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On the design of a fault compensation algorithm for consensus networks

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Abstract:
This paper is concerned with the consensus problem of a multi-agent system where some nodes inject unexpected values as a consequence of a fault. A fault compensation algorithm is proposed and its properties are thoroughly analyzed. Afterwards, closeness of system trajectories to the consensus value in the presence of compensated persistent faults is studied. Finally, an example on two different applications of the proposed strategy is shown and simulation results are reported to validate theoretical results developed in the paper.

Keywords: Consensus Networks, fault compensation, Multi-agent systems.

1. INTRODUCTION

The analysis and design of the collective behavior of a number of agents locally interacting between themselves is receiving a great research effort in the last years (see, e.g., Bullo et al. (2009), Olfati-Saber et al. (2007), Chen et al. (2013)). Recently, this challenging framework has been considered for platoons of vehicles (see, e.g., Pan (2009), Ghasemi and Azadi (2013)).

A key tool for the achievement of a variety of global properties is the ability of the group to reach a *consensus* on a given quantity. Consensus is reached by iteratively updating a value using the information shared with neighboring agents. A large number of practical applications have been developed based on this simple concept of consensus between agents (Olfati-Saber et al. (2007), Chen et al. (2013), Ren et al. (2006)). In the last years, consensus has been adopted as a basic tool for the self organization of mission objectives or formation control of platoons of vehicles (see, e.g., Porfiri et al. (2006), Wang et al. (2012)). The problem of the presence of a set of misbehaving nodes that perturb the network evolution has been widely studied in the literature and it has been considered from different perspectives. One significant branch of research on this topic has been developed since the eighties by computer scientists (see e.g. Lamport et al. (1982), Pease et al. (1980)) where the problem was set within the area of distributed computing and computer networks. Recently, the same problem arose in the framework of robotic networks, power systems and sensor networks (see e.g. Sundaram and Hadjicostis (2011), Pasqualetti et al. (2012), Pasqualetti et al. (2011)).

In the paper of Bauso et al. (2009), the authors propose a modified consensus protocol to face the problem of reaching a consensus under unknown but bounded noise. Another interesting paper on this topic is Chen et al. (2010) where the authors propose a protocol to make nodes react against packet losses. In the recent papers (Yucelen and Egerstedt (2012), Yang et al. (2011)) the consensus problem in the presence of exogenous disturbances is studied. Finally, in the recent paper (LeBlanc et al. (2013)) a promising concept of network robustness is studied as an important property for analyzing the behavior of resilient distributed algorithms using local information only.

The paper is organized as follows. In Section II the notation and some preliminaries about system model are given in order to state the problem clearly. In Section III the proposed control law is introduced. First the controller structure is given, then a proposition on the choice of the controller's gains to restore the initial consensus value in the presence of misbehaving inputs follows. Afterwards, the existence of time-invariant gains is discussed. Section IV is reserved to the convergence properties of the controlled system. In Section V a set of simulation results are given and discussed.

Notation: The notation adopted in this paper is fairly standard. Vectors are denoted with bold letters. Given a vector $\mathbf{v} \in \mathbb{R}^d$, we denote v_i the i-th component of $\mathbf{v}$ so that $\mathbf{v} = [v_1 \dots v_d]^T$. We denote with I_n the $n \times n$ identity matrix, and $\mathbf{e}_i$, $i \in \mathbb{N}$, denotes the i-th vector of the standard basis, e.g. $\mathbf{e}_1 = [1\ 0\ \dots\ 0]^T$. Vector $\mathbf{1}$ is the vector with all components equal to 1.

A graph $G = (V, E)$ is a pair where $V = \{1, 2, \dots n\}$ is the set of vertices and $E \subset V \times V$ is the edge set. A *path* between two nodes u and v is a collection of edges $\{(v_{i-1}, v_i) \in \mathcal{E}, i = 1.., n\}$ such that $v_0 = u$, $v_n = v$. We call *distance* $d(i, j)$ from i to j the minimum number of edges of a path going from i to j. Given a node i, we call a *neighbor* of i any node distant 1 from i and we denote with $\mathcal{N}_i$ the set of neighbors of node i defined as $\mathcal{N}_i = \{j \in V : (i, j) \in E\}$. A graph G is connected if any node of G can be connected to another node of G through a path.

[1] The research leading to these results has been funded by the italian PRIN 2010-2011 project MARIS: Marine Autonomous Robotics for InterventionS.

