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# Enhanced distinguishability in Supervisory Fault Tolerant Control

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**Abstract:** This paper deals with the design of an estimator-based supervisory Fault Tolerant Control (FTC) scheme for Linear Time Invariant (LTI) systems. A formal stability proof based on dwell-time conditions is presented when the state of the detection filter does not correspond to the plant state, as in classical (Luenberger) observer-based approaches. In this context, fault isolability could be improved leading to an enhanced distinguishability between all possible operating modes. Note that this paper should be understood like a preliminary work to provide a unified context for a norm-based performance optimization problem. The efficiency of the proposed technique is illustrated on a numerical example. *Copyright © 2014 IFAC*

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## 1. INTRODUCTION

In active Fault Tolerant Control (FTC), the interaction between Fault Detection and Isolation (FDI) and FTC algorithms is a key issue (Zhang & Jiang, 2008; Zolghadri *et al*., 2013). In fact, faults are detected and identified by a FDI system to allow a reconfiguration of control laws accordingly. Very often, it is assumed that a perfect FDI output is available (*i.e.* no detection delay, no false alarm, ...). It is however well-known that poor FDI performances can affect stability and performances of the overall system (Shin & Belcastro, 2006). In residual-based fault diagnosis, the two main design goals consist in: *i*) minimizing the influence of unknown inputs (noise, uncertainties, disturbances) on residuals and *ii*) maximizing the effect of fault(s) on them. If disturbance and fault act on the same orthogonal space, FDI performances of classical observer-based methods could be problematic for a satisfactory fault accommodation. One solution among others could consist in using $H_\infty/H_-$ optimization-based methods to manage these contradictory goals in order to guarantee a certain level of FDI performances (Ding, 2008). However, inherent imperfections of FDI part still exist and make the need of reconfigurable strategies taking into account the interactions between FDI and FTC parts of great importance to safe operation.

In order to address the above problem, supervisory control concept (Liberzon, 2003; Hespanha *et al*., 2003, Yoon *et al*., 2007) has received considerable attention for the development of FTC strategies. To cope with the delay due to fault isolation, an unfalsified supervisory FTC scheme is proposed in (Yang *et al.*, 2009). The idea consists in using the switching algorithm to simultaneously perform fault isolation and FTC. More precisely, a given switching sequence of controllers is performed, until the appropriate one is found. (Jain *et al.*, 2012) presents a supervisory scheme based on behavioural point of view. The method is referred as a model-free approach by the authors and thus seems to be very attractive. However, the need to compute on-line the inverse of controllers can make it problematic. In (Efimov *et al*., 2013), the global stability of a supervisory FTC based on dwell-time conditions is proposed for the case of disturbed systems subjected to multiplicative/additive faults. The dwell-time is the minimal time interval between two switches guaranteeing stability. The system remains stable if the time interval between any two consecutive switching instants is not smaller than the dwell time. The work reported in (Efimov *et al*., 2013) seems to be particularly appealing since *i*) global stability is proved even if the bank of estimators - that plays the role of FDI unit - fails to identify the correct faulty operating mode (in this case, it is shown that a chattering phenomenon may exist) and *ii*) it is shown how FDI and FTC performances interact and can be managed to get a global optimal solution under a given criterion.

In order to overcome the chattering phenomenon underlined in (Efimov *et al*., 2013), one solution consists in improving the distinguishability property. The distinguishability concerns the capacity of discerning all operating modes between them (Takrouni *et al*., 2011; Lou & Si, 2009). In switching control theory, this feature is usually referred to mode-observability (Baglietto *et al*., 2013; Caravani & De Sentis, 2012). Mode observability can also be formulated as the classical fault isolability derived from FDI community where it is possible to take into account the performances of FDI algorithm on residuals (Basseville, 2001).

In this paper, a bank of pre-defined estimators is used for FDI purpose. The estimator having the smallest estimation error enables to identify the current operating mode. In (Efimov *et al*. 2013), all proofs have been done for a bank of Luenberger estimators. Here, the contribution is the establishment of a stability proof in the case of more general detection filters, which could lead to an enhancement of the distinguishability.

The paper is organised as follows. Section 2 introduces the notations used in this paper. Section 3 gives the problem statement in FTC context. Section 4 is devoted to the formal stability proof when the state of the detection filter does not correspond to the plant state is given and section 4 presents the efficiency of the technique on a numerical example.

## 2. NOTATIONS

The set of real numbers is denoted by $\Re$. A square matrix $X \in \Re^{n\times n}$ is called Hurwitz if all of its eigenvalues possess a strictly negative real part. For any $x \in \Re$ the symbol $|x|$ denotes its absolute value, if $x \in \Re^n$ then $|x|$ states for the Euclidean norm. For a matrix $X \in \Re^{n\times m}$ the symbol $|X|$ denotes the corresponding induced norm $|X| = \max_{1\le i\le m} \sqrt{\lambda_i(X^T X)}$ where $\lambda_i(X^T X)$ is the $i^{th}$ eigenvalue of the matrix $(X^T X)$. For a measurable function $d : \Re_+ \to \Re^d$ the corresponding $L_\infty$ norm is defined as $\|d\|_{[0,T)} = ess\sup_{0\le t\le T} |d(t)|$, $\|d\| = \|d\|_{[0,+\infty)}$. The set of all such functions with the property $\|d\| < +\infty$ is denoted as $L_\infty^d$. The symbol $\wedge$ corresponds to the logic "and".

