mathbb{R}$ such that $\alpha_i(\rho) \geq 0$, $\sum_{i=1}^N \alpha_i(\rho) = 1$, $\forall \rho \in \Gamma$.

It is assumed that the pairs (A_i, C_i) are detectable and the pairs (A_i, B_iF_j), for $i = 1, \ldots, N$ and $j = 0, 1, \ldots, M$ are stabilisable. In addition, $\left(\begin{bmatrix} A_i & 0 \\ C_v & I \end{bmatrix}, \begin{bmatrix} B_iF_j \\ 0 \end{bmatrix}\right)$ are stabilisable, for $i = 1, \ldots, N$ and $j = 0, 1, \ldots, M$. (This is required for the VA design.)

We will further assume that the process disturbance and the measurement noise satisfy $w(k) \in \mathcal{W}$ and $\eta(k) \in \mathcal{N}$ for all time instants $k \geq 0$, where the bounding sets are defined as [3] $\mathcal{W} \triangleq \{w \in \mathbb{R}^r : |w| \leq \overline{w}\}$ and $\mathcal{N} \triangleq \{\eta \in \mathbb{R}^p : |\eta| \leq \overline{\eta}\}$ for some nonnegative vectors $\overline{w} \in \mathbb{R}^r$ and $\overline{\eta} \in \mathbb{R}^p$.

3. NOMINAL LPV CONTROLLER

We consider the following observer-based, reference tracking LPV controller:

$$u_c = -K_s(\hat{x} - x_{\text{ref}}) + u_{\text{ref}}, \tag{2}$$

$$\hat{x}^+ = A_s\hat{x} + B_s u_c + L_s(y_c - C_s\hat{x}), \tag{3}$$

$$x_{\text{ref}}^+ = A_s\, x_{\text{ref}} + B_s u_{\text{ref}}, \tag{4}$$

where, under healthy conditions ($F = F_0 = I$), $u_c = u$, $y_c = y$ (u, y are the signals in the plant (1), see Figure 1). More generally, u, u_c, y and y_c are related through the virtual actuator selected by the switching logic (cf. (7)–(9)) according to the detected fault situation.

In (2)–(4), A_s, B_s, C_s, K_s and L_s are "self-scheduled" matrices which are calculated at each sample time when a new measurement of the varying parameter ρ is acquired. Given the functions $\alpha_i(\rho)$ for all $i = 1, \ldots, N$, defined in Section 2, the matrices A_s, B_s and C_s are calculated as $A_s = A(\rho) = \sum_{i=1}^N \alpha_i(\rho)A_i$, $B_s = B(\rho) = \sum_{i=1}^N \alpha_i(\rho)B_i$ and $C_s = C(\rho) = \sum_{i=1}^N \alpha_i(\rho)C_i$. Similarly, the gains K_s and L_s are computed as

$$K_s = K(\rho) = \sum_{i=1}^N \alpha_i(\rho)K_i, \quad L_s = L(\rho) = \sum_{i=1}^N \alpha_i(\rho)L_i.$$

The matrices K_i and L_i, $i = 1, \ldots, N$ are a set of stabilising controller and observer gains, whose computation will be explained later.

Remark 3.1. (Reference System). The reference system (4) generates a trajectory $(u_{\text{ref}}, x_{\text{ref}})$ which is designed such that it is bounded and the output $C_v x_{\text{ref}}$, where C_v is the plant performance output matrix in (1c), asymptotically tracks a bounded external signal v^*; that is, such that $\lim_{k\to\infty}[\,C_v x_{\text{ref}}(k) - v^*(k)\,] = 0$. The signal v^* is a reference trajectory that we ultimately wish the plant output v in (1c) to track, in the absence of disturbances, under all possible fault situations. (The signal u_{ref} in (4) can be designed by a number of methods; for the example of Section 7 we employ a technique proposed by [7].) Given the designed reference system, it is easy to obtain constant vectors $u_{\text{ref}}^0 \in \mathbb{R}^m$ and $\overline{u_{\text{ref}}} \in \mathbb{R}^m$ such that $u_{\text{ref}}(k) \in \mathcal{U}_{\text{ref}} = \{u \in \mathbb{R}^m : |u - u_{\text{ref}}^0| \leq \overline{u_{\text{ref}}}\}$ for all $k \geq 0$. ◦

