

# Decentralized PID Controller Tuning Based on Desired Dynamic Equations

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**Abstract:** The paper aims to show the ability of the desired-dynamic-equations (DDE) based decentralized PID controller tuning method with its application to the ALSTOM gasifier benchmark control problem. The DDE-based PID controller tuning method was deduced from a kind of nonlinear adaptive controller with relative degree of two, which behaves good tracking performance and robustness by using an extended state observer and DDE to estimate and compensate the uncertainty and disturbance. The tunable parameters of DDE-based PID controller have explicit and distinctive physical meanings, thus can be tuned separately rather than iteratively as traditional PID controller does. The tuning method is firstly applied to the ALSTOM gasifier benchmark control problem with linear model, simulation shows that it exceeds output limits only twice (the least in literature) at 0% load with the step and sinusoidal disturbance; then the same controller is applied without any modification to nonlinear gasifier model, and the simulations show that it not only meets all control specifications, but also follows load change rapidly and shows good adaptability to the coal quality change.

**Keywords:** desired dynamic equation, PID controller tuning, multivariable control, gasifier control.

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## 1. INTRODUCTION

As the most commonly used feedback controller, PID controller behaves the advantages of simple structure, ease of operation, and fairly good control performance. It has dominated over 90% of industrial control loops for several decades. Many PID controller tuning formula (Aidan, 2003) have been proposed since last century to facilitate its application and improve its performance. Some are widely accepted and used in practical industry, but these tuning formulas are often suitable to some special kinds of process such as one or two order process with time delay, so model reduction or approximation is often needed before tuning, which may result in deteriorate performance. Thus it is beneficial to explore a wide-spectrum PID controller tuning method, especially for multi-input-multi-output process.

Except the tuning formula, there are some tuning methods suitable to more general process control. Wang (2003) considered the impact of time delay and non-minimum phase on desired closed-loop transfer function, and obtained the PID controllers after simplification of decoupled diagonal and non-diagonal high-order controllers. Based on advanced control theory such as  $H_\infty$  or IMC, Tan et al(2002) and Dong et al(1997) designed PI or PID controllers by truncating the low order of Maclaurin series of high-order decouple controllers. Similar to the problem brought by model reduction aforementioned, the low-order PI or PID controller could not guarantee the same performance or even the stability with the original high-order controller, thus performance deterioration is still a problem. Huang, et al (2003) proposed a decentralized PID controller tuning method based on effective open-loop process and obtained good performance, but the calculation becomes difficult with

the increase of system dimension. Li, et al(2007) also proposed a decentralized PID controller tuning method, which can achieve the optimal performance and suitable to high dimension system without limitation on model orders, but the numerical calculation load is as heavy as numerical optimization method always shows.

In recent years, a kind of adaptive decentralized controller based on dynamic compensation mechanism and desired dynamic equation (DDE) was proposed (Tornambe, 1994) to stabilize a class of multi-input-multi-output system, which is named as TC controller in this article. The TC controller has achieved good performance and robustness demonstrated by the application and simulation in many industrial systems, such as boiler-turbine coordinated system (Shan,2007), spacecraft attitude control (Chen,2007), and hydro-turbine regulating system (Ning, 2006), whereas, its industrial application is rarely reported up to now owing to its complicated control structure.

Based on TC and desired dynamic equation, this paper explored a kind of decentralized PID tuning method for a wide-spectrum process, and then applied it to the linear and nonlinear ALSTOM gasifier benchmark control problem to verify its feasibility. Simulations show its ease of use and good performance.