Some features of the graph are described by suitable matrices. In this paper we use the *weighted Laplacian* of the graph G, that is a matrix $\tilde{L} \in \mathbb{R}^{n\times n}$ defined $(\tilde{L})_{ij} = -l_{ij}$ if $i \neq j$ and $(\tilde{L})_{ii} = \sum_{j=1,j\neq i}^{n} l_{ij}$ where l_{ij} is a positive weight if $(i,j) \in E$ and $l_{ij} = 0$ if $(i,j) \notin E$.

2. PROBLEM STATEMENT

In this paper we consider a group of n agents each one described by a first order discrete-time integrator dynamics $x_i(t+1) = x_i(t) + u_i(t)$, $i = 1,..,n$, and we assume that each agent follows an agreed common protocol $u_i(t) = \sum_{j\in\mathcal{N}_i} l_{ij}(x_j(t) - x_i(t))$ but some nodes may *fail* during their activity and inject a different value with respect to the agreed protocol, namely $u_i(t) = \sum_{j\in\mathcal{N}_i} l_{ij}(x_j(t) - x_i(t)) + \bar{u}_i(t)$, $\bar{u}_i(t) \neq 0$.

If all the nodes assign correct input values, then each node asymptotically reaches the *consensus value* $f(\mathbf{x}(0)) = \frac{\sum_{i=1}^{n} \gamma_i x_i(0)}{\sum_{i=1}^{n} \gamma_i} = \frac{\boldsymbol{\gamma}^T \mathbf{x}(0)}{\boldsymbol{\gamma}^T \mathbf{1}}$, where $\boldsymbol{\gamma} = [\gamma_1 \ \gamma_2 \ \ldots \ \gamma_n]^T$ is the eigenvector of $(I - \tilde{L})^T$ associated to the eigenvalue 1.

Consider now the presence of some m faulty agents with labels $i_1, i_2, \ldots, i_m$. The evolution of the group can be compactly written

$$\mathbf{x}(t+1) = A\mathbf{x}(t) + \sum_{\ell=1}^{m} \mathbf{e}_{i_\ell} \bar{u}_{i_\ell}(t), \text{ where } A = (I - \tilde{L}). \quad (1)$$

The presence of misbehaving nodes in a consensus network even for few instants can have a wasteful impact on the control objectives; the consensus value deviates from $f(\mathbf{x}(0)) = \dfrac{\gamma^T \mathbf{x}(0)}{\gamma^T \mathbf{1}}$ to

$$f(\mathbf{x}(0), \bar{u}(\cdot)) = \frac{\gamma^T \mathbf{x}(0) + \sum_{\ell=1}^{m} \gamma_{i_\ell} \sum_{j=0}^{t} \bar{u}_{i_\ell}(j)}{\gamma^T \mathbf{1}} \quad (2)$$

so, even if a misbehaving agent is located and excluded, the consensus value is corrupted by the past values of the faulty nodes.

The main goal of this paper is to design self counteractions of the network against this phenomenon. To reach this goal, we assume that some compensation nodes are allowed to inject corrective signals into the network. We denote the set of *compensation nodes* with $\mathcal{V}_o = \{i_{o_1}, i_{o_2}, \ldots, i_{o_q}\}$, and we assume that each node of $\mathcal{V}_o$ can communicate with any other node of $\mathcal{V}_o$ and is able to perform elaborations based on its own state and the received signals. More precisely, we assume that any node $j \in \mathcal{V}_o$ can send a message ν to any node $l \in \mathcal{V}_o$ and the message ν is received after a temporal delay τ_{lj}. We denote with $\nu_{lj}(t)$ a message sent from node j to node l at time t and $\hat{\nu}_{lj}(t)$ a message received from node l at time t, so $\hat{\nu}_{lj}(t) = \nu_{lj}(t - \tau_{lj})$. We assume that each compensation node ℓ can elaborate its own value $x_\ell(t)$ and all messages received from any other compensation node $\hat{\nu}_{\ell j}(t) = \nu_{\ell j}(t - \tau_{\ell j}), j \in \mathcal{V}_o$. Based on such elaboration, any compensation node of $\mathcal{V}_o$ has the capability of driving the time evolution of the group with an additional input $\psi(\hat{\nu}_{ij}(t))$ so that the evolution of the network takes the form

$$\mathbf{x}(t+1) = A\mathbf{x}(t) + \sum_{\ell=1}^{m} \mathbf{e}_{i_\ell} \bar{u}_{i_\ell}(t) + \sum_{j\in\mathcal{V}_o} \mathbf{e}_j \psi(\hat{\nu}_{ij}(t)) \quad (3)$$

with $A = (I - \tilde{L})$. The main goal of this paper is a suitable design of such functions $\psi(\hat{\nu}_{ij}(t))$ to overcome or mitigate the effect of faulty nodes on network evolution.