The augmented system, with estimation error $\tilde{x} \triangleq x - \hat{x}$ and tracking error $z \triangleq x - x_{\text{ref}}$ as the states, under nominal (healthy) conditions $F = F_0 = I$, $u_c = u$ and $y_c = y$, is

$$\begin{bmatrix} \tilde{x}^+ \\ z^+ \end{bmatrix} = \phi_s \begin{bmatrix} \tilde{x} \\ z \end{bmatrix} + \begin{bmatrix} E \\ E \end{bmatrix} w + \begin{bmatrix} -L_s \\ 0 \end{bmatrix} \eta, \tag{5}$$

where

$$\phi_s \triangleq \begin{bmatrix} A_s - L_sC_s & 0 \\ B_sK_s & A_s - B_sK_s \end{bmatrix} = \sum_{i=1}^N \sum_{j=1}^N \sum_{h=1}^N \alpha_i(\rho)\alpha_j(\rho)\alpha_h(\rho)\phi_{ijh} \triangleq \phi(\rho)$$

and where

[2] The dependence of variables on discrete time k will be omitted when clear from the context.

[3] Inequalities and absolute values are taken elementwise.

$$\phi_{ijh} = \begin{bmatrix} A_i - L_h C_i & 0 \\ B_i K_j & A_i - B_i K_j \end{bmatrix}. \tag{6}$$

There are several methods that can be used to calculate the controller and observer gains (respectively, K_j and L_h in (6)) which stabilise the augmented system (5) in the entire parameter domain. For example, Theorem 3 of [9] can be applied leading to bilinear matrix inequality (BMI) conditions that can be solved numerically.

4. BANK OF VIRTUAL ACTUATORS

We will consider a bank of VAs with integral action [3]. The ℓth self-scheduled VA in the bank, associated with the fault matrix F_ℓ, $\ell = 0, \ldots, M$ (with $F_0 = I$), is given by

$$\begin{bmatrix} \theta_\ell^+ \\ \sigma_\ell^+ \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ tC_v & I \end{bmatrix} \begin{bmatrix} \theta_\ell \\ \sigma_\ell \end{bmatrix} + \begin{bmatrix} B_s \\ 0 \end{bmatrix} u_c - \begin{bmatrix} B_s \\ 0 \end{bmatrix} F_\ell u_\ell, \tag{7}$$

$$u_\ell = -M_s^{F_\ell} \begin{bmatrix} \theta_\ell \\ \sigma_\ell \end{bmatrix} + N_s^{F_\ell} u_c + d^{F_\ell}, \tag{8}$$

$$y_\ell = y + C_s \theta_\ell, \tag{9}$$

where $\theta_\ell \in \mathbb{R}^n$ is the VA state; $\sigma_\ell \in \mathbb{R}^q$ is the integral action state; $t > 0$ is an arbitrary scalar. The matrices $M_s^{F_0}$ and $N_s^{F_0}$ in (8) satisfy (in order to recover the nominal control action for $\ell = 0$)

$$M_s^{F_0} = 0, \qquad N_s^{F_0} = I. \tag{10}$$

For $\ell \neq 0$, partitioning the self-scheduled VA feedback gains $M_s^{F_\ell}$ as $M_s^{F_\ell} = \begin{bmatrix} M_{s,\theta}^{F_\ell} & M_{s,\sigma}^{F_\ell} \end{bmatrix}$, the ℓth VA closed-loop matrices $\mathbb{A}_s^{F_\ell}$ take the form

$$\mathbb{A}_s^{F_\ell} \triangleq \begin{bmatrix} A_s + B_s F_\ell M_{s,\theta}^{F_\ell} & B_s F_\ell M_{s,\sigma}^{F_\ell} \\ tC_v & I \end{bmatrix}. \tag{11}$$

Applying the convex properties to $\mathbb{A}_s^{F_\ell}$, we have

$$\mathbb{A}_s^{F_\ell} = \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i(\rho) \alpha_j(\rho) \mathbb{A}_{ij}^{F_\ell} \tag{12}$$

where

$$\mathbb{A}_{ij}^{F_\ell} = \begin{bmatrix} A_i + B_i F_\ell M_{j,\theta}^{F_\ell} & B_i F_\ell M_{j,\sigma}^{F_\ell} \\ tC_v & I \end{bmatrix}. \tag{13}$$

The self-scheduled matrices $N_s^{F_\ell} = N^{F\ell}(\rho) = \sum_{i=1}^{N} \alpha_i(\rho) N_i^{F\ell}$ in (8) are in principle arbitrary matrices but could be chosen to satisfy some desired design specifications. The signals d^{F_ℓ} in (8) are constant vectors that represent degrees of freedom in the design and satisfy

$$B_i F_\ell d^{F_\ell} = 0 \text{ for } \ell \in \{0, 1, \ldots, M\},\ i \in \{1, \ldots, N\}. \tag{14}$$