## 3. PROBLEM STATEMENT

Assume that the system under consideration operates in *N* possible modes such that there exist *N*-1 faulty modes and 1 fault-free one. It is considered that each system operating mode (indexed $i^{th}$) can be modelled according to

$$\dot{x} = A_i x + B_i u + G_i d, \quad y = Cx, \quad i = 1,...,N, \quad N > 1 \tag{1}$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$ and $d \in \Re^d$ are state, control, output and disturbance vectors respectively. The index $i$=1 corresponds to the nominal (fault-free) situation and the *N*-1 others are devoted to fault situations. $A_i$, $B_i$ and $G_i$ refer to the $i^{th}$ operating mode. In this work, it is assumed that the plant shares the same state and same measurements.

For the sake of simplicity, the measurement noise is not considered in (1). However, it can be verified that the main theorem developed in the following still yields. Interested readers can refer to (Cieslak *et al*., 2014) for more details.

Let the pre-computed controllers $K_i$ be given by

$$\dot{\tilde{x}} = \tilde{A}_i \tilde{x} + \tilde{B}_i (y_{ref} - y), \quad u_i = \tilde{C}_i \tilde{x}, \; i = 1,...,N \tag{2}$$

where $\tilde{x} \in \Re^{\tilde{n}_i}$, $y_{ref} \in \Re^p$ and $u_i \in \Re^m$ are state of (2), reference and control signals respectively. The following assumption is made.

**Assumption 1:** *The matrices*

$$H_i = \begin{bmatrix} A_i & B_i\tilde{C}_i \\ -\tilde{B}_i C & \tilde{A}_i \end{bmatrix} \tag{3}$$

*are Hurwitz for all* $i = 1,...,N$. □

The matrices $H_i$ for all $i = 1,...,N$ defined in (3) describe the dynamics of the closed-loop. In the matched case (the indexes of (1) and (2) are the same), all matrices $H_i$ guarantee pre-defined performance levels (disturbance attenuation, tracking, …). In the unmatched case (the index of the plant (1) and the control (2) are different, *i.e.* $j \ne i$), the closed-loop behaviour may become unstable. In others words, each controller (2) is designed to achieve the best performance level in the $i^{th}$ operating mode, without additional design constraints due to switching control concept. It is a point of great importance in practice.

Both systems (1) and (2) define a family of linear systems with the index $i \in I$. Estimator-based supervisory FTC concept can be considered to achieve fault tolerance (Liberzon, 2003; Efimov *et al.*, 2013; Cieslak *et al*., 2014). The estimator (referred as detection filter in Fig. 1) having the smallest estimation error in the Euclidean norm sense enables to identify the current operating mode. In (Efimov *et al.*, 2013), a bank of pre-defined Luenberger estimators has been considered. Here, let the fault detection filters be defined in a more general structure by

$$\dot{x}_{F_i} = A_{F_i} x_{F_i} + B_{F_{u_i}} u + B_{F_{y_i}} y, \quad e_i = C_{F_i} x_{F_i}, \quad i = 1,...,N \tag{4}$$

where $x_{F_i} \in \Re^{n_{F_i}}$ is the states of (4). $A_{F_i}$ are Hurwitz. $B_{F_{u_i}}, B_{F_{y_i}}$ and $C_{F_i}$ are matrices of appropriate dimensions. $e_i \in \Re^{n_e}$ is the enhanced error signal that is close to zero in the matched case (*i.e.* the indexes of (1) and (4) are the same) and different to zero in all other situations. In this work, the case of an augmented-order filter (4) is investigated, *i.e.* $n_{F_i} > n$. This situation is the most interesting configuration for the improvement of distinguishability since the addition of some objectives (disturbance attenuation, etc.) in the detection filter design helps to identify more accurately the current operating mode, but leads generally to augmented-order filters (see for example (Garcia *et al*., 2002)). For the case $n_{F_i} = n$, the interested reader can refer to (Efimov *et al.*, 2013).

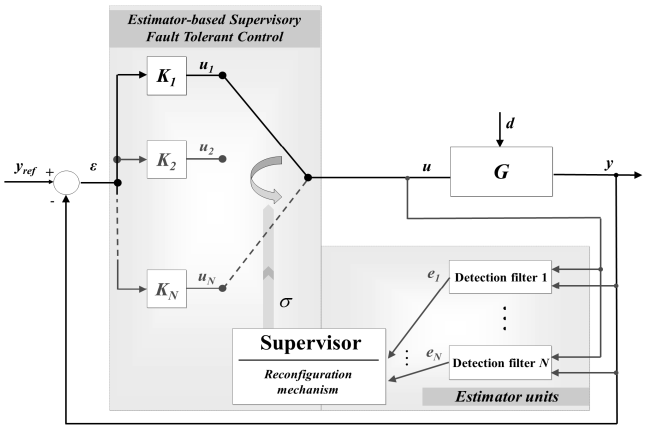


Fig. 1. Estimator-based supervisory FTC architecture

Now, consider the reconfiguration mechanism as depicted in Fig. 1. Thanks to the bank of detection filters (4), the operating mode can be identified. This available information is next used by a supervisor to switch on the appropriate controllers (2). This switching logic is formulated using the following decision map

