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Power Balancing of Internal Combustion Engines – A Time and Frequency Domain Analysis ⋆

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Abstract: Power balancing of internal combustion engines is studied in time and frequency domains. Speed measurements at the flywheel and the load side are used to estimated the torque applied to the flywheel. The power contributions during the cylinder-wise work phases are estimated by integrating the reconstructed torque, and a controller which adjusts the fuel injections to reduce power unbalances is applied. A convergent power balancing algorithm is presented for multi-cylinder engines, where the work phases of the individual cylinders may overlap. This time-domain method is verified against previous frequency-domain approaches. The analysis shows that for engines with flexible crankshafts, the time-domain method performs in general differently from frequency-domain cylinder balancing approaches. The feasibility of the power balancing algorithm is investigated by simulations and full-scale tests on a large diesel power plant engine.

Keywords: cylinder balancing, diesel engines, power balancing, vibration control

1. INTRODUCTION

During the last decades, intelligent control of large-scale internal combustion engines has been subject to increased attention. Technological advances have, for example, enabled electrical control of cylinder fuel injectors, which is now a standard feature of common-rail engines and allows computerized on-line adjustment of both individual cylinder injection durations and timings. This procedure is commonly referred to as cylinder balancing, where the injection durations are adjusted to maintain optimal length with respect to a balancing criterion.

The objectives of cylinder balancing are diverse. A common target is to minimize torsional vibrations arising from unbalanced cylinders, which have several undesirable properties, such as load fluctuations and decreased passenger comfort on vessels. Secondly, cylinder balancing methods can ensure that the mechanical strain on the engine components, such as the crankshaft, is more evenly distributed. Thirdly, balancing of the cylinders can have a significant impact on exhaust gas properties such as the exhaust temperature and thus serve as a means of emission control.

Cylinder balancing methods can be divided into two categories: time domain and frequency domain. Time-domain methods rely on balancing the cylinder-wise powers, which are obtained as integrals of the estimated torque (Shim *et al.* (1996)). The time-domain approach to balancing has been studied within automotive industry, for example in vehicles with dual mass flywheel (Walter *et al.* (2008)) and in multi-cylinder SI engines (Li and Shen (2011)).

In frequency-domain methods, cylinder balancing is performed by minimizing individual frequency components of the reconstructed torque, of which the low orders are of particular interest. Frequency-domain balancing studies include, for example, DFT-based control of cylinder injections in turbo-charged diesel engine (Macian *et al.* (2006)), torsional vibration reduction by adjusting low-order components of reconstructed torque (Östman and Toivonen (2008a); Östman and Toivonen (2008b)) and pattern-recognition of crankshaft angular speed waveforms using artificial neural networks (Desbazeille *et al.* (2010)).

As time-domain cylinder balancing methods do not involve calculation of frequency components, they are typically computationally less demanding than frequency-domain methods. Secondly, time-domain methods do not require specific knowledge about engine phase-angle diagrams, which the frequency-based methods do. On the other hand, in frequency-domain methods the harmful vibration frequency components are reduced directly, whereas this is achieved only indirectly in time-domain methods.

In this paper, a convergent time-domain power balancing approach is presented for a large scale medium-speed engine-generator set, which, in contrast to typical automotive applications, features overlapping cylinder work phases and for which the flexibility of the crankshaft is not negligible. In order to study the impact of the flexible

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crankshaft on torsional vibration frequencies, the power balancing results are analyzed in frequency domain. The balancing algorithm is evaluated using simulations and full-scale tests on a 3.5MW seven-cylinder diesel engine. The results show that power balancing does not imply the elimination of vibration frequency components when applied to an engine with flexible crankshaft.

2. POWER BALANCING PROBLEM

The problem considered in this paper is power balancing of medium-speed combustion engines utilizing speed measurements obtained from the engine and generator. The objective is to minimize variations in the cylinder-wise work, which can be performed by adjusting injection durations in each cylinder. The mechanical model discussed here is a power system driven by a medium-speed internal combustion engine, used either in a marine installation or as a stand-alone power system. The model can be divided into three main parts: the engine, a flexible coupling and a generator connected to an infinite grid. In automotive cylinder balancing applications, it is often sufficient to include the engine only, as the engine-load coupling can be assumed stiff (Kiencke and Nielsen (2005)). However, for large medium-speed engines, commonly applied in power or marine systems, the dynamics of the coupling between engine and load can in general not be ignored (Östman and Toivonen (2008a)). For these engines, two-mass models which include the dynamics of the flexible coupling between engine and load have been applied (Östman and Toivonen (2008a); Östman and Toivonen (2008b)).

