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Joint optimization approach of maintenance planning and production Scheduling for a multiple-product manufacturing system

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Abstract: In this paper, we deal with the problem of maintenance planning and production planning for a multiple-product manufacturing system. The manufacturing system under consideration consists of one machine which is subject to random failures and produces several products in order to satisfy some random demands. At any given time, the machine can only produce one type of product. The purpose of this study is to establish an economical production planning followed by an optimal maintenance strategy, taking into account the influence of production rate on the system degradation. Analytical models are developed in order to minimize sequentially the production/storage costs and the total maintenance cost. Finally, a numerical example is presented to illustrate the usefulness of the proposed approach.

1. INTRODUCTION

The joint maintenance and production policies for manufacturing system, which is subject to uncertainties such as machine failures, demand fluctuations, etc., has attracted the attention of several researchers. The development of industrial strategies (maintenance and production) has become very important for industrial companies in order to reduce their costs. In this context, Dehayem et al. (2011) developed a method to find the optimal production, replacement/repair and preventive maintenance policies for a degraded manufacturing system. Gharbi and kenné (2007) assumed that failure frequencies can be reduced through preventive maintenance, and developed joint production and preventive maintenance policies depending on produced part inventory levels. An analytical model and a numerical procedure which allow determining a joint optimal inventory control and an age based on preventive maintenance policy for a randomly failing production system was presented by Rezg et al. (2008). Several reviews have been published to summarize the development in this area; (Aghezzaf et al. 2007, and Dhouib et al. 2012).

This paper examined a problem of the optimal production planning formulation of a manufacturing system consisting of one machine producing several products in order to meet several random demands. The stochastic nature of the system is due to the fact that demands are random and the machine is subject to random breakdowns. We consider that the finite production horizon is divided into sub-periods. At any given sub-period, the machine can only produce one type of product.

This problem was studied by (Kenné et al. 2003). They presented the analysis of the production control and corrective maintenance rate problem in a multiple-machine, multiple-product manufacturing system. They obtained a near optimal control policy of the system through numerical techniques by controlling both production and repair rates. Wei Feng et al. (2012) developed a multi-product manufacturing systems problem with sequence dependent setup times and finite buffers under seven scheduling policies. Sloan and Shanthikumar (2000) presented a Markov decision process model that simultaneously determines maintenance and production schedules for a multiple-product, single-machine production system, accounting for the fact that equipment condition can affect the yield of different product types differently. Filho (2005) developed a stochastic dynamic optimization model to solve a multi-product, multi-period production planning problem with constraints on decision variables and finite planning horizon.

The considered equipment is subject to random failures. The failure rate increases with time and according to the production rate. The machine undergoes a preventive maintenance policy in order to reduce the occurrence of failures. In the literature, the influence of the production rate on the materiel degradation is rarely studied. In this study, we take into consideration this influence in order to establish the optimal maintenance strategy.

Based on the works of Hajej et al. (2009, 2010, 2011), the objective of this study is to determine an economical production plan followed by an optimal maintenance strategy. Firstly, for a given randomly demand, we established an optimal production plan which minimizes the average total storage and production costs. Secondly, using the optimal production plan obtained and its influence on the manufacturing system failure rate, we established an optimal maintenance scheduling which minimizes the maintenance total cost.

This paper is organized as follows: In section 3, we develop the production policy. The maintenance strategy is stated in section 4. A numerical example is presented in section 5. Finally, the conclusion is given in section 6.

2. NOTATIONS

Cp_i	The unit production cost of product i
Cs_i	The unit holding cost of product i during Δt
st_i	The Setup cost of product i
Mc	Cost of corrective maintenance
Mp	Cost of preventive maintenance
U_{imax}	Maximal production rate of product i during Δt
H	The total number of production periods
n	The total number of products
p	The total number of sub-periods
Δt	Length of periods
$\delta_{(.)}$	Length of the sub-period
$d_{i,k}$	Demand of product i in period k
$\sigma(d_{i,k})$	Standard deviation of demand of product i at period k
θ_i	Probabilistic index (related to customer satisfaction) of product i
$S_{i,(.)}$	Stock level of product i at the end of the sub-period
$Z(.)$	The total expected cost of production and inventory over the finite horizon $H.\Delta t$
$\psi(.)$	The total cost of maintenance
$\lambda_{(.)}$	Failure rate function
$\lambda_n(.)$	Nominal failure rate
$\omega(.)$	The average number of failures
T	Intervention period for preventive maintenance actions
$U_{i,j,k}$	Production rate of product i in sub-period j of the period k
$y_{i,j,k}$	A binary variable equal to 1 if the product i is produced in sub-period j of the period k, and 0 otherwise
up	Unit produced
mu	Monetary unit

