Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Control design for a multi input single output hydraulic cylinder system

Thomas Schwarzgruber * **Thomas Ernst Passenbrunner** * **Luigi del Re** *

* *Institute for Design and Control of mechatronical Systems, Johannes Kepler University Linz, 4040 Linz, Austria (email:* {thomas.schwarzgruber,thomas.passenbrunner,luigi.delre}@jku.at)

Abstract: This work discusses several approaches to the multi input, single output problem in hydraulic control. The multi input problem arises from the fact, that the actuator force in a hydraulic cylinder is the result of a combination of two pressure states, which is not uniquely defined. With each individual pressure state there may be associated a subsystem with a control input. To achieve a desired actuator force there are inifinitely many combinations of pressure values possible, and it is therefore unclear how to chose the tracking reference values for the pressure subsystems. Secondary objectives are defined, with time optimality of the step response of the subsystems being one of them, that can be used to solve the multi input problem by treating it as an optimization problem. The practical implications of the optimal solutions are discussed and a new solution is proposed, motivated by the results of the optimization problems. The proposed solution is used to develop a backstepping control for a hydraulic multi input single output system, and its performance is shown in simulation.

1. INTRODUCTION

A popular control design methodology for hydraulic piston-cylinder systems is the backstepping method (Krstić et al. [1995], Choux et al. [2009], Kaddissi et al. [2007]). It is especially useful in the design of a tracking controller, as the design method implicitely provides a combined feedforward and feedback structure and is straight forward to apply to strict-feedback systems in a single input single output configuration. Typically this configuration appears in hydraulic systems with a servo driven pump, where the rotational speed of the pump can be viewed as the system input (cf. Habibi and Goldenberg [2000], Ahn et al. [2013]) or in systems with a fixed speed pump and a servo valve, where the valve opening acts as the input to the system (cf. Ayalew and Kulakowski [2006],Loukianov et al. [2009]). In those systems there is a single input (valve position or pump speed) to single output (position, speed or force) relationship and the backstepping design method applies straight forwardly, resulting in various extensions of the backstepping having been applied in literature, ranging from adpative methods (Ahn et al. [2013], Choux et al. [2009]) to problem tailored lyapunov functions (Kaddissi et al. [2007]).

For systems that combine both inputs, a servo driven pump and a servo valve, the application of the backstepping method, or any other method that includes a feedforward part that relies on the reference trajectory, faces the problem of the non-unique relationship between the (reference or desired) force and the cylinder pressures. This problem is adressed in this work by defining secondary objectives that allow to cast this problem, under reasonable assumptions, into an optimization problem. Two different approaches are taken to cast the multi input problem as an optimization problem, aiming for different objectives. Motivated by the results of the optimization problems, a solution is proposed, that shows some characteristics of the optimal solutions and has advantages in the implementability.

The paper is structured as follows: First the hydraulic plant model is introduced that shows a two inputs, one output structure and that is used to design the control system. In the following the multi input problem is highlighted as the first steps of the backstepping control design are shown and different approaches are discussed, based on the formulation as an optimization problem. Motivated by those results a new approach to the multi input problem is proposed, which is used to design the control in the succeeding section. The penultimate section presents the simulation model and the simulation results of the proposed controller. Finally the conclusions conclude the paper, not without giving an outlook to possible follow up work.

2. PLANT MODEL

The schematic of the hydraulic plant that should be considered in this work is shown in Figure 1. The hydraulic system consists of a hydraulic fluid reservoir, under atmospheric pressure, a pump driven by a servo, which allows to control the pump speed and rotation direction (bidirectional pump), and a servo-valve, in the schematic shown as a 4/3 way valve to show the possible connections, but the valve can take all intermediate positions. The actuator of the hydraulic system is a hydraulic cylinder which acts against a load force F_L. The pump rotational speed and the valve position are considered as inputs u_1, resp. u_2, where positive values in u_2 represent the connection of A–T and B–P, negative values A–P, B–T.

The system model (1) of the hydraulic plant in state space form is derived using standard hydraulic equations for the flow through an orifice and Newton's Law for the movement of a mass. The valve shows by construction different opening characteristics for each of the four channels A–T, B–P, A–P and B–T, which are represented by the positive parameters $s_{AP} < s_{BP} < s_{AT} < s_{BT}$. Besides that the flow through the valve is characterized by the nominal flow Q_N at a nominal pressure drop p_N.