A two-mass model is often sufficient for describing the dynamics of an engine with relatively low number of cylinders, in which the flexible crankshaft can be considered rigid (Desbazeille *et al.* (2010)). Moreover, in frequency-domain cylinder balancing methods, such as the ones considered in (Östman and Toivonen (2008a); Östman and Toivonen (2008b)), it is usually sufficient to consider low frequency components, for which rigid crankshafts can be assumed. In the time-domain power balancing methods studied in this paper, the effect of higher frequency components can, however, not be ignored. Therefore, the dynamics of the crankshaft will be modelled using a lumped-mass model, in which the crankshaft is represented with a number of lumped masses interconnected with springs characterized by their stiffness and damping (Genta (1999)). The crankshaft model used in the simulations is depicted in Figure 1. The model consists of ten elastically connected masses (flange, seven cranks, gear, flywheel), and a flexible coupling between the flywheel and the generator.

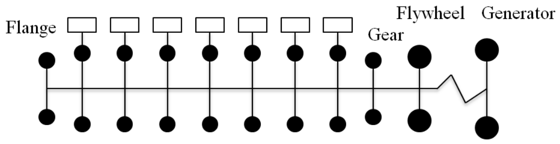

Fig. 1. Lumped mass model of crankshaft including flexible coupling and generator.

The model is described by the coupled differential equations

$$J_i(\varphi_i)\ddot{\varphi}_i + \frac{1}{2}\frac{\mathrm{d}J_i(\varphi_i)}{\mathrm{d}\varphi_i}\dot{\varphi}_i^2 + C_{i-1,i}\Delta\dot{\varphi}_{i,i-1} - C_{i,i+1}\Delta\dot{\varphi}_{i+1,i}$$
$$+K_{i-1,i}\Delta\varphi_{i,i-1} - K_{i,i+1}\Delta\varphi_{i+1,i} = M_i(t), \qquad (1)$$

$i = 1, 2, \ldots, N_{\mathrm{mass}}$, where $\Delta\varphi_{i,i-1} = \varphi_i - \varphi_{i-1}$, $\Delta\dot{\varphi}_{i,i-1} = \dot{\varphi}_i - \dot{\varphi}_{i-1}$, $J_i(\varphi_i)$ is the mass moment of inertia of mass number i (including oscillating mass), $K_{i-1,i}$ and $C_{i-1,i}$ are the stiffness and damping between masses $i-1$ and i, and $M_i(t)$ is the total torque applied at mass i.

The purpose of the power balancing method considered in this paper is to make the cylinder-wise work contributions at the flywheel equal. As direct cylinder torque measurements are known to be difficult, the torque is reconstructed using measurements of flywheel and generator positions and speeds. Using equation (1) the torque can be estimated from the relation

$$M(t) = J_1\ddot{\varphi}_{\mathrm{f}} + C_{\mathrm{g,f}}(\dot{\varphi}_{\mathrm{f}} - \dot{\varphi}_{\mathrm{g}}) + K_{\mathrm{g,f}}(\varphi_{\mathrm{f}} - \varphi_{\mathrm{g}}) \qquad (2)$$

where J_1 is the constant mass moment of inertia of the flywheel, and the subscripts 'f' and 'g' refer to the flywheel and generator, respectively. For an engine with a stiff crankshaft, $M(t)$ is the total engine torque if J_1 is taken as the mass moment of inertia of the engine.

