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Input Signal Generation for Constrained Multiple-Input Multple-Output Systems ⋆

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Abstract: We consider a recent method for generating an input signal with a desired auto correlation while satisfying both input and output constraints for the system it is to be applied to. This is an important problem in system identification, since the properties of the identified model depend on the used excitation signal while on real processes, due to actuator saturation and safety considerations, it is important to constraint the process inputs and outputs. Here, we extend an earlier method to work for longer input horizons and to the multiple-input multiple-output case. This corresponds to solving a fourth order multivariate polynomial in each time step. Two different methods for solving this problem are considered: one based on convex relaxation and the other based on a cyclic algorithm. The performance of the algorithm is successfully verified by simulations and the effects of the input horizon length are discussed.

Keywords: System identification; input signals; numerical methods; predictive control; MIMO.

1. INTRODUCTION

The problem of generating signals with specific second order (auto correlation) properties arises in many fields. In system identification, it is well known that the quality of the identified model depends on the auto correlation function of the applied input signal. Therefore, an essential part of an identification experiment is the choice and the realization of the excitation signal.

Input design is available in a variety of flavors but the common central idea is that the statistical properties of the estimates can be influenced by the choice of input signal through its covariance. Initially, input design was formulated as optimization of some measure of the covariance matrix directly. While this is still often the case in practice, research focus has since shifted to consider quality in terms of the intended use of the model. Gevers and Ljung (1986); Gevers (1991); Hjalmarsson et al. (1996) have introduced *identification for control*, Bombois et al. (2006) proposed *least costly identification* and Hjalmarsson (2009) looked at *applications oriented input design*. In these later methods, the input is designed in terms of the power spectrum and a time signal then has to be realized. Including time domain constraints on signals in these design is therefore difficult. However, in applications it is often vital to limit input and output amplitudes and rates of change. This is due to actuator saturations, equipment strain and the desire to keep the system within an operating region. Related to this, Rivera et al. (2003) introduced the *plant-friendly identification* techniques where signal variances are low and actuator strain is kept to a minimum.

Many methods for generating signals with time domain constraints have been proposed. Hannan (1970); Rojas et al. (2007); He et al. (2012) limit the design to binary signals where it is easy to control the input amplitudes. Schoukens et al. (1991); Pintelon and Schoukens (2001) propose considering sums of sinusoids, where the amplitude is controlled using the phase of the sinusoidal components. Further, one can choose to exclude the time domain constraints in the input design and enforce them when the time domain signal is generated. For example, white noise can be filtered to have the correct correlation properties and then “clipped” to the right amplitude. This approach is simple but changes the spectrum of the signal which can lead to suboptimal results, see Hannan (1970).

This contribution presents a signal generation idea applicable to the latter case. We consider time domain realization of an amplitude constrained signal such that the signal has a prescribed auto-correlation. The algorithm tries to match the auto-correlation while maintaining input and output constraints. Larsson et al. (2013) introduced the idea for single-input single-output (SISO) systems and Hägg et al. (2013) discussed extensions of the algorithm to the case of uncertain system knowledge. Here, we extend the algorithm to work for multiple-input multiple-output (MIMO) systems and more general constraints.

The structure of the paper is as follows. In Section 2 the receding horizon signal generation problem is set up and extended to a more general setting. Section 3 and Section 4 presents two numerical approaches to solve the receding

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horizon optimization problem and the differences between the two methods are discussed in Section 5. The proposed method is evaluated through simulations in Section 6 while Section 7 and Section 8 conclude the paper.

2. RECEDING HORIZON SIGNAL GENERATION

In this section the method by Larsson et al. (2013) is extended to the multiple-input multiple-output (MIMO) case. The objective is to generate N samples of the p signals

$$u_t = [u_t(1), \ldots, u_t(p)]^T, \quad t = 1, \ldots N,$$

with a prescribed auto-correlation

$$R^d(\tau) = E\left\{u_t u_{t-\tau}^T\right\}, \quad \tau = 0, \ldots, n.$$

That is, the first $n+1$ lags of the desired auto correlation function are matched. Furthermore, when the input signals are applied to the p input and m output MIMO system

$$\begin{aligned} x_{t+1} &= Fx_t + Gu_t, \\ y_t &= Hx_t, \end{aligned} \tag{1}$$

the input and the output constraints

$$\begin{aligned} &u_t \in \mathcal{U}, \\ &y_t \in \mathcal{Y}, \quad t = 1, \ldots, N, \end{aligned} \tag{2}$$

should be satisfied. Here we will only consider amplitude constraint *i.e.,* $|u_t| \leq u_{max}$ and $|y_t| \leq y_{max}$ but both state constraints and other convex constraints are possible. Note that the output constraints can be rewritten as time varying constraints on u_t since y_t is linear in u_t.