3. PRODUCTION POLICY

3.1Problem formulation

The aim of this section is to develop an analytical model that allows us to determine the optimal production plan U^*, $\left(U^* = U^*_{i,j,k}\ \forall\{i=1...n\},\{j=1...p\},\{k=1...H\}\right)$, consequently, to determine the quantity and the type of products to produce in each sub-period. We recall that n represents the total number of products, p the number of sub-period and H the total number of production periods. Figure below shows an example of a production plan.

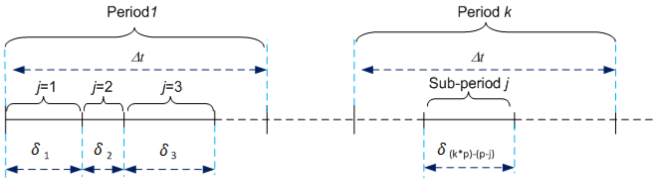

Fig.1. Repartition of the production plan

To develop this section, the following assumptions are specifically made:

- The setup time is negligible;
- Holding and production costs of each product are known and constant;
- Only a single product can be produced in each sub-period;
- The standard deviation of demands $\sigma(d_{i,k})$ and the average demand $\hat{d}_i$ mean for each product i and each period k are known and constant. These two data allow us to obtain the demand of each product in each period.

In this study, we assume that the horizon is divided into H equal periods and each period is divided into p sub-periods with different lengths. We consider that $p = n$ (the total number of products). *"Fig. 1"* shows the distribution of the production plan for the finite horizon $H \cdot \Delta t$. At any given sub-period, the machine can only produce one type of product. The demand of each product i, $\{i=1\ldots n\}$ is satisfied at the end of each period k, $\{k=1\ldots H\}$.

The mathematical formulation of the proposed problem is based on the extension of the model described by Hajej et al. (2011), for the one product case study.

Formally, the stochastic production problem is defined as follows:

$$Min\, E\left\{\left(Z(U)^2\right)\right\}$$
$$U = U_{i,j,k}\ \forall\{i=1...n\},\{j=1...p\},\{k=1...H\}$$

With:

$$Z(U) = \sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n}\left[\begin{array}{l} y_{i,j,k}\cdot\left(st_i + Cp_i\cdot U_{i,j,k}\right) \\ +Cs_i\cdot\dfrac{\delta_{(k\cdot p)-(p-j)}}{\Delta t}\cdot S_{i,(k\cdot p)-(p-j)}\end{array}\right] \quad (1)$$

Where: $E\{\ .\ \}$: The mathematical expectation

Under the following constraints:

$$S_{i,(k\cdot p)-(p-j)} = S_{i,(k\cdot p)-(p-j)-1} + y_{i,j,k}\cdot U_{i,j,k} - Int\left[\frac{j}{p}\right]\cdot d_{i,k}$$
$$\forall\{i=1...n\},\{j=1...p\},\{k=1...H\} \quad (2)$$

Where $Int[\cdot]$: Integer part

$$Prob\left(S_{i,(k\cdot p)} > 0\right) \ge \theta_i\ \ \forall\{i=1...n\},\{k=1...H\} \quad (3)$$

$$0 \le U_{i,j,k} \le \frac{\delta_{(k\times p)-(p-j)}}{\Delta t}\times U_{imax}\ \forall\{i=1...n\},\{j=1...p\},\{k=1...H\} \quad (4)$$

$$\sum_{j=1}^{p}\delta_{(k\times p)-(p-j)}=\Delta t \qquad \forall \{k=1\ldots H\} \tag{5}$$

The first constraint denotes the inventory balance equation for each product *i*, $\{i=1\ldots n\}$ during each period *k*, $\{k=1\ldots H\}$. The equation (3) refers to the satisfaction level of demand of product *i* in each period *k*. The constraint (4) defines the upper production rate of the machine for each product *i*. The aim of (5) is to divide each period into *p* different sub-periods.