The engine-generated torque $M(t) = M(\varphi(t))$ can be decomposed into a sum of cylinder-wise torques

$$M(t) = \sum_{n=1}^{N_{\mathrm{cyl}}} M_{\mathrm{cyl},n}(\varphi, u_n) \qquad (3)$$

where N_{cyl} is the number of cylinders and u_n is the fuel-injection duration for cylinder n, which determines the torque $M_{\mathrm{cyl},n}(\varphi, u_n)$ produced by the cylinder during its work phase. The successive work phases of the cylinders give rise to a periodically time-varying torque. In cylinder balancing, the objective is to make the cylinder-wise work contributions equal.

Following (Kiencke and Nielsen (2005)), the integral between two consecutive ignitions of the torque estimate,

$$E_n = \int_{\mathrm{TDC}_n}^{\mathrm{TDC}_{n+1}} M(\varphi)\mathrm{d}\varphi, \;\; n = 1, 2, \ldots, N_{\mathrm{cyl}} \qquad (4)$$

is taken as a measure of the work contributions of the cylinders. Here TDC_n denotes the angular position corresponding to the top dead center (TDC) of the work phase of cylinder number n. Notice that, as the deflection of the crankshaft is very small, the same crank angle φ can be applied in (4) to all cylinders.

The power balancing problem studied in this paper is to determine the fuel-injection durations u_n in such a way that the torque integrals are equal, i.e.,

$$E_n = \bar{E}, \;\; n = 1, 2, \ldots, N_{\mathrm{cyl}} \qquad (5)$$

where $\bar{E}$ is the mean cylinder-wise work,

$$\bar{E} = \frac{1}{N_{\mathrm{cyl}}} \sum_{n=1}^{N_{\mathrm{cyl}}} E_n \qquad (6)$$

In addition, it is required that the power balancing algorithm is decoupled from the speed/load controller, i.e., the algorithm should affect only the power unbalance, but not the total work E_{tot}. The power balancing problem can be summarized as follows.

Power balancing problem
Determine the fuel-injection durations u_n so that during stationary operation the following conditions hold:

Power balance condition: The torque integrals E_n satisfy (5).
Decoupling condition: The total work $E_{\mathrm{tot}} = N_{\mathrm{cyl}}\bar{E}$ is not affected, but can be determined independently of the power balancing.

By (3) and (4), the work contributions $\{E_n\}$ depend on the fuel-injection durations $\{u_n\}$. If the work phases of the cylinders do not overlap, E_n depends on u_n only, but for a multi-cylinder engine with overlapping work phases, it will also depend on $u_{n-1}, \ldots$ (assuming that the cylinder ordering is taken as the firing order). In the next section, a control law which achieves power balancing for a multi-cylinder engine with possibly overlapping work phases will be presented.

3. POWER BALANCING ALGORITHM

For notational convenience, we introduce the vector of torque integrals,

$$\mathbf{E} = [\, E_1 \quad \cdots \quad E_{N_{\mathrm{cyl}}} \,]^{\mathrm{T}} \tag{7}$$

and the $N_{\mathrm{cyl}} \times 1$ vector $\mathbf{1}$ with unit elements,

$$\mathbf{1}^{\mathrm{T}} = [\, 1 \quad 1 \quad \cdots \quad 1 \,] \tag{8}$$

Then (6) takes the form

$$\bar{E} = \frac{1}{N_{\mathrm{cyl}}}\mathbf{1}^{\mathrm{T}}\mathbf{E} \tag{9}$$

and the power balancing condition (5) can be expressed compactly as

$$\mathbf{E} - \frac{1}{N_{\mathrm{cyl}}}\mathbf{1} \cdot \mathbf{1}^{\mathrm{T}}\mathbf{E} = 0 \tag{10}$$

In view of the power balancing condition it is convenient to decompose the power integral vector $\mathbf{E}$ as

$$\mathbf{E} = \mathbf{E}_{\mathrm{ub}} + \mathbf{E}_{\mathrm{bal}} \tag{11}$$

where

$$\mathbf{E}_{\mathrm{ub}} = P_{\mathrm{ub}}\mathbf{E}, \;\; P_{\mathrm{ub}} = \mathbf{I} - \frac{1}{N_{\mathrm{cyl}}}\mathbf{1} \cdot \mathbf{1}^{\mathrm{T}} \tag{12}$$

where P_{ub} is the orthogonal projection onto the nullspace of $\mathbf{1}^{\mathrm{T}}$, $\mathbf{I}$ is the N_{cyl}-by-N_{cyl} dimensional identity matrix, and