Defining the (biased) sample auto-correlation of the signals $u(t)$ for $t = 1, \ldots, N$ as

$$R_N(\tau) = \frac{1}{N} \sum_{i=\tau+1}^{N} u_i u_{i-\tau}^T. \tag{3}$$

it is natural to formulate the constrained signal generation problem as the following optimization problem

$$\begin{aligned} \underset{\{u_t\}_{t=1}^N}{\text{minimize}} \quad & \sum_{\tau=0}^{n} \|R_N(\tau) - R^d(\tau)\|_F^2, \\ \text{subject to} \quad & (1) \text{ and } (2) \end{aligned}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. However, as noted by Larsson et al. (2013), even for the SISO case, this optimization problem is non convex. Larsson et al. (2013) simplified the problem by reducing the number of optimization variables by introducing a receding horizon solution to the problem. A natural extension of this to the MIMO case is, at sample t, solve

$$\begin{aligned} \underset{\{u_k\}_{k=t}^{t+N_u-1}}{\text{minimize}} \quad & \sum_{\tau=0}^{n} \|R_{t+N_u-1}(\tau) - R^d(\tau)\|_F^2, \\ \text{subject to} \quad & \hat{x}_{k+1} = F\hat{x}_k + Gu_k, \\ & \hat{y}_k = H\hat{x}_k, \\ & \hat{x}_1 = x_t, \\ & \begin{aligned} u_k &\in \mathcal{U} \\ y_k &\in \mathcal{Y}, \end{aligned} \quad k = 1, \ldots, N_y \end{aligned} \tag{4}$$

where N_y is the output horizon, *i.e.,* how far in the future we consider that the output should satisfy the constraints and N_u is the input horizon. The optimization is performed over the whole input horizon but only the first sample, $u_1^\star$, is implemented, *i.e.,* $u_t = u_1^\star$, and the optimization is performed iteratively in receding horizon fashion. If the output horizon is longer than the input horizon, *i.e.,* $N_y > N_u$, we set $u_{k+N_u} = \cdots = u_{k+N_y} = 0$. This approach can be seen as an MPC where we try to follow a correlation reference while satisfying input and output constraints of the considered system.

Since the sample auto correlation $R_{t+N_u-1}(\tau)$ is quadratic in u, the optimization problem (4) corresponds to minimizing a constrained, multivariate, fourth order polynomial. Larsson et al. (2013) showed that for the SISO case with input horizon $N_u = 1$ the optimization problem (4) can be solved analytically. However, in the general case, the problem is non convex and no analytic solution exists. In fact, in the extreme case of $N_u = N$ it is as hard to solve as the original problem, but for a shorter input horizon N_u the problem is small and can be solved numerically. In this paper, we consider two different numerical approaches to solve the optimization problem (4).

3. SDP RELAXATION

Solving the problem in (4) means finding the global minimizer of a real-valued, multivariate polynomial of degree four over an admissible set, which is in general non convex and difficult to solve. However, recent developments by Lasserre (2000) on optimization of polynomials over a set defined by polynomial inequalities offer a route to a numerical solution to this problem. The theory is based on the theory of moments and representation of polynomials that are strictly positive on a compact, semi algebraic set. It is shown that a family of convex LMI relaxations has an associated increasing sequence of lower bounds converging to the global optimal value. Here, we briefly present the idea; for a complete treatment of the theory and related references we refer to the work of Lasserre (2000).