$$\mathcal{H} : \Re^m \times \Re^p \times \Re^{\sum_{i=1}^{N} n_{F_i}} \to I \quad (5)$$

that generates the switching signal

$$\sigma(t) = \mathcal{H}(u, y, x_{F_1}, ..., x_{F_N}) \quad (6)$$

to control the plant (1) subjected to faults.

## 4. MAIN RESULT

The goal of this section is to establish the stability of the whole system (see Fig. 1) for the case $d \in L_\infty^d$ and $y_{ref} \in L_\infty^{y_{ref}}$ by means of dwell-time conditions. In the interest of brevity, the case of constant index $i \in I$ is considered here. However, note that the main result can be extended to determine the minimum admissible time interval between two consecutive faults $T_D$ using materials reported in (Efimov *et al.*, 2013; Wu *et al.*, 2013; Cieslak *et al.*, 2014).

For each fixed plant index $i \in I$ and according to the enhanced error signal definition of (4), the smaller enhanced error $e_i$ helps us to define the current operating mode. Then, consider the following switching logic:

$$t_0 = 0, t_{k+1} = \underset{t \geq t_k + \tau_D}{\arg\inf} \left\{ \left| e_{\sigma(t_k)}(t) \right| > \left| e_j(t) \right|, j = 1, ..., N, j \neq \sigma(t_k) \right\} \quad (7)$$

$$\sigma(t_k) = \arg\min_{1 \leq j \leq N} \left| e_j(t_k) \right|, k \geq 0 \quad (8)$$

$$\sigma(t) = \sigma(t_k) \text{ for all } t_k \leq t < t_{k+1}, k \geq 0 \quad (9)$$

where $t_k, k \geq 0$ are instants of switches and $\tau_D > 0$ is the dwell-time. For $t_0 = 0$ the switching signal is initialized as $\sigma(0) = \arg\min_{1 \leq j \leq N} \left| e_j(0) \right|$. The rule (8) is next used for all time instants $t_k$. The time instant of switch $t_{k+1}$ is calculated in (7) as the first time instant after $t_k + \tau_D$ when one enhanced error signal of (4) becomes smaller than the current one used to control the plant.

**Theorem 1:** *Let assumption 1 holds and $i(t) = cst$ for all $t \geq 0$. Let us consider the following state vector $\varsigma = [\chi_k^T \ \tilde{x}^T \ x^T \ z_k^T \ x_{q_k}^T \ x_{F_1}^T \cdots x_{F_N}^T]^T$ where $x_{F_1}^T \cdots x_{F_N}^T$ part does not contain $x_{F_k}^T$. $\chi_k$, $z_k$, $\pi$ and $x_{q_k}$ will be defined in the proof. There exists $\tau_D > 0$ such that for any $\varsigma(0) \in \Re$, $d \in L_\infty^d$ and $y_{ref} \in L_\infty^{y_{ref}}$, the solutions of system (1), (2), (4) and (6) give for all $t \geq 0$*

$$|\varsigma(t)| \leq v_i e^{-\mu_i t / \tau_D} |\varsigma(0)| + \upsilon_i \left\{ \|\delta\|_{[0,t)} + \|d\|_{[0,t)} + \|y_{ref}\|_{[0,t)} \right\} \quad (10)$$

*with $v_i > 0$, $\mu_i > 0$, $\upsilon_i > 0$ and where*

$$\delta(t) = \begin{cases} C[\pi_i(t) - \pi_{\sigma(t_k)}(t)] \ if \ t \in [t_k, t_k + \tau_D) \wedge |e_i(t)| < \left| e_{\sigma(t_k)}(t) \right| \\ 0 \ otherwise \end{cases}$$ ■

**Proof:** Based on the observer-based structures (Alazard & Apkarian, 1999), the detection filters defined by (4) can be rewritten without loss of generality according to

$$\begin{cases} \begin{pmatrix} \dot{z}_i \\ \dot{x}_{q_i} \end{pmatrix} = \begin{pmatrix} A_i - L_i C & 0 \\ -B_{q_{y_i}} C & A_{q_i} \end{pmatrix} \begin{pmatrix} z_i \\ x_{q_i} \end{pmatrix} + \begin{pmatrix} J_i \\ B_{q_{u_i}} \end{pmatrix} u + \begin{pmatrix} L_i \\ B_{q_{y_i}} \end{pmatrix} y & \begin{matrix} (11a) \\ (11b) \end{matrix} \\ e_i = \begin{pmatrix} W_i & C_{q_i} \end{pmatrix} \begin{pmatrix} z_i \\ x_{q_i} \end{pmatrix} & (11c) \end{cases}$$