Constraints below should also be taken into account:

$$\sum_{i=1}^{n}y_{i,j,k}=1 \qquad \forall \{j=1\ldots p\} For\{k=1\ldots H\} \tag{6}$$

$$\sum_{j=1}^{p}y_{i,j,k}=1 \qquad \forall \{i=1\ldots n\} For\{k=1\ldots H\} \tag{7}$$

$$y_{i,j,k}\epsilon \qquad \forall\{i=1\ldots n\},\{j=1\ldots p\},\{k=1\ldots H\} \tag{8}$$

The equations (6) and (7) mention that only one product *i* can be produced in sub-period *j* of period *k*. The constraint (8) states that $y_{i,j,k}$ is a binary variable. We note that $y_{i,j,k}$ equal to 1 if the product *i* is produced in sub-period *j* of the period *k*, and 0 otherwise.

3.2 The deterministic production model

We admit that the function $f_{(\mathrm{i,j,k})}\forall\{i=1\ldots n\},\{j=1\ldots p\},\{k=1\ldots H\}$ represents the cost of storage and production which is relative to the proposed plan and $E\{.\}$ denotes the value of the mathematical expectation. The quantity stocked of product *i* at the end of the sub-period *j* of period *k* is denoted $S_{i,(k\times p)-(p-j)}$. The production rate required to satisfy the demand of product *i* at the end of period *k* is $U_{i,j,k}$, where *j* is the sub-period during which the product *i* is produced.

Thus, the problem formulation can be presented as following:

$$U^{*}=Min\left[E\left\{\sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n}f_{(i,j,k)}\left(U_{i,j,k},S_{i,(k.p)-(p-j)}\right)^{2}\right\}\right] \tag{9}$$

So our problem is to determine the decision variables $\left(U_{i,j,k},y_{i,j,k}\text{ and }\delta_{(k.p)-(p-j)}\right)$, required to satisfy economically the various demands under the constraints seen in the previous paragraph.

- The inventory balance equation

The stochastic inventory balance equation is:

$$S_{i,(k\cdot p)-(p-j)}=S_{i,(k\cdot p)-(p-j)-1}+y_{i,j,k}\cdot U_{i,j,k}-Int\left[\frac{j}{p}\right]\cdot d_{i,k}$$

We suppose that the mean and variance of demand are known and constant for each product *i* in each period *k*.

$E\{d_{i,k}\}=\hat{d}_{i,k}$ and $Var\{d_{i,k}\}=\left(\sigma_{d_{i,k}}\right)^{2} \quad \forall\{i=1...n\},\{k=1...H\}$

$Var\{d_{i,k}\}$ is the demand variance of product *i* at period *k*.

The inventory variable $S_{i,(k\times p)-(p-j)}$ is statistically described by its mean:

$$E\left\{S_{i,(k\times p)-(p-j)}\right\}=\hat{S}_{i,(k\times p)-(p-j)}\ \forall\{i=1...n\},\{j=1...p\},\{k=1...H\}$$

We note that $E\{U_{i,j,k}\}=\hat{U}_{i,j,k}=U_{i,j,k}$ because $U_{i,j,k}$ is constant for each interval $\delta_{(k\times p)-(p-j)}$.

Then, the balance equation (2) can be converted into an equivalent inventory balance equation:

$$\hat{S}_{i,(k\cdot p)-(p-j)}=\hat{S}_{i,(k\cdot p)-(p-j)-1}+y_{i,j,k}\cdot U_{i,j,k}-Int\left[\frac{j}{p}\right]\cdot\hat{d}_{i,k}$$
$$\forall i=\{1...n\},\ j=\{1...p\},k=\{1...H\} \tag{10}$$

- The service level constraint:

The second step for transformed our problem into deterministic equivalent formulation is to transform the service level constraint into deterministic equivalent constraint by specifying certain minimum cumulative production quantities that depend on the service level requirements.