## 2. PROBLEM DESCRIPTION

Consider a process with  $n$  inputs and  $n$  outputs with diagonal domination, described by transfer function matrix  $G(s)$  as

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix}, \quad (1)$$

where,  $g_{ij}(s)$ ,  $i, j = 1, 2, \dots, n$  represents the transfer function from the  $j$ -th input to the  $i$ -th output. The decentralized PI/PID controller  $C(s)$  is designed as:

$$C(s) = \begin{bmatrix} c_1(s) & 0 & \cdots & 0 \\ 0 & c_2(s) & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & c_n(s) \end{bmatrix}, \quad (2)$$

where  $c_i(s)$ ,  $i=1, 2, \dots, n$  represent PI/PID controllers of  $i$ -th loop.

$$c_i(s) = K_{pi} + \frac{K_{li}}{s}, \quad i = 1, 2, \dots, n.$$

or

$$c_i(s) = K_{pi} + \frac{K_{li}}{s} + K_{di}s, \quad i = 1, 2, \dots, n.$$

The final decentralized PI/PID control system is as follows:

$$\begin{cases} Y(s) = G(s)U(s) \\ U(s) = C(s)(R(s) - Y(s)) \end{cases} \quad (3)$$

where,  $Y(s) = [y_1(s), y_2(s), \dots, y_n(s)]^T$  is output vector,  $U(s) = [u_1(s), u_2(s), \dots, u_n(s)]^T$  is manipulated vector, and  $R(s) = [r_1(s), r_2(s), \dots, r_n(s)]^T$  is set-point vector.

The goal of controller design is to tune the parameters of decentralized PI/PID controllers properly so that the system remains stable and the performance specification is as good as possible.

### 3 DDE-BASED PID CONTROLLERS TUNING

#### 3.1 PID controller tuning based on DDE

For a linear single-input-single-output (SISO) process with relative degree of two, a nonlinear control law (Tornabe, 1994) can be modified to have a simple formula as follows (Wang, 2009):

$$\begin{cases} u = (-h_0(y - y_r) - h_1\dot{y} - \hat{f}) / l \\ \hat{f} = \xi + k\dot{y} \\ \dot{\xi} = -k\xi - k^2\dot{y} - klu \end{cases}, \quad (4)$$

In which,  $u$  is the controller output,  $y$  is the system output to be controlled,  $y_r$  is the desired trajectory of  $y$ ,  $\hat{f}$  is the output of an extended state observer, which estimates and compensates the process uncertainties,  $h_1, h_0$  determines the response speed of the system and are chosen to meet the desired dynamic equation (DDE)  $\ddot{y}_2 + h_1\dot{y} + h_0y = h_0y_r$ ,  $\xi$  is an intermediate variable;  $k$  is the tunable parameter, which determines the stability of the system (Tornabe, 1994),  $l$  is

a proper positive constant number.

The controller (4) can be rewritten as:

$$u(t) = [(h_0 + kh_1)(y_r - y) + kh_0 \int (y_r - y)dt - (h_1 + k)\dot{y} - kh_1y_r] / l, \quad (5)$$

and it is similar to a two-degree-of-freedom PID controller:

$$u(t) = K_p e + K_I \int edt + K_D \dot{e} - [(h_1 + k)\dot{y}_r + kh_1y_r] / l, \quad (6)$$

if the following relation stands:

$$\begin{aligned} e &= y_r - y, \quad K_p = (h_0 + kh_1) / l, \\ K_I &= kh_0 / l, \quad K_D = (h_1 + k) / l. \end{aligned} \quad (7)$$

If the set-point  $y_r$  is a constant value ( $\dot{y}_r = 0$ ) or a step signal ( $|\dot{y}_r| = \inf$  at the step time, and after that  $\dot{y}_r = 0$ ), the item  $(h_1 + k)\dot{y}_r$  could be ignored, thus the controller (6) is equivalent to a two-degree-of-freedom PID controller, while the controller output is  $u(t) = K_p e + K_I \int edt + K_D \dot{e} - kh_1y_r / l$ .