$$\mathbf{E}_{\mathrm{bal}} = P_{\mathrm{bal}}\mathbf{E}, \;\; P_{\mathrm{bal}} = \frac{1}{N_{\mathrm{cyl}}}\mathbf{1} \cdot \mathbf{1}^{\mathrm{T}} \tag{13}$$

where P_{bal} is the orthogonal projection onto the range of $\mathbf{1}$. In power balancing, only the unbalance component $\mathbf{E}_{\mathrm{ub}}$ is controlled, whereas the balanced component $\mathbf{E}_{\mathrm{bal}}$ is controlled by the speed controller.

The powers generated by the cylinders are controlled by adjusting the fuel-injection durations u_n. A power balancing algorithm can be constructed to achieve the power balancing condition (5) by sequentially updating the cylinder-wise fuel-injection durations $u_n(k)$ at iteration k using the computed power integrals $\mathbf{E}(k)$. As the power unbalance depends only the component $\mathbf{E}_{\mathrm{ub}}$ of the torque integral vector, we can apply an integrating control law for the fuel injection updates according to

$$\mathbf{u}(k) = \mathbf{u}(k-1) + \mathbf{K}P_{\mathrm{ub}}\mathbf{E}(k) \tag{14}$$

where $\mathbf{u}(k) = [\, u_1(k) \quad u_2(k) \quad \cdots \quad u_{N_{\mathrm{cyl}}}(k) \,]^{\mathrm{T}}$ and $\mathbf{K}$ is a gain matrix, which will be discussed below. The class of power balancing algorithms to be considered can be summarized as follows.

Power balancing algorithm
Stage 0. Initialization. Set iteration index $k = 1$.
Stage 1. Measure the speeds $\dot{\varphi}_{\mathrm{f}}$ and $\dot{\varphi}_{\mathrm{g}}$ of the engine flywheel and the generator over a number of engine cycles.
Stage 2. If required, preprocess the measured speeds by low-pass filtering.
Stage 3. Estimate the periodic torque applied on the crankshaft as a function of the crank angle φ_1 according to Eq. (2).
Stage 4. Calculate the cylinder-wise work contributions $E_n(k)$ by integrating the estimated torque, eq. (4).
Stage 5. Adjust the cylinder injections $u_n(k)$ based on the work estimates according to (14).
Stage 6. Set $k \leftarrow k+1$ and continue from stage 1.

In order to determine a control law to achieve power balancing, we assume that locally, the cylinder-wise powers depend on the fuel injections linearly, so that

$$\mathbf{E}(k+1) = \mathbf{E}(k) + \mathbf{A}\boldsymbol{\Delta}\mathbf{u}(k) \tag{15}$$

where $\boldsymbol{\Delta}\mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$. The proportionality matrix $\mathbf{A}$ can be determined experimentally, or using known gas torque data and the engine model (1).

From (15) it follows that the unbalance component $\mathbf{E}_{\mathrm{ub}}(k)$ of the power integral vector is given by

$$\mathbf{E}_{\mathrm{ub}}(k+1) = \mathbf{E}_{\mathrm{ub}}(k) + P_{\mathrm{ub}}\mathbf{A}\boldsymbol{\Delta}\mathbf{u}(k) \tag{16}$$

Introducing the linear update law (14), the closed loop is then described by

$$\mathbf{E}_{\mathrm{ub}}(k+1) = P_{\mathrm{ub}}\left(\mathbf{I} + \mathbf{A}\mathbf{K}\right)\mathbf{E}_{\mathrm{ub}}(k)$$

where we have used the fact $P_{\mathrm{ub}}\mathbf{E}_{\mathrm{ub}}(k) = \mathbf{E}_{\mathrm{ub}}(k)$). Convergence of the algorithm is determined by the matrix $\mathbf{I} + \mathbf{A}\mathbf{K}$. If $\mathbf{A}$ is known, an obvious choice is to take $\mathbf{K} = -\mu\mathbf{A}^{-1}$, where μ is a positive steplength parameter. Then,

$$\mathbf{E}_{\mathrm{ub}}(k+1) = (1-\mu)\mathbf{E}_{\mathrm{ub}}(k)$$

and it is seen that convergence to $\mathbf{E}_{\mathrm{ub}}(k) = 0$ is obtained for $0 < \mu < 2$.