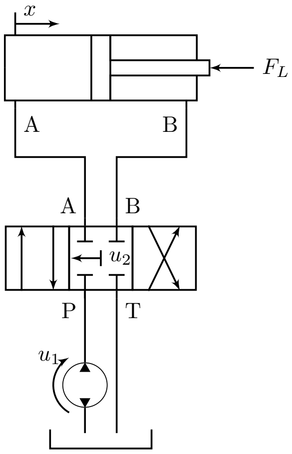

Fig. 1. Schematic of the hydraulic circuit in a multi input configuration, with inputs u_1 and u_2.

$$\dot{x}_1 = x_2 \tag{1a}$$

$$\dot{x}_2 = \frac{1}{m}\left(x_3 A_1 - x_4 A_2 - F_L\right) \tag{1b}$$

$$\dot{x}_3 = \frac{E}{V_{01} + A_1 x_1}\left(-A_1 x_2 - \sigma\left(u_2 - s_{BT}\right) Q_N \hat{\sqrt{\frac{x_3 - p_T}{p_N}}} + \sigma\left(-u_2 - s_{BP}\right) Q_N \hat{\sqrt{\frac{x_5 - x_3}{p_N}}}\right) \tag{1c}$$

$$\dot{x}_4 = \frac{E}{V_{02} - A_2 x_1}\left(+A_2 x_2 + \sigma\left(u_2 - s_{AP}\right) Q_N \hat{\sqrt{\frac{x_5 - x_4}{p_N}}} - \sigma\left(-u_2 - s_{AT}\right) Q_N \hat{\sqrt{\frac{x_4 - p_T}{p_N}}}\right) \tag{1d}$$

$$\dot{x}_5 = \frac{E}{V_{0P}}\left(V_S \eta_v u_1 - \sigma\left(u_2 - s_{AP}\right) Q_N \hat{\sqrt{\frac{x_5 - x_4}{p_N}}} - \sigma\left(-u_2 - s_{BP}\right) Q_N \hat{\sqrt{\frac{x_5 - x_3}{p_N}}}\right) \tag{1e}$$

The states of the model in (1) are as follows: $x_1 = x$ is the position of the cylinder, $x_2 = v$ is the translational speed of the cylinder, $x_3 = p_1$ is the pressure associated with chamber A and cylinder area A_1, $x_4 = p_2$ is associated with chamber B (area A_2) and $x_5 = p_P$ is the pressure between pump and valve. The parameter E represents the compressibility of the hydraulic fluid, and V_{01} resp. V_{02} the volume of chamber A resp. B for the cylinder position $x_1 = x = 0$, while V_{0P} is the volume of the connection pump to valve. The pump is characterized by the nominal volume V_S per rotation and an efficiency factor η_v, and the reservoir pressure is $p_T = 10^5\,\mathrm{Pa}$. The function $\sigma(\cdot)$ is limiting the opening of an individual valve channel to values between 0 (channel closed) and 1 (channel fully open – nominal orifice behaviour) and is defined as in (2),

$$\sigma(x) = \begin{cases} 1 & x > 1 \\ x & 0 \leq x \leq 1 \\ 0 & x < 0 \end{cases} \tag{2}$$

and $\hat{\sqrt{\cdot}}$ represents the signed square root, as defined in (3), where sgn is the signum function.

$$\hat{\sqrt{x}} = \operatorname{sgn} x \sqrt{|x|} \tag{3}$$

3. MULTI INPUT PROBLEM

The objective of the control system is to follow a given reference trajectory with the hydraulic cylinder. To highlight the multi input problem we detail the first steps in the backstepping design procedure for the tracking control design of the particular hydraulic plant. We define therefore the first error variable as the deviation from the reference position

$$z_1 = x_{1,ref} - x_1$$

and the associated lyapunov function $V_1(z_1) = \frac{1}{2} z_1^2$. By a proper choice of the virtual input $x_{2,des}$, the derivative $\dot{V}_1$ along a trajectory can be rendered negative definite, this is achieved e.g. by $x_{2,des} = c_1 z_1 + \dot{x}_{1,ref}$, with c_1 being a positive tuning parameter. This allows to augment the error system with the second error state

$$z_2 = x_{2,des} - x_2$$

resulting in the augmented lyapunov function $V_2(z_1, z_2) = \frac{1}{2}\left(z_1^2 + z_2^2\right)$. In this step of the procedure, the virtual input is the actuator force $F_a = x_3 A_1 - x_4 A_2$, and by the choice of

$$F_{a,des} = m\left(\left(1 - c_1^2\right) z_1 + \left(c_1 + c_2\right) z_2 + \ddot{x}_{1,ref}\right) + F_p$$

with $c_2 > 0$ the derivative of V_2 is rendered negative definite.

At this stage the MISO structure of the system shows – the actuator force F_a is not a state of the system, but is an algebraic expression of two individual system states. The desired value $F_{a,des}$ can be achieved now by various combinations of $x_{3,des}$ and $x_{4,des}$, and the split up is not defined a priori. The question on how to split up the desired force into pressure values $x_{3,des}$ and $x_{4,des}$ can be tackled from different perspectives, depending on the objectives of the system:

- If the main purpose of the system is fast motion and short travel time, it is assumed that $F_{a,des}$ shows large transients. To be able to follow $F_{a,des}$ it is necessary that a large portion of $F_{a,des}$ is due to the pressure with the faster dynamics. The exact split up ratio is then defined by the necessary dynamics due to the reference, and the system dynamics of the individual pressure states.
- If the main purpose of the system is exact tracking of slow references (relative to possible system dynamics) and good disturbance rejection, then it is advantageous that the slow dynamic pressure state is used to provide the main portion of $F_{a,des}$, and that the fast dynamic part is used to provide fast disturbance rejection.