A dynamic relation between a set of faulty nodes and the monitoring nodes is the state space description (1) together with $y_\ell(t) = x_\ell(t) = \mathbf{e}_\ell^T \mathbf{x}(t)$, $\ell = i_{o_1}, i_{o_2}, \ldots, i_{o_q}$. An equivalent description of the connection between a faulty node j and a compensation node ℓ is the *Auto Regressive Moving Average* (ARMA) model:

$$\begin{aligned} AR(y_\ell(t)) &= MA_{\ell,j}(u_j(t)) \qquad \text{where} \quad (4)\\ AR(y_\ell(t)) &= y_\ell(t) + a_1 y_\ell(t-1) + \cdots + a_n y_\ell(t-n)\\ MA_{\ell,j}(u_j(t)) &= b_1 u_j(t - \delta_{\ell j}) + .. + b_m u_\ell(t-n) \end{aligned}$$

where the temporal delay $\delta_{\ell j}$ depends only on the distance between the two nodes $\delta_{\ell j} = d(\ell, j) + 1$ while weights $l_{i,j}$ in (1) are related with coefficients a_i and b_i.

In the following, we denote with $\mathbf{b}_{\ell,j} \in \mathbb{R}^n$ the vector of the coefficients of the $MA_{\ell,j}$ elaboration with increasing order and zero entries in the first $(\delta_{\ell j} - 1)$ positions, i.e.

$$\mathbf{b}_{\ell,j} = [\underbrace{0 \ldots 0}_{\delta_{\ell j}-1} \ b_1 \, b_2 \ldots b_m \,].$$

There is a strict connection between the ARMA model (4) and the state space model (1); coefficients a_i, b_i in (4) are those of the polynomials of the transfer function between ℓ and j:

$$\begin{aligned} W_{\ell j}(z) &= C_\ell (zI - A)^{-1} e_j = \\ &= \frac{b_1 z^{n-\delta_{\ell j}} + b_2 z^{n-\delta_{\ell j}-1} + ..}{z^n + a_1 z^{n-1} + a_2 z^{n-2} + .. + a_n}. \quad (5) \end{aligned}$$

Notice that the structure of the $AR(\cdot)$ elaboration (i.e., its coefficients a_i) does not depend on the specific input or output node. Consider also that, even if the models (1) and (4) are equivalent mathematical descriptions of the evolution of the system, they give rise to different data elaboration, so one can be more suited than the other in some practical applications.

3. COMPENSATION OF THE FAULT EFFECTS ON THE CONSENSUS VALUE

3.1 Structure of the compensation algorithm

One key problem in the Intrusion Detection and Isolation is to separate the free response from the forced response caused by a misbehaving agent (see e.g.Sundaram and Hadjicostis (2011), Parlangeli (2010)). A first useful property of model (4) is that the $AR(\cdot)$ elaboration filters the free evolution out, so it is directly connected to the exogenous values of a faulty node.

Proposition 3.1. *Under the assumption that no faulty nodes are present in the network until time $t = n$, then the autoregressive $AR(\cdot)$ elaboration of the data collected from any output node is zero if data are coherent with an evolution without intruders.*

Remark As opposed to the classical problem of Intrusion Detection, notice that the present approach does not require the distinguishability of the free response from the forced one. Indeed, the main question we want to answer to is not to select the node that injected faulty values but to compensate for faulty inputs wherever the faulty node is. An interesting connection with the Intrusion Detection problem is that, if the set of monitoring nodes verify the necessary and sufficient structural conditions with respect to a given set of misbehaving nodes, then the proposed algorithm naturally performs an on-line recontruction of each faulty signal and injects a suitable compensation input. Anyway, the proposed algorithm can be applied also when these assumptions do not hold.