Lemma 1:

$$\sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right)\geq Var\left(d_{i,k}\right)\times\varphi^{-1}\left(\theta_{i}\right)+\hat{d}_{i,k}-\hat{S}_{i,(k-1)\times p}$$

$\forall i=\{1...n\},\ j=\{1...p\},k=\{1...H\}$

With:

$\varphi(\theta_i)$: Cumulative Gaussian distribution function

$\varphi^{-1}(\theta_i)$: Inverse distribution function

Proof: (Contact author)

- The expected total cost of production:

In this step, we proceed to a simplification of the expected cost of production and storage. Then, the expression of the expected total cost of production is represented as following:

Lemma 2:

$$\mathrm{Z}(U)=\sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n}\left[\begin{array}{l} y_{i,j,k}\cdot\left(St_{i}+Cp_{i}\cdot\left(U_{i,j,k}\right)^{2}\right)\\ +Cs_{i}\cdot\dfrac{\delta_{(k\cdot p)-(p-j)}}{\Delta t}\cdot\left(\begin{array}{l}\displaystyle\sum_{Q=1}^{k-1}\sum_{l=1}^{p}Int\left(\frac{l}{p}\right)\times\sigma^{2}\left(d_{i,Q}\right)\\ \displaystyle+\sum_{l=1}^{j}Int\left(\frac{l}{p}\right)\times\sigma^{2}\left(d_{i,k}\right)\\ +\left(\hat{S}_{i,(k\cdot p)-(p-j)}\right)^{2}\end{array}\right)\end{array}\right]$$

In summary:

The deterministic optimization problem becomes:

Objective function:

$$Min\left[\sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n}\left[\begin{array}{l} y_{i,j,k}\cdot\left(St_i+Cp_i\cdot\left(U_{i,j,k}\right)^2\right) \\ +Cs_i\cdot\dfrac{\delta_{(k\cdot p)-(p-j)}}{\Delta t}\cdot\left(\begin{array}{l}\displaystyle\sum_{Q=1}^{k-1}\sum_{l=1}^{p}Int\left(\frac{l}{p}\right)\times\sigma^2\left(d_{i,Q}\right) \\ \displaystyle+\sum_{l=1}^{j}Int\left(\frac{l}{p}\right)\times\sigma^2\left(d_{i,k}\right) \\ +\left(\hat{S}_{i,(k\cdot p)-(p-j)}\right)^2\end{array}\right)\end{array}\right]\right]$$

Under the constraints bellow:

- $\hat{S}_{i,(k\cdot p)-(p-j)}=\hat{S}_{i,(k\cdot p)-(p-j)-1}+y_{i,j,k}\cdot U_{i,j,k}-Int\left[\dfrac{j}{p}\right]\cdot\hat{d}_{i,k}$

  $\forall i=\{1...n\},j=\{1...p\},k=\{1...H\}$
- $\displaystyle\sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right)\geq Var\left(d_{i,k}\right)\times\varphi^{-1}\left(\theta_i\right)+\hat{d}_{i,k}-\hat{S}_{i,(k-1)\times p}$
- $0\leq U_{i,j,k}\leq\dfrac{\delta_{(k\times p)-(p-j)}}{\Delta t}\times U_{imax}\ \forall\{i=1\ldots n\},\{j=1\ldots p\},\{k=1\ldots H\}$
- $\displaystyle\sum_{j=1}^{p}\delta_{(k\times p)-(p-j)}=\Delta t\qquad\forall\ \{k=1\ldots H\}$

4. MAINTENANCE STRATEGY

4.1 Description

The aim of this strategy is to define the sub-periods of production that must be followed by preventive maintenance actions. Maintenance strategy adopted in this study is known as preventive maintenance with minimal repair. These preventive actions are put into practice in the period $\gamma\times T$ (γ = 1,2...). The replacement rule for this policy is to replace the system with a new system at each $\gamma\times$ T. If the system fails between preventive maintenance actions, only minimal repair is implemented.

In this study, we assume that:

- Maintenance actions have negligible durations;
- In the case of preventive maintenance, the system becomes as good as new;
- *Mp* and *Mc* costs incurred by the preventive and corrective maintenance actions are known and constant, with *Mc*>>*Mp*.

Generally, if λ *(t)* is the function of machine failure rate, the total maintenance cost per unit time is expressed as following:

$$\psi_{(T)}=\frac{M_p+M_c\times\int_0^T\lambda(t)\,dt}{T}\tag{11}$$

The aim of this maintenance strategy is to find the optimal period of preventive maintenance actions *T** minimizing the total cost per unit time over a given horizon $H\cdot\Delta t$.