All other parameters and variables are as in the plant and nominal controller equations (1)–(4). Combining (7), (8), and using (11)–(14), the dynamics of each VA satisfy

$$\begin{bmatrix} \theta_\ell^+ \\ \sigma_\ell^+ \end{bmatrix} = \mathbb{A}_s^{F_\ell} \begin{bmatrix} \theta_\ell \\ \sigma_\ell \end{bmatrix} + \begin{bmatrix} B_s \\ 0 \end{bmatrix} (I - F_\ell N_s^{F_\ell}) u_c. \tag{15}$$

In order to stabilise the LPV system (15) for $u_c = 0$ for each fault situation F_ℓ, a similar method to the one used for stabilising the augmented system (5) can be employed to calculate the gains $M_{j,\sigma}^{F_\ell}$ and $M_{j,\theta}^{F_\ell}$ in (12)–(13) (see the last paragraph of Section 3).

The VA signals u_ℓ and y_ℓ in (8)–(9) are fed back into the closed-loop system whenever the switching logic engages the ℓth VA according to the diagnosed fault situation; in particular $u = u_\ell$ is applied to the plant and $y_c = y_\ell$ to the nominal controller (see Figure 1). Also, whenever the switching logic selects the VA with index $\ell = 0$, its initial condition is reset as follows:

$$\begin{bmatrix} \theta_0(k_0) \\ \sigma_0(k_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \ \forall \text{ time } k_0 \text{ when } \ell = 0 \text{ is chosen.} \tag{16}$$

Note that (16) implies that the VA associated with the nominal condition $F = F_0$ becomes inactive when it is selected by the switching logic. Indeed, together with (10), the resetting (16) implies that $u = u_0 = u_c + d^{F_0}$ (with $B_i F_0 d^{F_0} = B_i d^{F_0} = 0$ for all $i \in \{1, \ldots, N\}$ according to (14)) and thus the plant (1a) receives the nominal control law under nominal conditions.

Fault hiding goal. The VAs have the property of "hiding" faults from the nominal controller, that is, they restore the controller signals u_c and y_c to their nominal trajectories [3]. To show this property, we define the variables

$$\xi_\ell \triangleq x + \theta_\ell, \quad \ell = 0, \ldots, M. \tag{17}$$

Using (1) and (7) we have

$$\xi_\ell^+ = A_s \xi_\ell + B_s (Fu - F_\ell u_\ell) + B_s u_c + Ew. \tag{18}$$

Suppose $F_\ell = F$ (i.e., the ℓth VA "matches" the actual fault situation) and $u = u_\ell$, $y_c = y_\ell$ (i.e., the ℓth VA is engaged in the closed-loop system by the switching logic, see Figure 1). Then, using (1b), (9) and (18), yields the following equations associated with (17):

$$\xi_\ell^+ = A_s \xi_\ell + B_s u_c + Ew, \tag{19}$$

$$y_c = C_s \xi_\ell + \eta. \tag{20}$$

System (19)–(20) coincides with the nominal plant dynamics ($F = F_0 = I$) with input u_c and output y_c (see (1)). This means that the VA has the effect of hiding the fault from the nominal controller (2)–(4) by always providing it with its nominal input y_c.

In Section 6, we will show that the reconfiguration of the closed-loop system by engaging the ℓth VA whenever the plant's fault matrix F in (1a) is $F = F_\ell$, leads to the preservation of closed-loop stability for all fault situations. In the following section we describe the role of the bank of virtual actuators in residual generation and FDI.

5. RESIDUAL GENERATION AND ROBUST FDI

In this section we first define residual signals directly generated by the bank of VAs. We then derive invariant "matching" sets, where the residuals associated with the VA that matches the plant fault situation evolve while this situation remains unchanged; and "after-change" sets, where these residuals "jump" when a change in the plant fault situation occurs. Finally, we propose a robust FTC algorithm that identifies the fault based on set membership of the residual signals, and reconfigures the controller's output using the corresponding VA dynamics.