We assume that the initial conditions of our system and the reference trajectory imply the first case: large transients in $F_{a,des}$ with fast dynamics. For a suitable performance of the closed loop system it is necessary that $F_{a,des}$ will be followed as close as possible, and the split up of the force demand onto the pressure values should be according to the respective subsystem dynamics – if one of the two subsystems is significantly faster than the other, then this should reflect in the demanded pressure value. A bad split up of $F_{a,des}$ onto the desired values $x_{3,des}$ and $x_{4,des}$ would result in one subsystem that may follow the desired value quite closely, whilst the other subsystem showing large deviations from the desired value. By this then, the desired force would not be achieved.

A way to treat the split up problem in the light of the mentioned effects, is to set up an optimization problem, with

$$\min_{\Delta x_3, \Delta x_4} J \tag{4}$$

$$J = \left(F_{a,des} - x_{3,des}A_1 + x_{4,des}A_2\right)^2 + \Delta x^{\mathrm{T}}{}_{3,4} R \Delta x_{3,4} \tag{5}$$

where we define

$$x_{3,des} = x_3 + \Delta x_3 \tag{6}$$

$$x_{4,des} = x_4 + \Delta x_4, \tag{7}$$

$\Delta x_{3,4} = (\Delta x_3, \Delta x_4)^{\mathrm{T}}$, and R is a diagonal weighting matrix $\mathrm{diag}(r_{11}, r_{22})$. The purpose of the cost function is to allow to find a combination of $x_{3,des}, x_{4,des}$ which reflects $F_{a,des}$, whilst minimizing the deviation from the actual values x_3, x_4. The weighting matrix R can then be used to take into account the different dynamics of the subsystems x_3 and x_4.

The optimization problem can be solved using the necessary condition for optimality of first order (Bryson and Ho [1975])

$$\nabla J = 0$$

$$\begin{bmatrix} -2\left(F_{a,des} - F_a - \Delta x_3 A_1 + \Delta x_4 A_2\right) A_1 + 2 r_{11} \Delta x_3 \\ 2\left(F_{a,des} - F_a - \Delta x_3 A_1 + \Delta x_4 A_2\right) A_2 + 2 r_{22} \Delta x_4 \end{bmatrix} = (0,0)^{\mathrm{T}}$$

yielding following expressions for $\Delta x_{3,4}$

$$\Delta x_4 = \frac{-2\left(F_{a,des} - F_a - \frac{2\left(F_{a,des} - F_a\right)A_1^2}{2r_{11} + 2A_1^2}\right) A_2}{\left(2r_{22} + 2A_2^2 - \frac{4A_1^2 A_2^2}{2r_{11} + 2A_1^2}\right)}$$

$$\Delta x_3 = \frac{2\left(F_{a,des} - F_a + \Delta x_4 A_2\right) A_1}{\left(2r_{11} + 2A_1^2\right)}$$

by which the optimal split up onto the desired pressure values is achieved, when substituting into equations (6) and (7).

While the optimization problem we set up shows a way how to split up the desired force, the choice of the weighting gains is strongly influencing the performance. Each weighting gain r_{11} and r_{22} should therefore reflect the system dynamics of the associated pressure state (in an indirect proportional way) very well, to assure a suitable split up of the desired force. By fixing the weighting gains to constant values, changes in the dynamics of the system are not reflected in the optimal split up, which may lead to deterioration of the performance. A way to circumvent this problem would be to introduce state dependent weighting gains, and while this is feasible from a mathematical point of view, this can lead to a demanding task when these parameters have to be tuned for the practical application, and makes it a less favorable solution.

Another approach can be taken by assuming that by the backstepping procedure (or any other control method) it can be assured for each subsystem $\dot{x}_3, \dot{x}_4$ that the step response performance is of first order, more specifically that

$$\dot{x}_3 = k_1\left(x_{3,des} - x_3\right) \tag{8}$$

$$\dot{x}_4 = k_2\left(x_{4,des} - x_4\right) \tag{9}$$

for any $x_{3,des} \in \left[-\overline{x_{3,des}}, \overline{x_{3,des}}\right], x_{4,des} \in \left[-\overline{x_{4,des}}, \overline{x_{4,des}}\right], \overline{x_{3,des}} > 0, \overline{x_{4,des}} > 0$ with positive constants k_1, k_2. The split up problem may then be viewed as a minimum time problem. By defining the desired values $x_{3,des}, x_{4,des}$ as a degree of freedom, we search for the optimal trajectories, which allows to follow a step in the desired force (stepping to a constant value $F_{a,des} > 0$) in minimum time:

$$\min_{x_{3,des}, x_{4,des}} \int_{t_0}^{t_f} 1 \, \mathrm{d}t$$

$$\begin{aligned} s.t. \quad & (8), (9) \\ & x_3\left(t_f\right) A_1 - x_4\left(t_f\right) A_2 = F_{a,des} \\ & x_3\left(t_0\right) A_1 - x_4\left(t_0\right) A_2 < F_{a,des} \leq \overline{x_{3,des}} A_1 + \overline{x_{4,des}} A_2 \end{aligned}$$