First, we introduce the structure of the proposed algorithm executed by a compensation node $\imath \in \mathcal{V}_o$, that is the mathematical structure of functions $\psi(\hat{\nu}_{ij})$ in (3). The algorithm is based on the $AR(\cdot)$ elaboration of local data $y_\imath(t)$ and the messages exchanged with the other compensation nodes $\hat{\nu}_{ji}$, $j \in \mathcal{V}_o$. The reasons of this choice are related to all the peculiarities reported in the end of the previous section, mainly because it cancels out the free evolution of the system without a large amount of computation.

We assume that each compensation node i chooses a corrective signal $\nu_i(t)$ according to the rule:

$$\nu_i(t) = r_i(t)\left[\varepsilon_\imath(t-\bar{\tau}_i) - \sum_{j\in\mathcal{V}_o} MA_{ij}(\hat{\nu}_{ij}(t-\bar{\tau}_i))\right] \qquad (6)$$

$$\varepsilon_i(\xi) := AR(y_i(\xi)) = y_i(\xi) + a_1 y_i(\xi-1) + \cdots + a_n y_i(\xi-n)$$

$$\bar{\tau}_i = \max\left\{0, \max_{j\in\mathcal{V}_o}[\tau_{ij} - \delta_{ij}]\right\}.$$

where $r_i(t)$ are some suitable time-varying gains, $\varepsilon_\imath(\xi)$ is the AR elaboration of local data available to each elaboration node, while the exchanged signals are processed with the $MA_{ij}(\cdot)$ elaboration. All the involved signals are delayed of $\bar{\tau}_i$ in order to build a causal elaboration (6).

A preliminary and simplified version of the above algorithm has been studied in (Parlangeli (2013b)) for the compensation of a single faulty node under the hypothesis that an underlying IDS sends information about the faulty node identity to the compensation node. In the previous paper Parlangeli (2013a) the fault compensation is based on an explicit monitoring by a suitable Intrusion Detection System.

A remark on the set of equations (6) is useful for the forthcoming analysis. A fault on a node at a single time instant, say $\hat{t}$, induces a finite-time reaction of the compensation nodes of span equal to $n-\delta_{ij}$. Indeed, by (4) it is straight to see that the $AR(y_\ell(t))$ is nonzero only in the interval $[\hat{t}+\delta_{\ell j}, \hat{t}+n]$, and so does $\varepsilon_\ell(t)$ for any $\ell \in \mathcal{V}_0$, so the action of a correction for a single faulty injection lasts n time steps.

3.2 Compensation of the consensus value: assignment of the compensation gains

The next result is a step toward the design of the time-varying gains in (6), namely the way to choose them in order to recover the effects of the faulty inputs on the objective function.

Theorem 3.2 (p faulty nodes)**.** *Let $p, q \in \{1, \ldots, n\}$. A set of p faulty nodes acting at the same time on system* (1) *can be effectively compensated by a set of q monitoring nodes running algorithm* (6)*. Each compensation node must inject a compensation signal of the form* (6) *where $r_\kappa(t)$ are time-varying gains (that can be computed off-line) satisfying the relations*

$$\begin{aligned}
&\gamma_{j_1} + \gamma_{\imath_{o_1}}\mathbf{b}_{11}\boldsymbol{\rho}_1(\ell) + \gamma_{\imath_{o_2}}\mathbf{b}_{12}\boldsymbol{\rho}_2(\ell) + \cdots + \gamma_{\imath_{o_q}}\mathbf{b}_{1q}\boldsymbol{\rho}_q(\ell) = 0\\
&\gamma_{j_2} + \gamma_{\imath_{o_1}}\mathbf{b}_{21}\boldsymbol{\rho}_1(\ell) + \gamma_{\imath_{o_2}}\mathbf{b}_{22}\boldsymbol{\rho}_2(\ell) + \cdots + \gamma_{\imath_{o_q}}\mathbf{b}_{2q}\boldsymbol{\rho}_q(\ell) = 0\\
&\quad\vdots \\
&\gamma_{j_p} + \gamma_{\imath_{o_1}}\mathbf{b}_{p1}\boldsymbol{\rho}_1(\ell) + \gamma_{\imath_{o_2}}\mathbf{b}_{p2}\boldsymbol{\rho}_2(\ell) + \cdots + \gamma_{\imath_{o_q}}\mathbf{b}_{pq}\boldsymbol{\rho}_q(\ell) = 0
\end{aligned} \qquad (7)$$

where $\ell \in \mathbb{N}$, $\boldsymbol{\rho}_\kappa(\ell) = [r_\kappa(\ell)\ r_\kappa(\ell+1)\ \ldots\ r_\kappa(\ell+n)]^T$, $\kappa \in [1,..,p]$ and $\mathbf{b}_{\ell,j}$ as in (4).