We define the residual associated with the ℓth VA as

$$r_\ell \triangleq y_\ell - C_s \hat{x}, \tag{21}$$

where y_ℓ is as in (9) and $\hat{x}$ is the nominal observer's state, see (3). Consider also the following error variables:

$$\tilde{\xi}_\ell \triangleq \xi_\ell - \hat{x}, \tag{22}$$

$$\zeta_\ell \triangleq \xi_\ell - x_{\text{ref}}, \tag{23}$$

for $\ell = 0, \ldots, M$, where ξ_ℓ is as defined in (17). From (18), (20) and (2)–(4), and the equivalent expression for the nominal control law (2) (see (22)–(23))

$$u_c = -K_s\zeta_\ell + K_s\tilde{\xi}_\ell + u_{\text{ref}}, \tag{24}$$

we obtain the following dynamics for the error variables:

$$\tilde{\xi}_\ell^+ = A_s\tilde{\xi}_\ell + B_s(Fu - F_\ell u_\ell) + Ew - L_s(y_c - C_s\hat{x}), \tag{25}$$

$$\zeta_\ell^+ = (A_s - B_sK_s)\zeta_\ell + B_s(Fu - F_\ell u_\ell) + B_sK_s\tilde{\xi}_\ell + Ew. \tag{26}$$

Also, using (9), (1b), (17) and (22) in (21) yields

$$r_\ell = C_s\tilde{\xi}_\ell + \eta. \tag{27}$$

Matching conditions. Suppose, without loss of generality, that $F_\ell = F$ (i.e., the ℓth VA "matches" the actual plant fault situation) and consider that the correct FDI decision is made so that $u = u_\ell$ and $y_c = y_\ell$; that is, the ℓth VA is engaged in the closed-loop system by the switching logic, see Figure 1 (note that this is not an assumption; in fact, in Section 6 we provide the required conditions to ensure that this matching situation is indeed satisfied, see Theorem 6.3). Using (20), (22), we then have that (25)–(26) become

$$\tilde{\xi}_\ell^+ = (A_s - L_sC_s)\tilde{\xi}_\ell + Ew - L_s\eta, \tag{28}$$

$$\zeta_\ell^+ = (A_s - B_sK_s)\zeta_\ell + B_sK_s\tilde{\xi}_\ell + Ew. \tag{29}$$

Note that (28)–(29) has the same dynamics as (5)–(6), which was designed to be stable (see the last paragraph of Section 3). Then, since the disturbance signals w and η are bounded, the trajectories of the above system are bounded.

To obtain attractive invariant sets $\tilde{\Xi}$ and $\mathcal{Z}$ associated with system (28) and (29), respectively, we employ a technique proposed in [8]. [4] Note that these sets are "centred" at zero since the same holds for the disturbance sets. From (27) we then have that, whenever $\tilde{\xi}_\ell \in \tilde{\Xi}$, and under the condition that the matching VA ($F_\ell = F$) is engaged in the closed-loop system ($u = u_\ell$, $y_c = y_\ell$), the residual signal associated with the matching VA satisfies the set membership ($\oplus$ denotes Minkowski sum)

$$r_\ell \in \mathcal{R}, \quad \text{where } \mathcal{R} \triangleq \text{Co}\left\{\bigcup_{i=1}^{N} C_i\tilde{\Xi}\right\} \oplus \mathcal{N}. \tag{30}$$

Note that $\mathcal{R}$ is centred at zero since the same holds for $\tilde{\Xi}$ and $\mathcal{N}$ ($\mathcal{N}$ was defined at the end of Section 2).

From (24), and using $\tilde{\Xi}$ and $\mathcal{Z}$, the nominal controller output satisfies

$$u_c \in \mathcal{U}_c \triangleq \text{Co}\left\{\bigcup_{i=1}^{N}(-K_i)\mathcal{Z}\right\} \oplus \text{Co}\left\{\bigcup_{i=1}^{N} K_i\tilde{\Xi}\right\} \oplus \mathcal{U}_{\text{ref}}, \tag{31}$$

(where $\mathcal{U}_{\text{ref}}$ is defined in Remark 3.1) whenever $\tilde{\xi}_i \in \tilde{\Xi}$ and $\zeta_i \in \mathcal{Z}$. The set $\mathcal{U}_c$ is centred at the reference offset u^0_{ref}, which is the centre of $\mathcal{U}_{\text{ref}}$ (see Remark 3.1).

For each VA, in particular the matching VA (associated to the current fault F_ℓ), we can use its associated dynamics (15) and the fact that the LPV matrix $\mathbb{A}_s^{F_\ell}$ was designed to be stable and u_c is bounded as in (31) to compute an attractive invariant set which will retain its states $(\theta_\ell, \sigma_\ell)$ whenever u_c remains in $\mathcal{U}_c$. Let us denote this set $\mathcal{S}_\ell$ and observe that, whenever $(\theta_\ell, \sigma_\ell) \in \mathcal{S}_\ell$ and $u_c \in \mathcal{U}_c$, the control output (8) associated with VA$_\ell$ satisfies

$$u_\ell \in \mathcal{U}_\ell \triangleq \text{Co}\left\{\bigcup_{i=1}^{N}(-M_i^{F_\ell})\mathcal{S}_\ell\right\} \oplus \text{Co}\left\{\bigcup_{i=1}^{N} N_i^{F_\ell}\mathcal{U}_c\right\} \oplus \{d^{F_\ell}\}. \tag{32}$$