Setting up the Hamiltonian $\mathscr{H} = 1 + \lambda_1 k_1\left(x_{3,des} - x_3\right) A_1 - \lambda_2 k_2\left(x_{4,des} - x_4\right) A_2$, the stationary condition $\frac{\partial \mathscr{H}}{\partial u} = 0$ does not reveal any information about the optimal desired values $u^* = \left(x_{3,des}^*, x_{4,des}^*\right)^{\mathrm{T}}$. With Pontryagin's minimum principle (Pontryagin et al. [1962]) it can be shown that the time optimal desired values are bang-bang signals, with the switching time depending on the adjoint states λ_1, λ_2

$$x_{3,des}^* = \begin{cases} \overline{x_{3,des}} & \lambda_1 < 0 \\ -\overline{x_{3,des}} & \lambda_1 > 0 \end{cases}, \qquad x_{4,des}^* = \begin{cases} \overline{x_{4,des}} & \lambda_2 > 0 \\ -\overline{x_{4,des}} & \lambda_2 < 0 \end{cases}$$

with

$$\dot{\lambda}_1 = -\frac{\partial \mathscr{H}}{\partial x_3} = \lambda_1 k_1 A_1 \tag{10}$$

$$\dot{\lambda}_2 = -\frac{\partial \mathscr{H}}{\partial x_4} = -\lambda_2 k_2 A_2 \tag{11}$$

With the terminal cost function being $\Phi(x_3, x_4) = 0$ it follows that $\lambda\left(t_f\right) = (0,0)^{\mathrm{T}}$, and that the first change of sign of λ_1, λ_2 appears at $t = t_f$. The adjoint equations (10), (11) reveal then that $\lambda_1(t_0) < 0$ and $\lambda_2(t_0) > 0$, by which the optimal desired values are defined for the interval $\left[t_0, t_f\right[$. Note that the optimization problem does not give any information about the optimal input for $t > t_f$. This can be achieved by introducing the additional constraint $F_a(t) = F_{a,des}, \forall t > t_f$. This constraint is fulfilled by $x_3(t) = x_3\left(t_f\right), x_4(t) = x_4\left(t_f\right), \forall t > t_f$, from which the following optimal desired values sequence can be determined:

$$x_{3,des}^*(t) = \begin{cases} x_3\left(t_f\right) & t > t_f \\ \overline{x_{3,des}} & t < t_f \end{cases}, \; x_{4,des}^*(t) = \begin{cases} x_4\left(t_f\right) & t > t_f \\ \overline{x_{4,des}} & t < t_f \end{cases}.$$

The time optimal solution gives the best performance in tracking a step in the desired force that is achieveable under the given assumptions. The practical implementation of this solution is difficult though, by two reasons. First, the demanded force will not be a step signal, but a continously changing signal, and second even if the required force signal is constant, determining the *exact* switching time t_f online (by monitoring the actual force) and determining the correct desired pressure values, to hold the current force, is difficult. Both – the continuously changing demanded force and deviations in the determined switching time and in pressure end values – will lead to a high frequency switching signal in the desired value, switching from the maximal to the minimal values and vice versa.

Although both optimization approaches have their drawbacks from an implementation/application point of view, they show some insights into the problem. In both approaches the importance of the subsystem dynamics to the optimal split up ratio becomes obvious, and from the time optimal formulation it can be concluded, that a higher than necessary demand on the individual states may be necessary for some time to improve the rise time in the step response. This motivates a more practical approach to the split up problem: As the main objective is to make the faster subsystem providing the bigger part of the desired force, the desired value of each of the pressure states is

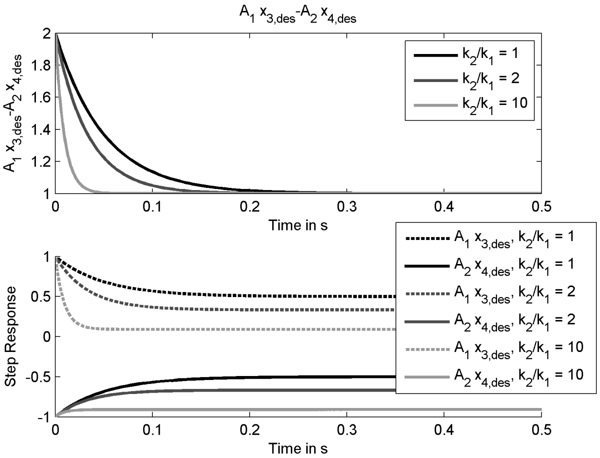

Fig. 2. Step responses of the transfer functions, for different ratios $k_2/k_1 = 1$ (black), 2 (grey), 10 (light gray).

set up, as if the full desired force would needed to be provided by the respective pressure state, while the other would be kept constant at the current value. This yields for the desired pressure values

$$x_{3,des} = \frac{F_{a,des} + x_4 A_2}{A_1} \tag{12}$$

$$x_{4,des} = \frac{x_3 A_1 - F_{a,des}}{A_2} \tag{13}$$

and by that, the demand on x_3 depends on the dynamics of x_4 (if x_4 is closer to $x_{4,des}$ then the demand on x_3 is reduced) and vice versa.

