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High gain observer for embedded Acrobot ⋆

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Abstract: Underactuated mechanical systems are systems with less actuators than degrees of freedom. Therefore, it is complicated to measure all states, i.e. angular positions or angular velocities, of the mechanical system. Alternative solution consists in an observer design such that unmeasurable states are estimated. For this purpose, a high gain observer for an Acrobot was introduced. By virtue of an Acrobot embedding into a 4-link model, the high gain observer was simply extended and applied to the 4-link model. The main aim of this paper consists in a coupling of a method of the Acrobot embedding into the 4-link model and the high gain observer design for the Acrobot. The coupling results in an observer for the 4-link model with the same structure as the already developed high gain observer for the Acrobot model. The control of the 4-link model using the embedding method is based on a definition of constraining functions for knees control whereas the remaining angle in the hip is controlled in the same way as it would be an Acrobot's angle. The ability of feedback tracking of the walking-like trajectory of the 4-link with observed geometry during a swing phase of a single step is demonstrated in simulations.

Keywords: Underactuated mechanical systems, Embedding, High gain observer.

1. INTRODUCTION

Underactuated walking robots, i.e. robots with less degrees of freedom than number of actuators, form subclass of walking robots. The simplest underactuated mechanical systems are an Acrobot and a Pendubot. Both of them consist from two links and one actuator. In the case of the Acrobot, the actuator is placed between its links whereas in the case of the Pendubot, the actuator is placed at the end of one link.

Despite the fact, that the Acrobot, alternatively also referred to as the underactuated Compass gait walker, is the simplest underactuated walking mechanism, theoreticaly able to walk. The walking ability is interesting problem, therefore, the Acrobot has been studied extensively in the control area during the past few decades, especially an efficient control of the Acrobot in application to reliable walking or running. There are many works in this direction. Dynamics of walking robots have been exploited in order to make passive walkers McGeer (1990), which have limited capabilities and walk down a slope, or have used $n-1$ actuators to execute motion on a flat terrain Spong (1998); Grizzle et al. (2005); Zikmund and Moog (2006).

However, in the real application, the Acrobot would not be able to walk by virtue of stumbing of the moving leg upon the ground. Therefore, it is necessary to bend the moving leg during the walk. The direct extension of the Acrobot depicts in addition of knees and its control during a walk. The recent state of the art in underactuated walking robots is reflected in Westervelt et al. (2007); Chevallereau et al. (2009).

In Čelikovský et al. (2013) an idea of an Acrobot embedding into the 4-link was presented. The Acrobot embedding consists in a definition of constraining functions for knees with dependence on angle in the hip whereas the angle in the hip is controlled in the same way as it would be an Acrobot's angle. By virtue of the embedding method, it is not necessary to develop a new control strategy for the 4-link, but, the 4-link could be controlled using already developed control strategies for the Acrobot and using constraining functions for bending of the swing leg and straighten of the stance leg during one step.

However, either the Acrobot or the 4-link are underactuated mechanical systems and, in generall, some states are not measureable due to lack of actuators or rotary resolvers. In the case of the Acrobot or the 4-link it is not obvious to know the angle between the first link and the ground. Nevertheless, for the full state feedback controller is necessary to measure or estimate all states of the system. Either, it is possible to use an indirect method of measurement of the underactuated angle, e.g. an laser beam sensor as it is suggested in Anderle and Čelikovský (2010b) or it is possible to estimate this angle using an observer. A high gain observer for the Acrobot was developed and used in Anderle and Čelikovský (2010a) and by virtue of

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embedding method the observer will be extended for the 4-link here.

The key idea of the novel approach presented here is based on the combination of the embedding method from Čelikovský et al. (2013) and the high gain observer from Anderle and Čelikovský (2010a) and its extension according to the embedding method in order to demonstrate the advantage of the embedding approach. It was not necessary to design a new observer for 4-link system. However, by virtue of embedding method, the already developed high gain observer could be successfully used.

The rest of the paper is organized as follows. The next section briefly presents the model of the 4-link model together with the main theoretical pre-requisites necessary for the further tracking analysis. The sections 3 describes the results from Čelikovský et al. (2013) whereas the sections 4 describes the high gain observer from Anderle and Čelikovský (2010a) and its extension. Simulations of the 4-link walking are presented in Section 5. Final section draws briefly some conclusions and discusses some open future research outlooks toward an efficient underactuated walking.