After-change sets and FDI logic. Let a change in the plant fault situation occur so that the fault matrix F in (1a) changes from $F = F_\ell$ to $F = F_j$, for some $j, \ell \in \{0, 1, \ldots, M\}$, $j \neq \ell$. Note that u is still equal to u_ℓ and y_c is still equal to y_ℓ at the time of the change since no reconfiguration has been made yet. Using (25) we then have that the "after-change" residual signal of the previously matching VA$_\ell$ satisfies

$$r_{\ell j}^+ \in \mathcal{R}_{\ell j}^+ \triangleq \text{Co}\left\{\bigcup_{i=1}^{N} C_i\Gamma_{\ell j}\right\} \oplus \mathcal{N}, \tag{33}$$

where

$$\Gamma_{\ell j} \triangleq \text{Co}\left\{\bigcup_{i=1}^{N}\bigcup_{h=1}^{N}(A_i - L_hC_i)\tilde{\Xi}\right\} \oplus$$

$$\text{Co}\left\{\bigcup_{i=1}^{N} B_i(F_j - F_\ell)\mathcal{U}_\ell\right\} \oplus E\mathcal{W} \oplus \text{Co}\left\{\bigcup_{h=1}^{N}(-L_h)\mathcal{N}\right\}$$

whenever $\tilde{\xi}_\ell \in \tilde{\Xi}$ and $u_\ell \in \mathcal{U}_\ell$. Notice that the second term in the definition of $\Gamma_{\ell j}$ above has the possibility to cause a shift of this set away from zero. From (31)–(32) this shift depends on the reference offset u^0_{ref} and the 'degree of freedom' signals d^{F_ℓ}, which can then be utilised as a mechanism to "separate" the matching and after-change sets. We thus consider the following condition.

Assumption 5.1. (Set Separation). For each $\ell \in \{0, ..., M\}$, the matching set $\mathcal{R}$ and the after-change sets $\mathcal{R}_{\ell j}^+$, for $j = 0, \ldots, M$, $j \neq \ell$, are all disjoint. ∘

We also consider the following fault scenario.

Assumption 5.2. (Fault Scenario). Between any two consecutive changes in the fault matrix F, sufficient time T elapses such that the system states converge to their respective attractive invariant sets. (The maximum time of convergence T to these sets can be estimated using techniques from [8].) ∘

Under Assumptions 5.1 and 5.2, a robust FDI algorithm can be implemented by monitoring the residual r_ℓ associated with the matching VA$_\ell$, as follows.

Algorithm 5.3. (Robust FTC algorithm).

(1) For the matching VA$_\ell$ compute its associated residual signal r_ℓ as in (21).
(2) If $r_\ell \in \mathcal{R}$ [c.f. (30)] go to step 1; if $r_\ell \in \mathcal{R}_{\ell j}^+$ for some $j \in \{0, 1, \ldots, M\}$, $j \neq \ell$ [c.f. (33)], then engage VA$_j$ in the loop by setting $u = u_j$ and $y_c = y_j$.
(3) Wait a time period T before performing any action.
(4) Go to step 1.

The waiting time T at step (3) is required for the reconfigured system's states to converge to the relevant invariant sets so that new changes in the fault situation can be correctly diagnosed.

[4] The results of [8] apply to switching systems for which the coefficient functions $\alpha_i \in \{0, 1\}$; however, it is easy to show that the results extend to the convex case where $\alpha_i \in [0, 1]$ and $\Sigma_{i=1}^N \alpha_i = 1$.

6. CLOSED-LOOP PROPERTIES

The first result in this section shows the stability properties of the closed-loop system under matching conditions.

Lemma 6.1. (Matching Properties). Suppose that $F = F_\ell$ in (1a) and let $u = u_\ell$, $y_c = y_\ell$, that is, the matching ℓth VA is engaged in the closed-loop system of Figure 1, thus consisting of the plant (1), nominal controller (2)–(4) and VA_ℓ (7)–(9). Then all closed-loop system variables are bounded and, if the disturbance signals w, η are zero, the error variables (22)–(23) asymptotically converge to zero.