Recalling the assumption that a control exists that provides a closed loop behaviour of each subsystem of the form

$$\frac{x_3(s)}{x_{3,des}(s)} = \frac{1}{\frac{s}{k_1}+1} \tag{14}$$

$$\frac{x_4(s)}{x_{4,des}(s)} = \frac{1}{\frac{s}{k_2}+1} \tag{15}$$

where $\cdot(s)$ denotes a laplace transformed quantity $\mathscr{L}\{\cdot(t)\}$, we can analyze the behaviour of the desired quantities in dependency of the (closed loop) dynamics of the subsystems. Substituting (14) and (15) into (12) respectively (13) and rearranging yields following transfer function, from the input $F_{a,des}$ to the output $\overline{F}_{a,des} = A_1 x_{3,des}(s) - A_2 x_{4,des}(s)$:

$$\frac{\overline{F}_{a,des}(s)}{F_{a,des}(s)} = \frac{A_1 x_{3,des}(s) - A_2 x_{4,des}(s)}{F_{a,des}(s)}$$
$$= \frac{1 - \frac{1}{s/k_2+1}}{1 - \frac{1}{s/k_1+1}\frac{1}{s/k_2+1}} - \frac{\frac{1}{s/k_1+1} - 1}{1 - \frac{1}{s/k_1+1}\frac{1}{s/k_2+1}}$$

This transfer function represents the response of the combination of the desired pressure values to a change in the desired force value. The step response of this transfer function is shown in Figure 2, where also the (scaled) step responses of the individual desired values ($A_1 x_{3,des}$, $A_2 x_{4,des}$) are shown, for different ratios of $k_2/k_1 \in \{1,2,10\}$. This figure shows that in the transient phase the desired pressure values represent an equivalent desired force value that is up to two times the desired force, and it also shows that when the real pressure states of the system approaches the desired value (by the first order dynamics in (14),(15)) the desired pressure values approaches in the same dynamics the desired force value. Additionally this figure shows that the split-up ratio between the desired pressures relates to the ratio between the pressure subsystem dynamics – for the ratio of $k_2/k_1 = 10$ the desired pressure are split-up according to $x_{3,des}/-x_{4,des} = 1/10$.

4. BACKSTEPPING CONTROL

Having defined the desired value for the states x_3 and x_4, we proceed with the backstepping design method. Looking at the system structure we can identify two different subsystems: the subsystem associated with the state x_3 is the first branch of the model, while the subsystem associated with x_4 can be seen as a second branch. The branches show either the strict (x_3) or the pure (x_4) feedback structure, when considered solitary, and allow the backstepping method to be applied to each of them. The interconnections between the branches can be considered in the design as measured disturbances.

Starting with the first branch we define the deviation

$$z_{31} = x_{3,des} - x_3 \tag{16}$$

as subsequent error state, with the augmented lyapunov function $V_{31}(z_1, z_2, z_{31}) = \frac{1}{2}\left(z_1^2 + z_2^2 + z_{31}^2\right)$. The state x_3 can be expressed as

$$x_3 = x_{3,des} - z_3$$

which we can substitute in $\dot{z}_2$ resulting in the derivative of the lyapunov function

$$\dot{V}_{31} = z_1\dot{z}_1 + z_2\dot{z}_2 + z_{31}\dot{z}_{31}$$
$$= -c_1 z_1^2 - c_2 z_2^2 + z_{31}\left(\frac{1}{m}A_1 z_2 + \dot{z}_{31}\right)$$

which needs to be rendered negative definite by a proper design of $\dot{z}_{31}$. E.g. rendering $\dot{z}_{31} = -c_{31} z_{31}^2 - \frac{1}{m}A_1 z_2$, with a positive design constant c_{31}, would fulfill this requirement. The input to this first branch appears in

$$\dot{z}_{31} = \dot{x}_{3,des} + \dot{x}_3$$

and depending on the actual value of u_2 the main input to drive $\dot{z}_{31}$ will be either u_2 itself for any $u_2 > s_{AP}$ or the virtual input x_5 for any $u_2 < -s_{AT}$. In the following we will consider the case $u_2 > s_{AP}$, the other case can then be treated analogously. Due to the interconnection to the second branch via the desired value $x_{3,des} = x_{3,des}(x_4)$ the input u_2 also enters via $\dot{x}_{3,des}$ which has to be considered. It is then possible to find an expression for u_2 such that $\dot{z}_{31} = -c_{31} z_{31}^2 - \frac{1}{m}A_1 z_2$ and the derivative of the lyapunov function is

$$\dot{V}_{31} = -c_1 z_1^2 - c_2 z_2^2 - c_{31} z_{31}^2$$

by which the system is asymptotically stable.