Consider the polynomial optimization problem

$$g^\star = \left\{ \min_{x \in \mathbf{R}^n} g_0(x) : g_i(x) \geq 0, i = 1, \ldots, m \right\},$$

where g_i are multivariate polynomials. First note that $g_0(x) - g^\star$ is a positive polynomial on the constraint set and that the problem of finding a sum of squares polynomial

$$p(x) = g_0(x) - g^\star = q_0(x) + \sum_{i=1}^{m} g_i(x) q(x)$$

can be represented as an LMI if the degrees of the polynomials $q_i(x)$ are fixed. The primal formulation is constructed as a minimization over moments with support on the constraint set. Then a condition by Putinar (1993) on the moment and localizing matrices of the polynomials can be used to construct a family of LMI relaxations with the increasing sequence of lower bounds $p_k^\star$. The dual LMI corresponds to the condition that the polynomial $p(x)$ is a sum of squares polynomial, denote the optimum $d_k^\star$. It is proved by Lasserre (2000) that if the constraint set is compact, under mild conditions, $p_k^\star = d_k^\star \leq g^\star$ and that

$$\lim_{k \to \infty} p_k^\star = g^\star.$$

The price to be paid is that the size of the LMI relaxations quickly grows very large. In fact, for a fixed number of polynomial variables n, the number of primal and dual variables grow polynomially in $\mathcal{O}(\delta^n)$ and $\mathcal{O}(m\delta^n)$, respectively, where δ is half the polynomial degree. However, it has been noted that in practice the convergence is often very fast and therefore the theory has proven to be useful also for numerical implementation.

4. CYCLIC ALGORITHM

Cyclic algorithms have been successfully applied to many signal generation problems, for example by He et al. (2012) for applications to radar beam pattern generation and by Jansson and Medvedev (2013) for stimulus design for eye-tracking identification. Common to these applications is that a complete sequence of input signals is generated and only input constraints are considered. We use the cyclic algorithm in a new setting, to solve the receding horizon signal generation with both input and output constraints.

To be able to fit our problem into the cyclic algorithm algorithm suggested by Stoica et al. (2008) we do not directly solve the optimization problem (4) but instead solve a related problem. Consider the cost function

$$\min_{\{u_k\}_{k=t}^{t+N_u}} \quad \|\tilde{R}_{t+N_u-1} - \tilde{R}^d\|_F^2. \tag{5}$$

where $\tilde{R}_{t+N_u-1}$ is a block Toeplitz matrix with first (block) row equal to

$$[R_{t+N_u-1}(0)\ R_{t+N_u-1}(1)\ \cdots\ R_{t+N_u-1}(n)]$$

and $\tilde{R}^d$ defined analogously. This cost functions satisfies

$$\sum_{\tau=0}^{n} \|R_{t+N_u-1}(\tau) - R^d(\tau)\|_F^2 \le \|\tilde{R}_{t+N_u-1} - \tilde{R}^d\|_F^2.$$

Note that the optimum of this cost function need not be the optimum of the original problem, However, if the cyclic algorithm makes the cost of the right hand side small then the cost of the original problem is also small. This can be seen as a re-weighting of the original optimization problem such that it is more important to match the correlation for small lags (small τ) than for large lags.

Using the correlation matrices in (3), the Toeplitz matrix containing these matrices can be written recursively as

$$\tilde{R}_{t+N_u-1} = \frac{1}{t+N_u-1}\left((t-1)\tilde{R}_{t-1} + \Phi_U^T\Phi_U - Q\right)$$

where

$$\Phi_U = \begin{bmatrix} u_{t+N_u-1}^T & & 0 \\ \vdots & \ddots & \\ u_{t-n}^T & & u_{t+N_u-1}^T \\ & \ddots & \vdots \\ 0 & & u_{t-n}^T \end{bmatrix}$$

and Q is a block Toeplitz matrix with first (block) row

$$\left[\sum_{p=-n}^{-1} u_{t+p}u_{t+p}^T, \ldots, \sum_{p=-1}^{-1} u_{t+p}u_{t+p-n+1}^T, 0\right].$$

The optimization problem (5) can now be written on recursive form as

$$\min_{\{u_k\}_{k=t}^{t+N_u}} \left\| \frac{1}{t+N_u-1}\left((t-1)\tilde{R}_{t-1} + \Phi_U^T\Phi_U - Q\right) - \tilde{R}^d \right\|_F^2.$$

Rescaling this problem, the optimization at each time is

$$\begin{aligned} &\underset{\{u_k\}_{k=t}^{t+N_u}}{\text{minimize}} && \|\Phi_U^T\Phi_U - Z_t\|_F^2, \\ &\text{subject to} && u_k \in \mathcal{U}_{t+k}, \quad k = 0, \ldots, N_u - 1, \end{aligned} \tag{6}$$

where we have rewritten the output constraints as time variable input constraints to shorten the notation. Note that $-Z_t = (t-1)\tilde{R}_{t-1} - Q - (t+N_u-1)\tilde{R}^d$ only depends on the previous inputs up to time t.