Proof. If $F = F_\ell$ in (1a) and $u = u_\ell$, $y_c = y_\ell$, then the error variables (22), (23) of the matching VA_ℓ, satisfy (28)–(29) and are therefore bounded as discussed in Section 5 (and converge to zero if w and η are zero). Thus, we have from (23) that, since x_{ref} is bounded, then ξ_ℓ is bounded. Since ξ_ℓ is bounded, it follows from (22) that $\hat{x}$ is bounded. Finally, since θ_ℓ is bounded (in fact, all VAs have bounded states, see (15), (24) and recall that $\mathbb{A}_s^{F_\ell}$ in (11) is designed to be stable in the entire uncertainty domain), we have from (17) that x is bounded. In a similar way, it can be proved from (15), (17), (22) and (23) that the error variables $\tilde{\xi}_i$ and ζ_i for the non-matching VAs are also bounded. That is, all internal variables in the closed-loop system remain bounded, thus proving the result. □

We next establish the fault tolerant properties of the overall scheme, under the following initialisation assumption. [5]

Assumption 6.2. (Initialisation). Before the first change in the plant fault situation, the matching VA_ℓ (that is, $F_\ell = F$ in (1a) and (7), $u = u_\ell$ and $y_c = y_\ell$) is engaged in the closed-loop system, and the error variables of VA_ℓ, defined in (22) and (23) are in their attractive invariant sets $\tilde{\Xi}$ and $\mathcal{Z}$, respectively. In addition, the VA states (θ_j, σ_j), for $j = 0, \dots, M$, are in the attractive invariant sets $\mathcal{S}_j$. ∘

Theorem 6.3. (Fault Tolerance). Suppose that Assumption 6.2 holds. Then, under the set separation condition of Assumption 5.1 and the fault scenario of Assumption 5.2, the states of the closed-loop system represented in Figure 1, encompassing the plant (1), the nominal tracking controller (2)–(4) and the bank of virtual actuators (7)–(9), reconfigured by Algorithm 5.3, are bounded under all considered fault situations.

Proof. By Assumption 6.2, before any change in the plant fault situation the matching VA_ℓ (that is, $F_\ell = F$, $u = u_\ell$ and $y_c = y_\ell$) is engaged in the closed-loop system and thus all closed-loop system states are bounded, as shown in Lemma 6.1. Moreover, also by Assumption 6.2 all relevant variables are in their respective invariant sets and hence the analysis of Section 5 under the matching conditions is validated. In particular, the residual r_ℓ associated with the matching VA satisfies (30) and it is thus sensitive to any subsequent change in the plant fault situation, which will cause r_ℓ to satisfy (33) one time step after the change occurs. Hence, Assumption 5.1 ensures that Algorithm 5.3 makes the correct decision and controller reconfiguration and, due to the waiting timer of its third step and the fault scenario of Assumption 5.2, the initialisation conditions of Assumption 6.2 are recovered after a finite number of time steps. The same arguments can be applied for subsequent changes in the fault situation, concluding that the closed-loop system states remain bounded at all times. □

[5] This is a natural assumption that holds if the system has evolved with the matching VA engaged in the loop for sufficiently long time before any change of the fault situation occurs. This is a reasonable assumption since the system will typically start operating under perfectly known actuator conditions.

7. SIMULATION EXAMPLE

We consider an LPV model of the form (1) with matrices

$$A(\rho) = \begin{bmatrix} 0.25 & 1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.6+\rho \end{bmatrix}, \; B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$E = I$, $C_v = [0\ 0\ 1]$, where ρ is a sinusoidal sequence bounded between 0 and 0.05. The matrix $A(\rho)$ has the polytopic form $\alpha_1(\rho)A_1 + \alpha_2(\rho)A_2$, where $A_1 = A(0)$, $A_2 = A(0.05)$, $\alpha_1 = (0.05-\rho)/0.05$ and $\alpha_2 = \rho/0.05$. We assume that each component w_i of the disturbance vector is bounded as $|w_i| \le 10^{-3}$, and the measurement noise components η_i satisfy $|\eta_i| \le 10^{-5}$. The fault situations considered for the FTC design correspond to the matrices

$$F_0 = I, \quad F_1 = \begin{bmatrix} f & 0 \\ 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1 & 0 \\ 0 & f \end{bmatrix}, \tag{34}$$

where $f = 0.1$. The tracking controller (2)–(4) employs the following gains, designed using the conditions in [9]:

$$K_1 = \begin{bmatrix} -0.1259 & -0.9886 & 0.2647 \\ 0.1141 & 0.9695 & -0.0497 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.1201 & -0.6704 & 0.4071 \\ 0.1209 & 0.6805 & -0.0361 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.2995 & -0.0012 \\ 0.0079 & -0.0034 \\ -0.0011 & 0.8688 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.1918 & -0.2678 \\ -0.0058 & -0.0359 \\ -0.0037 & 0.8419 \end{bmatrix}.$$