2. THE MODEL OF THE 4-LINK

The 4-link is a special case of n-link chain with $n-1$ actuators attached by one of its ends to a pivot point through an unactuated rotary joint. Therefore it belongs in a class of underactuated walking robots. The 4-link depicted in Figure 1 has four degrees of freedom and three actuators placed among its rigid links, roughly speaking, the 4-link has two leg with knees and three actuators. Two actuators are placed in knees and one actuator is placed between legs. Mechanical systems with one-sided constraint are in the literature usually called Lagrangian hybrid systems.

Walking robots or underactuated walking robots are typical representatives of hybrid systems, i.e. systems having continuous-time and discrete-time dynamics. The continuous-time dynamics is described by differential equations whereas the discrete-time dynamics is described using an equation. Both dynamics are covered by a general hybrid system model in the form

$$\dot{x} \in F(x,u), \quad x \in C(x), \tag{1}$$

$$x^+ \in G(x,u), \quad x \in D(x), \tag{2}$$

where $x \in \mathbb{R}$, $F(x)$ is a set-valued mapping, C is a subset of $\mathbb{R}$, $G(x)$ is a set-valued mapping, D is a subset of $\mathbb{R}$ and u is an input. The general model (1,2) describes wide variety of systems not only mechanical systems, but e.g. switching systems, hybrid system automata, discrete events in biological systems etc.

The continuous part of the 4-link robot occurs during a walking, i.e. when one leg, usually called *swing leg* is in air. The discrete part occurs when the swing leg touches the ground. The collision between the swing leg and the ground is called an impact. In general, the impact causes a discontinuous change in velocities while positions remain unchanged. Only for completeness, the second leg, which is in contact with ground, is usually called *stance leg*.

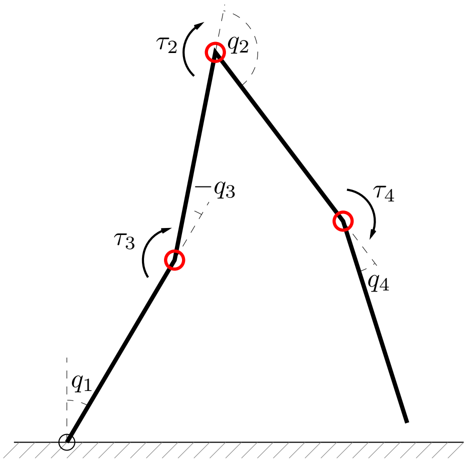

Fig. 1. The 4-link.

2.1 Continuous-time dynamics of the 4link

The continuous part, when the swing leg of the 4-link is in air, is modelled by usual Lagrangian approach. The resulting Euler-Lagrange equation is

$$\begin{bmatrix} \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} - \frac{\partial \mathcal{L}}{\partial q_1} \\ \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_2} - \frac{\partial \mathcal{L}}{\partial q_2} \\ \vdots \\ \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_4} - \frac{\partial \mathcal{L}}{\partial q_4} \end{bmatrix} = u = \begin{bmatrix} 0 \\ \tau_2 \\ \vdots \\ \tau_4 \end{bmatrix}, \tag{3}$$

where u stands for vector of external controlled forces. The system (3) is the so-called **underactuated** mechanical system having the degree of the underactuation equal to one, Spong (1998). Moreover, the underactuated angle is at the pivot point. Equation (3) leads to a dynamic equation in the form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u, \tag{4}$$

where $D(q)$ is the inertia matrix which depends on variables q_2, q_3 and q_4, $C(q,\dot{q})$ contains Coriolis and centrifugal terms, $G(q)$ contains gravity terms and u stands for vector of external forces, see Fantoni and Lozano (2002).

The configuration of the 4-link is described by the generalized coordinates q and is bounded by one-sided constrait represents the limitation that, in general, two solid bodies do not penetrate each other. In our case, the limitation means that the 4-link's swing leg cannot goes under the ground, i.e. the height of the swing leg's end-point has to be $h_{endpoin}(q) > 0$.

2.2 Discrete-time dynamics of the 4-link

When the swing leg of the 4-link touches the ground, i.e. $h_{endpoint}(q) = 0$, the impact occurs. The result of this event is instantaneous jump in angular velocities $\dot{q}$ while angles q remain unchanged.