Sketch of the proof: The detailed proof is omitted for the sake of space, a sketch of the proof is here reported.

For the ease of presentation, consider two faulty nodes and two monitoring nodes. Let j_1, j_2 be the two faulty nodes and i_{o_1}, i_{o_2} the two monitoring ones.

According to (3) and (4), it is possible to rearrange the time evolution of the objective function $\gamma^T\mathbf{x}(t)$ as a function of the faulty inputs. After some algebraic manipulations, it can be written:

$$\begin{aligned}
\gamma^T\mathbf{x}(t) = \gamma^T\mathbf{x}(0) + \sum_{m=1}^{2}\sum_{\ell=1}^{n+\bar{\tau}_m}\gamma_{j_m}\bar{u}_m(t-\ell) + \qquad (8)\\
+\sum_{m=1}^{2}\sum_{\ell=0}^{t-\bar{\tau}_m-n}\left[\gamma_{j_m}+\gamma_{\imath_{o_1}}\mathbf{b}_{m1}\boldsymbol{\rho}_1(\ell) + \gamma_{\imath_{o_2}}\mathbf{b}_{m2}\boldsymbol{\rho}_2(\ell)\right]\bar{u}_m(\ell) + \\
+\sum_{\kappa=1}^{2}\sum_{m=1}^{2}\varphi_\kappa(\bar{u}_m(t), \bar{u}_m(t-1), \ldots, \bar{u}_m(t-\delta_{\ell,m}))
\end{aligned}$$

where, for the sake of readability, vectors $\boldsymbol{\rho}_1(t) = [r_1(t+\bar{\tau}_1)\ r_1(t+\bar{\tau}+1)\ \ldots\ r_1(t+\bar{\tau}_1+n)]^T$, and $\boldsymbol{\rho}_2(t)$ accordingly, have been introduced and vectors $\mathbf{b}_{m\kappa}$, $m, \kappa = 1, 2$ contain the coefficients of $MA_{\imath_{o_m}, j_\kappa}(\cdot)$ according to the convention described in the comments after equation (4); functions $\varphi_\kappa(\cdot)$ are explained hereafter.

Equation (8) describes the time evolution of the consensus value and it deserves some comments. Three terms are associated to the faulty inputs: the first is the influence of faults before a faulty signal reaches the output of any compensation node and no compensation action has started still. The second term is the effect that each faulty input induces to the consensus value together with the reaction induced by each faulty input on the compensation nodes. It involves the compensation gains at different time steps because each faulty injection induces a reaction of $n-\delta_{j\ell}$ to each compensation node. The last term is related to those faulty injections that are being compensated at time t but the compensation action is not finished already and some other terms are expected in the very next steps. Functions $\varphi_\kappa(\cdot)$ encode this last transient signal; for the sake of completeness, considering a given compensation node κ and a faulty node j_m, each function $\varphi_\ell(\cdot)$ has the following expression

$$\varphi_\kappa(\bar{u}_m) = \sum_{\ell=t-n+1}^{t-\delta_{\kappa,j_m}} \left(\gamma_{j_m} + \sum_{\varsigma=\ell}^{t-\delta_{\kappa,j_m}} r(\varsigma + \delta_{\kappa,j_m})(\mathbf{b}_{\kappa m})_{\varsigma-\ell+1} \right) u_m(\ell). \tag{9}$$

In view of equation (8), it is clear that it is possible to choose the gains $r_1(t)$ and $r_2(t)$ to make ineffective the action of the misbehaving nodes on the consensus value. This is possible if they are chosen in order to zero all the coefficients of each faulty injection so that any faulty input cannot influence the consensus value:

$$\begin{cases} \gamma_{j_1} + \gamma_{\iota_{o_1}} \mathbf{b}_{11} \boldsymbol{\rho}_1(\ell) + \gamma_{\iota_{o_2}} \mathbf{b}_{12} \boldsymbol{\rho}_2(\ell) \ = \ 0 \\ \gamma_{j_2} + \gamma_{\iota_{o_1}} \mathbf{b}_{21} \boldsymbol{\rho}_1(m) + \gamma_{\iota_{o_2}} \mathbf{b}_{22} \boldsymbol{\rho}_2(m) = 0 \end{cases}$$

where $\ell, m \in \mathbb{N}$.