Similarly, when the process relative degree is one, a two-degree-of-freedom PI controller can be deduced for a given desired dynamic equation  $\dot{y} + hy = hy_r$ :

$$\begin{aligned} u(t) &= [(h+k)(y_r - y) + \int kh(y_r - y) - ky_r] / l \\ &= K_p e + K_I \int edt - ky_r / l, \end{aligned} \quad (8)$$

$$\text{where } K_p = (k+h) / l, \quad K_I = kh / l. \quad (9)$$

Thus, the PI/PID controller parameters can be determined easily by using (7) or (9) based on the parameter  $k, l, h_0, h_1$  in TC controller. It should be noticed that the tuning of one-degree and two-degree TC controller is much easier than that of PI/PID controller directly, since the parameter  $k, l, h_0, h_1$  have explicit and distinctive physical meaning and can be tuned separately. Parameter  $h_0, h_1$  should be tuned firstly according to the desired dynamic equation, then parameter  $k$  and  $l$  could be tuned to meet the stability and performance specifications.

#### 3.2 Closed-loop performance analysis

For a linear system with relative degree two,

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = f + lu \\ y = z_1 \end{cases} \quad (10)$$

Using controller (4), the dynamic of closed-loop output is:

$$\ddot{y} + h_1\dot{y} + h_0y = h_0y_r + f - \hat{f}. \quad (11)$$

If the process model is known exactly, i.e.  $f = \hat{f}$ , the desired dynamic equation can be implemented perfectly; otherwise, the actual output  $y$  will not follow accurately the desired  $y_r$ . For tracking error  $e = y_r - y$ , we have

$\ddot{e} + h_1\dot{e} + h_0e = \hat{f} - f$ , so the tracking error is determined by  $\hat{f} - f$ .

In frequency domain, we have the follows from (4) and (10),

$$\hat{f}(s) = \frac{k}{s+k} f(s). \quad (11)$$

So a larger  $k$  is preferred to reduce the tracking error. This conclusion is also valid if we use  $r$ -order TC controller to control a process of relative degree  $r$ . It has been proved that there exists a constant value  $k^*$  such that the closed-loop system is asymptotically stable for any  $k \geq k^*$  (Tornambe, 1994). But it is completely different if a two-order TC controller is adopted to control a process with relative degree unknown or more than two, which is common in practice.

For a linear system with relative degree  $r$  ( $r > 2$ ),

$$\begin{cases} \dot{z}_i = z_{i+1}, & i = 1, \dots, r-1, \\ \dot{z}_r = f + lu, \\ y = z_1 \end{cases} \quad (12)$$

Also use controller (4), the dynamic of closed-loop output is:

$$y^{(r)} + h_1\dot{y} + h_0y = h_0y_r + f - \hat{f}. \quad (13)$$

With a tracking error dynamic:

$$\ddot{e} + h_1\dot{e} + h_0e = \hat{f} - f + y^{(r)} - \ddot{y}. \quad (14)$$

In frequency domain, we have

$$\hat{f}(s) = \frac{k}{s+k} (f(s) + s^2Y(s) - s'Y(s)). \quad (15)$$

Substitute (15) to (14),

$$\ddot{e} + h_1\dot{e} + h_0e = -\hat{f} / k \quad (16)$$

Since  $\hat{f}$  is a function of  $k$ , the influence of parameter  $k$  on tracking error is not as direct as in process of relative degree two, and there exists the optimal value  $k$  for better tracking.

Here, parameter  $l$  can be regarded as a scaling factor of control effort  $u$ , so it also has great impact on the performance of closed-loop system. Larger  $l$  results in weaker control effort  $u$ .

### 3.3 DDE based PID controller tuning procedure

The PI/PID controller tuning based on DDE can be carried out as follows:

- (1) Select desired dynamic equation parameter  $h$  or  $h_0, h_1$ ;
- (2) Select proper value of  $k$  in its stable region, then choose  $l$  so as to make the stable boundary of parameter  $|k|$  equals to  $2k$  approximately.
- (3) Calculate the PID controller parameters using (9) or (7), then evaluate the control performance, if not satisfactory, return to step (1).

Note: If the stable region of the process is unknown, the parameter  $l$  in step (2) can be tuned by experiments, that is, gradually decrease it from a value large enough till the performance is satisfactory.