The idea of the cyclic algorithm for solving optimization problems on the form (6) is the following. If Z_t is a positive semidefinite matrix then the class of signals $\{u_k\}_{k=t}^{t+N_u}$ that satisfy $\Phi_U^T\Phi_U = Z_t$ is given by $\Phi_U^T = Z_t^{1/2}U^T$, where U is an arbitrary unitary matrix ($U^TU = I$) and $Z_t^{1/2}$ is a Hermitian square root of Z_t. The optimization problem (6) can then be reformulated as

$$\begin{aligned} &\underset{\{u_k\}_{k=t}^{t+N_u}, U}{\text{minimize}} && \|\Phi_U - UZ_t^{1/2}\|_F^2, \\ &\text{subject to} && u_k \in \mathcal{U}_k, \quad k = 0, \ldots, N_u - 1. \end{aligned} \tag{7}$$

This problem is still non convex. However, for a fixed U it is straightforward to find the optimal $\{u_k\}_{k=t}^{t+N_u}$ and vice versa for a fixed $\{u_k\}_{k=t}^{t+N_u}$. The idea is to alternate between solving for $\{u_k\}_{k=t}^{t+N_u}$ and U while keeping the other variable fixed. This is repeated until convergence. One can show that if the cost function for this optimization problem is small then the cost function for the original problem will also be small. For more details and properties of the cyclic algorithm we refer to Tropp et al. (2005). The steps in the cyclic algorithm are:

Step 0: Initialize U to an arbitrary matrix.
Step 1: Find the vector $\{u_k\}_{k=t}^{t+N_u}$ that minimizes (7) for U fixed. Since Φ_U is an affine function of $\{u_k\}_{k=t}^{t+N_u}$ the problem becomes a quadratic optimization problem with convex constraints. This problem can be solved efficiently and accurately using numerical optimization.
Step 2: For $\{u_k\}_{k=t}^{t+N_u}$ fixed, find a unitary U that minimizes (7) disregarding the constraints. Defining the singular value decomposition (SVD) of $Z_t^{1/2}\Phi_U^T = \bar{U}\Sigma\tilde{U}^T$, the optimal solution is given by $U_{opt} = \tilde{U}\bar{U}^T$.
Step 3: Check if the solution satisfies the stopping criterion, if not, goto Step 1.

The algorithm alternates between solving the two simpler problems in step 1 and 2. Since this only involves solving a quadratic optimization problem and an SVD it is possible to solve relatively large problems on a standard computer.

4.1 Non positive semidefinite Z_t

The matrix Z_t is required to be positive semidefinite which we cannot guarantee. Therefore, we first project Z_t on to the space of positive semidefinite matrices, *i.e.,* we find

$$Z_t^+ = \underset{X \in S_n^+}{\text{argmin}} \|X - Z_t\|.$$

where S_n^+ is the set of positive semidefinite $n \times n$ matrices. This projection is easily done using spectral decomposition (Henrion and Malick, 2012). Consider $Z_t = V\,\text{Diag}(\lambda_1, \ldots, \lambda_n)V^T$, where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of Z_t and V is the corresponding matrix with eigenvectors. The projection can then be written

$$Z_t^+ = V\,\text{Diag}(\max(0, \lambda_1), \ldots, \max(0, \lambda_n))V^T,$$

and Z_t^+ replaces Z_t in the relevant expressions.

4.2 Termination

We stop the algorithm when the relative error of the cost between two iterations is smaller than a threshold ϵ.

5. COMPARISON

The two methods outlined in this paper have different advantages and disadvantages. For the SDP relaxation method it is possible to certify numerically and find the global optimum to the constrained polynomial optimization problem in many cases. However, the number of variables in the relaxed problem grows exponentially with the length of the input horizon N_u and the number of inputs, p. Due to memory limitations it is therefore only feasible to solve problems where pN_u is in the order of 10 on a standard desktop computer.