The reference signal x_{ref} is designed using the state feedback control proposed in [7] with gains

$$K_{01} = \begin{bmatrix} 0.2485 & 0.9939 & -0.5967 \\ -0.2490 & -0.9961 & -0.0006 \end{bmatrix},$$

$$K_{02} = \begin{bmatrix} 0.2485 & 0.9940 & -1.0952 \\ -0.2490 & -0.9961 & -0.0011 \end{bmatrix}.$$

The reference design is such that the last state tracks a constant setpoint $v^* = 0.75$. The virtual actuator gains are calculated via the condition in [9] as follows:

$$M_1^{F_0} = 0, \quad M_2^{F_0} = 0,$$

$$M_{1,\sigma}^{F_1} = \begin{bmatrix} -12.1249 & 0.5048 \end{bmatrix}^\mathsf{T}, \; M_{2,\sigma}^{F_1} = -\begin{bmatrix} 2.2790 & 0.4419 \end{bmatrix}^\mathsf{T},$$

$$M_{1,\theta}^{F_1} = \begin{bmatrix} 2.2006 & 10.0583 & -7.4000 \\ -0.2255 & -1.0068 & -0.0036 \end{bmatrix},$$

$$M_{2,\theta}^{F_1} = \begin{bmatrix} 2.3960 & 9.9400 & -7.3488 \\ -0.2283 & -0.9974 & 0.0017 \end{bmatrix},$$

$$M_{1,\sigma}^{F_2} = \begin{bmatrix} -1.0515 & 4.4372 \end{bmatrix}^\mathsf{T}, \; M_{2,\sigma}^{F_2} = -\begin{bmatrix} 0.2121 & 5.0869 \end{bmatrix}^\mathsf{T},$$

$$M_{1,\theta}^{F_2} = \begin{bmatrix} 0.2053 & 0.9831 & -0.8033 \\ -2.3219 & -9.9840 & 0.7863 \end{bmatrix},$$

$$M_{2,\theta}^{F_2} = \begin{bmatrix} 0.2308 & 0.9894 & -0.7522 \\ -2.2234 & -9.9137 & 0.0915 \end{bmatrix}.$$

We further take fixed values for matrices N^{F_ℓ} as follows:

$$N^{F_0} = I, \quad N^{F_1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad N^{F_2} = \begin{bmatrix} 0 & -0.3 \\ 0.2 & 0 \end{bmatrix}.$$

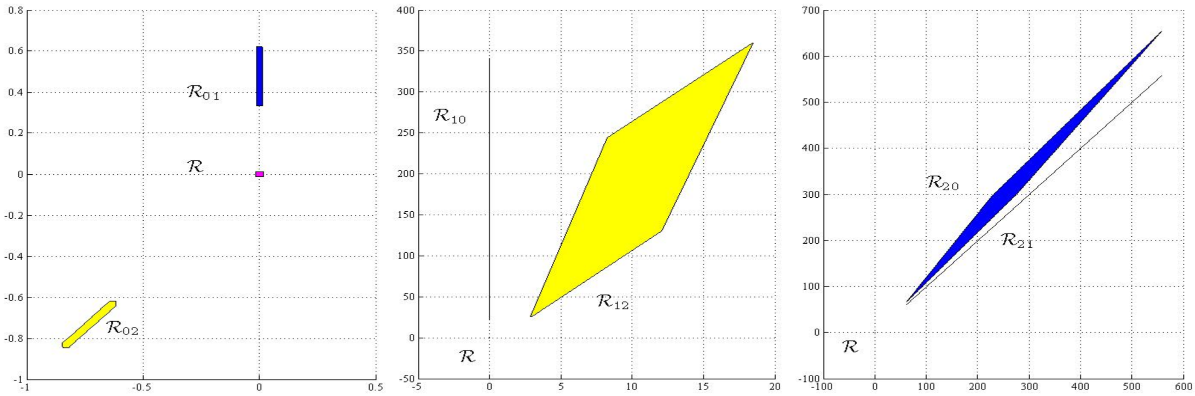

Fig. 2. Separation of the matching sets $\mathcal{R}$ (small sets centred at zero) and after-change sets $\mathcal{R}_{ij}$ (larger sets away from zero) associated with the ith observer: corresponding to F_0 (left), F_1 (centre) and F_2 (right).