Observe that in the case when the cylinder-wise work phases do not overlap, assuming identical cylinders and ignoring the crankshaft dynamics, the matrix in (15) is diagonal, $\mathbf{A} = a\mathbf{I}$, and the control law (14) reduces to

$$\mathbf{u}(k) = \mathbf{u}(k-1) - (\mu/a)\mathbf{E}_{\mathrm{ub}}(k) \tag{17}$$

or

$$u_n(k) = u_n(k-1) + c\Big(E_n(k) - \frac{1}{N_{\rm cyl}} \sum_{m \neq n} E_m(k)\Big),$$

$n = 1, 2, \ldots, N_{\rm cyl}$, where $c = -\mu/a$ This is the cylinder balancing control algorithm given in Walter *et al.* (2008). Even when there is overlap between the cylinder work phases, but $\mathbf{A}$ is diagonally dominant, it can be shown that the step-length parameter μ can be selected so that (17) gives convergence.

4. FREQUENCY-DOMAIN ANALYSIS

As torque imbalances often give rise to undesired torsional vibrations in internal combustion engines, it is natural to pay attention to the relevant frequency components of the periodically time-varying torque. Consequently, various frequency-domain approaches have been applied to cylinder balancing Macian *et al.* (2006); Östman and Toivonen (2008a); Östman and Toivonen (2008b). For these reasons it is of interest to analyze frequency-domain properties of the power balancing method based on torque integrals presented in section 3.

In order to relate the power balancing method to cylinder balancing approaches in which certain frequency components of the reconstructed torque are suppressed, the torque integrals (4) will be expressed in terms of the torque frequency components. For a four-stroke engine, the torque $M(\varphi)$ is 4π-periodic with respect to the crank angle, and it has a Fourier-series expansion,

$$M(\varphi) = \sum_{k=-K}^{K} \hat{M}(k)\, {\rm e}^{{\rm j}k\varphi/2} \tag{18}$$

where K corresponds to the highest frequency component considered. Here, the frequency components $p = k/2, k = 1, 2, \ldots,$ which are integer multiples of the work cycle frequency, are commonly referred to as frequency orders.

Then, the sequence $\{E_n\}$ of power integrals defined by (4) has the Fourier-series expansion

$$E_n = \sum_{l=-L}^{L} \hat{E}_l\, {\rm e}^{{\rm j}2\pi l(n-1)/N_{\rm cyl}}, \quad n = 1, 2, \ldots, N_{\rm cyl} \tag{19}$$

where

$$L = \begin{cases} N_{\rm cyl}/2, & \text{if } N_{\rm cyl} \text{ is even} \\ (N_{\rm cyl}-1)/2, & \text{if } N_{\rm cyl} \text{ is odd} \end{cases}$$

and (see Appendix)

$$\hat{E}_0 = \hat{M}(0)\ 4\pi/N_{\rm cyl} \tag{20}$$

$$\hat{E}_l = \sum_{m:|l+mN_{\rm cyl}| \leq K} \hat{M}(l + mN_{\rm cyl}) \frac{{\rm e}^{{\rm j}2\pi l/N_{\rm cyl}} - 1}{{\rm j}(l + mN_{\rm cyl})/2}, \ l \neq 0 \tag{21}$$

The cylinder balancing criterion (13) is equivalent to the frequency-domain condition

$$\hat{E}_l = 0, \ \ l = \pm 1, \pm 2, \ldots, \pm L \tag{22}$$

By (21), this implies that certain linear combinations of the torque frequency components $\hat{M}(l)$ and their alias components $\hat{M}(l + mN_{\rm cyl})$ vanish. In general it does not follow from (22) that individual torque frequency orders vanish. An exception is the case with a rigid crankshaft and identical equidistant cylinders, in which case the torque consists of a superposition of cylinder-wise torques which depend on the corresponding fuel injections u_n,

$$M(\varphi) = \sum_{n=1}^{N_{\rm cyl}} M_{\rm cyl}(\varphi - {\rm TDC}_n; u_n)$$