With the cyclic algorithm on the other hand, larger problems can be solved. In each iteration one singular value decomposition of a real matrix of size $(n+1)pN_u + 2n$ needs to be calculated and a constrained quadratic optimization problem with pN_u variables needs to be solved. Thus the number of variables pN_u can be in the order of 1000. However, this method does not solve the original problem but instead a related problem and not much can be guaranteed in terms of the optimality of the solution to the original problem, except that if the cost function of the solved problem is small then the original cost function is also small. Nevertheless, this method has been shown to give good performance in many applications, see for example the work by He et al. (2012) and their references.

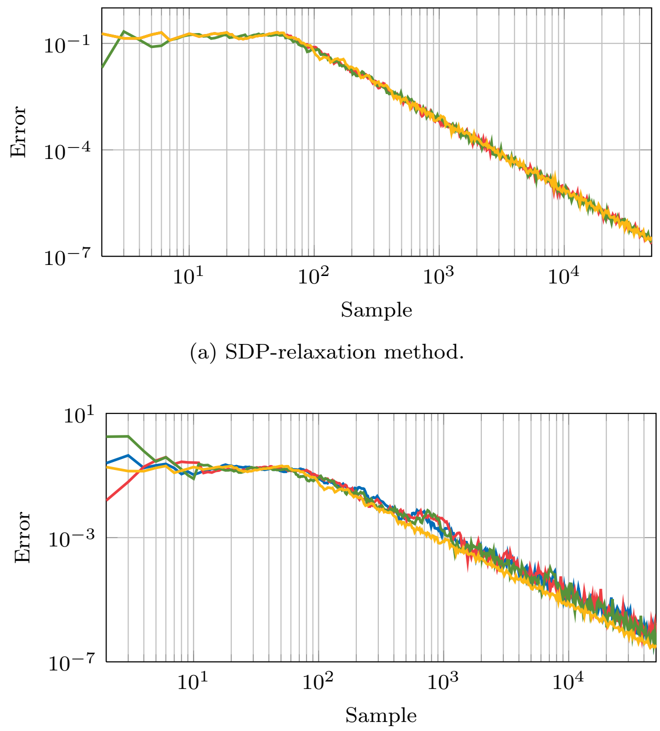
6. EXAMPLES

In this section we present three simulation examples. The first example illustrates the effect of the length of the input horizon on the quality of the generated input signal when no output constraints are considered. In the second example we look at an example with output constraints where the choice of input horizon is important. In the final example we apply the algorithm to the four tank process, a MIMO system with input and output constraints.

Throughout the examples, we set the relative error stopping criterion for the cyclic method to $\epsilon = 10^{-5}$ and the constrained quadratic optimization problem is solved with the Matlab command *quadprog*. The SDP relaxation solution to the multivariate polynomial of degree four is calculated using *GlotiPoly* by Henrion et al. (2009), a polynomial global optimization tool for Matlab.

6.1 Pseudo random white noise

We look at the effect of longer input horizons on the signal generation performance of the two numerical methods. The results are compared to the optimal solution with input horizon $N_u = 1$ introduced by Larsson et al. (2013), where it is possible to find the global optimum analytically. We generate a white noise sequence with unit variance by matching $n = 50$ correlation lags. The input constraints are $|u_t| \leq 1.5$. Here we are only interested in the signal generation and thus we do not consider any output constraints. Three different input horizons are



(a) SDP-relaxation method.

(b) Cyclic algorithm.

Fig. 1. Convergence rate of the method for input horizons, $N_u = 1$ (——), $N_u = 2$ (——) and $N_u = 5$ (——). The analytic solution with $N_u = 1$ is shown as (——).

considered, $N_u = 1, 2, 5$, for both methods. $N = 50000$ samples are generated and the convergence rate of the two methods for the different settings are compared.

In Figure 1, the cost $\sum_{\tau=0}^{n} \|R_N(\tau) - R^d(\tau)\|_F^2$ is plotted as a function of N. The SDP relaxation method is able to find the global optimum for all cases and for all time instances except at the first sample. Hence it is expected that the analytically optimal solution for $N_u = 1$ should be close to the SDP relaxation solution. Looking at Figure 1, this is indeed the case. One should also note that the performance for all input horizons are very close to each other. For the cyclic method we loose a factor of about two in performance compared to the analytically optimal solution for the case $N_u = 1$ due to the suboptimality of the solutions. However, the convergence rate seems to be the same. Again, the length of the input horizon does not seem to have any significant effect on the performance.