□

A key point for the design of the proposed control strategy is the existence of a solution for (7). We now consider a special case of control law (6), namely the time invariant one.

Time-invariant output injection gains. The following Lemma is about a technical result which is the building block for the existence of a solution for a set of time-invariant gains. The proof is omitted for the sake of space.

Lemma 3.3. *Consider system (3) and assume that the communication graph is connected. Let* $G_{ij}(z) = \dfrac{n_{ij}(z)}{(z-1)p(z)}$ *the tranfer function from any node j to any node i. The quantity $n_{ij}(1)$ does not depend on the output node i and it is equal to*

$$n_{ij}(1) = \frac{\gamma_j p(1)}{\sum_{j=1}^{n} \gamma_j}.$$

Remark : The above result has a strong impact on the existence of a solution for equation (7). We now prove that, based on the previous result, it is always possible to find constant solutions of (7) for any $p, q \in \mathbb{N}$.

Proposition 3.4. *[Existence condition for constant gains] For any $p, q \in \mathbb{N}$ it is always possible find a set of constant gains $r_i(t) = r_i, i \in \{1, \dots, q\}$ satisfying (7).*

4. CONVERGENCE ANALYSIS

The main goal of this section is to quantify the evolution of a consensus system (1) subject to faults and to find an estimation of the asymptotic limit set of system (1) under the action of a set of faulty nodes managed with the compensation strategy (6). Indeed, a persistent fault induces a persistent transient to system evolution and it is necessary to develop tools to understand how far the system trajectories are from the consensus value. In order to quantify this, in this section an estimation of the uncertainty region where system trajectories converge in the presence of unknown but bounded faults is sought. The results of this section can be the basic tool for the decision logic about an eventual reconfiguration of the network.

Here, in view of the previous results and for the sake of clarity, we make the analysis considering constant gains r_κ. It is matter of simple computation to extend the results in this section to the time-varying case.

To suitably perform this analysis, consider a decomposition of $\mathbf{x}$ along the $\mathbf{1}$ direction and the projection of $\mathbf{x}$ on the plane orthogonal to $\boldsymbol{\gamma}$, say $\pi_{\boldsymbol{\gamma}}$. According to Olfati-Saber et al. (2007), we call them respectively the *agreement component* and the *disagreement* one.

Now, consider such decomposition with respect to both the initial state vector $\mathbf{x}(0)$ and the current state vector $\mathbf{x}(t)$: $\mathbf{x}(0) = \alpha_c \mathbf{1} + \delta(0)$, $\mathbf{x}(t) = \alpha(t)\mathbf{1} + \delta(t)$, $\alpha(t) = \alpha_c + \alpha_f(t)$ where α_c is the component along $\mathbf{1}$ of $\mathbf{x}(0)$ and it is the unbiased consensus value in the absence of faults. When faulty nodes influence the evolution of the system, the value of α deviates from the correct value α_c by a quantity $\alpha_f(t)$ uniquely depending on the intensity of faults.

Based on these considerations, the next Proposition can be proven, which gives the upper bound of a convergence region of the state vector in the presence of compensated faults.

Proposition 4.1. *Consider system (1) subject to a set of bounded exogenous inputs $|\bar{u}_\kappa(t)| \le M_\kappa$, $\kappa = 1, \dots, p$ and the compensation injection signals (6), (7) for a given choice of coefficients r_j.*

State trajectories converge to a ball centered in $\dfrac{\boldsymbol{\gamma}^T \mathbf{x}(0)}{\boldsymbol{\gamma}^T \mathbf{1}} \cdot \mathbf{1}$ *and radius $R = [\Omega^2 + \Delta^2 + \Omega\Delta \sin(\theta)]^{\frac{1}{2}}$ where Δ and Ω are defined later and θ is the angle between γ and $\mathbf{1}$.*