The existence of an optimal preventive maintenance period *T**, is proved in the literature. (Lyonnet, 2000) proved that *T** exists if the failure rate is increasing.

In this section we will optimize the maintenance strategy adopted which is a preventive maintenance with minimal repair. From the production plan developed during the time horizon $H\cdot\Delta t$, we determine the optimal number of sub-periods $\left((k\cdot p)-(p-j)\right)^*$, after which the preventive maintenance should be performed. We note that if $\left((k\cdot p)-(p-j)\right)^*$ exceeds $H\cdot\Delta t$, no preventive maintenance action is implemented.

4.2 Failure rate expression

Before determining the analytical model minimizing the total cost of maintenance, we need first, to develop the expression of the failure rate $\lambda_{(k\times p)-(p-j)}(t)$. Then, the average number of failures expression $\omega_{(T,U)}$, during the finite horizon $H\cdot\Delta t$.

$$\lambda_{(k\times p)-(p-j)}(t)=\lambda_{(k\times p)-(p-j)-1}\left(\delta_{(k\times p)-(p-j)-1}\right)+\sum_{i=1}^{n}\frac{U_{i,j,k}\times\Delta t}{U_{imax}\times\delta_{(k\times p)-(p-j)}}\times\boldsymbol{\lambda_n}(\boldsymbol{t})$$

$$\forall t\epsilon\ \ldots)-(p-j)\Big]\ \forall\{k=1\ldots H\}\ \{j=1\ldots p\}\tag{12}$$

After simplifying, the failure rate expression becomes:

Lemma 3:

$$\lambda_{(k\times p)-(p-j)}(t)=\lambda_0+\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\sum_{i=1}^{n}\frac{U_{i,l,Q}\times\Delta t}{U_{imax}\times\delta_{(Q\times p)-(p-l)}}\times\boldsymbol{\lambda_n}\left(\boldsymbol{\delta_{(Q\times p)-(p-l)}}\right)+\sum_{l=1}^{j-1}\sum_{i=1}^{n}\frac{\boldsymbol{U_{i,l,k}}\times\Delta t}{U_{imax}\times\delta_{(k\times p)-(p-l)}}\times\boldsymbol{\lambda_n}\left(\boldsymbol{\delta_{(k\times p)-(p-l)}}\right)+\sum_{i=1}^{n}\frac{\boldsymbol{U_{i,j,k}}\times\Delta\boldsymbol{t}}{\boldsymbol{U_{imax}}\times\boldsymbol{\delta_{(k\times p)-(p-j)}}}\times\boldsymbol{\lambda_n}(\boldsymbol{t})$$

$$\boldsymbol{t}\epsilon\ \ldots)-(\boldsymbol{p}-\boldsymbol{j})\Big]\quad\forall\{k=1\ldots H\}\ \{j=1\ldots p\}$$

Proof: contact the author

4.3 The expression of average number of failures:

Generally the average number of failures in the case of maintenance with minimal repair is expressed during a defined period and under operating conditions assumed to be constant over time. Under these assumptions, the average number of failures for a period T is expressed by the relation bellow:

$$\varphi(T,U)=\int_0^T\lambda(t)\,dt\tag{13}$$

The intervention period of our model is defined by:

$$T = \sum_{Q=1}^{k-1}\sum_{l=1}^{p} \boldsymbol{\delta}_{(Q\times p)-(p-l)} + \sum_{l=1}^{j} \boldsymbol{\delta}_{(k\times p)-(p-l)} \tag{14}$$

Thus,

$$\omega_{(T,U)} = \int_0^{\delta_1} \lambda_1(t)\,dt + \int_0^{\delta_2} \lambda_2(t)\,dt + \ldots + \int_0^{\delta_{(k.p)-(p-j)}} \lambda_{(k\times p)-(p-j)}(t)\,dt \tag{15}$$

Hence the number of failures during the interval $[0,T]$ is expressed as following:

$$\omega_{(T,U)} = \sum_{Q=1}^{k-1}\sum_{l=1}^{p}\int_0^{\delta t} \lambda_{(Q\times p)-(p-l)}(t)\,dt + \sum_{l=1}^{j}\int_0^{\delta t} \lambda_{(k\times p)-(p-l)}(t)\,dt$$