The "degree of freedom" signals d^{F_ℓ} are selected such that (14) holds as follows: $d^{F_0} = [0\ 0]^T$, $d^{F_1} = [0.1\ 0]^T$, $d^{F_2} = [0\ 0.2]^T$. Fig. 2 shows the separation of the matching sets (small sets centred at zero) and after-change sets (larger sets away from zero) for each of the three VAs. The left plot corresponds to the VA associated with the fault matrix F_0, the middle plot to F_1 and the right plot to F_2. It follows from the separation of these sets for each VA that Assumption 5.1 holds for this example and thus correct FDI is guaranteed for the given fault scenario.

The scheme was simulated under the following fault scenario: F_1 was used between 140s and 340s, F_2 between 590s and 790s, and F_0 at all other times. The FDI Algorithm 5.3 detects, identifies and accommodates the actuator fault one step after each change in the fault situation. The upper plot of Fig. 3 presents the evolution of the performance variable v using $f = 0.1$ in (34). It is observed that v evolves close to the desired setpoint $v^* = 0.75$ in all fault situations, with a small offset under F_1 and F_2, which cannot be fully compensated due to the LPV nature of the system. To test robustness agains errors in the FDI decision, we repeated the test using $f = 0.05$ (Fig. 3, middle plot) and $f = 0.15$ (Fig. 3, bottom plot) in (34) as the 'actual' plant fault magnitudes. In both these cases the FTC algorithm 'believes' f to be $f = 0.1$ and reconfigures the VA with this value. Note that the performance slightly deteriorates and the parameter variation becomes apparent, but the signal remains bounded within acceptable margins.

8. CONCLUSIONS

In this paper, a new LPV actuator fault tolerant control scheme is presented which integrates a set-separation approach to FDI together with a controller reconfiguration method based on a bank of virtual actuators. Each VA is designed to accomodate an abrupt (common/most critical) actuator fault over a finite range. In designing the VAs, the LPV uncertainty of the plant has been taken into account applying convex polytopic properties. As a result, each VA, together with the nominal controller and observer are self-scheduled at each time when a new measurement of the linear varying parameter is obtained. A switching rule engages the suitable VA from the bank and is based on sets defined for residual signals constructed directly from the

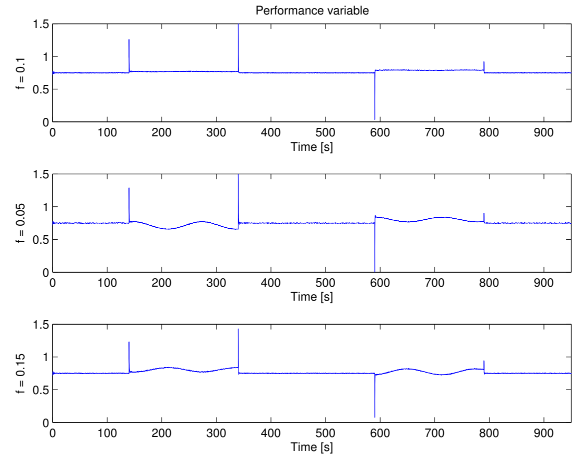

Fig. 3. Performance variable v under different fault magnitudes.

virtual actuators signals. We have shown that the overall robust scheme guarantees closed-loop boundedness under all considered fault situations.

REFERENCES

[1] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, Springer, 2nd Edition, 2006.
[2] S. Ding, *Model-Based Fault Diagnosis Techniques*, Springer, 2008.
[3] T. Steffen, *Control Reconfiguration of Dynamical Systems*, Springer, 2005.
[4] M.M. Seron, J.A. De Doná and J. Richter, Bank of virtual actuators for fault tolerant control, *in Proceedings of 18th IFAC World Congress*, 2011, Milano, Italy.
[5] R. Nazari, M. Seron and J. De Doná, Fault-tolerant control of systems with convex polytopic linear parameter varying model uncertainty using virtual-sensor-based controller reconfiguration, *Annual Reviews in Control*, Vol. 37, 2013, pp. 146-153.
[6] W.J. Rugh, J.S. Shamma, Research on gain scheduling, *Automatica*, Vol. 36, 2000, pp. 1401–1425.
[7] S. Montes de Oca, and V. Puig, Fault-tolerant control design using a virtual sensor for LPV systems, *in Proceedings of the IEEE Conf. on Control and Fault Tolerant Systems, SysTol 2010*, Nice, France, 2010.
[8] H. Haimovich and M.M. Seron, Bounds and invariant sets for a class of discrete-time switching systems with perturbations, *International Journal of Control*, Vol. 87(2), 2014, pp. 371–383.
[9] J. Daafouz and J. Bernussou, Parameter dependent Lyapunov function for discrete time systems with time varying parametric uncertainties, *Systems and Control Letters*, Vol. 43, 2001, pp. 355–359.