where $(A_{q_i}, [B_{q_{u_i}} \ \ B_{q_{y_i}}], C_{q_i})$, $i = 1, ..., N$ are the proper state-space representations of a Youla parameter $Q_i$ for each system operating modes. $z_i \in \Re^n$ and $x_{q_i} \in \Re^{(n_{F_i} - n)}$ are the states of (4) reformulated by means of observer-based structures (Alazard & Apkarian, 1999) for the case of augmented-order filters, *i.e.* $n_{F_i} > n$. $L_i$ is an estimation gain. $J_i$ and $W_i$ are matrices of appropriate dimensions. In the interest of brevity, the mechanization equations leading to this reformulation are omitted here. The interested reader can refer to (Alazard & Apkarian, 1999), (Cieslak *et al.*, 2010) for some guidelines on this reformulation.

To find the stability proof, let us introduce the following fictitious dynamics

$$\dot{\chi}_i = (A_i - L_i C)\chi_i + J_i P_i u + L_i y \quad (12)$$

where the only difference from (11a) is due to the introduction of a matrix $P_i$. Now, let $\pi_i$ be an error signal given by $\pi_i = x - \chi_i$. From (1) and (12), it follows:

$$\dot{\pi}_i = (A_i - L_i C)\pi_i + (B_i - J_i P_i)u + G_i d \quad (13)$$

Since $A_{F_i}$ in (4) are Hurwitz, $(A_i - L_i C)$ involved in (11a) and (13) have the same property. Thus, equation (13) is asymptotically stable and has bounded solutions ($d \in L_\infty^d$) if the quantity $(B_i - J_i P_i)$ is null. Based on that observation, let $P_i$ be designed according to the following proposition:

**Proposition 1:** *Let us consider $B_i \in \Re^{n \times m}$ and $J_i \in \Re^{n \times m}$ for $i = 1, ..., N$. $P_i$ can be computed by solving*

$$M_i^T B_i - M_i^T J_i P_i = 0 \quad (14)$$

*where $M_i \in \Re^{n \times m}$ is of appropriate dimension. In the following, the particular solution $M_i = B_i$ will be used to guarantee the stability of (13) in the matched cases.* □

*Remark 1:* Error signals (13) could be more corrupted than the enhanced error signal of (11c), *e.g.* in terms of disturbance attenuation. This is why the switching logic (7)-(9) is only based on $e(t)$ and that the property of error signals $\pi_i = x - \chi_i$ will be only used in the stability proof.

The switched system (1), (2), (6), (11) and (12) equipped with the switching logic (7)–(9) has continuous solutions defined for all $t \geq 0$ and all $d \in L_\infty^d$. Indeed, it is a continuous linear system on all intervals $[t_k, t_{k+1})$, $k \geq 0$. The switches happen at isolated time instants $t_k$, $k \geq 0$ due to dwell-time presence. For each fixed plant index $i \in I$ and according to the error signal definition of (13)-(14), there exist $C_1 > 0$, $C_2 > 0$ and $\eta > 0$ such that $|\pi_i(t)| \leq C_1 |\pi_i(0)| e^{-\eta t} + C_2 \|d\|$ for all $t \geq 0$. Hence, we can write $|C\pi_i(t)| \leq \left( C_1 |\pi_i(0)| e^{-\eta t} + C_2 \|d\| \right) |C|$. Since $(A_i - L_i C)$ of (13) is Hurwitz in the matched case, we have for all $t \in [t_k + \tau_D, t_{k+1})$, $k \geq 0$:

$$\left| C\pi_{\sigma(t_k)}(t) \right| \leq |C\pi_i(t)| \leq \rho_1 |\pi_i(0)| e^{-\eta t} + \rho_2 \|d\|, \rho_1 = C_1 |C|, \rho_2 = C_2 |C|$$

Therefore, the sequence $\left| C\pi_{\sigma(t_k)}(t) \right|$ is bounded on those intervals. However, the behaviour of the error $\left| C\pi_{\sigma(t_k)}(t) \right|$ for $t \in [t_k, t_k + \tau_D)$ is hard to evaluate, but since the signal $\left| C\pi_{\sigma(t_k)}(t) \right| \leq |C\pi_i(t)| + |\delta(t)|$ for all $t \geq 0$ due to the definition of $\delta$, we can obtain the following estimate for $d \in L_\infty^d$:

$$\left|C\pi_{\sigma(t_k)}(t)\right| \le \rho_1 |\pi_i(0)| e^{-\eta t} + \rho_2 \|d\| + \|\delta\|, t \ge 0$$