The impact is modelled as a contact between two rigid bodies. Crucial in the impact mapping is extended inertia matrix $D_e(q_e)$. In order to obtain the matrix, it is necessary to extend the original model of the 4-link (4) by Cartesin coordinates of the end of the stance leg and apply the identical procedure as in the previous case, i.e. in obtaining the dynamical equation of the 4-link, especially the matrix $D(q)$.

There are different ways the impact can be modelled in the literature Brogliato (1996); Hurmuzlu and Marghitu (1994); Brach (1989); Keller (1986); Grizzle et al. (2001); Chevallereau et al. (2009).

Nevertheless, the discrete part of the 4-link's dynamics is not taken into an account during the high gain observer design because the estimation of the underactuated angle is done only during a continuos part of the step. The impact has no influence on the estimation of the underactuated angle q_1 because 4-link's angles q remain unchanges during the impact. Relabelling legs due to change of the leg is necessary only.

3. ACROBOT EMBEDDING INTO THE 4-LINK AND ITS CONTROL

The key idea of the Acrobot embedding into the 4-link model is straightforward. The angle in the hip of the 4-link, q_2 is controlled in the same way as it would be an Acrobot's angle whereas the remaining angles in knees are controlled according to constraining functions $\phi_3(q_2), \phi_4(q_2)$ for knees control. The idea of Acrobot embedding into the 4-link model was firstly introduced in Čelikovský et al. (2013).

Dependencies of angles q_3 and q_4 on angle q_2 represented by constraining functions $\phi_3(q_2), \phi_4(q_2)$ for knees control and new coordinates as $\bar{q}_1, \ldots, \bar{q}_4, \dot{\bar{q}}_1, \ldots \dot{\bar{q}}_4, \bar{\tau}_2, \ldots, \bar{\tau}_4$ are crucial for the embedding method.

The coordinate change taking the "old" coordinates in (3, 4) into new coordinates is defined as follows:

$$\begin{aligned}
\bar{q}_1 &= q_1,\ \bar{q}_2 = q_2,\\
\dot{\bar{q}}_1 &= \dot{q}_1,\ \dot{\bar{q}}_2 = \dot{q}_2,\ \bar{\tau}_2 = \tau_2,\\
\bar{q}_3 &= q_3 - \phi_3(q_2),\\
\dot{\bar{q}}_3 &= \dot{q}_3 - \frac{\partial\phi_3(q_2)}{\partial q_2}\dot{q}_2,\\
\bar{\tau}_3 &= \ddot{q}_3 - \frac{\partial\phi_3(q_2)}{\partial q_2}\ddot{q}_2 - \frac{\partial^2\phi_3(q_2)}{\partial q_2^2}\dot{q}_2^2,\\
\bar{q}_4 &= q_4 - \phi_4(q_2),\\
\dot{\bar{q}}_4 &= \dot{q}_4 - \frac{\partial\phi_4(q_2)}{\partial q_2}\dot{q}_2,\\
\bar{\tau}_4 &= \ddot{q}_4 - \frac{\partial\phi_4(q_2)}{\partial q_2}\ddot{q}_2 - \frac{\partial^2\phi_4(q_2)}{\partial q_2^2}\dot{q}_2^2,
\end{aligned} \tag{5}$$

where $\ddot{q}_2, \ddot{q}_3, \ddot{q}_4$ are substituted from (4). The definition of constraining functions $\phi_3(q_2), \phi_4(q_2)$ for knees control will be discussed later. In Čelikovský et al. (2013) is shown that transformation of coordinates (5) is invertible. For more details see Čelikovský et al. (2013).

3.1 Partial exact feedack linearization of the embedded Acrobot

The **partial exact feedback linearization** method is based on a system transformation into a new system of coordinates that displays linear dependence between some auxiliary output and new (virtual) input Isidori (1996). The partial exact feedback linearization is generated by the suitable output function having the relative degree r yields a linear subsystem of dimension r.

In Grizzle et al. (2005) it was shown that if the generalized momentum conjugates to the cyclic variable is not conserved (as it is the case of Acrobot or the embedded Acrobot) then there exists a set of outputs that defines a one-dimensional exponentially stable zero dynamics.