The second branch introduces a second system, with the same primary error states z_1, z_2 augmented by

$$z_{32} = x_{4,des} - x_4$$

which leads by similar steps as above to

$$\dot{V}_{32} = z_1\dot{z}_1 + z_2\dot{z}_2 + z_{32}\dot{z}_{32}$$
$$= -c_1 z_1^2 - c_2 z_2^2 + z_{32}\left(-\frac{1}{m}A_2 z_2 + \dot{z}_{32}\right)$$

which can be rendered negative definite by rendering $\dot{z}_{32}$ to $-c_{32} z_{32} + \frac{1}{m}A_2 z_2$ with $c_{32} > 0$. In this case (and for $u_2 > s_{AP}$) the input u_1 does not appear in $\dot{z}_{32}$, but we set x_5 as our virtual input. Through the interconnection to the first branch via $\dot{x}_{4,des}$ u_2 appears again in the expression $\dot{z}_{32}$, but as it is already set to fulfill the requirements of the first branch, we treat it here as a measured disturbance. As the virtual input x_5 is an argument of the signed square root, we need to make a distinction of cases to get the sign of $x_{5,des}$ right. To do this we first determine the value the signed square root needs to have to assure that $\dot{z}_{32} = -c_{32}z_{32} + \frac{1}{m}A_2z_2$. We denote this desired value for the signed square expression $\hat{\sqrt{x_5 - x_4}}_{des}$. Depending on the sign of this value we need to set $x_{5,des}$ to

$$x_{5,des} = \begin{cases} \left(\hat{\sqrt{x_5 - x_4}}_{des}\right)^2 + x_4 & \hat{\sqrt{x_5 - x_4}}_{des} \geq 0 \\ -\left(\hat{\sqrt{x_5 - x_4}}_{des}\right)^2 + x_4 & \text{else} \end{cases}$$

The last error state that is introduced is the deviation from the desired value of x_5

$$z_4 = x_{5,des} - x_5 \tag{17}$$

with the augmented lyapunov function $V_4 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_{32}^2 + z_4^2\right)$ for the complete error system. Substituting $x_5 = x_{5,des} - z_4$ in $\dot{z}_{32}$ yields:

$$\dot{z}_{32} = -\dot{x}_{4,des} - \frac{E}{V_{02} - A_2x_1}\left(+A_2x_2 + \sigma\left(u_2 - s_{AP}\right)Q_N \hat{\sqrt{\frac{x_{5,des} - z_4 - x_4}{p_N}}}\right)$$

We can then upper bound the signed square root term by

$$\operatorname{sgn}\gamma \hat{\sqrt{\frac{x_{5,des} - z_4 - x_4}{p_N}}} \leq$$

$$\operatorname{sgn}\gamma\left(\hat{\sqrt{\frac{x_{5,des} - x_4}{p_N}}} + c_5\frac{\operatorname{sgn}z_4 + \operatorname{sgn}\gamma}{2} \cdot \sqrt{\frac{|z_4|}{p_N}}\right)$$

for any $\gamma \in \mathbb{R}$ and $c_5 \geq 1.5$. Setting $\gamma = z_{32}$ this yields for the derivative of the lyapunov function

$$\begin{aligned} \dot{V}_4 &= z_1\dot{z}_1 + z_2\dot{z}_2 + z_{32}\dot{z}_{32} + z_4\dot{z}_4 \\ &\leq -c_1z_1^2 - c_2z_2^2 - c_{32}z_{32}^2 + z_4\dot{z}_4 - \Gamma(z_4)z_{32} \end{aligned}$$

where $\Gamma(z_4)$ is

$$\Gamma(z_4) = \frac{E}{V_{02} - A_2x_1}\sigma\left(u_2 - s_{AP}\right)Q_Nc_5\frac{\operatorname{sgn}z_4 + \operatorname{sgn}z_{32}}{2} \cdot \sqrt{\frac{|z_4|}{p_N}}.$$

In $\dot{z}_4$ the input u_1 appears and we can design therefore its value to the following expression

$$\dot{z}_4 = -c_4z_4 - \frac{\Gamma(z_4)^2}{4z_4} \tag{18}$$

where we use $|z_4| = z_4 \operatorname{sgn}z_4$ and get for the expression

$$\begin{aligned} &\frac{\Gamma(z_4)^2}{4z_4} = \\ &= \left(\frac{E}{V_{02} - A_2x_1}\sigma\left(u_2 - s_{AP}\right)Q_Nc_5\frac{\operatorname{sgn}z_4 + \operatorname{sgn}z_{32}}{2}\right)^2 \cdot \frac{\operatorname{sgn}z_4}{4p_N} \end{aligned}$$

The resulting derivative of the lyapunov function is then

$$\begin{aligned} \dot{V}_4 &\leq -c_1z_1^2 - c_2z_2^2 - c_{32}z_{32}^2 - \Gamma(z_4)z_{32} - \frac{\Gamma(z_4)^2}{4} - c_4z_4^2 \\ &= -c_1z_1^2 - c_2z_2^2 - \tilde{c}_{32}z_{32}^2 - \left(z_{32} + \frac{\Gamma(z_4)}{2}\right)^2 - c_4z_4^2 \end{aligned}$$

and negative definite with $c_{32} = \tilde{c}_{32} + 1$, $\tilde{c}_{32} > 0$, $c_4 > 0$.