6.2 AR(1) process

We generate a signal which is to be applied to the SISO system

$$G(z) = \frac{z + 1.25}{z^2 + 0.4z + 0.6}$$

with a non minimum phase zero at $z_{nmp} = -1.25$. The goal is to identify the location of this zero, and the input signal that minimizes the variance of this estimate is shown by Mårtensson et al. (2005) to be the AR(1) process with auto correlation function $r^d(\tau) = a^{|\tau|}(1 - a^2)^{-1}$, where $a = z_{nmp}^{-1} = -0.8$. The generated signal should also satisfy $|u(t)| \leq 1.5$ and $|y(t)| \leq 1.2$. The output horizon is chosen as $N_y = 5$ and for each of the two methods we study the

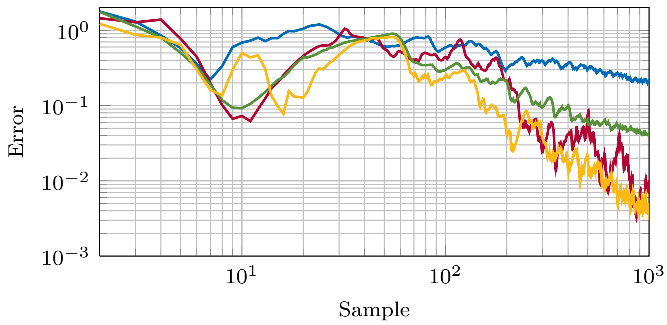


Fig. 2. Convergence rate for the SDP relaxation method with $N_u = 1$ (——), $N_u = 5$ (——) and for the cyclic method with $N_u = 1$ (——), $N_u = 5$ (——).

two cases, $N_u = 1$ and $N_u = N_y = 5$. We set $N = 1000$ and the number of auto correlation lags to match to $n = 50$.

The output constraints are never violated for either case. However, looking at the cost function in Figure 2, it is seen that by using a longer input horizon $N_u = 5$ instead of $N_u = 1$, the performance is improved. This is due to the added flexibility from the longer input horizon. Instead of having $u_{t+1} = \cdots = u_{t+4} = 0$, these variables can be used to control the future predicted output of the system, making it possible to generate a sequence with properties closer to the desired ones. Hence, in certain applications it is beneficial to consider longer input horizons.

6.3 The quadruple tank system

We consider a quadruple tank system. Its main components are two lower tanks, two upper tanks and two electrical pumps. Each pump delivers water to one of the upper tank and one of the lower tanks. Water also flows from the upper tanks into the lower tanks and from the lower tanks into the water basin through small holes at the bottom of each tank. The two input signals to the system are the applied voltages to the pumps and the outputs are the water level in each of the four tanks. See Johansson et al. (1999) for details about the considered process.

From a nonlinear model of the plant from Johansson et al. (1999), a linearized continuous time model of the process, around a working point x^0, u^0, is given by

$$\dot{x}_t = \begin{bmatrix} \tau_1 & 0 & -\tau_3 & 0 \\ 0 & \tau_2 & 0 & -\tau_4 \\ 0 & 0 & \tau_3 & 0 \\ 0 & 0 & 0 & \tau_4 \end{bmatrix} x_t + \frac{1}{A} \begin{bmatrix} k_1\gamma_1 & 0 \\ 0 & k_2\gamma_2 \\ 0 & k_2(1-\gamma_2) \\ k_1(1-\gamma_1) & 0 \end{bmatrix} u_t,$$

$$y_t = x_t,$$

where $\tau_i = -\frac{a_i}{A}\sqrt{\frac{g}{2x_i^0}}$. The parameter values are taken from Larsson (2011). This model is discretized using zero order hold at a sampling rate of 1 Hz, see Larsson (2011).

We want to excite the system with an input signal with the low pass FIR spectrum with $n = 10$ lags shown in Figure 3 as suggested by Larsson (2011). To keep the water levels close to the working point, we constrain the four outputs to satisfy $|y_t| \leq 1.3$ cm. We also require that the input signal amplitude should be less than 2 V. $N = 1000$ input samples are generated using both solution methods.