Using the failure rate expression, the average number of failures can be presented as follows:

Lemma 4:

$$\omega_{(T,U)} = \sum_{Q=1}^{k-1}\sum_{l=1}^{p}\int_0^{\delta_{(Q\times p)-(p-l)}} \begin{pmatrix} \lambda_0 \\ +\sum_{d=1}^{Q-1}\sum_{\mu=1}^{p}\sum_{i=1}^{n} \frac{U_{i,\mu,d}\times \Delta t}{U_{imax}\times \delta_{(d\times p)-(p-\mu)}} \times \lambda_n\left(\delta_{(d\times p)-(p-\mu)}\right) \\ +\sum_{\mu=1}^{l-1}\sum_{i=1}^{n} \frac{U_{i,\mu,Q}\times \Delta t}{U_{imax}\times \delta_{(Q\times p)-(p-\mu)}} \times \lambda_n\left(\delta_{(Q\times p)-(p-\mu)}\right) \\ +\sum_{i=1}^{n} \frac{U_{i,j,}\times \Delta t}{U_{imax}\times \delta_{(Q\times p)-(p-l)}} \times \lambda_n(t) \end{pmatrix} dt$$

$$+\sum_{l=1}^{j}\int_0^{\delta_{(k\times p)-(p-l)}} \begin{pmatrix} \lambda_0 \\ +\sum_{d=1}^{k-1}\sum_{\mu=1}^{p}\sum_{i=1}^{n} \frac{U_{i,\mu,d}\times \Delta t}{U_{imax}\times \delta_{(d\times p)-(p-\mu)}} \times \lambda_n\left(\delta_{(d\times p)-(p-\mu)}\right) \\ +\sum_{\mu=1}^{l-1}\sum_{i=1}^{n} \frac{U_{i,\mu,k}\times \Delta t}{U_{imax}\times \delta_{(d\times p)-(p-\mu)}} \times \lambda_n\left(\delta_{(d\times p)-(p-\mu)}\right) \\ +\sum_{i=1}^{n} \frac{U_{i,l,k}\times \Delta t}{U_{imax}\times \delta_{(k\times p)-(p-l)}} \times \lambda_n(t) \end{pmatrix} dt$$

We recall that $\lambda_n(t)$ is the nominal rate of failures and the failure rate $\lambda_{(k\cdot p)-(p-j)}(t)$ depends on the production rate $U_{i,j,k}\ \forall\{i=1...n\},\{j=...p\},\{k=1...H\}$.

The decision variables sought in this policy are: the period k^* and the sub-period j^*, after which we must intervene for a preventive maintenance action. These variables allow us to determine the optimal period of preventive maintenance T. T's formulation is represented in (14).

So our objective function is:

$$Min\left[\Psi\left(k^*, j^*\right)\right]$$

$$\Rightarrow Min\left[\frac{Mp + \omega_{(T,U)}\cdot Mc}{\sum_{r=1}^{k-1}\sum_{\mu=1}^{p}\delta_{(r\cdot p)-(p-\mu)} + \sum_{\mu=1}^{j}\delta_{(k\cdot p)-(p-\mu)}}\right] \tag{16}$$

5. NUMERICAL EXAMPLE

Let us consider a system that produces three products to meet the random demands below. Using the models described in previous sections, we will determine the optimal production plan.

We will determine then the optimal number of preventive maintenance minimizing the total cost of maintenance over a finite planning horizon: H=9 trimesters. We consider that the length of periods Δt = 3 months. We supposed that the standard deviation of demand of product i, is the same for all periods, $(\sigma(d_{i,k})= \sigma(d_{i,k+1})= \sigma(d_i))$. The data required to run this model are given in sequence.