Let us now consider the equations of the system (1), (2), (6), (11) and (12) (for simplicity we use shorthand notation $\sigma(t_k) = k$). The proposed supervisory FTC architecture gives the following equations for $j = 1,...N, j \ne k$:

$$\begin{aligned}
\dot{x} &= A_i x + B_i u + G_i d - L_i y + L_i y \\
&= (A_i - L_i C)x + B_i \widetilde{C}_k \widetilde{x} + L_i (C\pi_k + C\chi_k) + G_i d \\
\dot{\widetilde{x}} &= \widetilde{A}_k \widetilde{x} + \widetilde{B}_k (y_{ref} - y) = \widetilde{A}_k \widetilde{x} - \widetilde{B}_k C\pi_k - \widetilde{B}_k C\chi_k + \widetilde{B}_k y_{ref} \\
\dot{\chi}_k &= (A_k - L_k C)\chi_k + J_k P_k \widetilde{C}_k \widetilde{x} + L_k (C\pi_k + C\chi_k) \\
&= A_k \chi_k + B_k \widetilde{C}_k \widetilde{x} + L_k C\pi_k \\
\dot{z}_k &= (A_k - L_k C) z_k + J_k \widetilde{C}_k \widetilde{x} + L_k (C\chi_k + C\pi_k) \\
\dot{x}_{q_k} &= A_{q_k} x_{q_k} - B_{q_{y_k}} C z_k + B_{q_{u_k}} \widetilde{C}_k \widetilde{x} + B_{q_{y_k}} (C\pi_k + C\chi_k) \\
\dot{x}_{F_1} &= A_{F_1} x_{F_1} + B_{F_{u_1}} \widetilde{C}_k \widetilde{x} + B_{F_{y_1}} (C\pi_k + C\chi_k) \\
&\vdots \\
\dot{x}_{F_N} &= A_{F_N} x_{F_N} + B_{F_{u_N}} \widetilde{C}_k \widetilde{x} + B_{F_{y_N}} (C\pi_k + C\chi_k)
\end{aligned}$$

where the substitution $y = C\pi_k + C\chi_k$ has been introduced in the aforementioned equations. Introducing $\varsigma_k = [\chi_k{}^T \;\; \widetilde{x}^T \;\; x^T \;\; z_k{}^T \;\; x_{q_k}{}^T \;\; x_{F_1}{}^T \cdots x_{F_N}{}^T]^T$ such that $x_{F_1}{}^T \cdots \;\; x_{F_N}{}^T$ part does not contain $x_{F_k}{}^T$. The system (1), (2), (6), (11) and (12) can be written according to

$$\dot{\varsigma}_k = W_{k,i}\varsigma_k + V_{k,i} C\pi_k + \widetilde{G}_k d + \overline{G}_k y_{ref} \tag{15}$$

with $V_{k,i} = [L_k{}^T \;\; -\widetilde{B}_k{}^T \;\; L_i^T \;\; L_k{}^T \;\; B_{q_{y_k}}{}^T \;\; B_{F_{y_1}}{}^T \cdots \;\; B_{F_{y_N}}{}^T]^T$, $\widetilde{G}_k = [0 \;\; 0 \;\; G_i^T \;\; 0 \;\; 0 \;\; 0 \cdots 0]^T$, $\overline{G}_k = [0 \;\; \widetilde{B}_k{}^T \;\; 0 \;\; 0 \;\; 0 \;\; 0 \cdots 0]^T$.

The matrix $W_{k,i}$ is lower triangular matrix where its blocks in the main diagonal are

$$H_k, A_i - L_i C, A_k - L_k C, A_{q_k}, A_{F_1}, \cdots, A_{F_N} \tag{16}$$

according to the proposition 1. All matrix blocks of (16) are Hurwitz with the previous definitions. Hence, the matrix $W_{k,i}$ has the same property.

Based on the definition of (4), there exists a single minimum among estimation errors, *i.e.*:

$$|e_i(t)| < |e_j(t)| \text{ for all } t \ge T \text{ and } j = 1,\cdots,N, j \ne i \tag{17}$$

Then due to (7)-(9), the switching logic stops after a finite number of switches $k \ge 0$ and $\sigma(t_k) = i$. To calculate the required $\tau_D$ guaranteeing overall stability, let us consider the following estimates for $t \in [t_k, t_{k+1})$ of (15)

$$\begin{aligned}
|\varsigma(t)| \le \beta_{\sigma(t_k),i} |\varsigma(t_k)| e^{-\alpha_{\sigma(t_k),i}(t-t_k)} + \gamma_{\sigma(t_k),i} \left\|C\pi_{\sigma(t_k),i}\right\|_{[t_k,t)} \\
+ \psi_{\sigma(t_k),i} \|d\| + \varpi_{\sigma(t_k),i} \|y_{ref}\|
\end{aligned} \tag{18}$$

$$\left\|C\pi_{\sigma(t_k)}\right\|_{[t_k,t)} = ess\sup_{t_k \le \tau < t} \left|C\pi_{\sigma(t_k)}(\tau)\right| \le \rho_1 |\pi_i(0)| e^{-\eta t_k} + \rho_2 \|d\| + \|\delta\|_{[t_k,t)}$$