In the case of the embedded Acrobot there are two independent functions with relative degree 3 which transform the original system into the desired partial linearized form with one dimensional zero dynamics, namely

$$\sigma = \frac{\partial\mathcal{L}}{\partial\dot{\bar{q}}_1}, \tag{6}$$

$$p = \bar{q}_1 + \int_0^{\bar{q}_2} \bar{d}_{11}(s)^{-1}\bar{d}_{12}(s)\mathrm{d}s. \tag{7}$$

In Čelikovský et al. (2008) it was shown that using the set of functions with maximal relative degree, the following transformation

$$\xi = \mathcal{T}(\bar{q}, \dot{\bar{q}}): \quad \xi_1 = p,\ \ \xi_2 = \sigma,\ \ \xi_3 = \dot{\sigma},\ \ \xi_4 = \ddot{\sigma} \tag{8}$$

can be defined. In Čelikovský et al. (2013) particular form of (11) was defined, namely

$$\begin{aligned}
\xi_1 &= \bar{q}_1 + \int_0^{\bar{q}_2} \bar{d}_{11}(s)^{-1}\bar{d}_{12}(s)\mathrm{d}s,\\
\xi_2 &= \bar{d}_{11}(\bar{q}_2)\dot{\bar{q}}_1 + \bar{d}_{12}(\bar{q}_2)\dot{\bar{q}}_2,\\
\xi_3 &= -\overline{G}_1(\bar{q}),\\
\xi_4 &= -\frac{\partial\overline{G}_1}{\partial\bar{q}_1}(\bar{q})\dot{\bar{q}}_1 - \frac{\partial\overline{G}_1}{\partial\bar{q}_2}(\bar{q})\dot{\bar{q}}_2.
\end{aligned} \tag{9}$$

The bar above q, $\dot{q}$ represents new coordinates (5) and the same bar above dynamic equation's matrices (4) represents the dynamics of the embedded Acrobot.

Notice, that by (6,7) and some straightforward but laborious computations the following relation holds

$$\dot{p} = \bar{d}_{11}(\bar{q}_2)^{-1}\sigma, \tag{10}$$

where $\bar{d}_{11}(\bar{q}_2)$ is the corresponding element of the inertia matrix $\bar{D}$ of the embedded Acrobot. Applying (8), (10) to (4) we obtain the dynamics of the embedded Acrobot in partial exact linearized form as follows

$$\begin{aligned}
&\dot{\xi}_1 = \bar{d}_{11}(\bar{q}_2)^{-1}\xi_2,\ \ \dot{\xi}_2 = \xi_3,\ \ \dot{\xi}_3 = \xi_4,\\
&\dot{\xi}_4 = \alpha(\bar{q}, \dot{\bar{q}})\tau_2 + \beta(\bar{q}, \dot{\bar{q}}) = w
\end{aligned} \tag{11}$$

with the new coordinates ξ and the input w being well defined wherever $\alpha(\bar{q}, \dot{\bar{q}})^{-1} \neq 0$.

The system (11) can be used either for a design of a reference step as follows Čelikovský et al. (2008) which results in pseudo-passive reference trajectory or for design exponentially stable state feedback to track a given reference trajectory.

Assume, the following reference system is used to generate the reference trajectory

$$\dot{\xi}_1 = \bar{d}_{11}(\bar{q}_2)^{-1}\xi_2,\ \ \dot{\xi}_2 = \xi_3,\ \ \dot{\xi}_3 = \xi_4,\ \ \dot{\xi}_4 = 0. \tag{12}$$

To obtain the exponentially stable state feedback, subtract the original system (11) and the reference one (12) and apply Taylor expansion

$$
\begin{aligned}
\dot{e}_1 &= \mu_1(t)e_1 + \mu_2(t)e_2 + \mu_3(t)e_3 + o(e), \\
\dot{e}_2 &= e_3, \quad \dot{e}_3 = e_4, \quad \dot{e}_4 = w,
\end{aligned} \tag{13}
$$

where $e := \xi - \xi^r$. Definition of $\mu_{1,2,3}(t)$ is given in Čelikovský et al. (2008); Čelikovský et al. (2013).

By virtue of embedded Acrobot into the 4-link it is possible to use the control approach developed for the Acrobot from Čelikovský et al. (2008); Anderle et al. (2010); Anderle and Čelikovský (2009, 2011a) to stabilize the tracking error dynamics (13), i.e. to control the 4-link in a way resembling the walking.