If the cylinder-wise torque have identical Fourier series expansions

$$M_{\rm cyl}(\varphi; u_n) = \sum_{k=-K}^{K} \hat{M}_{\rm cyl}(k; u_n)$$

the Fourier-series coefficients of the total torque $M(\varphi)$ can be expressed as

$$\hat{M}(k) = \sum_{n=1}^{N_{\rm cyl}} \hat{M}_{\rm cyl}(k; u_n)\, {\rm e}^{-{\rm j}k{\rm TDC}_n/2} \quad {\rm e}^{{\rm j}k\varphi/2}$$

As

$$\sum_{n=1}^{N_{\rm cyl}} {\rm e}^{-{\rm j}k{\rm TDC}_n/2} = \sum_{n=1}^{N_{\rm cyl}} {\rm e}^{-{\rm j}2\pi k(n-1)/N_{\rm cyl}} = 0$$

it follows that selecting the fuel injections so that

$$\hat{M}_{\rm cyl}(k; u_n) = \hat{M}_{\rm cyl}(k; u_m), \ \ n, m = 1, 2, \ldots, N_{\rm cyl},$$

all k, then all torque frequency orders are zero, $\hat{M}(k) = 0$, and the power balancing criterion (22) is achieved. However, in the general case with a flexible crankshaft, and possible discrepancies between the cylinders, the power balancing condition does not imply suppression of individual torque orders.

5. EXAMPLES

In this section simulations and full scale engine tests are presented to demonstrate the performance of the power balancing procedure applied to an engine with a flexible crankshaft. The method is specifically evaluated for a seven-cylinder 3.5MW common-rail Wärtsilä W7L32CR engine-generator set running at 750 rpm.

5.1 Simulations

The simulations were done on the lumped-mass model (1) of the engine-generator set using engine-specific gas-torque data (Östman and Toivonen (2008a)) for the cylinder-wise torques in equation (3). The firing order of the cylinders is 1-3-5-7-6-4-2, where cylinder 1 is taken as the one closest to the flywheel (see Fig. 1), and the simulations were performed at 50% load.

For a seven cylinder engine, the cylinder phases are $720/7 = 102.9$ degrees apart, and there is therefore some overlap between the work phases of two consecutive cylinders, so that the model (15) takes the form (assuming that the cylinder are numbered according to firing order)

$$E_n(k+1) = E_n(k) + \mathbf{A}_{nn}\Delta u_n(k) + \mathbf{A}_{n,n-1}\Delta u_{n-1}(k),$$

$n = 1, 2, \ldots, N_{\rm cyl}$, where $\mathbf{A}_{nn} > \mathbf{A}_{n,n-1} > 0$, and with the understanding that $\Delta u_0 = \Delta u_{N_{\rm cyl}}$. For simplicity, the controller (17) was applied, where the gain μ was selected to achieve convergence.

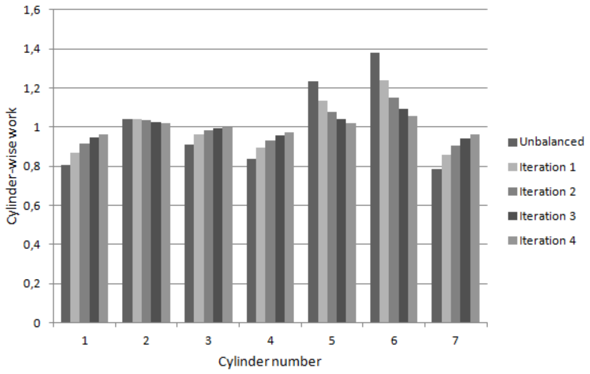

Fig. 2. Normalized cylinder-wise torque integrals E_n when the power balancing procedure is applied to simulated example. The figure shows four iterations of the algorithm applied to each cylinder.