where $\alpha_{j,i}$ is the minimal in norm real part of the matrix $W_{j,i}$ eigenvalues, $\psi_{j,i} = \left|W_{j,i}{}^{-1} \widetilde{G}_i\right|$, $\gamma_{j,i} = \left|W_{j,i}{}^{-1} V_{j,i}\right|$, $\beta_{j,i} = \sup_{t \ge 0} \left|\exp(W_{j,i} t)\right|$ and $\varpi_{j,i} = \left|W_{j,i}{}^{-1} \overline{G}_i\right|$ for $j = 1,\cdots,N, j \ne i$. Using previous definitions, $|e_i(0)| \le |\varsigma(0)|$ and $\alpha_{j,i} \le \eta$ for all $j = 1,\cdots,N$. From standard results on dwell-time conditions on switched system stability (Liberzon, 2003; Efimov *et al.*, 2013), $\tau_D$ can be computed by

$$\tau_D = \max_{1 \le j \le N} - \alpha_{j,i}{}^{-1} \ln(\lambda \beta_{j,i}{}^{-1}) \tag{19}$$

where $0 < \lambda < 1$ is a design constant defined such that $e^{-\eta t_k} \le \lambda^k, k \ge 0$. Using the same mathematical development given in the proof of theorem 1 of (Efimov *et al.*, 2013), the use of (19) leads to the following upper bound of the switched system (1), (2), (4) and (7)-(9) for all $t \ge 0$:

$$\begin{aligned}
|\varsigma(t)| \le e\Big(\overline{\beta}_i (1 - 4\ln(\lambda)^{-1} e^{-1} \overline{\gamma}_i \rho_1 \lambda^{-1}) + \overline{\gamma}_i \rho_1\Big) e^{0.25\ln(\lambda) t \tau_D{}^{-1}} |\varsigma(0)| \\
+ \left(1 + \overline{\beta}_i (1-\lambda)^{-1}\right)\left(\overline{\gamma}_i \|\delta\| + \overline{\psi}_i \|d\| + \overline{\varpi}_i \|y_{ref}\|\right)
\end{aligned} \tag{20}$$

with $\overline{\psi}_i = \max_{1 \le j \le N} (\gamma_{i,j}\rho_2 + \psi_{i,j})$, $\overline{\beta}_i = \max_{1 \le j \le N} \beta_{i,j}$, $\overline{\gamma}_i = \max_{1 \le j \le N} \gamma_{i,j}$ and $\overline{\varpi}_i = \max_{1 \le j \le N} \varpi_{i,j}$. The identification of (10) and (20) leads to the following algebraic relations:

$$\nu_i = \overline{\beta}_i e + (e - 4\ln(\lambda)^{-1} \overline{\beta}_i \lambda^{-1}) \overline{\gamma}_i \rho_1 \tag{21}$$

$$\mu_i = -0.25\ln(\lambda) \tag{22}$$

$$\upsilon_i = \max\{\overline{\gamma}_i, \overline{\psi}_i, \overline{\varpi}_i\}(1 + \overline{\beta}_i (1-\lambda)^{-1}) \tag{23}$$

Equations (21)-(23) complete the proof. ■

The proof of theorem 1 gives the dwell-time constant $\tau_D$ (see (19)) and $\nu_i$, $\mu_i$ and $\upsilon_i$ are derived from (20). Using a bank of augmented-order detection filters, the enhanced error signals in a matched case should be less corrupted than those provided by classic Luenberger structures (Efimov *et al.*, 2013) in terms of disturbance attenuation and robustness to model uncertainty. This leads to an enhanced capability of estimator-based supervisory scheme (4), (7)-(9) to identify the correct operating mode and *de facto* reduce the possible chattering phenomenon.

## 5. ACADEMIC EXAMPLE

This section provides some numerical simulation to illustrate the proposed methodology. The model is taken from (Hou *et al.*, 2010) and corresponds to the Highly MAneuverable Technology (*HiMAT*) aircraft-based benchmark. Here, we consider the linear model derived from the flying operating point that corresponds to an altitude of 2500 [ft] with a Mach number of 0.29 (see (Hou *et al.*, 2010) for more details). The aircraft has two sensors to measure the pitch rate $y_q$ [rad/s] and the angle of attack $y_\alpha$ [rad]. It also possesses three inputs: the elevons $\delta_e$, elevators $\delta_s$, and canard flaps $\delta_c$. Two actuator faults are considered. Both are an abnormal gain variation (effectiveness loss of 50%) of one control channel leading to the following switched system

$$\dot{x}(t) = A_i x(t) + B_i u(t), \; i = 1,2,3 \tag{24}$$

where the state $x$ is composed by the angle of attack and the pitch rate and $u = (\delta_s, \delta_e, \delta_c)^T$. Each measure is corrupted by a noise of 0.1°. The matrices $A_i$ and $B_i$ denote the state-space representations of the aircraft in fault-free ($i = 1$) and fault operating modes ($i = 2,3$). The index $i = 2$ and $i = 3$

correspond to elevator and elevon malfunctions respectively. In healthy situations ($i = 1$), matrices are given by:

$$A_1 = \begin{bmatrix} -1.0772 & 0.96528 \\ 9.068 & -1.5077 \end{bmatrix}, B_1 = \begin{bmatrix} -0.17211 & -0.12245 & -0.01431 \\ -7.9948 & -4.955 & 5.0369 \end{bmatrix}$$