The cost function, $\sum_{\tau=0}^{n} \|R_N(\tau) - R^d(\tau)\|_F^2$, for the generated sequences are $4.4 \cdot 10^{-3}$ and $9.7 \cdot 10^{-3}$, respectively compared to the norm of the desired auto correlation sequence, $\sum_{\tau=0}^{n} \|R^d(\tau)\|_F^2 \approx 0.38$. Again the SDP relaxation method performs slightly better. For comparison, a white Gaussian noise sequence filtered through a spectral factor of the FIR-spectrum gives an average cost of $7.7 \cdot 10^{-2}$ over 100 realizations. Hence the properties of the signals generated with the proposed method are better than for filtered white noise. Moreover, the output constraints are never violated for the proposed method while there are in average 25 violations for the filtered white noise.

Figure 3 shows the spectra of the generated signals and the desired spectrum. The spectra for the generated input signals are calculated using a Hanning window of width 50 (Stoica and Moses, 2005). Both methods successfully generate an input signal with the desired properties.

7. ROBUSTNESS ISSUES

The method in this paper requires perfect knowledge of the true system to predict the effect of the input on the output, which is is not possible in real applications since the model will often have some uncertainties due to under modeling and noise. To overcome this, a robust and adaptive implementation of the method for the SISO case with input horizon one was proposed by Hägg et al. (2013). The method is made robust to model uncertainties by using tools from robust MPC as presented by Maciejowski (2002) and the input signal is generated to satisfy the output constraints for all possible models in some uncertainty region. If the uncertainties are large, the input could be quite conservative to satisfy the robust output constraints. Furthermore, the propagation of the uncertainty in time will eventually force the input to be zero.

The sample by sample nature of the proposed method does however allow for real time implementation. One sample of the generated input can be applied to the system and the resulting output measured. Measured data can be used to estimate the state of the system and a new input sample can be generated using this new information, reducing the propagated uncertainty. Hägg et al. (2013) suggest re-identifying the model recursively to reduce the model uncertainties. These results should be directly applicable to the proposed method making it practically useful.

8. CONCLUSIONS

We extended the signal generation method proposed by Larsson et al. (2013) to the MIMO case and to general input horizon lengths. This is formulated as an receding horizon algorithm where the objective is to generate a signals over a finite horizon such that the signal has a specified sample auto covariance while satisfying input and output constraints of the system which the signal is to be applied to. The resulting optimization problem is a constrained minimization of a polynomial of degree four, which is in general non convex. Two methods from the literature for solving such problems were investigated: a convex relaxation method and a cyclic algorithm.

With the convex relaxation method it is in many cases possible to find the global optimum. However, due to the exponential growth of the complexity, the method is only feasible for small problems. The method using the cyclic

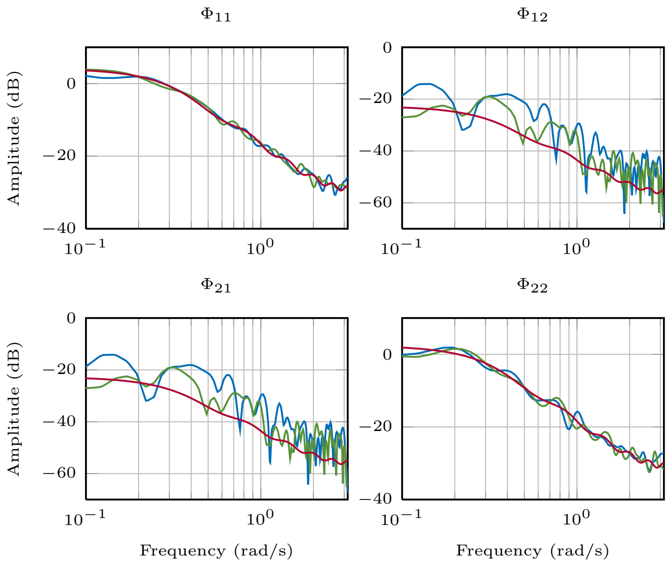


Fig. 3. The estimated spectra of generated for SDP relaxation (——) and cyclic method (——). Desired input spectrum is shown as (——). Φ_{ij} is the cross spectrum between $u(i)$ and $u(j)$.

algorithm works for larger problems but not much can be said about the convergence of the method. Still, it has shown good performance in many applications.

Three simulation examples illustrate the properties of the methods. First, the convergence rate was investigated when the length of the input horizon is varied. It was noted that quality of the generated signal is relatively unaffected by the length of the input horizon. This was also noted for the binary input case by Rojas et al. (2007). However, as noted in the second example, the longer input horizon could improve the performance if output constraints are considered. In the third and final example, the MIMO capabilities of the methods were shown.

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