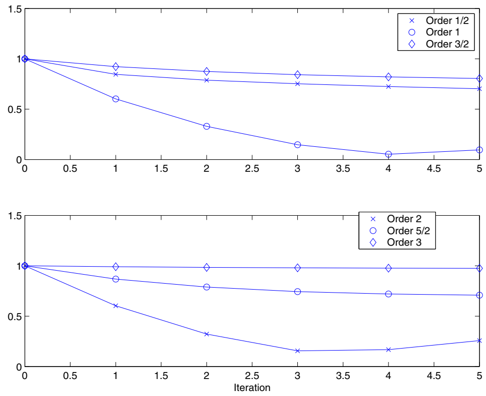

Fig. 3. Normalized torque order magnitudes below ignition frequency in power balancing simulation.

Figure 2 shows the results of a simulation of the cylinder balancing method. The initial fuel injections were taken so that the initial power unbalances match those in the full-scale test (cf. section 5.2). The figure shows the propagation of the normalized torque integrals $E_n/\bar{E}$. It is seen that the cylinder-wise powers at the flywheel are well balanced in less than five iterations of the algorithm, resulting in a 82% reduction of the standard deviation of the power unbalances $E_n - \bar{E}$. The magnitudes of the torque order frequency components below ignition frequency are shown in Figure 3. It is seen that although all frequency components are reduced, due to the crankshaft dynamics the reductions are much smaller than that of the power unbalance.

5.2 Engine tests

The proposed method was tested on a Wärtsilä W7L32CR diesel engine at the Wärtsilä Vaasa test laboratory. The engine was connected to a synchronous generator through a flexible coupling. Speed data were obtained both from the engine and generator side. Engine speed data was represented by timer ticks, established between every third hole on the engine flywheel. This corresponds to 40 timer ticks per rotation, equalling a measurement every 9th crank degree. The timer resolution was 167 kHz. On the generator side, measurements were down-sampled to 120 pulses per revolution from a HBM 7200 pulses/rev speed encoder. The test was run at 50% engine load.

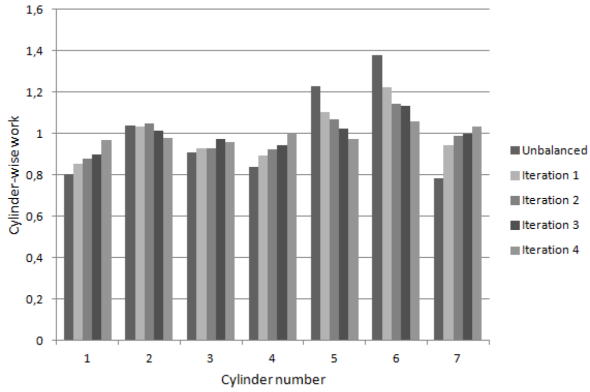

Fig. 4. Normalized cylinder-wise torque integrals E_n in power balancing test. The figure shows four iterations of the algorithm applied to each cylinder.

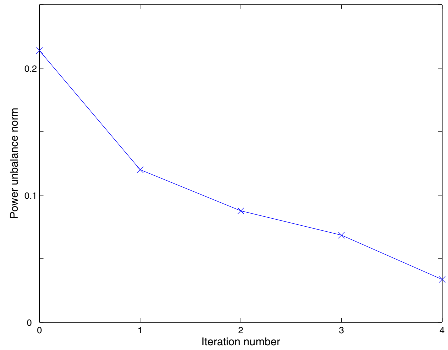

Fig. 5. Normalized standard deviation of power unbalance in power balancing test.

The results from a full-scale test of the power balancing algorithm are shown in Figures 4 and 5, which show the propagation of the cylinder-wise torque integrals and the standard deviation of the power unbalances $E_n - \bar{E}$ during four iterations of the algorithm. It is seen that the reductions achieved in power unbalance match well with those obtained in simulations with the corresponding initial power unbalances (Figure 2). The results demonstrate a clear decrease in the cylinder-wise work unbalances, with the standard deviation of the cylinder unbalance decreasing by 85% after four iterations, cf. Figure 5.

6. CONCLUSION

Power balancing algorithms for multi-cylinder internal combustion engines have been studied. The procedure uses speed measurements to reconstruct the torque at the flywheel. The integrals of the reconstructed torque over the cylinder-wise work phases are taken as a measure of the powers at the flywheel associated with the different cylinders. The cylinder-wise fuel injections are controlled in such a way that the estimated powers are balanced, while not affecting the total power. A convergent balancing algorithm has been presented for the general case, where the different cylinder work phases may overlap. Simulations and full-scale engine tests on a 7-cylinder diesel engine demonstrate the efficiency of the algorithm.