The proof is constructive and here it is omitted for space limits. Here, we report the explicit expressions of the upper bounds

$$\Omega = \sum_{m=1}^{p} (n + \bar{\tau}_m) \cdot q \cdot \gamma_{j_m} M_m + \sum_{m=1}^{p} \sum_{\ell=1}^{q} \bar{\varphi}_\ell(\bar{u}_m),$$

$$\bar{\varphi}_\ell(\bar{u}_m) = \sum_{\zeta=1}^{n-\delta_{\ell m}} \gamma_\ell |(\mathbf{b}_{\ell m})_\zeta| (n - \delta_{\ell m} - \zeta) \bar{r}_\ell M_m$$

$$\Delta := \sum_{j=1}^{m} \frac{|\lambda_2|^{-n} M_j B_{fj}}{|1 - \lambda_2|} \qquad where$$

$$B_{fj} = |\lambda_2|^n \|\mathbf{g}_j\| + \sum_{i=0}^{n-\delta_{mj}-\bar{\tau}_j} |\lambda_2|^{i-\bar{\tau}_j} \|\mathbf{g}_{io_j}\| |r_j (\mathbf{b}_{\ell j})_i|$$

with $\|\mathbf{g}_\ell\| = n \dfrac{\gamma_\ell^2}{\left(\sum_j \gamma_j\right)^2} + 1 - \dfrac{2\gamma_\ell}{\left(\sum_j \gamma_j\right)}$ for any vector $\mathbf{g}_\ell$.

5. SIMULATION RESULTS

In this section we show some simulations that confirm the theoretical results developed in the paper and the effectiveness of the proposed control strategy. Consider a group of eight agents connected as in Fig. 1, and assume that all weights l_{ij} are chosen equal to one. The eigenvalue of the system matrix $(I - \tilde{L})$ (see eq. (1)) with second largest modulus is $\lambda_2 = 0.6548$, and the characteristic polynomial is $z^8 - 1.4z^7 - 0.2z^6 + 0.8z^5 - 0.1z^4 - 0.11z^3 +$

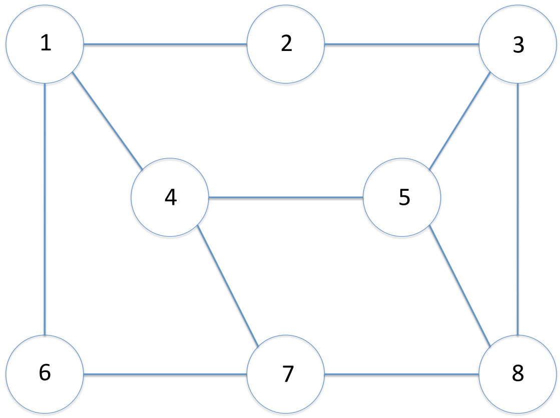

Fig. 1. Topology of the communication graph.

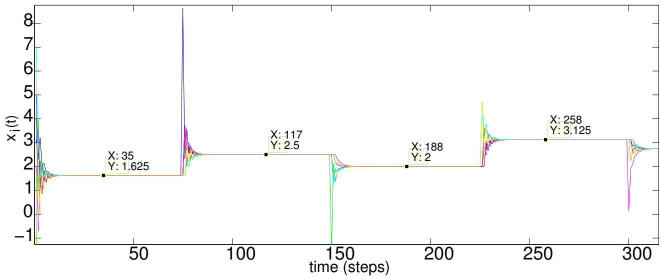

Fig. 2. Evolution of the consensus network in Fig. 1 with uncompensated faulty nodes.

$0.0125z^2 + 0.004z$ so the $AR(\cdot)$ elaboration performed by each compensation node is $AR(y_\ell(t)) = y_\ell(t) - 1.4y_\ell(t-1) - 0.2y_\ell(t-2) + 0.8y_\ell(t-3) - 0.1y_\ell(t-4) - 0.11y_\ell(t-5) + 0.0125y_\ell(t-6) + 0.004y_\ell(t-7)$.

In a first simulation, we consider a scenario where nodes $1, 3, 8$ and 6 inject each one a single faulty value of amplitude one at the time steps $t = 75, 150, 225, 300$.

In the first figure, Figure 2, the detrimental effect of faulty nodes in a consensus network is shown. The small windows inside the picture show the variations of the consensus value as a consequence of faults, and jumps at the consensus value as a consequence of a fault can be easily seen.

Figure 3 shows the evolution of the system with a single compensation node, namely node 1. Here, according to the theoretical results, the consensus value is promptly adjusted by the feedback of the compensation node.

In the next figure, Fig. 4, simulations with two compensation nodes are shown. The interesting point that can be appreciated in this figure is that the control effort of the compensation node is significantly lower than the single compensation node case.

A second simulation has been performed considering all nodes as faulty nodes. The faulty signal is an unknown but bounded signal, namely a random variable identically distributed in $[0; 0, 2]$.