5. SIMULATION RESULTS

The designed backstepping controller is implemented in simulation with the parameters of the hydraulic plant set to the values in table 1 and with the hydraulic cylinder acting against a spring with the unloaded position at $x_{s0} = 0.25\,m$ and a stiffness factor of $k_s = 10^5\,\mathrm{N/m}$. Additionally to the nominal plant the simulated plant models friction with $F_R = \operatorname{sgn} v\left(k_rv + F_{R0}\right)$, and the (first order) dynamics of the underlying closed loops for the pump rotational speed and the valve position. Because of that the determined control inputs u_1, u_2 will not become applied directly to the plant, but the values $\tilde{u}_1, \tilde{u}_2$ – which are underlying the following dynamics:

$$\dot{\tilde{u}}_1 = k_p\left(u_1 - \tilde{u}_1\right), \qquad \dot{\tilde{u}}_2 = k_v\left(u_2 - \tilde{u}_2\right)$$

The parameters k_p, k_v are as well available in table 1 as the friction parameters and the control parameters. Besides the dynamics of the servo-pump system the servo drive is limited in simulation to a max rev-speed of $u_{1,max} = 25\,\mathrm{Hz}$ and the power of the servo is limited to 500 W.

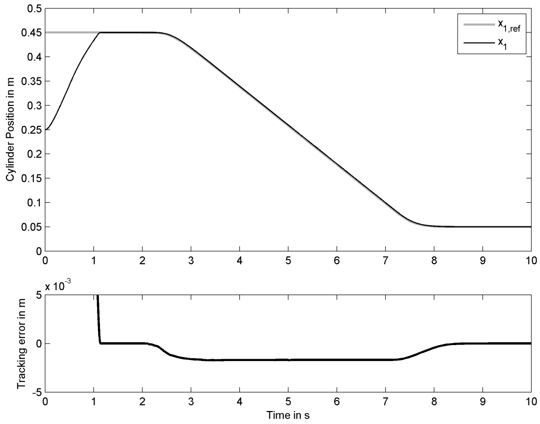

Fig. 3. The top figure shows the reference and the cylinder position and the bottom figure shows the tracking error.

The tracking performance of the system is shown in Figure 3: The initial position is set to the unloaded spring position, the pressure states of the hydraulic system are set to atmospheric pressure; the initial position of the reference starts at the fully moved out position of the cylinder, resulting in a large initial error that is rejected as fast as possible, by applying full pump speed, that is only limited by the maximum rotational speed and by the maximum power, and full valve opening as shown in Figure 4. Note that in this initial phase, we almost recover the minimum time solution proposed in section 3 – though we do not see the bang-bang behaviour in the desired quantities, the effect of the desired pressure values on the system inputs results in this bang-bang behaviour in the inputs for the initial step, only limited by the input dynamics and saturations.

The effect of the split-up strategy shows in the initial phase in Figure 5 too, where the pressure states and their desired values are shown (scaled by the according cylinder areas). The desired force cannot be achieved instantly because of the large initial error and the necessary pressure to achieve the force. Because of that the desired value of x_3 is initially large, as well as the desired value of x_4 which is far in the negative values. The negative values cannot be achieved by x_4 due to system limitations, but the pressure x_3 is approaching its desired value. Following our discussion from section 3, this can be interpreted as x_4 having very poor dynamics in following its reference, with the effect of the demand on x_3 being adjusted with the same dynamics (it is basically kept constant), while x_3 approaches its

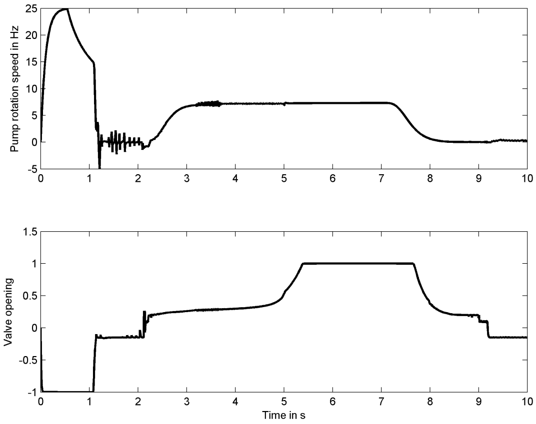

Fig. 4. Pump rotation speed (top) and valve opening (bottom).