4. HIGH GAIN OBSERVER FOR 4-LINK

The tracking method in previous section based on the partial exact linearization was developed under the assumption that all state variables are available for measurement. However, in the case of the 4-link this assumption is not fulfilled because the underactuated angle between the 4-link's stance leg and the ground is not measured due to lack of actuator or rotary resolver in this point.

In Anderle and Čelikovský (2010b) an idea of an observer for the Acrobot was presented based on additional measurement of a distance between a fixed point on the stance leg and the ground. Whereas in Anderle and Čelikovský (2010a) the idea of the high gain observer based on q_2 and $\dot{q}_1$ measurement was shown. In Anderle and Čelikovský (2011b), the high gain observer was used in periodic walking of the Acrobot. The stability analysis of the periodic walking was done numerically using the Poincaré method.

Let us here extend the high gain observer developed for the Acrobot in order to estimate the underactuated angle q_1 of the 4-link. By virtue of method of Acrobot embedding into the 4-link, the extension of the high gain observer is straightforward.

Assume that all states except the underactuated angle q_1 are measured. The angular velocity $\dot{q}_1$ could be measured using a gyroscope and remaining angular positions $q_{2,3,4}$ and velocities $\dot{q}_{2,3,4}$ are measured by virtue of actuators placed between each two links. The high gain observer from Anderle and Čelikovský (2010a) is able to estimate the angular velocity $\dot{q}_2$ and the extended high gain observer is able to estimate the angular velocity $\dot{q}_2$ as well, however, the knowledges of angular velocities $\dot{q}_{3,4}$ is essential for transformation of coordinates (5) for embedding the Acrobot into the 4-link. Therefore we have admitted angular velocities measurement.

In Anderle and Čelikovský (2010a) a slightly adapted coordinate change (9) was defined, namely

$$
\eta_1 = p - \bar{q}_1, \quad \eta_2 = \sigma, \quad \eta_3 = \dot{\sigma}, \quad \eta_4 = \ddot{\sigma}, \tag{14}
$$

then $\dot{\eta}_1 = \dot{p} - \dot{\bar{q}}_1$, so following the same path as when deriving (11) one has in new coordinates

$$
\begin{aligned}
\dot{\eta}_1 &= \bar{d}_{11}(\bar{q}_2)^{-1}\,\eta_2 - \dot{\bar{q}}_1, \\
\dot{\eta}_2 &= \eta_3, \\
\dot{\eta}_3 &= \eta_4, \\
\dot{\eta}_4 &= \beta(\eta) + \alpha(\eta_1, \eta_3)\,\bar{\tau}.
\end{aligned} \tag{15}
$$

η_1 is measurable because $\bar{q}_2, \dot{\bar{q}}_1$ are outputs of the system, therefore, high gain observer can take the form

$$
\begin{aligned}
\dot{\hat{\eta}}_1 &= -L_1(\eta_1 - \hat{\eta}_1) + \bar{d}_{11}(\bar{q}_2)^{-1}\,\hat{\eta}_2 - \dot{\bar{q}}_1, \\
\dot{\hat{\eta}}_2 &= -L_2(\eta_1 - \hat{\eta}_1) + \hat{\eta}_3, \\
\dot{\hat{\eta}}_3 &= -L_3(\eta_1 - \hat{\eta}_1) + \hat{\eta}_4, \\
\dot{\hat{\eta}}_4 &= -L_4(\eta_1 - \hat{\eta}_1) + \beta(\hat{\eta}) + \alpha(\eta_1, \hat{\eta}_3)\,\bar{\tau}.
\end{aligned} \tag{16}
$$

Denoting the observation error as $\tilde{e} = \hat{\eta} - \eta$, one has

$$
\begin{aligned}
\dot{\tilde{e}}_1 &= L_1\,\tilde{e}_1 + \bar{d}_{11}(\bar{q}_2)^{-1}\,\tilde{e}_2, \\
\dot{\tilde{e}}_2 &= L_2\,\tilde{e}_1 + \tilde{e}_3, \\
\dot{\tilde{e}}_3 &= L_3\,\tilde{e}_1 + \tilde{e}_4, \\
\dot{\tilde{e}}_4 &= L_4\,\tilde{e}_1 + \beta(\hat{\eta}) - \beta(\eta) + (\alpha(\eta_1, \hat{\eta}_3) - \alpha(\eta_1, \eta_3))\,\bar{\tau}.
\end{aligned} \tag{17}
$$