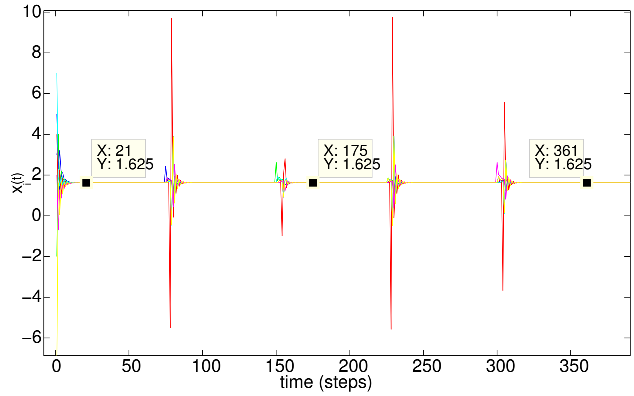

Fig. 3. Evolution of the consensus network with one compensation node.

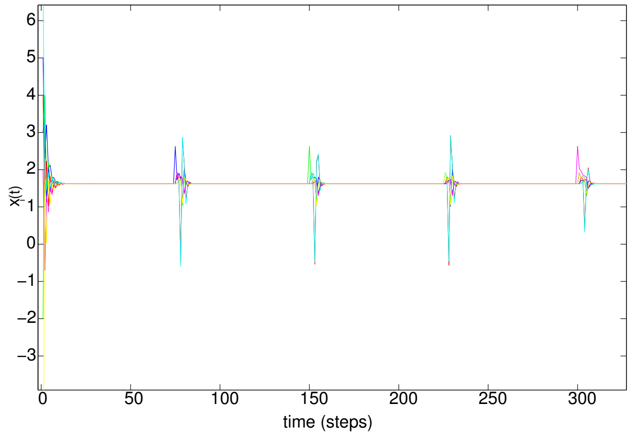

Fig. 4. Evolution of the consensus network with two compensation nodes.

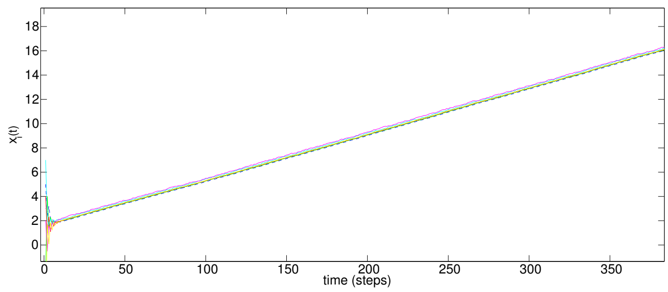

Fig. 5. Evolution of the consensus network with uncompensated unknown but bounded noise

In the first figure of this simulation, Fig. 5, an unstable behavior of the group is reported. This phenomenon is a straight consequence of an uncompensated disturbance action on a consensus network and it was well described in the paper Bauso et al. (2009).

In Fig. 6 the behavior of the system with one compensation node is reported. The figure shows that the system evolution is confined within a bounded strip of the consensus value, and this is in accordance with Proposition 4.2.

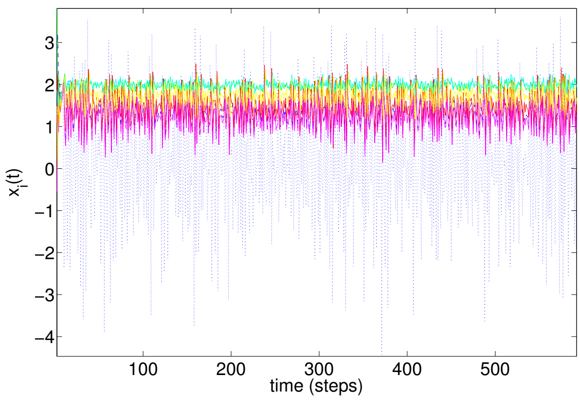

Fig. 6. Evolution of the consensus network in Fig. 1 subject to unknown but bounded noise and one compensation node

6. CONCLUSIONS

In this paper the consensus problem for a multi-agent system where some nodes inject unexpected values as a consequence of a fault is considered. A fault compensation algorithm is proposed and its properties are thoroughly analyzed. Unbiasness of the consensus value in the presence of a set of faults is studied and the existence of a corrective injection by a prescribed set of monitoring nodes is discussed. Afterwards, closeness of system trajectories to the consensus value in the presence of compensated persistent faults is studied. Finally, an example on two different applications of the proposed strategy is shown and simulation results are reported to validate theoretical results developed in the paper.

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