Three control laws possessing integral action are designed using a linear quadratic approach (Cieslak *et al.* 2014). Let the three detection filters (4) be now designed by means of an augmented Kalman filter (Garcia *et al.*, 2002) to be robust against some flexible mode effects. Due to space limitation, we only provide the state-space representations of each filter:

$$A_{F_1} = A_{F_2} = A_{F_3} = \begin{bmatrix} -10 & 0 & 1 & 0 & 0 \\ 0 & -10 & 0 & 1 & 1 \\ 0 & 0 & -1.4017 & -0.0252 & -0.9905 \\ 0 & 0 & 8.143 & -4.331 & -2.823 \\ 0 & 0 & 0.934 & 0.3116 & -10.688 \end{bmatrix}$$

$$B_{F_1} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ -0.172 & -0.1225 & -0.0143 & 0.3245 & 0.9905 \\ -7.995 & -4.955 & 5.037 & 0.9249 & 2.823 \\ 0 & 0 & 0 & 0.0656 & 0.6884 \end{bmatrix}$$

$$B_{F_2} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ -0.086 & -0.1225 & -0.0143 & 0.3245 & 0.9905 \\ -3.997 & -4.955 & 5.037 & 0.9249 & 2.823 \\ 0 & 0 & 0 & 0.0656 & 0.6884 \end{bmatrix}$$

$$B_{F_3} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ -0.172 & -0.0612 & -0.0143 & 0.3245 & 0.9905 \\ -7.995 & -2.477 & 5.037 & 0.9249 & 2.823 \\ 0 & 0 & 0 & 0.0656 & 0.6884 \end{bmatrix}$$

$$C_{F_1} = \begin{bmatrix} 21 & 0 & 0 & 0 & 0 \\ 0 & 21 & 0 & 0 & 0 \end{bmatrix}, C_{F_2} = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \end{bmatrix}, C_{F_3} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \end{bmatrix}$$

Following equations (11a)-(11c), the three above detection filters are reformulated. We obtain:

$$A_{q_1} = \begin{bmatrix} -10 & 0 & 0.3303 \\ 0 & -10 & 1.0083 \\ 0 & 0 & -1.6739 \end{bmatrix}, \quad J_1 = \begin{bmatrix} 6.8993 & 4.2644 & -4.4372 \\ 13.5154 & 8.38 & -8.4863 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0. & 4.2672 \\ 0 & 12.162 \end{bmatrix}, \quad W_1 = \begin{bmatrix} -0.3972 & 0.0802 \\ -3.9482 & 1.4046 \end{bmatrix}, \quad C_{q_1} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \end{bmatrix},$$

$$B_{q_{y_1}} = \begin{bmatrix} -1 & 0.0342 \\ 0 & -1.0112 \\ 0.9824 & -8.7093 \end{bmatrix}, \quad B_{q_{u_1}} = \begin{bmatrix} 0.0788 & 0.0486 & -0.0515 \\ 0.3931 & 0.2412 & -0.2666 \\ -19.186 & -11.908 & 11.952 \end{bmatrix},$$

$$A_{q_2} = \begin{bmatrix} -10 & 0 & 0.3303 \\ 0 & -10 & 1.0083 \\ 0 & 0 & -1.6739 \end{bmatrix}, \quad J_2 = \begin{bmatrix} 3.4497 & 4.2644 & -4.4372 \\ 6.7577 & 8.38 & -8.4863 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0. & 4.2672 \\ 0 & 12.162 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -0.1135 & 0.0229 \\ -1.1281 & 0.4013 \end{bmatrix}, \quad C_{q_2} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \end{bmatrix},$$

$$B_{q_{y_2}} = \begin{bmatrix} -1 & 0.0342 \\ 0 & -1.0112 \\ 0.9824 & -8.7093 \end{bmatrix}, \quad B_{q_{u_2}} = \begin{bmatrix} 0.0394 & 0.0486 & -0.0515 \\ 0.1966 & 0.2412 & -0.2666 \\ -9.593 & -11.908 & 11.952 \end{bmatrix},$$

$$A_{q_3} = \begin{bmatrix} -10 & 0 & 0.3303 \\ 0 & -10 & 1.0083 \\ 0 & 0 & -1.6739 \end{bmatrix}, \quad J_3 = \begin{bmatrix} 6.8993 & 2.1322 & -4.4372 \\ 13.5154 & 4.19 & -8.4863 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 0. & 4.2672 \\ 0 & 12.162 \end{bmatrix}, \quad W_3 = \begin{bmatrix} -0.1891 & 0.0382 \\ -1.8801 & 0.6689 \end{bmatrix}, \quad C_{q_3} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \end{bmatrix},$$

$$B_{q_{y_3}} = \begin{bmatrix} -1 & 0.0342 \\ 0 & -1.0112 \\ 0.9824 & -8.7093 \end{bmatrix}, \quad B_{q_{u3}} = \begin{bmatrix} 0.0788 & 0.0243 & -0.0515 \\ 0.3931 & 0.1206 & -0.2666 \\ -19.186 & -5.9543 & 11.952 \end{bmatrix}.$$