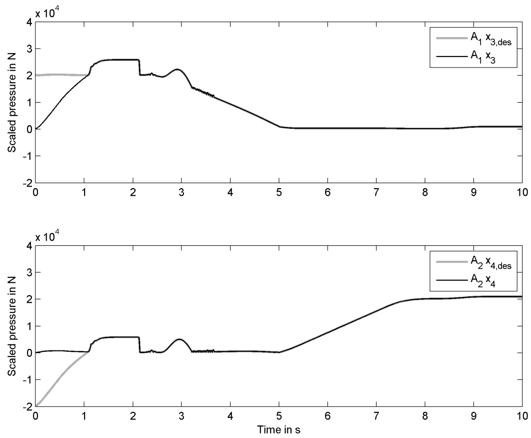

Fig. 5. Pressure values x_3 (top), x_4 (bottom) and their references, scaled by the according cylinder areas.

Table 1. Parameter values

Parameter	Value	Unit	Parameter	Value	Unit
A_1	19.64	cm^2	A_2	16.49	cm^2
V_{01}	0.02	dm^3	V_{02}	0.762	dm^3
m	10	kg	E	$1.4 \cdot 10^9$	Pa
p_T	10^5	Pa	p_N	$5 \cdot 10^5$	Pa
s_{AP}	0.1	–	s_{BP}	0.15	–
s_{AT}	0.19	–	s_{BT}	0.2	–
V_{0P}	0.075	dm^3	Q_N	0.53	dm^3/s
V_S	0.019	$\mathrm{dm}^3/\mathrm{rev.}$	η_v	0.95	–
k_r	10^3	$\mathrm{N/m/s}^2$	F_{R0}	10^2	N
k_v	100	–	k_p	10	–
c_1	10^2	–	c_2	10^2	–
c_{31}	$5 \cdot 10^4$	–	c_{32}	$5 \cdot 10^4$	–
c_4	$5 \cdot 10^4$	–	c_5	1.5	–

reference, by which the demand on x_4 gets relaxed, until the point where both x_3 and x_4 meet their respective requirements. Note that this shows, that the split-up strategy adapts the desired values to the dynamics of the subsystems, and is even able to cope with saturations in the pressure values, without the need of having any information about the subsystem dynamics and possible saturations in the first place.

6. CONCLUSION AND OUTLOOK

In this work a solution to the multi input problem is proposed, which is motivated by the results of casting the problem into an optimization problem. The proposed solution is used to develop a tracking control, applying the backstepping design method, and the suitability of the approach is shown in simulation, where the advantage of the method, the implicit consideration of the subsystem dynamics, shows. It could also be seen, that this also includes very poor dynamics, as they appear when one subsystem gets saturated. Future work can now be developed in two directions: First the designed control scheme can be expanded by using adaptive methods, by including friction estimators and/or load force estimators. The second direction may concentrate on the split-up strategy, which is per se not limited to the hydraulic control problem, and on the applicability of the strategy to different problem settings.

ACKNOWLEDGEMENT

This work has been partly carried out at LCM GmbH as part of a K2 project. K2 projects are financed using funding from the Austrian COMET-K2 programme. The COMET K2 projects at LCM are supported by the Austrian federal government, the federal state of Upper Austria, the Johannes Kepler University and all of the scientific partners which form part of the K2-COMET Consortium.

REFERENCES

K.K. Ahn, D.N.C. Nam, and M. Jin. Adaptive backstepping control of an electrohydraulic actuator. *Mechatronics, IEEE/ASME Transactions on*, PP(99):1–9, 2013. ISSN 1083-4435.

B. Ayalew and Bohdan T. Kulakowski. Cascade tuning for nonlinear position control of an electrohydraulic actuator. In *American Control Conference, 2006*, pages 6 pp.–, 2006.

A.E. Bryson and Yu-Chi Ho. *Applied optimal control: optimization, estimation, and control; Revised Printing*. Taylor & Francis, 1975.

M. Choux, H.R. Karimi, G. Hovland, M.R. Hansen, M. Ottestad, and M. Blanke. Robust adaptive backstepping control design for a nonlinear hydraulic-mechanical system. In *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pages 2460–2467, 2009.

S. Habibi and A. Goldenberg. Design of a new high-performance electrohydraulic actuator. *Mechatronics, IEEE/ASME Transactions on*, 5(2):158–164, 2000. ISSN 1083-4435.

C. Kaddissi, J.-P. Kenne, and M. Saad. Identification and real-time control of an electrohydraulic servo system based on nonlinear backstepping. *Mechatronics, IEEE/ASME Transactions on*, 12(1):12–22, 2007. ISSN 1083-4435.

Miroslav Krstić, Ioannis Kanellakopoulos, and Petar Kokotović. *Nonlinear and Adaptive Control Design*. John Wiley & Sons, Inc., New York, 1995.

Alexander G. Loukianov, Jorge Rivera, Y.V. Orlov, and E.Y. Morales Teraoka. Robust trajectory tracking for an electrohydraulic actuator. *Industrial Electronics, IEEE Transactions on*, 56(9):3523–3531, 2009. ISSN 0278-0046.

L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mischenko. *The Mathematical Theory of Optimal Processes*. Wiley Interscience, 1962.