Now, gains $L_{1,2,3,4}$ can be designed using the standard high-gain technique, namely, take any $\widetilde{L}_{1,2,3,4}$ such that the matrix

$$
\begin{bmatrix}
\widetilde{L}_1 & 1 & 0 & 0 \\
\widetilde{L}_2 & 0 & 1 & 0 \\
\widetilde{L}_3 & 0 & 0 & 1 \\
\widetilde{L}_4 & 0 & 0 & 0
\end{bmatrix} \tag{18}
$$

is Hurwitz. Define

$$
L_1 = \Theta\widetilde{L}_1, \quad L_2 = \Theta^2\widetilde{L}_2, \quad L_3 = \Theta^3\widetilde{L}_3, \quad L_4 = \Theta^4\widetilde{L}_4, \tag{19}
$$

then the system (17) is exponentially GAS for Θ large enough. Therefore $\tilde{e}(t) = \widehat{\eta}(t) - \eta(t) \to 0$, i.e. $\widehat{\eta}(t) \to \eta(t)$, as $t \to \infty$ and therefore (16) is the exponential observer for (15).

The estimation of angle $\bar{q}_1$ is possible to obtain from (14), namely

$$
\eta_3 = \dot{\sigma} = -\bar{G}_1(\bar{q}), \tag{20}
$$

after a simple modification.

5. SIMULATIONS

Simulations of the 4-link with the underactuated angle estimation are done for so called pseudo-passive trajectory, developed in Čelikovský et al. (2008). Pseudopassive trajectory is the one for which $w^r \equiv 0$, i.e. there is no input action in the exact feedback linearized coordinates.

The reference trajectory design is done in ξ coordinates. From the fixed initial and final positions $\xi_1^r(0, T)$ and $\xi_3^r(0, T)$ are determined. From the partially linearized form and from the meaning of the variables it can be seen that the above pseudo-passive feedback ensures the movement of the centre of mass of the 4-link horizontally forward with constant horizontal velocity. Therefore $\xi_4^r(t) \equiv (\xi_3^r(T) - \xi_3^r(0))/T$, where T is time of the step. From partially linearized coordinates $\xi_3^r(0)$, respectively $\xi_3^r(T)$, represents x-position in the beginning, respectively in the end, of the step. The remaining reference step parameter $\xi_2^r(0)$ is determined by simple numerical tuning.

Constraining functions defining dependencies of angles q_3 and q_4 on angle q_2 represented by functions $\phi_3(q_2), \phi_4(q_2)$ are defined as follows. The initial and final values of the functions result from initial and final posture of the 4-link, i.e. both of them are bent. During the step, the swing leg is bended whereas the stance leg is straighten. Moreover, both functions are continuous.

The corresponding simulations for the reference trajectory tracking with estimated angular positions q_1 and with

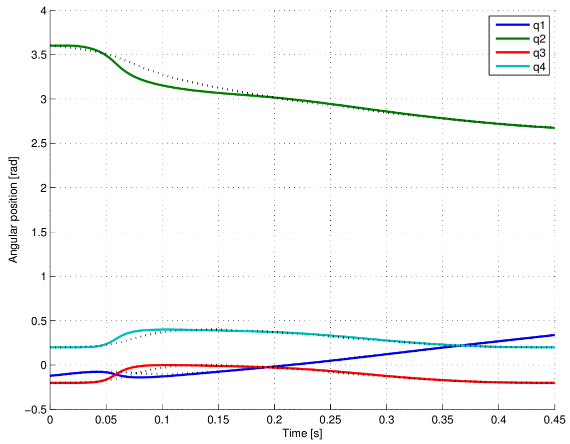

Fig. 2. Angular positions q_1, q_2, q_3, q_4 and references (dotted line) for 1 steps walking using the feedback controller with the high gain observer.