Matrices $P_i$ , $i = 1,2,3$ are next computed by using the Proposition 1. We have:

$$P_1 = \begin{bmatrix} -22.0115 & -13.4353 & 14.95 \\ -2.7425 & -1.8606 & 1.2296 \\ -36.8222 & -22.6509 & 24.43 \end{bmatrix}, P_2 = \begin{bmatrix} -35.83 & -43.981 & 48.236 \\ 8.4879 & 10.3476 & -11.853 \\ -19.679 & -24.22 & 26.112 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} -22.4976 & -7.1832 & 15.3283 \\ -4.0977 & -0.5318 & 1.379 \\ -36.9114 & -11.4108 & 24.4996 \end{bmatrix}$$

Following the developments given in section 4, it can be easily verified that the stability of (13) for the three matched cases hold with the previous matrices $P_i$. Hence, the dwell-time can be evaluated by means of (19). By taking the design constant $\lambda = 0.95$, we get $\tau_D = 3.16$ [s]. Note that the minimization of dwell-time is not considered in this example to focus on the main steps of the considered work. The interested can refer to (Efimov *et al.*, 2013) for a mutual performance optimization problem over $\lambda$, controllers and detection filters.

The supervisory FTC architecture depicted in Fig. 1 is then designed to assess the proposed scheme. The initial operating point is fixed to a reference attack angle of 7° and a reference pitch rate null. At $t$=20$s$, the reference of attack angle goes to 2°. Elevon fault has been simulated between [15, 40] $s$ and elevator malfunction between the time interval [70, 93] $s$. To emphasize the benefit of the proposed scheme, same simulations are performed when the system is only controlled by the nominal controller $K_1$ (no FTC) and when the system is controlled by the proposed supervisory FTC approach. As it can be seen in Fig. 2, when the supervisory FTC architecture is not in place, the controlled system goes unstable for elevator fault. Furthermore, when supervisory FTC algorithm performs, the controlled system is stable and keeps acceptable performance level (null static error). In addition, it can be seen in Fig. 3 that there is no wrong decision of the bank of estimators to identify the current operating mode.

## 6. CONCLUSION

The problem of designing active FTC systems by means of an estimator-based supervisory control approach is studied. The main contribution of the proposed approach is the establishment of a formal stability proof based on dwell-time conditions when the current operating mode is identified by a bank of augmented-order detection filters, *i.e.* when the state of the detection filters does not correspond to the plant state as in the classical observer-based approaches. This feature could help to avoid chattering phenomenon by an enhanced distinguishability between all possible operating modes. Further investigations are necessary to minimize the dwell-time value within the proposed supervisory system and assess the proposed scheme on more representative benchmarks.

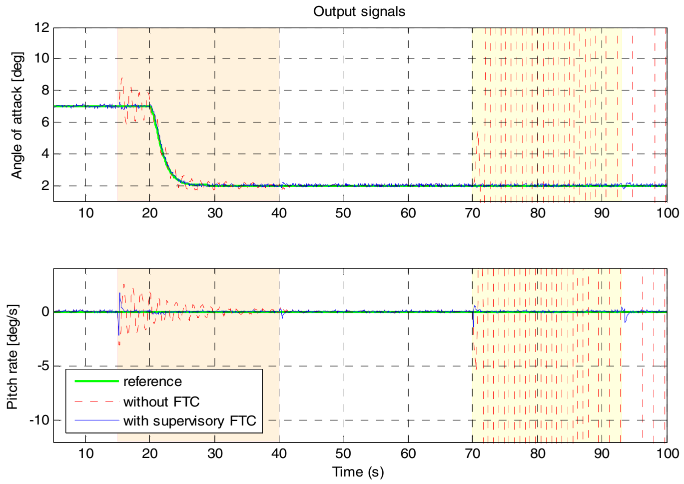


Fig. 2 Angle of attack (top) and pitch rate (bottom) signals

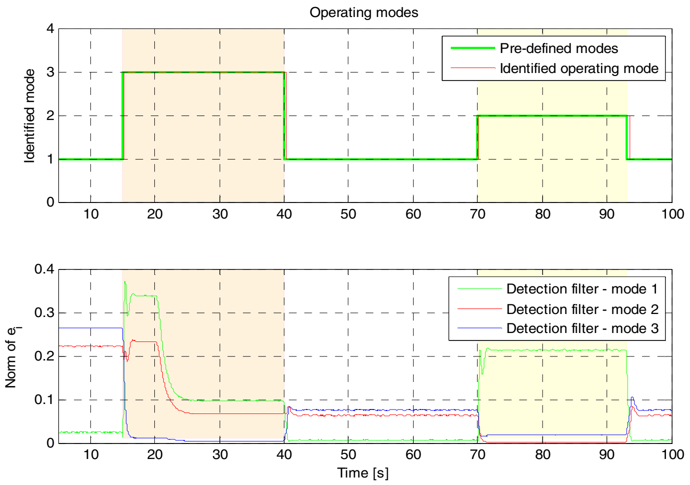


Fig. 3 Identified mode (top) and enhanced error (bottom)

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