Generally from the analysis of two order process, the parameter  $h_0, h_1$  in DDE can be selected as:

$$h_1 = (8 \sim 25) / t_{sd}, \quad h_0 = h_1^2 / 4 \quad (17)$$

for PID controller, and for one order process,

$$h = (4 - 12) / t_{sd} \quad (18)$$

for PI controller, where  $t_{sd}$  is expected regulation time.

The decentralized PI/PID controller can be obtained by tuning PI/PID controller of each loop.

## 4. SIMULATION

### 4.1 ALSTOM gasifier benchmark control problem

The ALSTOM gasifier benchmark control problem provided linear and nonlinear model under three load conditions. The controller should operate the outputs within their constraints under downstream pressure disturbances, load test and model error test, as described in Dixon, et al (2000, 2004). The decentralized control diagram of ALSTOM gasifier benchmark problem is shown in Fig 1, which was suggested by Asmar et al. (2000) after numerical simulation and results analysis for four difference possible schemes. The manipulated inputs are char extraction flow rate  $W_{chr}$ , inlet air flow rate  $W_{air}$ , inlet coal flow rate  $W_{coal}$ , limestone flow rate  $W_{ls}$ , and inlet steam flow rate  $W_{stm}$ . The outputs are syngas caloric value  $Cv_{gas}$ , the bedmass  $Mass$ , syngas pressure  $P_{gas}$  and syngas temperature  $T_{gas}$ . In addition, the regulation of downstream gas turbine inlet valve will bring disturbance to gasifier pressure, represented as  $Psink$ . The limestone mass flow rate  $W_{ls}$  should keep a fixed ratio (10% in this model) of coal flow rate to capture the sulfur in the coal, this leaves the gasifier model a four-input-four-output system. The parameters  $h_1, h_2, h_3, h_4$  are the DDE parameters for loop of  $Cv_{gas}, Mass, P_{gas}, T_{gas}$ , respectively, and parameters  $k_1, k_2, k_3, k_4$  are their controller parameters. Let  $l=1$ , the controller parameters are tuned as shown in Table 1. The parameters of PI controllers are then calculated from  $h_1, h_2, h_3, h_4$  and  $k_1, k_2, k_3, k_4$  by using (9).

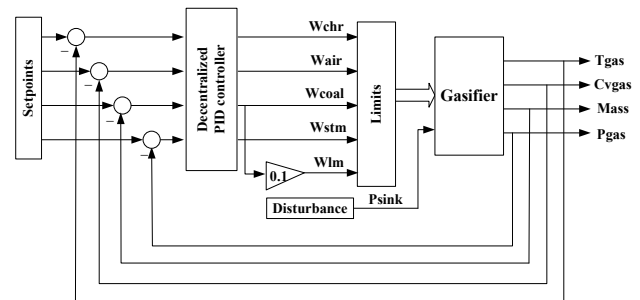


Fig. 1. Decentralized control structure of gasifier system

Table 1. Controller parameters

Loop	$h$	$k$
<i>Wchr-Tgas</i>	0.0001	2
<i>Wair-Cvgas</i>	0.0001	-2
<i>Wcoal-Mass</i>	0.015	0.045
<i>Wstm-Pgas</i>	0.0003	0.00001

#### 4.2 Simulation for linear gasifier model

The control system simulation based on linear model is carried out at 100%, 50%, 0% load respectively. It is found that the decentralized PI controller proposed can meet the output constraints at 100% and 50% load for all the disturbance test; at 0% load, three loops meet the output specifications except the pressure loop(Fig.1, Appendix). When compared with published control method for ALSTOM gasifier linear model on the total violation number under all required test, as listed in Table 2, it is found that proposed DDE-based PI control system has minimum violations, which indicate its better disturbance rejection ability and robustness. We also noticed that the violation is inevitably for all published method. This implicates the strong nonlinearity of gasifier system.