In practice, the proposed algorithm should be applied during steady-state operation with a constant load. After load changes, it may be necessary to rerun the algorithm to eliminate possible power imbalances.

Whereas frequency-domain cylinder balancing requires on-line computation of a Fourier transform and information about phase angles associated with the relevant frequency components, the time-domain power balancing method requires only numerical integration of the estimated torque and information about the ignition points of the cylinders.

However, some of the advantages of the time-domain approach are lost if the cylinder balancing should be done using the lowest frequency components only, for example because the crankshaft dynamics cannot be ignored at higher frequencies. A frequency-domain analysis of the procedure shows that power balancing implies suppression of torsional frequency components only in the special case where the cylinders are identical and the crankshaft dynamics can be ignored.

A possible approach to engines with flexible crankshafts would be to apply the power-balancing algorithm to low-pass filtered signals. However, when considering only the lowest frequency components the cylinder-wise torque contributions will be spread out in time (or crank angle). This spreading of the torque contributions will complicate the time-domain power balancing procedure. The development of time-domain power balancing methods to such cases will be left for future studies.

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Appendix A. PROOF OF (20), (21)

Consider a T-periodic signal $y(t)$ with Fourier-series expansion

$$y(t) = \sum_{k=-\infty}^{\infty} \hat{Y}(k)\, \mathrm{e}^{\mathrm{j}2\pi kt/T}$$

Define the integrals

$$E_n = \int_{(n-1)T/N}^{nT/N} y(t)\mathrm{d}t, \quad n = 1, 2, \ldots, N$$

Then,

$$\begin{aligned} E_n &= \int_{(n-1)T/N}^{nT/N} \left(\sum_{k=-\infty}^{\infty} \hat{Y}(k)\, \mathrm{e}^{\mathrm{j}2\pi kt/T} \right) \mathrm{d}t \\ &= \hat{Y}(0)T/N + \sum_{k=-\infty, k\neq 0}^{\infty} \hat{Y}(k)\, \frac{\mathrm{e}^{\mathrm{j}2\pi k/N} - 1}{\mathrm{j}2\pi k/T}\, \mathrm{e}^{\mathrm{j}2\pi k(n-1)/N} \end{aligned}$$

As

$$\mathrm{e}^{\mathrm{j}2\pi(k+mN)/N} = \mathrm{e}^{\mathrm{j}2\pi k/N}, \quad \text{all integer } m$$

we can decompose the sum by setting $k = l + mN$, where l takes values $\leq N/2$ (excluding $l = 0$), and m from $-\infty$ to $+\infty$. This gives

$$\sum_{k=-\infty, k\neq 0}^{\infty} \hat{Y}(k) \frac{\mathrm{e}^{\mathrm{j}2\pi k/N} - 1}{\mathrm{j}2\pi k/T}\, \mathrm{e}^{\mathrm{j}2\pi k(n-1)/N} =$$

$$\sum_{l=-L, l\neq 0}^{L} \sum_{m=-\infty}^{\infty} \hat{Y}(l + mN) \frac{\mathrm{e}^{\mathrm{j}2\pi l/N} - 1}{\mathrm{j}2\pi(l + mN)/T}\, \mathrm{e}^{\mathrm{j}2\pi l(n-1)/N}$$

and it follows that

$$E_n = \sum_{l=-L}^{L} \hat{E}_l\, \mathrm{e}^{\mathrm{j}2\pi l(n-1)/N}$$

where

$$\hat{E}_l = \sum_{m=-\infty}^{\infty} \hat{Y}(l + mN) \frac{\mathrm{e}^{\mathrm{j}2\pi l/N} - 1}{\mathrm{j}2\pi(l + mN)/T}, \ l \neq 0$$

and

$$\hat{E}_0 = \hat{Y}(0)\ T/N$$

from which (20) and (21) follow by setting $T = 4\pi$ and $N = N_{\mathrm{cyl}}$.