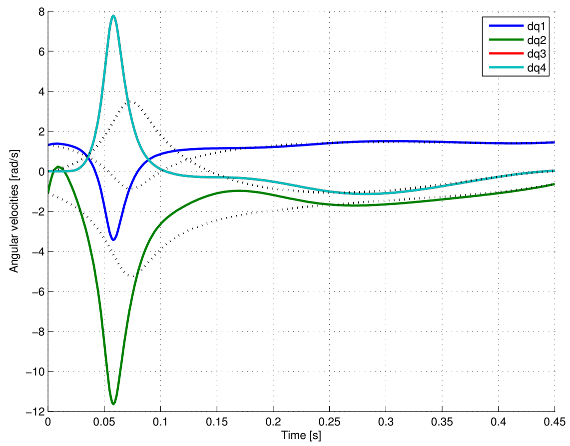

Fig. 3. Angular velocities $\dot{q}_1$, $\dot{q}_2$, $\dot{q}_3$, $\dot{q}_4$ and references (dotted line) for 1 steps walking using the feedback controller with the high gain observer. The course of velocities of $\dot{q}_3$ and $\dot{q}_4$ are the same and therefore their curves overlap.

an initial error in angular positions and velocities are shown in Figs. 2, 3. One can easily see the convergence to the reference angular positions and velocities depicted in figures with dotted line. The estimation of angle q_1 was used as an input for the state feedback controller.

A detail of underactuated angle q_1 estimation using extended high gain observer is depicted in Fig. 4. The blue curve is the real course of angle q_1 whereas the black curve is its estimation.

Animation of the corresponding 4-link walking is shown in Fig. 5.

6. CONCLUSION

We extended the feedback controller with the high gain observer for the tracking of the pseudo-passive reference trajectory in order to demonstrate the Acrobot walking without angle q_1 measurement. The numerical simulations and animation of one step show nicely convergence of the tracking error.

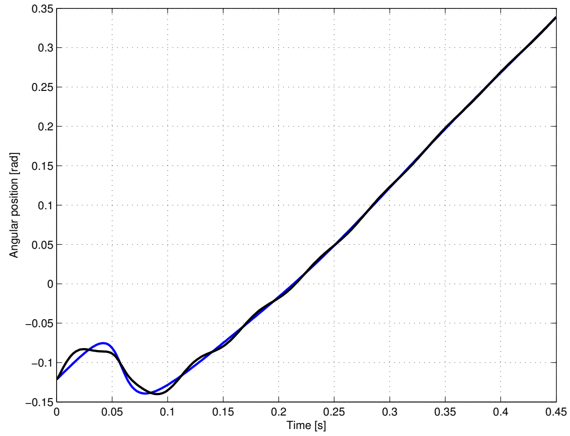

Fig. 4. An detail of estimation of angle q_1 using extended high gain observer. The blue curve is the real cource of angle q_1 whereas the black curve is its estimation.

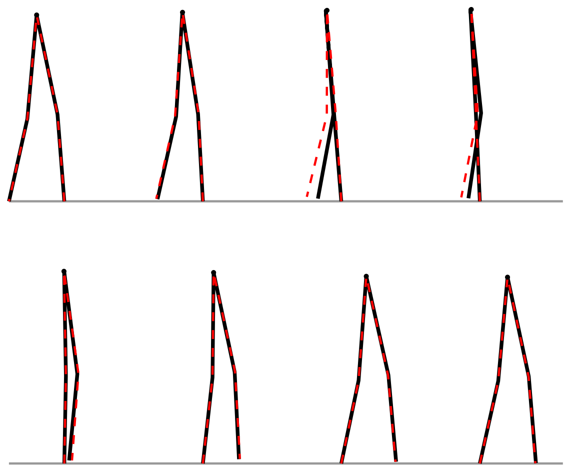

Fig. 5. The animation of one step shown in time moments with gaps $\Delta t = 0.08\,\mathrm{s}$ between them. Dashed line is reference, the full one represents the actual 4-link.

The advantage of the embedding approach was demonstrated. By virtue of embedding method, the already developed high gain observer for the Acrobot was successfully used for the underactuated angle estimation of the 4-link.

A high gain observer is quite sensitive to a noise. Therefore its direct connection with real sensors usually failed. However, the high gain observer could be connect to a Kalman fiter which eliminates the sensor's noise.

A numerical test of walking stability using method of Poincare will be done immediately the multi-step walking trajectory will be finished. Its design is current scope of research.

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