Table 2. Comparison of linear gasifier control

Control Method	Total Violation
Proposed DDE-based PI control	2
Adaptive nonlinear control (Wang, 2007)	2
Mixed-sensitivity $H_\infty$ design (Prempain,2000)	3
PI-Plus control (Pike, et al.1998)	3
PI control based on process engineering approach (Taylor, et al, 2000)	4
PI control design using multi-objective optimization (Liu, et al, 2000)	5
Predictive control plus a simple control law (Rice, et al, 2000)	5

#### 4.3 Simulation for nonlinear model

The decentralized PI controller designed based on linear gasifier model is adopted to the nonlinear gasifier model without any modification. The simulation demonstrates that the proposed DDE-based PI controller meets all the input and output constraints under three load conditions. When the load rises from 50% to 100% load, the actual load follows the demanded load well. The dynamic response under  $P_{sink}$  disturbance at 0% condition and load change are shown in Fig. 2 of Appendix.

The control method based on nonlinear gasifier model is compared in Table 3. Most of these control strategies meet the specification well, but all of them are redesigned or retuned to meet the nonlinear model specifications except the proposed controller, which indicate its better robustness.

Table 3. Comparison of nonlinear gasifier control

Method	Total
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	Violation
Proposed DDE-based PI controller	0
Adaptive nonlinear control design optimized using Genetic Algorithm (Wang, et al, 2007)	0
Decentralized PI controller tuning with multi-objective optimization (Xue, et al, 2005)	0
Multi-variable PID controllers based on genetic algorithms (Frag, et al, 2006)	0
Decentralized PI controller using multi-objective optimization (Dixon, et al, 2004)	1
Combined $H_\infty$ loop-shaping controller and $H_\infty$ optimized anti-windup compensator (Gatley, et al,2004)	2
Multivariable PI-plus control (Taylor, et al, 2004)	0
State estimation-based feed-forward control (Wilson, et al,2006)	1
Model predictive control (Al Seyab, et al, 2006)	0

The scope of permitted coal quality change is listed in Table 4, which is obtained by gradually increasing or decreasing the coal quality and simulating the system 300 seconds under the combination of all disturbances and load conditions until output limits violation occurs. The results show that the proposed control system works well under a wide range of coal quality variation, which is crucial for operation under fuel uncertainty.

Table 4. Permitted coal quality variation

Load	Lower Limit(%)	Upper Limit(%)
100%	-18	15
50%	-18	18
0%	-18	18

## 5. CONCLUSION

Based on a kind of nonlinear adaptive controller with dynamic compensation mechanism and desired dynamic equation, a PI/PID controller tuning method is developed and analyzed. The control parameters have explicit and distinctive physical meanings, thus can be tuned separately.

The PI controller tuning method is applied to the tuning of linear gasifier benchmark problem and later applied to nonlinear gasifier control without any modification. Simulations show that for linear model, the control performance of 100% and 50% load is satisfactory, only the pressure loop exceeds its upper limit at 0% load, which is the best in all published results; for nonlinear model, all the control specifications are satisfied, the system can follow the demanded load rapidly from 50% to 100% load, and shows some adaptability to coal quality variation. This is now the first control strategy that can be extended from linear gasifier model to nonlinear gasifier model with good performance and without any modification, which indicates better robust in practice. The simulation application shows its promising future to control practice.