- The data relating to production:

The mean demands:
$\hat{d}_1 = 200,\ \sigma(d_1) = 12$, $\hat{d}_2 = 100,\ \sigma(d_2) = 7$, and $\hat{d}_3 = 300,\ \sigma(d_3) = 15$

	$d_{i,1}$	$d_{i,2}$	$d_{i,3}$	$d_{i,4}$	$d_{i,5}$	$d_{i,6}$	$d_{i,7}$	$d_{i,8}$	$d_{i,9}$
$i=1$	210	189	217	194	210	175	208	197	199
$i=2$	102	90	93	100	101	99	95	97	99
$i=3$	315	310	292	288	280	302	330	325	310

The other data are presented as following:

	$S_{i,0}$	$U_{i,max}$	Cp_i	Cs_i	St_i	Θ_i
$i=1$	30	450	15	4	65	92
$i=2$	100	330	22	7	80	87
$i=3$	80	620	10	3	75	90

- The data relating to system reliability:

System reliability, costs and times related to maintenance actions are defined by the following data:

  - The law of failure characterizing the nominal conditions is Weibull. It is defined by:

- Scale parameter (β) : 20
- Shape parameter (α) : 2
- Position parameter ($\boldsymbol{\gamma}$) :0

  - The initial failure rate: $\lambda_0 = 0$

These parameters provide information on the evolution of the failure rate in time.
This failure rate is increasing and linear over time. Thus the function of the nominal failure rate is expressed by:

$$\lambda_n(t) = \frac{\alpha}{\beta}\times\left(\frac{t}{\beta}\right)^{\alpha-1} = \frac{2}{20}\times\left(\frac{t}{20}\right)$$

- The obtained production plan:

	Period 1			Period 2			Period 3		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9
	0.23	1.09	1.62	1.33	0.11	1.56	1	1.28	0.72
P1	0	0	272	223	0	0	0	190	0
P2	98	0	0	0	0	188	0	0	110
P3	0	357	0	0	211	0	327	0	0

	Period 4			Period 5			Period 6		
	δ_{10}	δ_{11}	δ_{12}	δ_{13}	δ_{14}	δ_{15}	δ_{16}	δ_{17}	δ_{18}
	0.23	1.09	1.62	1.2	1.27	0.71	0.27	1	1.63
P1	0	241	0	0	0	190	0	0	283
P2	0	0	194	175	0	0	0	134	0
P3	213	0	0	0	396	0	292	0	0
	Period 7			Period 8			Period 9		
	δ_{19}	δ_{20}	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{25}	δ_{26}	δ_{27}
	1.33	1.11	0.56	1.62	1.33	0.11	1.56	1	0.28
P1	0	269	0	0	201	0	0	129	0
P2	159	0	0	125	0	0	0	0	107
P3	0	0	258	0	0	93	249	0	0

- The obtained maintenance strategy:

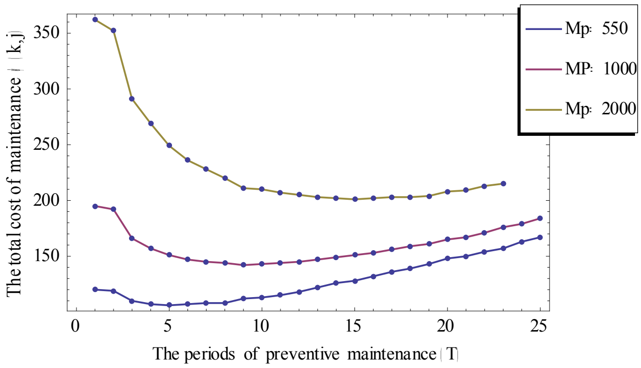

Fig. 2. The total cost of maintenance depending to sub-periods.

The preventive maintenance cost (*mu*)	500	1000	2000
The optimal period of intervention	5	9	17

To illustrate the robustness of the proposed approach, we made a sensitivity study on the preventive maintenance cost *Mp*. The corrective maintenance cost *Mc* is fixed at 2500 *mu*, then we change the preventive maintenance cost value. We deduce that if we increase *Mp*, the intervention period of preventive maintenance *T* increases too.

6. CONCLUSION

In this paper we considered a manufacturing system composed in one machine which produces several products in order to meet several random demands. The machine is subject to random failures, then, preventive maintenance actions are considered in order to improve its reliability. At failure, a minimal repair is carried out to restore the system into the operating state without changing its failure rate.

It's noted that the use of the optimal production plan in the maintenance cost formulation is justified by the significant the influence of the production plan on the system deterioration.

The primary objective of the study was to determine the optimal production rates and when to perform the preventive maintenance. Firstly, we have formulated a stochastic production problem in order to obtain an optimal production plan. Secondly, using the optimal production plan in the maintenance problem formulation, we established an optimal maintenance scheduling.

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