## ACKNOWLEDGEMENT

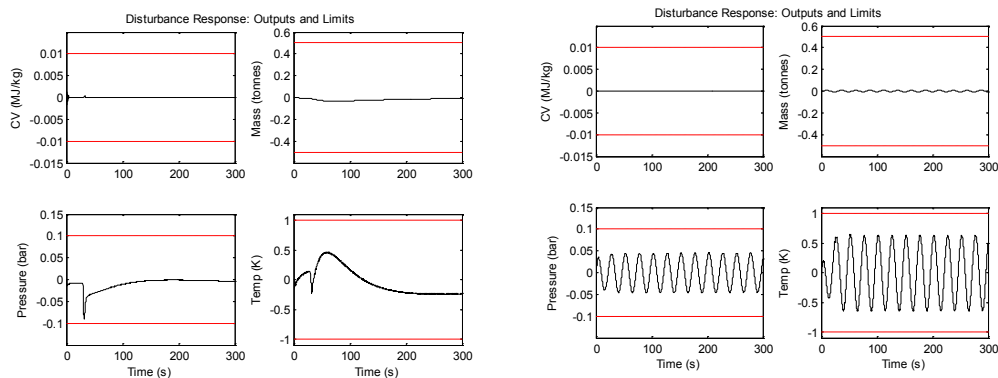
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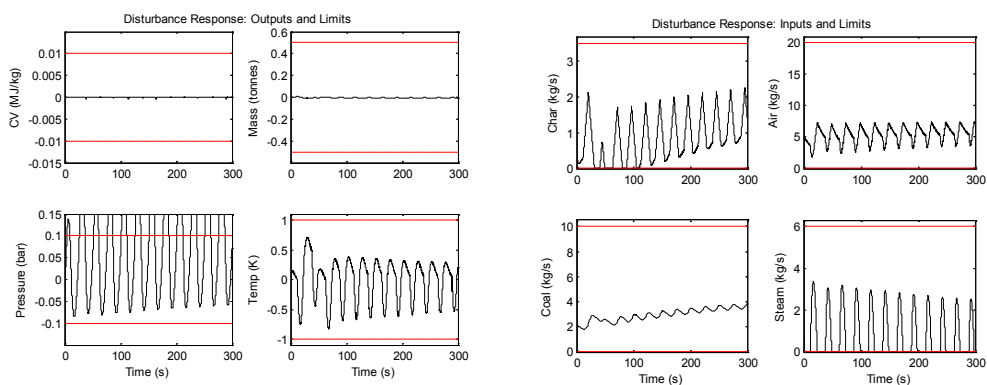
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## Appendix . Dynamic response of ALSTOM gasifier benchmark control

(The solid line represents each input and output dynamic response, and the dashed line shows their permitted scope)

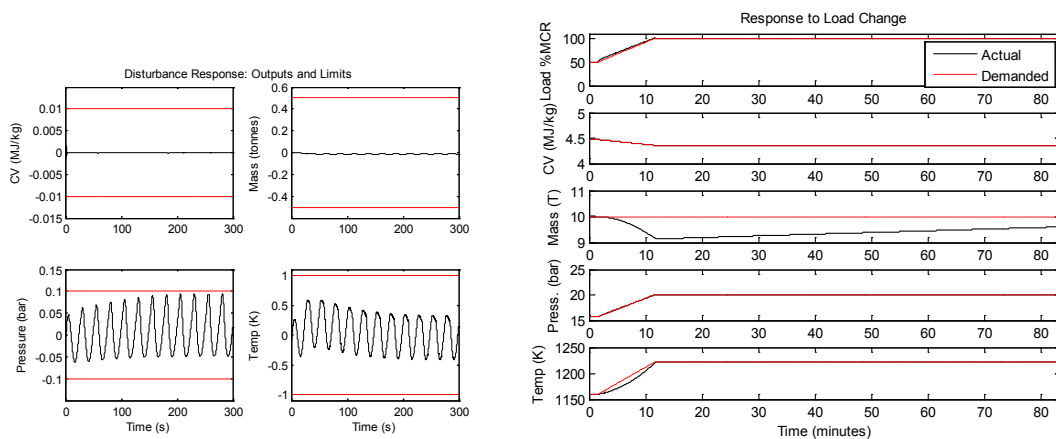


(a). Dynamic response at 100% load with  $P_{sink}$  step and sine disturbance



(b) Dynamic response at 0% load with  $P_{sink}$  sine disturbance

Fig. 1. Dynamic response for linear gasifier system



(a) Dynamic response at 0% load with  $P_{sink}$  sine disturbance

(b) Dynamic response under load change test (from 50%~100%)

Fig. 2. Dynamic response for nonlinear gasifier system